SOLUTION
CONTINUITY & DERIVABILITY

EXERCISE-I (MHT CET LEVEL)

Q.1 (2)

$f(\pi/2)=3$. Since f(x) is continuous at $x=\pi/2$

$$\Rightarrow \lim_{x\to\pi/2}\left(\frac{k\cos x}{\pi-2x}\right)=f\left(\frac{\pi}{2}\right)\Rightarrow\frac{k}{2}=3\Rightarrow k=6.$$

Q.2 (1)

Here $\lim_{x\to0+} f(x)=k,\ \lim_{x\to0-} f(x)=-k$ and $f(0)=k$

But f(x) is continuous at x =0, therefore k must be zero.

Q.3 (d)

$$\underset{x\to0}{\text{Lt}}\frac{\left(e^x-1\right)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$$

$$\underset{x\to0}{\text{Lt}}\frac{\frac{e^x-1}{x}.\,x^2}{\frac{x}{a}\frac{\sin\left(\frac{x}{a}\right)}{\left(\frac{x}{a}\right)}.\frac{\log\left(1+\frac{x}{4}\right)}{\frac{x}{4}}.\frac{x}{4}}$$

$$\Rightarrow a=3$$

Q.4 (b)

Since f(x) is continuous at x = 2

$\therefore$ f(2) = $\lim_{x\to2^+} f(x)\Rightarrow 1-\lim_{x\to2^+}(ax+b)$

$\therefore$ 1 = 2a + b ......(1)

Agian f(x) is continuous at x = 4,

$\therefore$ f(4) = $\lim_{x\to4^-} f(x)\Rightarrow 7-\lim_{x\to4^-}(ax+b)$

$\therefore$ 7 = 4a + b ...... (2)

Solving (1) and (2), we get a = 3, b = – 5

Q.5 (b) $\lim_{x\to-5} f(x)=\frac{(x-2)(x+5)}{(x+5)(x-3)}=\frac{-7}{-8}=\frac{7}{8}$

Q.6 (b)

The function can be continuous only at those points for which

$\sin x=\cos x\Rightarrow x=n\pi+\frac{\pi}{4}$

Q.7 (c)

Here $f\left(\frac{3\pi}{4}\right)=1$ and $\lim_{x\to\frac{3\pi}{4}^-} f(x)=1$

$$\lim_{x\to\frac{3\pi}{4}^+} f(x)=\lim_{h\to0}2\sin\frac{2}{9}\left(\frac{3\pi}{4}+h\right)$$

$$=2\sin\frac{\pi}{6}=1$$

Hence f(x) is continuous at $x=\frac{3\pi}{4}$.

Q.8 (c)
Q.9 (a)
Q.10 **(c)**
Q.11 **(a)**
Q.12 (c)

Q.13 (3) $f(0)=0$

$\lim_{x\to0-} f(x)=\lim_{h\to0} e^{-1/h}=0$ and

$\lim_{x\to0+} f(x)=\lim_{h\to0} e^{1/h}=\infty$

Hence function is discontinuous at x =0.

Q.14 (2)

$f(x)=\frac{x+1}{(x-3)(x+4)}$.

Hence the points are
$3,-4$.

Q.15 (2)

$\lim_{x\to3} f(x)=\lim_{x\to3}\frac{x^2-9}{x-3}=\lim_{x\to3}(x+3)=6$

and $f(3)=2(3)+k=6+k$

$\because$ f is continuous at x= 3; $\therefore$ 6+k = 6

$\Rightarrow k=0.$

Q.16 (4)

By L-Hospital's rule $\lim_{x\to0} f(x)$ is 2. Therefore, for f(x) to be continuous, the value of function should be 2.

Q.17 (3)

$$\text{L.H.L.} = \lim_{x\to 0^-} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} = k$$

$$\text{R.H.L.} = \lim_{x\to 0^+} (2x^2+3x-2) = -2$$

Since it is continuous, L.H.L = R.H.L $\Rightarrow k = -2$.

Q.18 (1)

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases} \text{ continuous at } x = 0$$

$$\lim_{x\to 0^+} f(x) = f(0) \Rightarrow \lim_{x\to 0} \frac{2.\sin^2 x/2}{x} = k$$

$$\Rightarrow \lim_{x\to 0} \frac{2\sin^2 x/2}{\left(\frac{x}{2}\right)^2}.\frac{x}{4} = k \Rightarrow k = 0.$$

Q.19 (3)

For continuity at all $x \in R$, we must have

$$f\left(-\frac{\pi}{2}\right) = \lim_{x\to(-\pi/2)^-} (-2\sin x) \lim_{x\to(-\pi/2)^+} (A\sin x + B)$$

$\Rightarrow 2 = -A + B$ .....(i)

and $f\left(\frac{\pi}{2}\right) = \lim_{x\to(\pi/2)^-} (A\sin x + B) = \lim_{x\to(\pi/2)^+} (\cos x)$

$\Rightarrow 0 = A + B$ .....(ii)

From (i) and (ii), A = –1 and B = 1.

Q.20 (1) $f(5) = \lim_{x\to 5} f(x) = \lim_{x\to 5} \dfrac{x^2-10x+25}{x^2-7x+10}$

$$= \lim_{x\to 5} \frac{(x-5)^2}{(x-2)(x-5)} = \frac{5-5}{5-2} = 0.$$

Q.21 (c)

The function lig|x| is not defined

at x = 0, f(x) to be defined, log |x| $\neq 0 \Rightarrow x \pm 1$.

Hence, 0, 1, –1 are three points of discontinuity.

Q.22 (1) $f(x) = x^p \sin\dfrac{1}{x}, x \neq 0$ and $f(x) = 0, x = 0$

Since at x = 0, f(x) is a continuous function

$$\therefore \lim_{x\to 0} f(x) = f(0) = 0 \Rightarrow \lim_{x\to 0} x^p \sin\frac{1}{x} = 0 \Rightarrow p > 0.$$

is differentiable at , if exists

$$\Rightarrow \lim_{x\to 0} \frac{x^p \sin\frac{1}{x} - 0}{x-0} \text{ exists}$$

$$\Rightarrow \lim_{x\to 0} x^{p-1} \sin\frac{1}{x} \text{ exists}$$

$\Rightarrow p - 1 > 0$ or $p > 1$

If $p \le 1$, then $\lim_{x\to 0} x^{p-1} \sin\left(\frac{1}{x}\right)$ does not exist and at x = 0 f(x) is not differentiable.

$\therefore$ for $0 < p \le 1$ f(x) is a continuous function at $x = 0$ but not differentiable.

Q.23 (d)

$$Lf'(1) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{(1-h+a)-(1+a)}{-h} = \lim_{h\to 0} \frac{-h}{-h} = 1$$

$$Rf'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\left[a(1+h)^2 + 1\right] - (1+a)}{h}$$

$$= \lim_{h\to 0} (ah + 2a = 2a)$$

Q.24 (a)

$$f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$$

$$= \lim_{h\to 0} \frac{f(5+h) - f(5+0)}{h}$$

$$= \lim_{h\to 0} \frac{f(5).f(h) - f(5)f(0)}{h}$$

($\because f(x+y) = f(x).f(y)$ for all x, y)

$$= \left(\lim_{h\to 0} \frac{f(h) - f(0)}{h}\right).f(5) = f'(0)(5)$$

$= 3 \times 2 = 6$

Q.25 (4)

As $Lf'(2) \neq Rf'(2)$.

Q.26 (1,3,4) $x \le x^2 \Rightarrow x(1-x) \le 0 \Rightarrow x(x-1) \ge 0$

$$\Rightarrow x \le 0 \text{ or } x \ge 1; \therefore h(x) = \begin{cases} x & : x \le 0 \\ x^2 & : 0 < x < 1 \\ x & : x \ge 1 \end{cases}$$

h(x) is continuous for every x but not differentiable at

$x = 0$ and 1. Also $h'(x) = \begin{cases} 1 & x<0 \\ \text{not exists} & x=0 \\ 2x & 0<x<1 \\ \text{not exists} & x=1 \\ 1 & x>1 \end{cases}$

$\therefore$ $h'(x) = 1$ for all $x > 1$.

Q.27 (3)

$\lim_{h\to 0^-} 1+(2-h)=3$,

$\lim_{h\to 0^+} 5-(2+h)=3$, $f(2)=3$

Hence, f is continuous at x = 2

Now $Rf'(x) = \lim_{h\to 0}\frac{5-(2+h)-3}{h} = -1$

$Lf'(x) = \lim_{h\to 0}\frac{1+(2-h)-3}{-h} = 1$

$\because Rf'(x) \neq Lf'(x)$;

$\therefore$ f is not differentiable at x = 2

Q.28 (2) A continuous function may or may not be differentiable. So (b) is not true.

Q.29 (2)

Let $x<0 \Rightarrow |x| = -x$

$\Rightarrow f(x) = \frac{d}{dx}\left(\frac{x}{1-x}\right) = \frac{1}{(1-x)^2}$

$\Rightarrow [f'(x)]_{x=0} = 1$. Again $x>0 \Rightarrow |x| = x$

$f(x) = \frac{d}{dx}\left(\frac{x}{1+x}\right) = \frac{1}{(1+x)^2} \Rightarrow [f'(x)]_{x=0} = 1$

$\Rightarrow f'(0) = 1$.

Q.30 (4)

Since function $|x|$ is not differentiable at $x=0$

$\therefore |x^2-3x+2| = |(x-1)(x-2)|$

Hence is not differentiable at $x=1$ and 2

Now $f(x) = (x^2-1)|x^2-3x+2|\cos(|x|)$ is not differentiable at x =2

For $1<x<2$,

$f(x) = -(x^2-1)(x^2-3x+2)+\cos x$

For $2<x<3$,

$f(x) = +(x^2-1)(x^2-3x+2)+\cos x$

$Lf'(x) = -(x^2-1)(2x-3)-2x(x^2-3x+2)-\sin x$

$Lf'(2) = -3-\sin 2$

$Rf'(x) = (x^2-1)(2x-3)+2x(x^2-3x+2)-\sin x$

$Rf'(2) = (4-1)(4-3)+0-\sin 2 = 3-\sin 2$

Hence $Lf'(2) \neq Rf'(2)$.

Q.31 (2)

$$y' = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}.\frac{2(1+x^2)-4x^2}{(1+x^2)^2} = \frac{2(1-x^2)}{\sqrt{(1-x^2)^2.(1+x^2)}}$$

$$\Rightarrow y' = \begin{cases} \frac{2}{1+x^2} & \text{for } |x|<1 \\ \frac{-2}{1+x^2} & \text{for } |x|>1 \end{cases}$$

Hence for $|x|=1$, the derivative does not exist.

Q.32 (3)

Since the function is defined for $x \geq 0$ i.e. not defined for x< 0. Hence the function neither continuous nor differentiable at x = 0.

Q.33 (3) It is fundamental concept.

Q.34 (1)

$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$; As function is differentiable so it is continous as it is given that $\lim_{h\to 0}\frac{f(1+h)}{h} = 5$ and hence f(1) =0. Hence $f'(1) = \lim_{h\to 0}\frac{f(1+h)}{h} = 5$.

Q.35 (d)

Q.36 **(a)**

Q.37 (4)

$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \leq \lim_{x\to y}|x-y|$ or $|f'(x)| \leq 0$

$\Rightarrow f'(x) = 0 \Rightarrow f(x)$ is constant, As $f(0)=0$

$\therefore f(1) = 0$.

Q.38 (2)

Let a function be $g(x) = f(x) - x^2$

$\Rightarrow$ g(x) has at least 3 real roots which are x =1, 2, 3

$\Rightarrow$ g'x) has at least 2 real roots in $x \in (1,3)$

$\Rightarrow$ g”(x) has at least 1 real roots in $x \in (1,3)$

$\Rightarrow f'(x) = 2$ for at least one $x \in (1,3)$.

Q.39 (4)

We have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f(x)+f(h)-f(x)}{h}$$

$[\because f(x+y) = f(x)+f(y)]$

$$= \lim_{h\to 0} \frac{f(h)}{h} = \lim_{h\to 0} \frac{h^2 g(h)}{h} = 0.g(0) = 0$$

[$\because$ g is continuous therefore $\lim_{h\to 0} g(h) = g(0)$].

Q.40 (2)

$$\frac{d}{dx}(\log\tan x) = \frac{1}{\tan x}\sec^2 x = \frac{\cos x}{\cos^2 x \sin x}$$

$$= \frac{2}{2}\frac{1}{\cos x \sin x} = 2\operatorname{cosec} 2x$$

Q.41 (c)

$$f'(t) = \frac{d}{dt}\left[\frac{1-t}{1+t}\right] = \frac{(1+t)(-1)-(1-t)\times 1}{(1+t)^2}$$

$$= \frac{-1-t-1+t}{(1+t)^2} = \frac{-2}{(1+t)^2}$$

$$f'[1/t] = \frac{-2}{\left(1+\frac{1}{t}\right)^2} = \frac{-2t^2}{(t+1)^2}$$

Q.42 **(c)**

Given $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides, we get

$x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 + x^2y - xy^2 = 0$

$\Rightarrow (y-x)(x+y+xy) = 0$

$\Rightarrow y = x$ or $y(1+x) = -x \Rightarrow y = x$ or $y = -\frac{x}{1+x}$

$$\Rightarrow \frac{dy}{dx} = \frac{-(1+x).1+x.1}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

Q.43 (c)
Q.44 (a)
Q.45 **(c)**
Q.46 **(b)**
Q.47 **(a)**
Q.48 **(a)**
Q.49 (1)

Here $z = a - \frac{1}{y} \Rightarrow \frac{dz}{dy} = \frac{1}{y^2} = (a-z)^2$

Q.50 (1)

$$\frac{d}{dx}\left(x^2 e^x \sin x\right) = x^2\frac{d}{dx}\left(e^x \sin x\right) + e^x \sin x \frac{d}{dx}(x^2)$$

$$= xe^x(2\sin x + x\sin x + x\cos x)$$

Q.51 (3)

$$\frac{d}{dx}[\cos(1-x^2)^2] = -\sin(1-x^2)^2 \frac{d}{dx}(1-x^2)^2$$

$$= 4x(1-x^2)\sin(1-x^2)^2$$

Q.52 (4)

Since $\frac{dy}{dx} = -\sin(\sin x^2).\cos x^2.2x$

Therefore, at $x = \sqrt{\frac{\pi}{2}}$, $\cos x^2 = \cos\frac{\pi}{2} = 0 \Rightarrow \frac{dy}{dx} = 0$

Q.53 (1)

$$\frac{d}{dx}(e^x \log\sin 2x) = e^x \log\sin 2x + 2e^x \frac{1}{\sin 2x}\cos 2x$$

$$= e^x \log\sin 2x + e^x 2\cot 2x = e^x(\log\sin 2x + 2\cot 2x).$$

Q.54 (2)

$y = \sin\{\cos(\sin x)\}$

$$\Rightarrow \frac{dy}{dx} = -\cos\{\cos(\sin x)\}\sin(\sin x)\cos x$$

Q.55 (1)

Rationalising,

$$y = \frac{2x^2 + 2\sqrt{x^4-1}}{2} = x^2 + (x^4-1)^{1/2}$$

$$\Rightarrow \frac{dy}{dx} = 2x + \frac{2x^3}{\sqrt{x^4-1}}.$$

Q.56 (1)

$y = (x\cot^3 x)^{3/2}$

$$\therefore \frac{dy}{dx} = \frac{3}{2}(x\cot^3 x)^{1/2}[\cot^3 x + 3x\cot^2 x(-\operatorname{cosec}^2 x)]$$

$$= \frac{3}{2}(x\cot^3 x)^{1/2}[\cot^3 x - 3x\cot^2 x\operatorname{cosec}^2 x]$$

Q.57 (2)

$x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2at}{1+t^2}$

Q.58 (d)
Q.59 **(a)**
Q.60 **(a)**
Q.61 (4)

$\sqrt{1+\tan^2\theta} = |\sec\theta|$.

Q.62 (1)

$$y = a\sin^4\theta \Rightarrow \frac{dy}{d\theta} = 4a\sin^3\theta\cos\theta$$

and $x = a\cos^4\theta \Rightarrow \frac{dx}{d\theta} = -4a\cos^3\theta\sin\theta$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$$

$$\therefore \left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1$$

Q.63 (3)

Let $f(x) = x^6 + 6^x$. Then $f'(x) = 6x^5 + 6^x \log 6$.

Q.64 (d)

Consider $y = x^{x^2} \Rightarrow \text{In } y = x^2 \text{ In } x$

$$\frac{1}{y}\frac{dy}{dx} = x^{x^2}.x(1+2\text{In } x) = x^{x^2+1}(1+2\text{ In } x)$$

Q.65 (c)
Q.66 **(b)**
Q.67 **(d)**
Q.68 (2)

$x^y = y^x \Rightarrow y\log_e x = x\log_e y$

Differentiating *w.r.t. x* of *y*, we get

$$\log_e x\frac{dy}{dx} + \frac{y}{x} = \log_e y + x\frac{1}{y}\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{y(x\log_e y - y)}{x(y\log_e x - x)}$$

Q.69 (3)

$$y = x^{(x^x)} \Rightarrow \log y = x^x \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dz}{dx}.\log x + \frac{1}{x}.z\ ,$$

(where $x^x = z$)

$$\Rightarrow \frac{dy}{dx} = x^{(x^x)}\left[x^x(\log ex).\log x + x^{x-1}\right]\ ,$$

$$\left\{\because \frac{dz}{dx} = x^x\log ex\right\}$$

Q.70 (3) Let $y = \cos^{-1}\sqrt{x}$ and $z = \sqrt{1-x}$

$$\therefore \frac{dy}{dz} = \frac{\frac{-1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}}{-\frac{1}{2\sqrt{1-x}}} = \frac{1}{\sqrt{x}}.$$

Q.71 (c)
Q.72 **(b)**
Q.73 (4)

Let $y = \sin^{-1}\frac{1-x}{1+x} \Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{x}(1+x)}$ ....(i)

and $z = \sqrt{x} \Rightarrow \frac{dz}{dx} = \frac{1}{2\sqrt{x}}$ .....(ii)

Therefore by (i) and (ii) $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{-2}{1+x}$.

Q.74 (2)

$$y = \cos 4x + \cos 2x \Rightarrow \frac{d^{20}y}{dx^{20}} = 4^{20}\cos 4x + 2^{20}\cos 2x$$

Q.75 (3)

$$\sqrt{x} + \sqrt{y} = 1 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{1}{4},\frac{1}{4}\right)} = -1$$

Q.76 (d)
Q.77 **(c)**
Q.78 **(d)**
Q.79 **(c)**
Q.80 **(b)**
Q.81 **(1)**
Q.82 (2)

$$f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \tan^{-1}\left[\tan\frac{x}{2}\right] = \frac{x}{2}$$

$$\Rightarrow f'(x) = \frac{1}{2}.\ \text{Hence } f'\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Q.83 (4)

$$\frac{d}{dx}[\tan^{-1}(\cot x) + \cot^{-1}(\tan x)]$$

$$\frac{1(-\text{cosec}^2 x)}{1+\cot^2 x} - \frac{1(\sec^2 x)}{1+\tan^2 x} = -1-1 = -2.$$

Q.84 **(d)**

$$y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right) = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+\sqrt{x}.x}\right)$$

$$= \tan^{-1}\left(\sqrt{x}\right) - \tan^{-1}(x)$$

On differentiating w.r.t. x, we get

$$y' = \frac{1}{1+x}.\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$

$$\Rightarrow y'(1) = \frac{1}{2}.\frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$$

Q.85 (a)
Q.86 (a)
Q.87 (b)
Q.88 **(c)**
Q.89 (2)

$y = \cos^{-1}\cos(x-1), \quad x > 0$

$\Rightarrow y = x - 1, \quad x > 0 \text{ and } 0 < x - 1 < p$

we have, $1 < \frac{5\pi}{4} < \pi + 1$

$\therefore \quad y = x - 1, \ 1 \le x \le \pi + 1 \ \text{ and } \ \frac{5\pi}{4} \in [1, \pi + 1]$

$\left.\frac{dy}{dx}\right|_{x=\frac{5\pi}{4}} = 1\Big|_{x=\frac{5\pi}{4}} = 1$

Q.90 (1)

$y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

$= \cot^{-1}\left[\frac{2 + 2\cos x}{2\sin x}\right] = \cot^{-1}\left[\frac{1+\cos x}{\sin x}\right]$

$= \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

Q.91 (4)

$$f'''(x) = \begin{vmatrix} \frac{d^3}{dx^3}x^3 & \frac{d^3}{dx^3}\sin x & \frac{d^3}{dx^3}\cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = \begin{vmatrix} 6 & -\cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\therefore f'''(0) = \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = 0,$$

which is independent of *p*.

Q.92 (2)

$$D = \begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -p^3\cos px & p^4\sin px & p^5\cos px \\ -p^6\sin px & -p^7\cos px & p^8\sin px \end{vmatrix}$$

$$= p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ -\sin px & -p\cos px & p^2\sin px \end{vmatrix}$$

$$= -p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ \cos px & p\sin px & p^2\cos px \\ \sin px & p\cos px & -p^2\sin px \end{vmatrix} = 0.$$

Q.93 (2)

We have $\frac{dx}{dt} = 1 - \frac{1}{t^2}, \ \frac{dy}{dt} = 1 + \frac{1}{t^2}$

$\therefore \frac{dy}{dx} = \frac{t^2+1}{t^2-1} = \left(1 + \frac{2}{t^2-1}\right)$ and

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$

$= 2.\frac{-1}{(t^2-1)^2}.2t \times \frac{t^2}{t^2-1} = -\frac{4t^3}{(t^2-1)^3}.$

Q.94 (2)

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3}{2}t = \frac{3}{2}\sqrt{x} \Rightarrow \frac{d^2y}{dx^2} = \frac{3}{4\sqrt{x}} = \frac{3}{4t}$

Q.95 (b)

Given x = a sin θ and y = b cosθ

$\Rightarrow \frac{dy}{d\theta} = a\cos\theta$ and $\frac{dy}{d\theta} = -bsib\theta$

$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -\frac{b}{a}\tan\theta \Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a}\sec^2\theta$

Q.96 (a)

$\frac{dy}{dx} = \left(\frac{dx}{dy}\right)^{-1}$

$\Rightarrow \frac{d^2y}{dx^2} = -1\left(\frac{dx}{dy}\right)^{-2}\left\{\frac{d}{dx}\left(\frac{dx}{dy}\right)\right\}$

$\Rightarrow \frac{d^2y}{dx^2} = (-1)\left(\frac{dx}{dy}\right)^{-2}\left\{\frac{d}{dy}\left(\frac{dx}{dy}\right)\frac{dy}{dx}\right\}$

$= (-1)\left(\frac{dy}{dx}\right)^2\left\{\frac{d^2x}{dy^2}, \frac{dy}{dx}\right\}$

$= -\left(\frac{dy}{dx}\right)^3\left\{\frac{d^2x}{dy^2}\right\}$

$\Rightarrow \frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = 0$

Q.97 (a)

Given expression can be written as

$$y=1-\frac{1}{x+1}+1+\frac{1}{x}\Rightarrow\frac{dy}{dx}=\frac{1}{(1+x)^2}-\frac{!}{x^2}$$

$$\frac{d^2y}{dx^2}=-2(1+x)^{-3}+2x^{-3}=\frac{-2}{(1+x)^3}+\frac{2}{x^3}$$

Now, $\left.\frac{d^2y}{dx^2}\right|_{x=1}=\frac{-2}{(1=1)^3}+\frac{2}{(1)^3}=\frac{-2}{8}+2=\frac{7}{4}$

Q.98 (d)

$$y=e^{2x}\therefore\frac{dy}{dx}=2e^{2x}\text{ and }\frac{d^2y}{dx^2}=4e^{2x}$$

$$\frac{dx}{dy}=\frac{1}{2e^{2x}}=\frac{1}{2y}$$

$$\therefore\frac{d^2x}{dy}=-\frac{1}{2y^2}=-\frac{1}{2}e^{-4x}$$

$$\therefore\frac{d^2y}{dx^2}\cdot\frac{d^2x}{2y^2}=4e^{2x}\left(\frac{-e^{-2x}}{2e^{2x}}\right)=-2e^{-2x}$$

Q.99 (c)
Q.100 (b)
Q.101 **(b)**
Q.102 **(b)**
Q.103 **(a)**
Q.104 (3)

$$\frac{dx}{d\theta}=a\cos\theta\text{ and }\frac{dy}{d\theta}=-b\sin\theta$$

$$\Rightarrow\frac{dy}{dx}=\frac{-b}{a}\tan\theta\text{ and }\frac{d^2y}{dx^2}=\frac{-b}{a}\sec^2\theta\frac{d\theta}{dx}$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{-b}{a}\sec^2\theta\frac{1}{a\cos\theta}=\frac{-b}{a^2}\sec^3\theta.$$

Q.105 (3)

Here $y=t^{10}+1$ and $x=t^8+1$

$\therefore\ t^8=x-1\ \Rightarrow\ t^2=(x-1)^{1/4}$

So, $y=(x-1)^{5/4}+1$

Differentiate both sides *w.r.t. x*,

$$\frac{dy}{dx}=\frac{5}{4}(x-1)^{1/4}$$

Again, differentiate both sides *w.r.t. x*,

$$\frac{d^2y}{dx^2}=\frac{5}{16}(x-1)^{-3/4}$$

$$\frac{d^2y}{dx^2}=\frac{5}{16(x-1)^{3/4}}=\frac{5}{16(t^2)^3}=\frac{5}{16t^6}.$$

Q.106 (4) $y=a^xb^{2x-1}$

$$\frac{dy}{dx}=a^xb^{2x-1}\log a+2a^xb^{2x-1}\log b$$

$$=a^xb^{2x-1}(\log a+2\log b)$$

$$\frac{d^2y}{dx^2}=a^xb^{2x-1}(\log a+2\log b)^2$$

$$=a^xb^{2x-1}(\log ab^2)^2=y(\log ab^2)^2$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (1)

$$\lim_{x\to0}f(x)=f(0)$$

$$\lim_{x\to0}\frac{\cos(\sin x)-\cos x}{x^2}=a$$

$$\Rightarrow\ \frac{2}{x^2}\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)=a$$

$$\Rightarrow\quad a=\lim_{x\to0}2.\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\frac{\sin x+x}{2}}.\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\frac{x-\sin x}{2}}.$$

$$\frac{1}{4}\left(\frac{\sin x+x}{x}\right)\left(\frac{x-\sin x}{x}\right)$$

$$=\ 2.1\ .\ 1.\ \frac{1}{4}(1+1)\ (1-1)=0$$

Q.2 (2)

$$f(x)=\begin{cases}\dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1\le x\le0\\[2ex]\dfrac{2x+1}{x+2}, & 0\le x\le1\end{cases}$$

since it is cont, so,

$$\lim_{x\to0^-}f(x)=f(0)$$

$$\lim_{h\to0}\frac{\sqrt{1+p(-h)}-\sqrt{1-p(-h)}}{-h}=-\frac{1}{2}$$

$$\lim_{h\to0}\frac{(1-ph)-(1+ph)}{-h\left\{\sqrt{1-ph}+\sqrt{1+ph}\right\}}=-\frac{1}{2}$$

$$\frac{+2p}{2} = -\frac{1}{2}$$

p = –1/2

Q.3 (c)

$$f\left[(\pi/2)^-\right] = \lim_{h\to 0}\frac{1-\sin^3[(\pi/2)-h]}{3\cos^2[(\pi/2)-h]}$$

$$= \lim_{h\to 0}\frac{1-\cos^3 h}{3\sin^2 h} = \frac{1}{2}$$

$$f\left[(\pi/2)^+\right] = \lim_{h\to 0}\frac{q[1-\sin\{(\pi/2)+h\}]}{[\pi-2\{(\pi/2)+h\}]^2}$$

$$= \lim_{h\to 0}\frac{q(1-\cosh)}{4h^2} = \frac{q}{8}$$

$$\therefore p = \frac{1}{2} = \frac{q}{8} \Rightarrow p = \frac{1}{2}, q = 4$$

Q.4 (a)

We have, $f(x) = x - |x - x^2| = x - |x(1-x)|$

$= x - |x||1 - x|$,

$\therefore$ Continuity is to be checked at x = 0 and x = 1. At x = 0

LHS = $\lim_{h\to 0} f(0-h) = \lim_{h\to 0} -h - |-h||1-h|$

$\lim_{h\to 0} h - h(1+h) = 0$

RHS = $\lim_{h\to 0} f(0-h) = \lim_{h\to 0} h - |h||1-h|$

$\lim_{h\to 0} h - h(1+h) = 0$

and f(0) = 0

Since LHS = RHL = f(0),

$\therefore$ f(x) is cotinuous at x = 0.

At x = 1

LHS =

$\lim_{h\to 0} f(1-h) = \lim_{h\to 0}(1-h) - |1-h||1-(1-h)|$

$= \lim_{h\to 0}(1-h) - h)(1-h) = 1$

Similary $RHL = \lim_{h\to 0} f(1+h) = 1$

and f(1) = 1 – |1| . |1 – 1| = 1

$\therefore$ f(x) is continuous at x = 1

Hecne f(x) is continuous for all $x \in [-1, 1]$

Q.5 **(b)**

$$\lim_{x\to 0^+}\frac{e^{e/x} - e^{-e/x}}{e^{1/x} + e^{-1/x}}$$

$$= \lim_{x\to 0^+}\frac{e^{\frac{e-1}{x}}\left(1-e^{-2e/x}\right)}{\left(1+e^{-2/x}\right)} = +\infty$$

$$\lim_{x\to 0^-}\frac{e^{e/x} - e^{-e/x}}{e^{1/x} + e^{-1/x}} = \lim_{x\to 0^-}\frac{e^{-e/x}\left(e^{2e/x}-1\right)}{e^{-1/x}\left(e^{2/x}+1\right)}$$

$$\lim_{x\to 0^-} e^{-\left(\frac{e-1}{x}\right)}\left(\frac{e^{2e/x}-1}{e^{2/x}+1}\right) = -\infty$$

Limit doesn't exist, so f(x) not continuous at 0

Q.6 (b)

Q.7 (b)

Q.8 **(b)**

Q.9 **(c)**

Q.10 **(1)**

Q.11 **(d)**

Q.12 (4)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{1}{x} - \frac{2}{e^{2x}-1}\right)$$

$$= \lim_{x\to 0}\left(\frac{e^{2x}-1-2x}{x(e^{2x}-1)}\right) \text{ using L - hospital rule}$$

$$= \lim_{x\to 0}\left(\frac{e^{2x}-1-2x}{x(e^{2x}-1)}\right) = 1$$

Q.13 (4)

f(x) = |x –1| + |x – 2| + cos x

All three functions are cont. in [0, 4] so sum of all these functions is also a cont. funs.

Q.14 (4)

$$f(x) = \frac{|x-3|}{|x-2|} + \frac{1}{1+[x]}$$

$x \neq 2$ $\quad$ $1 + [x] = 0$

$[x] \neq -1$, $\quad$ $x \in [1, 0)$

And [x] will be disoint. at every integer

So $x \in R - \{(-1, 0) \cup n, n \in I\}$

Q.15 (2)

f(x) should be a constant function.

Q.16 (1)

$$RHL = \lim_{x\to 0^+}\frac{\sqrt{1+x}-\sqrt{1-x}}{x} = 1\text{(Rationlize)}$$

$$LHL = \frac{1}{\sqrt{2}} f(g(x))$$

$$= \lim_{x\to 0^-}\frac{1}{\sqrt{2}}\ \frac{|\sqrt{2}\cos x| - |\sqrt{2}\sin x|}{\cos 2x}$$

$= \lim_{x\to 0^-} \frac{1}{\cos x - \sin x} = 1$

cont. at x = 0

Q.17 *(3)*

$f\left(\frac{\pi^+}{4}\right) = \pi\left(\frac{\pi^+}{4}\right) + 1\pi \times 0 + 1 = 1$

$f\left(\frac{\pi}{4^-}\right) = f\left(\frac{\pi}{4}\right) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$

So jump = $1 - \frac{\pi}{4}$

Q.18 (a)

Let $f(x) = \frac{1}{\log|x|}$

The point of discontinuity of f(x) are those points where

F(x) is undefined or infinite. If is undefined Where x =0 and is infinite when

$\log|x| = 0, |x| = 1, i.e. x = \pm 1$

Q.19 **(b)**

Q.20 (2)

g(x) = x – [x] f(0) = f(1)

h(x) = f(g(x))

Let x = a ∈ I

$h(a^+) = \lim_{x\to a^+} f(\{x\}) = f(0)$

$h(a^-) = \lim_{x\to a^-} f(g(x)) = f(1)$

$h(a^+) = h(a^-)$ hence h(x) is continuous

Q.21 (4)

RHL = $\lim_{h\to 0} \sin[\ell nh] = [-1, 1]$

LHL = $\lim_{h\to 0} \sin[\ell n\, h] = [-1, 1]$

So DNE

Q.22 (3)

$f(x) = \text{Sgn}(4 - 2\sin^2 x - 2\sin x)$

$= \text{Sgn}[(\sin x + 2)(2 - 2\sin x)]$

f(x) = 0 when $x > \frac{\pi}{2}$

= 1 $x < \frac{\pi}{2}$

= –1 sinx >1 not possible

SO isolated point dicontinuity

Q.23 (1)

$g(x) = \tan^{-1}|x| - \cot^{-1}|x|$

$f(x) = \frac{[x]}{[x+1]}\{x\}$

h(x) = |g(f(x))|

$\lim_{x\to 0^-}(x)$ $|gf(0^-)|$

$= \frac{n}{2}$

$\lim_{x\to 0^+}(x) = |gf(0^+)|$

$= \frac{n}{2}$

h is continuous at x = 0

Q.24 (4)

$$\lim_{x\to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right) - x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right)}{x^2}$$

= 0

f(x) is cont at x = 0

Q.25 (2)

$f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$

$f(0^+) = \lim_{h\to 0} \frac{h\left(\sqrt{h} - \sqrt{h+1}\right)}{h}$

$= \lim_{h\to 0} \frac{h - h - 1}{\sqrt{h} + \sqrt{h+1}} = -1$

Q.26 (d)

$|x|$ is non-differentiable function at

$x = 0$ as L.H.D = –1 and R.H.D = 1

$\because |x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

But cos |h| is differentiable

∴ Any combination of two such functions will be non-differentiable. Hence option (a) and (b) are ruled out.

Now, consider sin |x| + |x|

$$L' = \lim_{h\to 0} \frac{\sin|-h| + |-h|}{-h}$$

$$= \lim_{h\to 0} \frac{\sin h}{-h} - 1 = -1 - 1$$

$$R' = \lim_{h\to 0} \frac{\sin|h| + |h|}{h}$$

$$= \lim_{h\to 0}\frac{\sin h}{h}+1=1+1=2$$

Consider sin |x| – |x|

$$L' = \lim_{h\to 0}\frac{\sin|-h|-|-h|}{-h}$$

$$= \lim_{h\to 0}\frac{\sin h}{-h}+1=0$$

$$R' = \lim_{h\to 0}\frac{\sin|h|-|h|}{h}$$

$$= \lim_{h\to 0}\frac{\sin h}{h}-1=0$$

Hence, sin |x| – |x| is diffentiable at x = 0.

Q.27 (c)
Continuous as well as differentiable,
so f'(1) =0

Q.28 (b)

We have; $f(x)=\begin{cases}(x-1)\sin\left(\dfrac{1}{x-1}\right) & \text{if } x\neq 1\\ 0\end{cases}$

$$Rf'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

if x=1

$$=\lim_{h\to 0}\frac{h\sin\frac{1}{h}-0}{h}=\lim_{h\to 0}\sin\frac{1}{h}$$

which does not exist.

$\therefore f$ is not differentiable at x=1 Also

$$f'(0)=\sin\frac{1}{(x-1)}-\frac{x-1}{(x-1)^2}\cos\left(\frac{1}{x-1}\right)\Bigg]\Bigg]_{x=0}$$

$$=-\sin 1+\cos 1$$

$\therefore f$ is differentiable at x=0

Q.29 (d)
Let

$$f(x)=\frac{x}{1+x}+\frac{x}{(x+1)(2x+1)}+\frac{x}{(2x+1)(3x+1)}+\ldots\infty$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left[\frac{x}{[(r-1)x+1]}-\frac{1}{rx+1}\right]$$

$$=\lim_{x\to\infty}\sum_{r=1}^{n}\left[\frac{x}{[(r-1)x+1]}-\frac{1}{rx+1}\right]$$

$$=\lim_{n\to\infty}\left[1-\frac{1}{nx+1}\right]=1$$

For x = 0, we have f(x) = 0

Thus, we have $f(x)=\begin{cases}1, & x\neq 0\\ 0, & x=0\end{cases}$

Clearly, $\lim_{x\to 0^-} f(x)=\lim_{x\to\infty} f(x)\neq f(0)$

So, f(x) is not continuous at x = 0.

Q.30 (d)

Let $f(x+y)=f(x)+f(y),\ \forall x,y\in R$

Put x = 0 = y

$\Rightarrow$ f(0) = f(0) + f(0)

$\Rightarrow$ f(0) = 0

Now, $f'(0)=\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$$f'''(0)=\lim_{h\to 0}\frac{f(h)}{h}$$

Now, $f(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$$

$$\Rightarrow f'(x)\lim_{h\to 0}\frac{f(h)}{h}=f'(0)$$

$$\Rightarrow f(x)=f'(0)+C$$

But f(0) = 0

$\therefore$ C = 0

Hence, $f(x)=f'(0),\ \forall x\in R$

Clear, f(x) is everwhere dcontinuous and differntialble and f'(x) is constant. $\forall x\in R$

Q.31 (a)
We have,

$$Lf'(0)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{-h\log\cosh}{-h\log(1+h^2)}$$

$$=\lim_{h\to 0}\frac{\log\ \cosh}{\log(1+h^2)}\left(\frac{0}{0}\ form\right)$$

$$= \lim_{h\to 0} \frac{-\tan h}{2h/(1+h^2)} = -1/2$$

$$Rf'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} \lim_{h\to 0} \frac{h \log \cosh}{h \log(1+h^2)}$$

$$= \lim_{h\to 0} \frac{\log \cos h}{\log(1+h^2)} \left(\frac{0}{0}\ form\right)$$

$$= \lim_{h\to 0} \frac{-\tan h}{2h/(1+h^2)} = \frac{-1}{2}$$

Since Lf'(0) = Rf'(0), therefore f(x) is differentiable at x = 0

Q.32 (2)

$$f(x) = \frac{x}{\sqrt{x+1}-\sqrt{x}} = \frac{x(\sqrt{x+1}+\sqrt{x})}{x+1-x}$$

$$f(x) = x\left(\sqrt{x+1}+\sqrt{x}\right)$$

Now, RHD

$$f(0^+) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h(\sqrt{h+1}-\sqrt{h})-0}{h}$$

$= 1$

since ve⁻ values are not in domain of f(x) hence differentiability calculated by RHD Since RHD is finit hence f(x) is differentiable

Q.33 *(4)*

$f(0) = \lim_{x\to 0} f(x) = 0 - 1 + 0.\sin(-1) = -1$

$f(0^+) = \lim_{x\to 0} f(x) = 0 + 0 + 0.\sin 0 = 0 = f(0)$

f(x) is not continuous at x = 0

at x = 2,

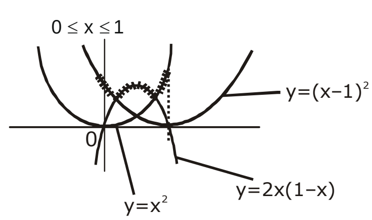
$f(2^+) = 2 + 2 + 2 \sin 2 = 4 + 2 \sin 2$

$f(2^-) = 2 + 1 + 2 \sin 1 = 3 + 2 \sin 1$

f(x) is not continuous at x = 2

Q.34 (3)

$f(x) = \max \{x^2, (x-1)^2, 2x(1-x)\}$



so, (c)

Q.35 (3)

$f(x) = x^3 - x^2 + x + 1$

$$g(x) = \begin{cases} \max(f(t))\,;\ 0 \le t \le x \ \text{ for } 0 \le x \le 1 \\ x^2 - x + 3\,;\ 1 < x \le 2 \end{cases}$$

max {f(t)} will be obtained when 't' would be max. so, t = x.

so, max $\{f(t)\} = x^3 - x^2 + x + 1$

$$f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2-(1+h)+3-2}{h}$$

= not defined

so not derivable

Now check cont by,

$$f(1^+) = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} (1+h)^2 - (1+h) + 3$$

= 3

& f(1) = 2

$f(1^+) \neq f(1)$

so f(x) is not continuous

Q.36 (2)

$$f'(2^-) = f'(2^+) = 2 \ \ \& \ f'(3^+) = f'(3^-) = \frac{21}{4}$$

Q.37 (1)

Q.38 (2)

Q.39 (4)

$f'(O^+) = p + q$ ....(1)

$f'(O^-) = -p + q$ ....(2)

$f'(O^+) = f'(O^-) \Rightarrow p + q = 0, r \in R$

Q.40 (2)

If f is differentiable everywhere.

then |f| will also be diff. everywhere.

and if two fns. are diff. then sum of then will also be diff. everywhere

Q.41 (b)

Q.42 (4)

$f(x + y) = f(x) . f(y), f(3) = 3$

$f'(0) = 11, f(3) = ?$

$$f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x)\cdot f(h)-f(x)}{h}$$

$$= f(x) . \lim_{h\to 0} \frac{f(h)-1}{(h)}$$

$$f'(3) = f(3) \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$f'(3) = f(3) . f'(0)$

$f'(3) = 3 \times 11 = 33$

$[\because\ f(0) = f(0) . f(0) \Rightarrow f(0) = 1]$

Q.43 (4)

$f(x + 2y) = f(x) + f(2y) + 2xy$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x)+f(h)-f(x)+2xy}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(0+h)-(0)}{h} + 2x$$

$f'(x) = f'(0) + 2$

Q.44 (4)

$f(x + y) = f(x).f(y)$

differentiate w.r.t. x

$f'(x + y) = f'(x) . f(y)$

put $x = 0,\ y = 5$

$f'(5) = f'(0) . f(5)$

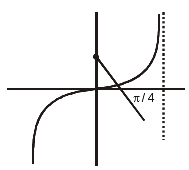
$= 3 . 2$

$\therefore\ f'(5) = 6$

Q.45 (2)

$2 \tan x + 5x - 2 = 0$

$$\tan x = -\frac{5x}{2} + 1$$



Q.46 (3)

By using L' Hospital rule

$$= \lim_{x\to 0} \frac{2f'(x)-f'(2x)+4f'(4x)}{2x}$$

$$\text{Again } = \lim_{x\to 0} \frac{2f''(x)-12f''(2x)+16f''(4x)}{2} = 12$$

Q.47 **(1)**

$f'(x) = \sqrt{2x^2-1}$, $y = f(x^2)$

$f'(x^2) = \sqrt{2x^4-1}$, $\frac{dy}{dx} = 2x.f'(x^2)$

$$\frac{dy}{dx} = 2x.\sqrt{2x^4-1}$$

$$\left(\frac{dy}{dx}\right)_{x=1} = 2$$

Q.48 (B)

$$y = e^x \Rightarrow \frac{dy}{dx} = e^x = y$$

Q.49 (b)

Q.50 (d)

Q.51 (d)

Q.52 (a)

Q.53 (3)

$y = x^3 - 8x + 7$ and $x = f(t)$

$$\frac{dy}{dt} = 2 \ \&\ x = 3 \text{ at } t = 0$$

$$\because \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Rightarrow \frac{dx}{dt} = \frac{dy/dt}{dy/dx}$$

$$\Rightarrow \frac{dx}{dt} = \frac{2}{3x^2-8}$$

$\because$ at $t = 0,\ x = 3$

$$\therefore \frac{dx}{dt} \text{ (at } t = 0) = \frac{2}{19}$$

Q.54 (2)

$\sin(xy) + \cos(xy) = 0$

$$\Rightarrow \cos(xy)\left(y + x\frac{dy}{dx}\right) - \sin(xy)\left(y + x\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\cos(xy).y - \sin(xy)\cdot y}{\cos(xy).x - \sin(xy).x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

Q.55 (3)

$y = f(x)$

$f(-x) = -f(x) \Rightarrow -f'(-x) = -f'(x)$

$f'(3) = f'(-3) = -2$

Q.56 (2)

$y = x - x^2$

$y^2 = x^2 + x^4 - 2x^3 \qquad u = y^2$

$u = x^2 + x^4 - 2x^3$

$$\frac{du}{dx} = 2x + 4x^3 - 6x^2$$

$v = x^2 \Rightarrow dv/dx = 2x$

$$\frac{du}{dv} = 2x^2 - 3x + 1$$

Q.57 (3)

$$x = \frac{1}{t^3} + \frac{1}{t^2}$$

$$\frac{dx}{dt} = \frac{-3}{t^4} - \frac{2}{t^3}, \quad \frac{dy}{dt} = \frac{3}{2}\left(\frac{-2}{t^3}\right) - \frac{2}{t^2}$$

$$\frac{dy}{dx} = \frac{-\frac{3}{t^3}-\frac{2}{t^2}}{-\frac{3}{t^4}-\frac{2}{t^3}}$$

$$\frac{dy}{dx} = t$$

so $x\left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = \frac{1+t}{t^3}.t^3 - t = 1$

Q.58 (3)

$y = x^{x^2}$

$y = e^{x^2 \ell n x}$

$\frac{dy}{dx} = e^{x^2 \ell n x}.(2x\ \ell n\ x + x)$

$= x^{x^2+1}\ (2\ \ell n\ x + 1)$

Q.59 (4)

$f(x) = |x|^{|\sin x|}$

at $x = \pi/4$, $|x| = x$ and $|\sin x| = \sin x$

$\therefore f(x) = x^{\sin x}$

$\Rightarrow \ell n(f(x)) = \sin x\ .\ \ell n x$

$\Rightarrow \frac{1}{f(x)} f'(x) = \cos x\ \ell n x + \frac{\sin x}{x}$

$\Rightarrow f'(\pi/4) = \left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(\frac{\sqrt{2}}{2}\ell n\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

Q.60 **(b)**

Q.61 **(1)**

$u = \sec^{-1}\frac{1}{(2x^2-1)}$; $v = \sqrt{1-x^2}$

$u = \cos^{-1}(2x^2-1)$;

differentiating w.r.t. to x

$\frac{du}{dx} = \frac{-1\times(4x)}{\sqrt{1-(2x^2-)^2}}$ & $\frac{du}{dx} = \frac{-x}{\sqrt{1-x^2}}$

$\frac{du}{dv} = \frac{-4x}{\sqrt{-4x^4+4x^2}} \times \frac{\sqrt{1-x^2}}{-x} = \frac{4}{2x}$

$\left|\frac{du}{dv}\right|_{x=1/2} = \frac{4}{2(1/2)} = 4$

Q.62 **(b)**

Q.63 **(d)**

Q.64 **(3)**

$x = e^{y+e^{y+.....to\ \infty}}$

$x = e^{y+x}$

$x = e^{(y+x)}\left\{\frac{dy}{dx}+1\right\}$

$\frac{dy}{dx} = \frac{1-e^{x+y}}{e^{x+y}} = \frac{1-x}{x}$

Q.65 **(4)**

$y = \sqrt{\sin x + y}$

squaring both side

$y^2 = \sin x + y$

$2yy' = \cos x + y'$

differentiating w.r.t. to x

$y' = \frac{\cos x}{2y-1}$

Q.66 (2)

$y = \cos^{-1}(\cos x)$, $\left.\frac{dy}{dx}\right|_{x=\frac{5\pi}{4}}$

$y' = \frac{-1}{\sqrt{1-\cos^2 x}} \times -\sin x = \frac{\sin x}{|\sin x|}$

$y'|_{x=\frac{5\pi}{4}} = -1$

Q.67 (3)

$y = \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$, $|x| > 1$

$\Rightarrow y = \frac{\pi}{2}$

$\frac{dy}{dx} = 0$

Q.68 (c)

Since, g is the inverse of function f. there fore,

$g(x) = f^{-1}(x)$

$\Rightarrow f[g(x)] = x$

$\Rightarrow fog(x) = x$, for all x

Differentiate both side, w.r.tx

$\Rightarrow \frac{d}{dx}\{fog(x)\} = \frac{d}{dx}(x)$, for all x

$\Rightarrow f'[g(x)]g'(x) = 1$, for all x

$\Rightarrow \sin\{g(x)\}g'(x) = 1$, for all x

(By defn of f'(x))

$\Rightarrow g'(x) = \frac{1}{\sin\{g(x)\}}$

Q.69 (b)

Let $f(x)=\cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$

Put log x = t in f(x)

$\therefore\ f(x)=\cos^{-1}\left[\frac{1-t^2}{1+t^2}\right]$

Now, put t = tanθ, we get

$f(x)=\cos^{-1}\left[\frac{1-\tan\theta^2}{1+\tan\theta^2}\right]$

$= \cos^{-1}[\cos 2\theta] = 2\theta = 2\tan^{-1} t = 2\tan^{-1}(\log x)$

Diff. both side w.r.t 'x' we get

$f(x)=2\cdot\frac{1}{1+(\log x)^2}\cdot\frac{1}{x}$

Now, $f'(e)=2\cdot\frac{1}{1+(\log e)^2}\cdot\frac{1}{e}=\frac{1}{e}$

($\because$ log e = 1)

Q.70 (b)

Q.71 **(b)**

Q.72 **(a)**

Q.73 **(d)**

Q.74 **(d)**

Q.75 **(2)**

Q.76 (4)

$y = \sin^{-1}(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}} + p$

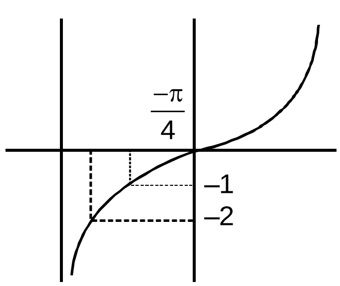
$y = \sin^{-1}(x) + \sin^{-1}(\sqrt{x})$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$

Q.77 **(3)**

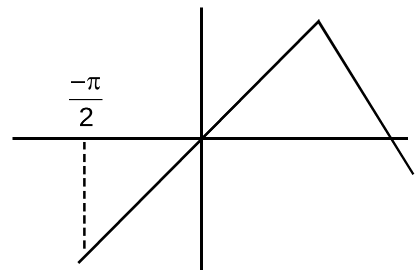
$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \cdot \left.\frac{dy}{dx}\right|_{x=-2}$

$x = \tan\theta \Rightarrow y = \sin^{-1}(\sin 2\theta)$



$\theta < -\frac{-\pi}{4}$

$2\theta < -\frac{-\pi}{2}$



$y = \pi - 2\theta = \pi - 2\tan^{-1} x$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-2}{(1+x^2)} = \frac{-2}{5}$

Q.78 **(2)**

$g(x) = f^{-1}(x)$ $g'(f(x)) = \frac{1}{f'(x)}$

$g'(f(x)) \cdot f'(x) = 1$ $= \frac{1+x^4}{x^5}$

$g'(f(g(2))) = \frac{1+a^4}{a^5}$

Q.79 (2)

F'(x) =

$\begin{vmatrix} f' & g' & h' \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix} + \begin{vmatrix} f & g & h \\ f'' & g'' & h'' \\ f'' & g'' & h'' \end{vmatrix} + \begin{vmatrix} f & g & h \\ g' & g' & h' \\ f''' & g''' & h''' \end{vmatrix} = 0$

Q.80 **(3)**

$f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 3x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$

differentiating w.r.t. to x

$f'(x) = \begin{vmatrix} \sin x & \cos x & -\sin x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$

$+ \begin{vmatrix} \cos x & \sin x & \cos x \\ -2\sin 2x & 2\cos 2x & -4\sin 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$

$+ \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ -3\sin 3x & 3\cos 3x & -9\sin 3x \end{vmatrix}$

$$f'\left(\frac{\pi}{2}\right) = \begin{vmatrix} -1 & 0 & -1 \\ -1 & 0 & -2 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 3 & 0 & 9 \end{vmatrix}$$

= –1(–2) + 0 –1(1) + 0 – 1 (–3) + 0

= 2 –1 +3 = 4

Q.81 (4)

$x = at^2$

$y = 2at$

$$\frac{dy}{dx} = \frac{2a}{2at}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{t^2}\ \frac{dt}{dx}$$

$$= -\frac{1}{t^2}\cdot\frac{1}{2at} = -\frac{1}{2at^3}$$

Q.82 (4)

$y = f(e^x)$

$y' = f'(e^x)\,.\,e^x \Rightarrow y'' = f''(e^x)\,e^{2x} + e^x\,f'(e^x)$

Q.83 (a)

Let $y = e^{-x}\cos x$

$$y_1 = -e^{-x}\sin x - e^{-x}\cos x = -e^{-x}\sin x - y$$

$$y_2 = -e^{-x}\cos x + e^{-x}\sin x - y_1$$

$$\Rightarrow y_2 = -y - y_1 e^{-x}\sin x = 2(y + y_{1)}$$

$$\Rightarrow y_3 = -2(y_1 + y_2) = -2(e^{-x}\sin x - y)$$

$$\Rightarrow y_4 = 4y_1 + 2y_2 = 4y_1 - 4y - 4y_1\alpha y_4 + 4y = 0$$

⇒ K=4

Q.84 (a)

$$y = (x + \sqrt{1+x^2})^n$$

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\left(1 + \frac{1}{2}(1+x^2)^{-1/2}\,2x\right);$$

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2} + x)}{\sqrt{1+x^2}}$$

$$= \frac{n(\sqrt{1+x^2} + x)^n}{\sqrt{1+x^2}}$$

$$\text{or}\ \sqrt{1+x^2}\,\frac{dy}{dx} = ny \ \text{ or}\ \sqrt{1+x^2}\,y_1 = ny\left(y_1 = \frac{dy}{dx}\right)$$

Squaring, $(1+x^2)y_1^2 = n^2y^2$

Differentiating,

$$(1+x^2)2y_1y_2 + y_1^2.2x = n^2.2yy_1$$

$$\text{or}\ (1+x^2)y_2 + xy_1 = n^2y$$

Q.85 **(b)**

$y = \sin x = e^x$

$$\Rightarrow \frac{dy}{dx} = \cos x + e^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos x + e^x} \qquad \ldots(i)$$

$$\therefore \frac{d^2x}{dy^2} = -\frac{1}{\left(\cos x + e^x\right)^2}\left[-\sin x + e^x\right]\frac{dx}{dy}$$

$$= -\frac{\left(e^x - \sin x\right)}{\left(\cos x + e^x\right)} \times \frac{1}{\cos x + e^x}$$

$$= \frac{-\left(e^x - \sin x\right)}{\left(\cos x + e^x\right)^3} = \frac{\sin x - e^x}{\left(\cos x + e^x\right)^3}$$

Q.86 (c)

Q.87 (c)

Q.88 (c)

Q.89 (d)

Q.90 (a)

Q.91 **(d)**

Q.92 **(a)**

Q.93 **(c)**

Q.94 **(b)**

Q.95 **(4)**

$$f'(4) = 5,\ \lim_{x\to 2}\frac{f(4) - f(x^2)}{2 - x}\qquad \left(\frac{0}{0}\right)$$

Apply L. Hospital rule

$$\lim_{x\to 2} 0 - \frac{f'(x^2).2x}{-1} \Rightarrow \lim_{x\to 2} 0 + f'(x^2)\,2x \Rightarrow f'(4)$$

$\cdot 2 \cdot 2$

$= f'(4)\cdot 4 = 20$

Q.96 (c)

Given,

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1$$

$$\Rightarrow f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \ldots + \frac{2x}{2} + 1 + 0$$

$[\because f(x) = x^n \Rightarrow f'(x) = nx^{n-1}]$

$\Rightarrow f'(x) = x^{99} + x^{98} + \ldots + x + 1$ ...(i)

Putting x = 1, we get

$$f'(1) = \underbrace{(1)^{99} + 1^{98} + \ldots + 1 + 1}_{100 \text{ times}} = \underbrace{1 + 1 + 1 \ldots + 1 + 1}_{100 \text{ times}}$$

$\Rightarrow f'(1) = 100$ ...(ii)

Agian, putting x = 0, we get

$f'(0) = 0 + 0 + \ldots + 0 + 1 \Rightarrow f'(0) = 1$ ...(iii)

From eqs. (ii) and (iii), we get; $f'(1) = 100f'(0)$

Hence, m = 100

Q.97 (b)
Q.98 (b)
Q.99 (b)
Q.100 **(c)**
Q.101 **(b)**
Q.102 **(2)**

$y = (1 + x)(1 + x^2) \ldots (1 + x^{2n})$

$$y = \frac{(1-x^2)(1+x^2)(1+x^4)\ldots(1+x^{2n})}{(1-x)}$$

$$y = \frac{1-x^{4n}}{1-x}$$

$$\frac{dy}{dx} = \frac{(1-x)(-4nx^{4n-1}) + (1-x^{4n})}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{-4nx^{4n-1} + 4nx^{4n} + 1 - x^{4n}}{(1-x)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{-4n \times 0 + 0 + 1 - 0}{1} = 1$$

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 **(0001)**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\{f(x) + f(h) - 2xh - 1\} - f(x)}{h}$$

(Using the given relation)

$$= \lim_{h\to 0} -2x + \lim_{h\to 0} \frac{f(h) - 1}{h}$$

$$= \lim_{h\to 0} -2x + \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

[Putting x = 0 = y in the given relation we find f(0) = f(0) + f(0) + 0 – 1 $\Rightarrow$ f(0) = 1]

$\therefore \ f'(x) = -2x + f'(0)$

$\Rightarrow f(x) = -x^2 - \sin\alpha . x + C$

$f(0) = -0 - 0 + C$

$\Rightarrow C = 1$

$\therefore \ f(x) = -x^2 - \sin\alpha . x + 1$

So, $f\{f'(0)\} = f(-\sin\alpha) = -\sin^2\alpha + \sin^2\alpha + 1$

$\therefore \ f\{f'(0)\} = 1$

Q.2 **(0000)**

Given that f(x) is a function satisfying

$f(-x) = f(x), \ \forall x \in R$ …(1)

Also $f'(0)$ exists

$\Rightarrow f'(0) = Rf'(0) = Lf'(0)$

Now, $Rf'(0) = f'(0)$

$$\Rightarrow \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = (0)$$

$$\Rightarrow \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = f'(0)$$

$$\Rightarrow \lim_{h\to 0} \frac{f(h) - f(0)}{h} = f'(0) \quad \ldots(2)$$

Again $Lf'(0) = f'(0)$

$$\Rightarrow \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = f'(0)$$

$$\Rightarrow \lim_{h\to 0} \frac{f(-h) - f(0)}{-h} = f'(0)$$

$$\Rightarrow \lim_{h\to 0} \frac{f(h) - f(0)}{h} = -f'(0) \quad \ldots(3)$$

[Using eq. (1)]

from equations (2) and (3) we get, $\Rightarrow f'(0) = 0$

Q.3 **(0001)**

$$f\left(\frac{1}{4^n}\right) = (\sin e^n)\, e^{-n^2} + \frac{n^2}{n^2+1}$$

$$\lim_{n\to\infty} f\left(\frac{1}{4^n}\right) = \lim_{n\to\infty}\left(\frac{\sin e^n}{e^{n^2}} + \frac{n^2}{1+n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{\sin e^n}{e^{n^2}} + \frac{1}{1+\frac{1}{n^2}}\right)$$

$\therefore \ f(0) = 0 + 1 = 1$

Q.4 (0001)

$t^2 f(x) = -2t\, f'(x) + f''(x) = 0$ has equal roots

Discriminant $= 4(f'(x))^2 - 4 f(x) f''(x) = 0$

$$\frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

$\ln(f'(x)) = \ln f(x) - \ln c$

$\Rightarrow f(x) = c\, f'(x)$

$f(0) = c\,f'(0) \Rightarrow c = \frac{1}{2}$

$\frac{f'(x)}{f(x)} = 2 \Rightarrow \ln f(x) = 2x + k \Rightarrow \ln f(0) = k \Rightarrow k = 0$

$\Rightarrow \ln f(x) = 2x$

$\therefore f(x) = e^{2x}$

$t^2 e^{2x} - 4te^{2x} + 4e^{2x} = 0$

$\Rightarrow t^2 - 4t + 4 = 0$

$t = 2$

$\left(\therefore \lim_{x\to 0}\frac{f(x)-1}{x} - \frac{t}{2}\right) =$

$\left(\lim_{x\to 0}\frac{e^{2x}-1}{2x} \times 2 - \frac{2}{2}\right)$

$= 2 - 1 = 1$

Q.5 (0008)

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x+0)}{h}$

$\Rightarrow \lim_{h\to 0}\frac{2f(x)+xf(h)+h\sqrt{f(x)}-2f(x)-xf(0)-0\sqrt{f(x)}}{h}$

as f(0) = 0

$\Rightarrow \lim_{h\to 0} x\left(\frac{f(h)-f(0)}{h-0}\right) + \sqrt{f(x)} \quad \Rightarrow f'(x) = \sqrt{f(x)}$

$\Rightarrow \int \frac{f'(x)}{\sqrt{f(x)}}dx = \int dx \quad \Rightarrow 2\sqrt{f(x)} = x + c$

$\Rightarrow f(x) = \frac{x^2}{4}$

when $\alpha = 0$ area is minimum

required minimum area =

$2\int_0^9 2\sqrt{y}\,dy = 4\left(\frac{y^{3/2}}{3/2}\right)_0^9 = 72\text{sq. unit.}$

Q.6 (0001)

$f(x) = x + \cos x + 2,\ f(0) = 3 \quad \Rightarrow g(3) = 0$

$g(f(x)) = x$

$\Rightarrow g'(f(x)).\, f'(x) = 1$ putting x = 0, $g'(3)\,.\,f(0) = 1$

Now, $f'(x) = 1 + \sin x \Rightarrow f'(0) = 1 \Rightarrow g'(3) = 1$.

Q.7 (0004)

$\frac{f(x+2y)}{3} = \frac{f(x)+2f(y)}{3}$

$\frac{1}{3}f'\frac{(x+2y)}{3} = \frac{f'(x)}{3} \quad \ldots(i)$

$\frac{2}{3}f'\left(\frac{x+2y}{3}\right) = \frac{2f'(x)}{3} \quad \ldots(ii)$

for (i & (ii) f′(x) = f′)y)

$\Rightarrow f'(x) = C = 1,\ f(x) = x + d$

As f(0) = 2

f(x) = x + 2

f(2) = 2 + 2 = 4

PREVIOUS YEAR'S

MHT CET

Q.1 (1)	**Q.2** (2)	**Q.3** (3)	**Q.4** (4)	**Q.5** (3)
Q.6 (2)	**Q.7** (4)	**Q.8** (3)	**Q.9** (1)	**Q.10** (3)
Q.11 (3)	**Q.12** (1)	**Q.13** (1)	**Q.14** (3)	**Q.15** (2)
Q.16 (3)	**Q.17** (4)	**Q.18** (3)	**Q.19** (3)	**Q.20** (2)
Q.21 (2)				

Q.22 (2)

Given, $f(x) = \begin{cases} \frac{3\sin \pi x}{5x}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{3\sin \pi x}{5x}\right)$

$= \frac{3}{5}\lim_{x\to 0}\left(\sin\frac{\pi x}{\pi x}\right) \times \pi = \frac{3}{5} \times 1 \times \pi = \frac{3}{5}\pi$

Also, f(0) = 2k

Since, f(x) is continuous at x = 0.

$\therefore f(0) = \lim_{x\to 0} f(x) \Rightarrow \quad 2k = \frac{3}{5}\pi \Rightarrow k = \frac{3\pi}{10}$

Q.23 (2)

we have, $f(x) = \begin{cases} \frac{\sin^3(\sqrt{3}).\log(1+3x)}{(\tan^{-1}\sqrt{x})^2(e^{5\sqrt{3}}-1)x}, & x \neq 0 \\ a, & x = 0 \end{cases}$

For continuity in [0, 1], $f(0) = \lim_{x\to 0} f(x)$ otherwise it is discontinuous.

$\therefore a = \lim_{x\to 0}\frac{\sin^3(\sqrt{x}).\log(1+3x)}{x(\tan^{-1}\sqrt{x})^2.(e^{5\sqrt{x}}-1)}$

$= \lim_{x\to 0}\left[\frac{3}{5}.\frac{\sin^3\sqrt{x}}{(\sqrt{x})^3}.\frac{(\sqrt{x})^3}{(\tan^{-1}\sqrt{x})^3}\right]$

Q.24 (2)

$\lim_{x\to 0}\frac{2^x - 2^{-x}}{x} = \lim_{x\to 0}\log 2 + 2^{-x}\log 2$

[using L' Hospital's rule]

$= \log 2 + \log 2 = \log 4$

Since, function is continuous at x = 0.

$\therefore \quad f(0) = \lim_{x\to 0} \frac{2^x - 2^{-x}}{x} = \log 4$

Q.25 (3)

Given, $f(x) = \begin{cases} ax+3, x \le 2 \\ a^2x - 1, x > 2 \end{cases}$

Continuity at x = 2,

LHL = $\lim_{x\to 2^-} f(x) = \lim_{x\to 2} (ax + 3) = 2a + 3$

RHL = $\lim_{x\to 2^+} f(x) = \lim_{x\to 2} (a^2 x - 1) = 2a^2 - 1$

Since, f(x) is continuous for all values of x.

$\therefore$ LHL = RHL

$\Rightarrow 2a + 3 = 2a^2 - 1$

$\Rightarrow 2a^2 - 2a - 4 = 0$

$\Rightarrow a^2 - a - 2 = 0$

$\Rightarrow a^2 - 2a + a - 2 = 0$

$\Rightarrow a(a-2) + 1(a-2) = 0$

$\Rightarrow (a + 1)(a - 2) = 0$

$\therefore$ a = – 1, 2

$\times \frac{\log(1+3x)}{3x} \cdot \frac{5\sqrt{x}}{3^{5\sqrt{x}} - 1}\Bigg]$

$= \frac{3}{5} \lim_{x\to 0} \frac{\sin^3}{(\sqrt{x})^3} \cdot \frac{(\sqrt{x})^3}{\tan^{-1}\sqrt{x}}$

$\times \frac{\log(1+3x)}{3x} \cdot \frac{5\sqrt{x}}{e^{\sqrt{x}} - 1} = \frac{3}{5}$

$\therefore \quad a = \frac{3}{5}$

Q.26 (4)

$\frac{\cos x}{|\cos x|}$ is not defined at x = (2n + 1) $\frac{\pi}{2}, \forall x \in I$.

Hence, it is discontinuous.

Q.27 (1)

Given, $f(x) = x - |x - x^2|$

at x = 1, f(1) = 1 – | 1 – 1| = 1

$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} [(1-h) - |(1-h) - (1-h)^2|]$

$= \lim_{h\to 0} [(1-h) - |h - h^2|] = 1$

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} [(1+h) - |1+h) - (1+h)^2|]$

$= \lim_{h\to 0} [1 + h - |-h^2 - h|] = 1$

$\because \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} = f(1)$

$\therefore$ f(x) is continuous at x = 1.

Q.28 (3)

Given f(x) = x |x| and g(x) = sin x

$gof(x) = \sin(x|x|) = \begin{cases} -\sin x^2, & x < 0 \\ \sin x^2, & x \ge 0 \end{cases}$

$(gof)'(x) = \begin{cases} -2\cos x^2, & x < 0 \\ 2x\cos x^2, & x \ge 0 \end{cases}$

Clearly, L (gof)'(0) = 0 = R (gof)'(0)

$\therefore$ gof is differentiable at x = 0 and also its derivative is continuous at x = 0.

Now, $(gof)''(x) \begin{cases} -2\cos x^2 + 4x^2 \sin x^2, & x < 0 \\ 2\cos x^2 - 4x^2 \sin x^2, & x > 0 \end{cases}$

$\therefore$ L (gof)'' (0) = – 2 and R (gof) '' (0) = 2

$\therefore$ L (gof)''(0) $\ne$ R (gof)'' (0)

$\therefore$ gof (x) is not twice differentiable at x = 0

Q.29 (2)

We have $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{for } x < 4 \\ a + b & \text{for } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{for } x > 4 \end{cases}$

$\because$ f(x) is continuous at x = 4

$\therefore \lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x) = f(4)$

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \frac{x-4}{|x-4|} + b = b + 1$ ...(i)

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \frac{x-4}{|x-4|} + a = a - 1$ ...(ii)

f(4) = a + b ...(iii)

Equating Eqs. (i) and (iii), we get a = 1

Equating Eqs. (ii) and (iii), we get b = – 1

Q.30 (2)	**Q.31** (1)	**Q.32** (1)	**Q.33** (2)	**Q.34** (3)
Q.35 (1)	**Q.36** (2)	**Q.37** (1)	**Q.38** (1)	**Q.39**(1)
Q.40(3)	**Q.41**(2)	**Q.42**(1)	**Q.43**(4)	**Q.44**(3)
Q.45(1)	**Q.46**(3)	**Q.47**(1)	**Q.48** (2)	**Q.49** (4)
Q.50 (1)	**Q.51** (4)	**Q.52** (1)	**Q.53** (4)	**Q.54** (1)
Q.55 (4)	**Q.56** (2)	**Q.57** (3)	**Q.58** (1)	**Q.59** (1)
Q.60 (1)	**Q.61** (3)	**Q.62** (1)	**Q.63** (3)	**Q.64** (1)
Q.65 (2)	**Q.66** (3)	**Q.67** (1)	**Q.68** (3)	**Q.69** (2)
Q.70 (2)	**Q.71** (4)	**Q.72** (3)	**Q.73** (2)	**Q.74** (1)
Q.75 (3)	**Q.76** (3)	**Q.77** (3)		

Q.78 (4)

$y = \tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$

$$y=\tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}}$$

$$y=\tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$y=\tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]$$

$$y=\frac{\pi}{2}-\frac{x}{2}\Rightarrow\frac{dy}{dx}=-\frac{1}{2}$$

Q.79 (2)

Let $u=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$, then

$$u=\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]=\tan^{-1}\left[\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right]$$

$$=\tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]=\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$=\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]=\tan^{-1}\left[\tan\frac{\theta}{2}\right]$$

$$\Rightarrow u=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x\quad\left[\because \tan^{-1}(\tan\theta)=\theta\right]$$

On differentiating both sides w.r.t.x, we get

$$\frac{du}{dx}=\frac{1}{2(1+x)^2}\left[\because\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}\right]\quad ...(i)$$

Also, let $v=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$, then we get

$$v=\sin^{-1}\left[\frac{2\tan\theta}{1+\tan\theta}\right]$$

$\Rightarrow$ $v = \sin^{-1}[\sin\theta]$

$\Rightarrow$ $v = 2q \Rightarrow v = 2\tan^{-1} x$

On differentiating both sides w.r.t.x, we get

$$\frac{dv}{dt}=\frac{2}{1+x^2}\quad ...(ii)$$

Now, $$\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}=\frac{1}{2(1-x^2)}\times\frac{(1+x^2)}{2}$$

[from Eqs. (i) and (ii)]

$$\therefore\ \frac{du}{dv}=\frac{1}{4}$$

Q.80 (1)

Given, $y=\sin^{-1}\left(6x\sqrt{1-9x^2}\right)$

$\Rightarrow$ $y=\sin^{-1}(2.3x\sqrt{1-(3x)^2}$

Put $3x = \sin q \Rightarrow y = \sin^{-1}(2\sin\theta.\cos\theta)$

$\Rightarrow$ $y = \sin^{-1}(\sin 2q)$

$\Rightarrow$ $y = 2q \Rightarrow$ $y = 2\sin^{-1}(3x)$

$$\Rightarrow\ \frac{dy}{dx}=\frac{2}{\sqrt{1-9x^2}}(3)$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{6}{\sqrt{1-9x^2}}$$

Q.81 (3)

Given, $y = (\sin x)^x + \sin^{-1} x$ ...(i)

Let $u = (\sin x)^x$ ...(ii)

Then, Eq. (i) becomes,

$y = u + \sin^{-1}\sqrt{x}$

On taking log both sides of Eq. (ii), we get

$\log u = x \log \sin x$

On differentiating both sides w.r.t.,x, we get

$$\frac{1}{u}\frac{du}{dx}=x\frac{d}{dx}(\log\sin x)+\log\sin x\frac{d}{dx}(x)$$

[by using product rule of derivative]

$$\Rightarrow\ \frac{du}{dx}=u\left[x\times\frac{1}{\sin x}\frac{d}{dx}(\sin x)+\log\sin x(1)\right]$$

$$\Rightarrow\ \frac{du}{dx}=(\sin x)^x\left[\frac{x}{\sin x}\times\cos\times\log\sin x\right]$$

$$\Rightarrow\ \frac{du}{dx}=(\sin x)^x[x\cot x+\log\sin x]\quad ...(iv)$$

On differentiating both sides of Eq. (iii) w.r.t.x, we get

$$\frac{dy}{dx}=\frac{du}{dx}+\frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}(\sqrt{x})$$

$$\therefore\ \frac{dy}{dx}(\sin x)^x[x\cot x+\log\sin x]+\frac{1}{\sqrt{1-x}}\times\frac{1}{2\sqrt{x}}$$

[from Eq. (iv)]

Q.82 (1)

Given, $y = x^{\sin x} + \sqrt{x}$

Let $y_1 = x^{\sin x}$ and $y_2 = \sqrt{x}$

Now, $y_1 = x^{\sin x} \Rightarrow \log y_1 = \sin x \log x$

Differentiating w.r.t. x, we get

$$\frac{1}{y_1}\frac{dy_1}{dx}=\cos x\log x+\frac{1}{x}\sin x$$

$$\Rightarrow \frac{dy_1}{dx} = x^{\sin x}\left[\cos x \log x + \frac{1}{x}\sin x\right]$$

$$\left(\frac{dy_1}{dx}\right)_{x=\frac{\pi}{2}} = \left(\frac{\pi}{2}\right)^{\sin\frac{\pi}{2}}\left[\cos\frac{\pi}{2}\log\frac{2}{\pi}\sin\frac{\pi}{2}\right] = \frac{\pi}{2}\times\frac{2}{\pi} = 1$$

Now, $y_2 = \sqrt{x} \Rightarrow \frac{dy_2}{dx} = 1/2\sqrt{x}$

$$\left(\frac{dy_2}{dx}\right)_{x=\frac{\pi}{2}} = \frac{1}{2\sqrt{\pi/2}} = \frac{1}{\sqrt{2\pi}}$$

Since, $y = y_1 + y_2$

$\therefore$ At $x = \frac{\pi}{2}, \frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx} \Rightarrow \frac{dy}{dx} = 1 + \frac{1}{\sqrt{2\pi}}$

Q.83 (4)

Given ,$y = \tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$

Put $x^2 = \cos 2\theta$ in the given equation,

$$\therefore\ y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}$$

$$= \tan^{-1}\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \tan^{-1}\frac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$$

$$= \tan^{-1}\frac{(1-\tan\theta)}{(1+\tan\theta)} = \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}$$

$$\Rightarrow y = \frac{\pi}{4}-\theta = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$$

$$\Rightarrow \frac{dy}{dx} = 0-\frac{1}{2}\left(\frac{-2x}{\sqrt{1-x^4}}\right) = \frac{x}{\sqrt{1-x^4}}$$

Q.84 (1)

Let $f(x) = ax^2 + bx + c$

$\because\ f(1) = f(-1)$

$\Rightarrow a + b + c = a - b + c$

$\Rightarrow\ b = 0$

$\therefore\ f(x) = ax^2 + c$

Differentiating w.r.t.x,

$\therefore\quad f'(a_1) = 2aa_1,$

$f'(a_2) = 2aa_2$

and $f'(a_3) = 2aa_3$

Assume that, $f'(a_1), f'(a_2)$ and $f'(a_3)$ are in AP, then

$2f'(a_2) = f'(a_1) + f'(a_3)$

$\Rightarrow\ 2.2aa_2 = 2aa_1 + 2aa_3$

$\Rightarrow\ 2a_2 = a_1 + a_3$

So, a_1, a_2, a_3 are also in AP.

$\therefore\ f'(a_1), f'(a_2), f'(a_3)$ are in AP.

Q.85 (2)

We have, $\log(x + y) = \log(xy) + 3$

$\Rightarrow\ \log(x + y) = \log x + \log y + 3$ ...(i)

On differentiating both sides of Eq. (i) w.r.t.x, we get

$$\frac{1}{x+y}\left[1+\frac{dy}{dx}\right] = \frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{x+y}+\frac{dy}{dx}\left(\frac{1}{x+y}\right) = \frac{1}{x}+\frac{1}{y}\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1}{x+y}-\frac{1}{y}\right] = \frac{1}{x}+\frac{1}{y}\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{y-x-y}{y(x+y)}\right] = \frac{x+y-x}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{-x}{y(x+y)}\right) = \frac{y}{x(x+y)} \Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^2$$

Q.86 (3)

We have,

$x = \sqrt{a^{\sin^{-1}t}}$ ...(i)

and $y = \sqrt{a^{\cos^{-1}t}}$ ...(ii)

On multiplying Eqs. (i) and (ii) we get

$$xy = \sqrt{a^{\sin^{-1}t}.a^{\cos^{-1}t}} = \sqrt{a^{\sin^{-1}t+\cos^{-1}t}}$$

$\Rightarrow\ xy = \sqrt{a^{\pi/2}}\ \left[\because \sin^{-1}x+\cos^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow\ xy = a^{\pi/4}$ ...(iii)

On differentiating both sides of Eq. (iii) w.r.t x, we get

$$x\frac{dy}{dx}+y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

JEE MAIN PREVIOUS YEAR'S

Q.1 (2)

Check continuity at x = 0, and also check continuity at those x where g (x) = 0

g (x) = 0 at x = 0, 2

$(fog)(0^+) = -1$

$(fog)(0^-) = 0$

Hence, discontinuous at x = 0,

$Fog(2^+) = 1$

$Fog(2^-) = -1$

Hence discontinuity at exactly two points.

Q.2 (3395)

$f(x) = (c+1)x^2 + (1-c^2)x + 2k$ ....(1)

& $f(x+y) = f(x) + f(y) - xy\ \forall\ x, y \in R$

$$\lim_{y\to 0}\frac{f(x+y)-f(x)}{y} = \lim_{y\to 0}\frac{f(y)-xy}{y} \Rightarrow f'(x) = f'(0) - x$$

$f(x) = -\frac{1}{2}x^2 + f'(0).x + \lambda$ but $f(0) = 0 \Rightarrow \lambda = 0$

$f(x) = -\frac{1}{2}x^2 + (1-c^2).\,x$ .....(2)

as $f'(0) = 1 - c^2$

Comparing equation (1) and (2)

We obtain, C = $-\frac{3}{2}$

$\therefore\ f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$

$|2\sum_{x=1}^{20} f(x)| = \sum_{x=1}^{20} x^2 + \frac{5}{2}.\sum_{x=1}^{20} x$

$= 2870 + 525 = 3395$

Q.3 **[7]**

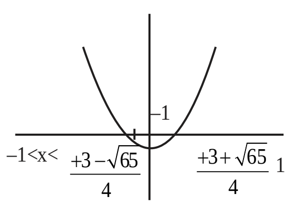
$$f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{if} \quad x \le -1 \\ [4x^2 - 1] & \text{if} \quad -1 < x < 1 \\ |x+1| + |x-2| & \text{if} \quad x \ge 1 \end{cases}$$

for $y = 2x^2 - 3x - 7$

Now $2x^2 - 3x - 7 = 0$

$$x = \frac{3 \pm \sqrt{65}}{4}$$

for $y = [4x^2 - 1] = [4x^2] - 1$;



$$y = \begin{cases} 3-1 = 2 & -1 < x \le \frac{-\sqrt{3}}{2} \\ 2-1 = 1 & \frac{-\sqrt{3}}{2} < x \le \frac{-1}{\sqrt{2}} \\ 1-1 = 0 & \frac{-1}{\sqrt{2}} < x \le \frac{-1}{\sqrt{2}} \\ 0-1 = -1 & \frac{-1}{2} < x < \frac{-1}{2} \\ 1-1 = 0 & \frac{1}{2} \le x < \frac{1}{\sqrt{2}} \\ 2-1 = 1 & \frac{1}{\sqrt{2}} \le x < \frac{\sqrt{3}}{2} \\ 3-1 = 2 & \frac{\sqrt{3}}{2} \le x < 1 \end{cases}$$

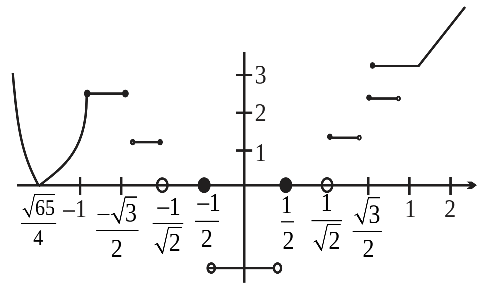
for $y = |x+1| + |x-2|$; $x \le 1$

$$y = \begin{cases} (x+1) - x + 2 & 1 \le x < 2 \\ x + 1 + x - 2 & 2 \le x \end{cases}$$

or

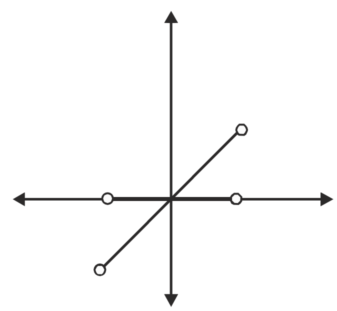
$$y = \begin{cases} 3 & 1 \le x < 2 \\ 2x - 1 & 2 \le x \end{cases}$$

so Graph will be



Total number of points where f(x) is discontinuous is 7.

Q.4 (3)



$\sin\frac{(x+2)}{x+2}$, $x \in (-2,-1)$

$f(x)$ [0 , $-1 < x < 0$
$2x$, $0 \le x < 1$

1 ,otherwise

maximum $(2x, 3[|x|]) \Rightarrow$

[0 , $-1 < x < 0$
$2x$, $0 \le x < 1$

$f(-1^-) = \sin 1$
$f(-1^+) = 0$] discontinuous at x = –1

$f(0^-) = 0$
$f(0^+) = 0$] continuous at x = 0 but not diff. at x = 0

$f(1^-) = 2$
$f(1^+) = 1$] discontinuous at x = 1

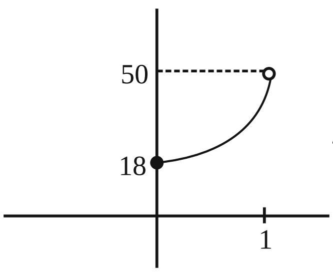
m = 2
n = 3] (m, n) = (2, 3)

Q.5 **[62]**

$f(x) = [x^2] + 1 \geq 1$

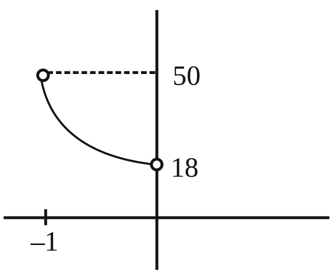
$$g(x) = \begin{cases} 2x-3, & x<0 \\ 2x+3, & x \geq 0 \end{cases}$$

Now, $fog(x) = [2(2x+3)^2] + 1$



49–18 = 31

$fog(x) = [2(2x-3)^2] + 1$



50–18–1 = 31

$\therefore$ 62 points of discontinuity

Q.6 **(2)**

$f(x) = \min\{1, 1 + x\sin x\}$

$$f(x) = \begin{cases} 1; & 0 \leq x \leq \pi \\ 1+\sin x; & \pi < x \leq 2\pi \end{cases}$$

at $x = 0$

$f(0) = \lim_{x \to 0^+} f(x) = 1$

at $x = \pi$

$f(\pi) = \lim_{x \to \pi} f(x) = \lim_{x \to \pi^+} f(x) = 1$

at $x = 2\pi$

$f(2\pi) = \lim_{x \to 2\pi} f(x) = 1$

function is continuous everywhere differentiability at $x = \pi$

$$f'(x) = \begin{cases} 0; 0 \leq x \leq \pi \\ x\cos x + \sin x; x < x \leq 2\pi \end{cases}$$

$$f'(\pi) = \begin{cases} 0; 0 \leq x \leq \pi \\ -\pi; \pi < x \leq 2\pi \end{cases}$$

L.H.D. $\neq$ R. H. D

f(x) is not differentiable at $x = \pi$

Q. 7 (4)

$$f(x) = \begin{cases} x+3 & ; & x < -3 \\ -(x+3) & ; & -3 \leq x < 0 \\ e^x & ; & x \geq 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 + k_1 x & ; & x < 0 \\ 4x + k_2 & ; & x \geq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} f(x) + k_1 f(x) & ; & f(x) < 0 \\ 4f(x) + k_2 & ; & f(x) \geq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} (x+3)^2 + k_1(x+3) & ; & x < -3 \\ (x+3)^2 - k_1(x+3) & ; & -3 \leq x < 0 \\ 4e^x + k_2 & ; & x \geq 0 \end{cases}$$

check continuity at x=0

$gof(0) = g(f(0^-)) = g(f(0^+))$

$4 + k_2 = 9 - 3k_1 = 4 + k_2$

$3k_1 + k_2 = 5$ ....(a)

differentiate

$$g(f(x))' = \begin{cases} 2(x+3) + k_1 & ; & x < -3 \\ 2\ (x+3) - k_1 & ; & -3 \leq x < 0 \\ 4e^x & ; & x \geq 0 \end{cases}$$

$6 - k_1 = 4$

$k_1 = 2$ .....(b)

$k_1 = 2$, $k_2 = -1$

$$gof(x) = \begin{cases} (x+3)^2 + 2(x+3) & ; & x < -3 \\ (x+3)^2 - 2(x+3) & ; & -3 \leq x < 0 \\ 4e^x - 1 & ; & x \geq 0 \end{cases}$$

$gof(-4) + gof(4) = 4e^4 - 2$

$\Rightarrow 2(2e^4 - 1)$

Q.8 (3)

f(x) is discontinuous at x = 1

For f(x) to be continuous at x = 0, a should be = 1

For f(x) to be continuous at x = 2, b + c should be = 1

a + b + c = 2

Q.9 (2)

Note : n should be given as a natural number

$$f = \begin{cases} -\dfrac{\sin(x-1)}{x-1} & x < -1 \\ -(\sin 2 + 1) & x = -1 \\ (\cos 2\pi x) & -1 < x < 1 \\ 1 & x = 1 \\ \dfrac{-\sin(x-1)}{x-1} & x > 1 \end{cases}$$

f(x) is discontinuous at x = –1 and x = 1

Q.10 (3)

$$f(x)=\begin{cases}\left|4x^2-8x+5\right|8x^2-4x-2x+1\ge 0\\ \left|4x^2-8x+5\right|8x^2-4x-2x+1<0\end{cases}$$

$$f(x)=\begin{cases}4(x-1)^2+1 \;\; x\le\frac{1}{4}\,\text{U}\, x<\frac{1}{2}\\ \left[4(x-1)^2+1\right] \;\; \frac{1}{4}<x<\frac{1}{2}\end{cases}$$

$$f(x)=\begin{cases}4(x-1)^2+1 & x\le\frac{1}{4}\cup x<\frac{1}{2}\\ 2 & 1-\frac{1}{\sqrt{2}}\le x<\frac{1}{2}\\ 3 & \frac{1}{4}<x<1-\frac{1}{\sqrt{2}}\end{cases}$$

$\Rightarrow$ f(x) is not diff. at $x\in\{\frac{1}{4},1-\frac{1}{\sqrt{2}},\frac{1}{2}\}$

3Ans.

Q.11 (4)

$$f(x)=\begin{cases}\dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ;\text{ if } x\ne 0\\ 10 & ;\text{ if } x=0\end{cases}$$

$$\lim_{x\to 0}\frac{\ln(1+5x)-\ln(1+\alpha x)}{x}=10$$

Using expansion

$$\lim_{x\to 0}\frac{(5x+...)(-\alpha x+...)}{x}=10$$

$5-\alpha=10\Rightarrow\alpha=-5$

Q.12 (79)

$$f(x)=4|2x+3|+9\left[x+\frac{1}{2}\right]-12[x+20]$$

$x\in(-20,20)$

f(x) is not Diff. at $x=1\in\{-19,-18,...0,...19\}=39$

at $x=I+\frac{1}{2}$, f(x) Non diff. at 39 points

Check at $x=\frac{-3}{2}$ Discontinuous at $x=\frac{-3}{2}$

$\therefore$ N.R. (1)

No. of point of non-differentiability

$=39+39+1=79$

Q.13 (2)

$f(x)=|x-1|\cos|x-2|\sin|x-1|+(x-3)|x^2-5x+4|$

$=|x-1|\cos|x-2|\sin|x-1|+(x-3)|x-1|\,|x-4|$

$=|x-1|\,[\cos|x-2|\sin|x-1|+(x-3)\,|x-4|]$

Non differentiable at x = 1 and x = 4

Q.14 (2)

$$\begin{array}{l} \\ \\ x\to\frac{x}{3}\\ x\to\frac{x}{3}\\ \\ \end{array}\left[\begin{array}{l}f(3x)-f(x)=x\\ f(x)-f\left(\frac{x}{3}\right)=\frac{x}{3}\\ f\left(\frac{x}{3}\right)-f\left(\frac{x}{3^2}\right)=\frac{x}{3^2}\\ \cdots\cdots\cdots\cdots\cdots\\ f\left(\frac{x}{3^{n-1}}\right)-f\left(\frac{x}{3^n}\right)=\frac{x}{3^n}\end{array}\right.$$

$$f(x)-f\left(\frac{x}{3^n}\right)=\frac{x}{3}\left[1+\frac{1}{3}+...+\frac{1}{3^{n-1}}\right]$$

$$=\frac{x}{3}\left[\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right]$$

$$=f(x)-f\left(\frac{x}{3^n}\right)=\frac{x}{3}\left[1+\frac{1}{3}+...+\frac{1}{3^{n-1}}\right]$$

$$=\frac{x}{3}\left[\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right]$$

$$=f(x)-f\left(\frac{x}{3^n}\right)=\frac{x}{2}\left[1-\frac{1}{3^n}\right]$$

Apply $\lim_{n\to\infty}$

$$f(x)-f(0)=\frac{x}{2}$$

Put $x=8\Rightarrow f(0)=3$

Put $x=14$

$f(14)=3+7=10$

Q.15 (1)

$f_0=k$

$f_0=f_{0^+}$

$$\lim_{x\to 0}\frac{\left[\frac{\log(x^4+x^2+1)}{(x^2+x^4)}\right](x^2+x^4)}{\left(\frac{1-\cos^2 x}{\cos x}\right)}$$

$$=\lim_{x\to 0}\frac{\left[\frac{\log\left(1+\left(x^2+x^4\right)\right)}{(x^2+x^4)}\cdot x^2(1+x^2)\right]}{\left(\frac{\sin^2 x}{x^2}\right)(x^2)}\times\cos x$$

$k = \frac{(1)}{(1)}(1) = 1$

Q.16 (4)

$g(f(2)) + f(1-2)$

$g[2] + f(-1)$

$(4+b) + (a-1)$

$a + b + 3$ ... (1)

Now f(x) is continuous $f(0^-) = f(0^+)$

$a = 4$

g(x) is continuous

$g(0^-) = g(0^+)$

$\Rightarrow 1 = b + 16$

$b = -15$

$\therefore$ Ans. $a + b + 3$

$4 - 15 + 3 = -8$

Q.17 **[2]**

f (x) is continuous at x = 0, $f(x) = \frac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{(729+qx)}-9}$

So $f(x) = \lim_{x\to 0} f(x)$

$f(0) = \lim_{x\to 0}, \frac{\sqrt[7]{p(729+x)-3}}{\sqrt[3]{(729+qx)-9}}, \frac{7\sqrt{p(729-3)}}{0} \Rightarrow$

$\sqrt[7]{p(729)} = 3 \Rightarrow P = 3$

$$f(0) \lim_{x\to 0} \frac{3\left((1+\frac{x}{729})^{\frac{1}{7}}-1\right)}{3^2\left((1+\frac{x^2}{729})^{\frac{1}{3}}-1\right)}$$

$$f(0) = \lim_{x\to 0}\frac{1}{3}\cdot\frac{1+\frac{x}{7.729}+....-1}{1+\frac{xq}{729.3}+....-1}$$

(using binomial expension)

$$f(0) = \lim_{x\to 0}\frac{1}{3}\cdot\frac{\frac{1}{7.729}}{\frac{q}{3.729}} = \frac{1}{3}\cdot\frac{3}{7q} = \frac{1}{7q}$$

$7q\, f(0)- 1 = 0 \Rightarrow 9.7\, q\, f(0) - 9 = 0$

$63q\, f(0) - p^2 = 0$

Q.18 **[248]**

$f(x+y) = 2^x f(y) + 4^y f(x)$ ...(1)

$x \Leftrightarrow y$

$f(y+x) = 2^y f(x) + 4^x f(y)$ ...(2)

(1) – (2)

$0 = f(x)(4^y - 2^y) + f(y)(2^x - 4^x)$

$$\Rightarrow \frac{f(x)}{(2^x-4^x)} = \frac{f(y)}{(2^y-4^y)} = \lambda(\text{say})$$

$\Rightarrow f(x) = \lambda(2^x - 4^x),\ f(y) = \lambda(2^y - 4^y)$

$f'(x) = \lambda[2^x \ln 2 - 4^x \ln 4]$

$$\frac{f'(4)}{f'(2)} = \frac{16\ln 2 - 256\ln 4}{4\ln 2 - 16\ln 4} = \frac{\ln 2[16-256\times 2]}{\ln 2[4-32]} = \frac{496}{28} = \frac{248}{14} \Rightarrow \text{Answer} = \frac{248}{14}\times 14 = 248$$

Q.19 (4)

$$\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$$

$$\cos^{-1}\left(\frac{y}{2}\right) = 5\log_e\left(\frac{x}{5}\right)$$

$$\frac{-1}{\sqrt{1-\frac{y^2}{4}}}\cdot\frac{y'}{2} = 5\cdot\frac{1}{\frac{x}{5}}\times\frac{1}{5}$$

$$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$$

$-xy' = 5\sqrt{4-y^2}$

$$-xy'' - y' = \frac{5}{2}\cdot\frac{1}{\sqrt{4-y^2}}(-2yy')$$

$$\Rightarrow xy'' + y' = \frac{5y'\cdot y}{\sqrt{4-y^2}}$$

$$xy'' + y' = 5\cdot\left(\frac{-5}{x}\right)y$$

$x^2y'' + xy' = -25y$

Q.20 **(3)**

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$f(x) = a\begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$$

$= a[1(a^2 + ax) + 1(ax + x^2)]$

$\Rightarrow f(x) = a(x+a)^2$

so, $f'(x) = 2a(x+a)$

as, $2f'(10) - f'(5) + 100 = 0$

$\rightarrow 2\times 2a(10+a) - 2a(5+a) + 100 = 0$

$\Rightarrow 40a + 4a^2 - 10a - 2a^2 + 100 = 0$

$2a^2 + 30a + 100 = 0$

$\Rightarrow a^2 + 15a + 50 = 0$

$(a+10)(a+5) = 0$

$a = -10$ or $a = -5$

Required $= (-10)^2 + (-5)^2 = 125$

Q.21 **[16]**

$y(x) = (x^x)^x$

$\ell n y(x) = x^2 \cdot \ell nx$

$$\frac{1}{y(x)} \cdot y'(x) = \frac{x^2}{x} + 2x \cdot \ell nx$$

$y'(x) = y(x)[x + 2x\,\ell nx]$

$y(1) = 1;\ y'(1) = 1$

$y''(x) = y'(x)[x + 2x \cdot \ell n(x)] + y(x)][1 + 2(1 + \ln x)]$

$y''(1) = 1[1+0] + 1(1+2) = 4$

$$\frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^3 \cdot \frac{d^2x}{dy^2}$$

$$\Rightarrow 4 = -\frac{d^2x}{dy^2},\quad \frac{d^2x}{dy^2} = -4$$

Ans. $-4 + 20 = 16$

Q.22 (1)

$f(x) = x^3 + x - 5 \Rightarrow \qquad f'(x) = 3x^2 + 1$

and $f(4) = 63$

$f(g(x)) = x \quad \therefore g(x) = f^{-1}(x)$

$$g'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow g'(f(x)) = \frac{1}{3x^2+1}$$

$$\Rightarrow g'(63) = \frac{1}{49},\ for\ x = 4$$

Q.23 (1)

$f(x) + f'(x) + f''(x) = x^5 + 64$

f is polynomial of degree 5

$f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 5x^4 + 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 20x^3 + 12ax^2 + 6bx + 2c$

$\therefore a + 5 = 0 \Rightarrow a = -5$

$b + 4a + 20 = 0 \Rightarrow b = 0$

$c + 3b + 12a = 0 \Rightarrow c = +60$

$d = -120$ and $e = 64$

$f(x) = x^5 - 5x^4 + 60x^2 - 120x + 64$

$= (x-1)(x^4 - 4x^3 - 4x^2 + 56x - 64)$

$$\therefore \lim_{x \to 1} \frac{f(x)}{x-1} = 1 - 4 - 4 + 56 - 64 = -15$$ Ans.

Q.24 (3)

$f(x) = \tan^{-1}(\sin x - \cos x)$

$$f'(x) = \frac{\cos x + \sin x}{(\sin x - \cos x)^2 + 1} = 0$$

$$\therefore\ x = \frac{3\pi}{4}$$

x	0	$\frac{3\pi}{4}$	π
f(x)	$-\frac{\pi}{4}$	$\tan^{-1}\sqrt{2}$	$\frac{\pi}{4}$

$f_{max} = \tan^{-1}\sqrt{2}$

$$f_{min} = -\frac{\pi}{4}$$

$$sum = \tan^{-1}\sqrt{2} - \frac{\pi}{4}$$

$$= \cos^{-1}\frac{1}{\sqrt{3}} - \frac{\pi}{4}$$

Q.25 (4)

$x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$

$$\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}$$

$y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$

$$\frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\frac{dy}{dx} = \tan 3t$$

$$\frac{dy}{dx} = 1 \text{ at } t = \frac{\pi}{4}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2\sqrt{2}}\sec 3t \cdot \sqrt{\sin 2t} = -3 \text{ at } t = \frac{\pi}{4}$$

$$\therefore \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{1+1}{-3} = -\frac{2}{3}$$

Q.26 (4)

$$\ell n2 \frac{d}{dx}\left(\frac{\ell n \operatorname{cosec} x}{\ell n \cos x}\right)$$

$$\ell n2\left\{\frac{\ell n(\cos x)\{-\cot x\} - \ell n(\operatorname{cosec} x)\{-\tan x\}}{(\ell n \cos x)^2}\right\}$$

$$x = \frac{\pi}{4}$$

$$\frac{\ell n2\left\{-\ell n\left(\frac{1}{\sqrt{2}}\right)+\ell n\left(\sqrt{2}\right)\right\}}{\left(\ell n\left(\frac{1}{\sqrt{2}}\right)\right)^2}$$

$$\ell n2\frac{\left\{\ell n\sqrt{2}+\ell n\left(\sqrt{2}\right)\right\}}{\left(-\frac{1}{2}\ell n2\right)^2}=4\text{Ans.}$$

Q.27 **[16]**

$(x^2+y^2-3)+(x^2-y^2-1)^5=2$

$(\alpha^2+\alpha^2-3)+(\alpha^2-\alpha^2-1)^5=0 \Rightarrow 2\alpha^2=4\ \alpha=\sqrt{2}$

Differentiate the equation (1)

$2x+2yy'+5(x^2-y^2-1)^4(2x-2yy')=0$

$2(x+yy')+5(x^2-y^2-1)^4.\ 2(x-yy')=0$

$x+yy'+5(x^2-y^2-1)^4.(x-yy')=0$ .... (i)

$\sqrt{2}+\sqrt{2}y'+5(-1)^4(\sqrt{2}-\sqrt{2}y')=0$

$y'=\frac{3}{2}$

Differentiate the equation [1]

$1+yy''+(y')^2+5.\ 4(x^2-y^2-1)^3(2x-2yy')(x-yy')+$

$5(x^2-y^2-1)^4(1-yy''-(y')^2)=0$

Put

$1+\sqrt{2}y''+\frac{9}{4}+20(-1)^3(2\sqrt{2}-2\sqrt{2}\,x\,\frac{3}{2})$

$(\sqrt{2}-\sqrt{2}\frac{3}{2})+5(-1)^4(1-\sqrt{2}y''-\frac{9}{4})=0$

$\frac{13}{4}+\sqrt{2}y''+40\times(-\frac{1}{2})+5-5\sqrt{2}y''-\frac{45}{4}=0$

$\left(\frac{13}{4}-\frac{45}{4}\right)-4\sqrt{2}y-15=0\Rightarrow y=-\frac{23}{4\sqrt{2}}$

$3y'-y^3y''=3\times\frac{3}{2}+2\sqrt{2}.\frac{23}{4\sqrt{2}}=\frac{32}{2}=16$

APPLICATION OF DERIVATIVES

EXERCISE-I (MHT CET LEVEL)

Q.1 (1)

Velocity, $v^2 = 2 - 3x$

Differentiating with respect to t, we get

$$2v\frac{dv}{dt} = -3.\frac{dx}{dt} \Rightarrow 2v\frac{dv}{dt} = -3v \Rightarrow \frac{dv}{dt} = -\frac{3}{2}$$

Hence acceleration is uniform.

Q.2 (1)

Displacements $s = -4t^2 + 2t$

Now velocity $v = -8t + 2$ and its acceleration $a = -8$

So $\left(\frac{ds}{dt}\right)_{t=1/2} = -8 \times \frac{1}{2} + 2 = -2$ and

$$\left(\frac{d^2s}{dt^2}\right)_{t=1/2} = -8$$

Q.3 **(2)**

Differentiating w.r.t. t: $\frac{dD}{dt} = \sqrt{2}\frac{da}{at}$

or $\frac{da}{dt} = \frac{1}{\sqrt{2}}\frac{da}{dt} = \frac{1}{\sqrt{2}} \times 0.5 \text{ cm/s}$

Let Area is denoted by A

$$\frac{dA}{dt} = 2a\frac{da}{dt} \quad ....(i)$$

when area A is 400 cm^2 then a = 20

$$\therefore \frac{dA}{dt} = 2 \times 20 \times \frac{0.5}{\sqrt{2}} = 10\sqrt{2} \text{ cm}^2/\text{sec}$$

Q.4 (1)

Let A sq. units in the area measure when the radius is r units. their $A = \pi r^2$

Differentiate both side w.r.t 't'

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} ....(i)$$

We have, $\frac{dA}{dt} = 3c\frac{dr}{dt}$

From eqn (i), we get

$$3c.\frac{dr}{dt} = 2\pi r.\frac{dr}{dt} \Rightarrow 3c = 2\pi r$$

Now, $c = \frac{2}{3}\pi(6) = 4\pi$ when $r = 6$

Q.5 (1)

We have, $a = \frac{d^2x}{dt^2} = -9.8$

The initial conditions are x(0)=19.6 and v(0)=0

So, $v = \frac{dx}{dt} = -9.8t + v(0) = -9.8t$

$$\therefore x = -4.9t^2 + x(0) = -4.9t^2 + 19.6$$

Now, the domain of the function is restricted since the ball hits the ground after a certain time. To find this time we set x=0 and solve for t.

Q.6 **(4)**

Q.7 **(4)**

Q.8 **(4)**

Q.9 **(1)**

Q.10 **(3)**

Q.11 **(2)**

Q.12 (1)

Given the rate of increasing the radius

$= \frac{dr}{dt} = 3.5 cm/sec$ and $r = 10cm$

Area of circle $= \pi r^2$, $A = \pi r^2$

$$\Rightarrow \frac{dA}{dt} = 2\pi r.\frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 2\pi \times 10 \times 3.5$$

$$\Rightarrow \frac{dA}{dt} = 220 cm^2/sec.$$

Q.13 (3)

Let $y = \sqrt{x^2 + 16}$ and $z = \frac{x}{x-1}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x^2+16)^{-1/2}(2x) \; \& \; \frac{dz}{dx} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$\therefore \frac{dy}{dz} = \frac{-x}{\sqrt{x^2+16}} \frac{1}{1/(x-1)^2}$$

$$\left(\frac{dy}{dz}\right)_{x=3} = \frac{-3(2)^2}{5} = \frac{-12}{5}$$

Q.14 (3)

$$\frac{dx}{dt}=2at+b\Rightarrow\frac{d^2x}{dt^2}=2a$$

Q.15 (4)

If displacement $\propto$ (velocity)^2 $s\propto v^2\Rightarrow v^2=2as$

Hence a is constant.

Q.16 (2)

$t=2$ for the point (2,–1)

$$\frac{dy}{dx}=\frac{4t-2}{2t+3}=\frac{6}{7}\text{ for } t=2$$

Q.17 (2)

$$y=\sin x\Rightarrow\frac{dy}{dx}=\cos x\Rightarrow\left(\frac{dy}{dx}\right)_{(\pi,0)}=-1$$

There fore the equation of tangent at (π, 0)
Is given by

$$y-0=-1(x-\pi)\Rightarrow x+y=\pi$$

Q.18 (2)

Differentiating the given equation of the curve

$$4x-6y.\left(\frac{dy}{dx}\right)=0\therefore\frac{dy}{dx}=\frac{2x}{3y}$$

$$\left(\frac{dy}{dx}\right)_{(3,2)}=\frac{2}{3}.\frac{3}{2}=1$$

Q.19 (4)

Slope of normal to y = f(x) at (3, 4) is $\frac{-1}{f'(3)}$.

Thus, $\frac{-1}{f'(3)}=\tan\left(\frac{3\pi}{4}\right)=\tan\left(\frac{\pi}{2}+\frac{\pi}{4}\right)$

$=-\cot\frac{\pi}{4}=-1\Rightarrow f'(3)=1.$

Q.20 (4)

The equation of the given curve is

$$y=\frac{1}{x-3},x\neq3$$

The slope of the tangent to the given curve at any point (x, y) is given by $\frac{dy}{dx}=\frac{-1}{(x-3)^2}$

For tangent having slope 2, we must have

$$2=\frac{-1}{(x-3)^2}$$

$$\Rightarrow2(x-3)^2=-1\Rightarrow(x-3)^2=-\frac{1}{2}$$

which is not possible as square of a real number cannot be negative. Hence, there is no tangent to the given curve having slope 2 .

Q.21 (2)

$$f(x)=\sqrt{x}(7x-6)=7x^{3/2}-6x^{1/2}$$

$$f'(x)=7\times\frac{3}{2}x^{1/2}-6\times\frac{1}{2}x^{-1/2}$$

When tangent is parallel to x axis f'(x) = 0

$$\propto,\frac{21}{2}x^{1/2}-3x^{-1/2}=0$$

$$\frac{21}{2}\sqrt{x}=\frac{3}{\sqrt{x}}$$

$$\propto,7x=2\Rightarrow x=\frac{2}{7}$$

Q.22 (1)
Q.23 (2)
Q.24 (4)
Q.25 (1)
Q.26 **(2)**
Q.27 **(1)**
Q.28 (3)

Given $y^2=2(x-3)$ .....(i)

Differentiate w.r.t. x, $2y.\frac{dy}{dx}=2\Rightarrow\frac{dy}{dx}=\frac{1}{y}$

Slope of the normal $=\frac{-1}{\left(\frac{dy}{dx}\right)}=-y$

Slope of the given line $=2$

$\therefore y=-2$

From equation (i), $x=5$

$\therefore$ Required point is $(5,-2)$.

Q.29 (2)

$y=2x^2-x+1$. Let the coordinates of P is (h, k), then

$$\left(\frac{dy}{dx}\right)_{(h,\,k)}=4h-1$$

Clearly $4h-1=3\Rightarrow h=1\Rightarrow k=2$;
$\therefore$ P is (1, 2).

Q.30 (1)

$$\sqrt{x}+\sqrt{y}=a\,;\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0,$$

$$\therefore\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$$

Hence tangent at (x, y) is $Y - y = -\frac{\sqrt{y}}{\sqrt{x}}(X - x)$

or $X\sqrt{y} + Y\sqrt{x} = \sqrt{xy}\left(\sqrt{x}+\sqrt{y}\right) = \sqrt{axy}$

or $\frac{X}{\sqrt{a}\sqrt{x}} + \frac{Y}{\sqrt{a}\sqrt{y}} = 1$.

Clearly its intercepts on the axes are $\sqrt{a}\sqrt{x}$ and $\sqrt{a}\sqrt{y}$.

Sum of the intercepts = $\sqrt{a}\left(\sqrt{x}+\sqrt{y}\right) = \sqrt{a}.\sqrt{a} = a$.

Q.31 (3)

Q.32 (3)

Q.33 (3) $x = a(t + \sin t)$, $y = a(1 - \cos t)$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a(\sin t)}{a(1+\cos t)} = \tan\frac{t}{2}$

Length of the normal = $y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$= a(1-\cos t)\sqrt{1+\tan^2(t/2)} = a(1-\cos t)\sec(t/2)$

$= 2a\sin^2(t/2)\sec(t/2) = 2a\sin(t/2)\tan(t/2)$.

Q.34 (4) $xy = c^2$ .....(i)

$\because$ Subnormal = $y\frac{dy}{dx}$

$\therefore$ From (i), $y = \frac{c^2}{x} \Rightarrow \frac{dy}{dx} = \frac{-c^2}{x^2}$

Subnormal = $\frac{y\times(-c^2)}{x^2} = \frac{-yc^2}{\left(\frac{c^2}{y}\right)^2} = \frac{-yc^2y^2}{c^4} = \frac{-y^3}{c^2}$

$\therefore$ Subnormal varies as y^3.

Q.35 (4)

$y = x^2 \Rightarrow \frac{dy}{dx} = m_1 = 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 2 = m_1$ and $x = y^2 \Rightarrow 1 = 2y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = m_2 = \frac{1}{2y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{1}{2}$

$\therefore$ Angle of intersection, $\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$

$= \frac{2-\frac{1}{2}}{1+2\times\frac{1}{2}} = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}(3/4)$

Q.36 (3)

Q.37 (3)

Let $f(x) = x^2 - x + 1$, $f'(x) = 2x - 1$

Obviously $f'(0) = -1$ and $f'(1) = 1$

Thus function is neither increasing nor decreasing.

Q.38 (3)

Let $f(x) = \sin x - bx + c$

$\therefore f'(x) = \cos x - b > 0$ or $\cos x > b$ or $b < -1$

Q.39 (3)

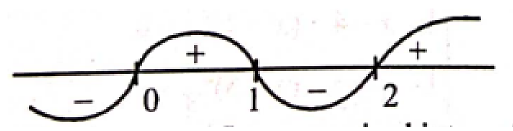
Here, $f(x) = (x(x-2))^2$

$\Rightarrow f'(x) = 4x(x-2)(x-1)$

For $f(x)$ as increasing, $f'(x) > 0$

So, $4x(x-1)(x-2) > 0$

$\Rightarrow x(x-1)(x-2) > 0$



From the above figure required interval is,

$(0,1)\cup(2,\infty)$

Q.40 (2)

Let f(x) = sin x – kx – c where k and c are constants.

f(x) = cos x – k

Thus, f(x) = sinx – kx – c decrease always

When $k \geq 1$

Q.41 (3)

(A) Graph of f(x) = cuts x-axis at infinite number of points. (5 of list II)

(B) Graph of f(x) = In x cuts x-axis in only one point. (4 of list II)

(C) Graph of f(x) = x^2 –5 + 4 cuts x axis in two points (2 of list II)

(4) Graph of f(x) = e^x cuts y-axis in only one point. (3 of list II)

Q.42 (3)

Since f (x) is an increasing function in [– 1,1] and it has a root in (–1,1).

$\therefore$ Only statement I is correct.

Q.43 (2)
A function f(x) is said to beincreasing function in [a, b] if $f'(x) > 0$ in [a, b].
Give $f(x) = x^x$ ....(i)
Differentiate equation (i)
$f'(x) = x^x(1 + \log x)$
Put $f'(x) = 0$
$0 = x^x(1 + \log x)$

$\Rightarrow x = 0, \log x = -1 \Rightarrow x = e^{-1} \Rightarrow x = \frac{1}{e}, 0$

Now, in $\left[0, \frac{1}{e}\right]$, $f'(x) > 0$

$\therefore$ f(x) is increasing in interval $\left[0, \frac{1}{e}\right]$

Q.44 (2)
Q.45 (1)
Q.46 **(1)**
Q.47 (4) $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow f'(x) = -6x^2 - 18x - 12$

To be decreasing $f'(x) < 0$, i.e., $-6x^2 - 18x - 12 < 0$

$\Rightarrow x^2 + 3x + 2 > 0 \Rightarrow (x+2)(x+1) > 0$
Therefore either $x < -2$ or $x > -1$

$\Rightarrow x \in (-1, \infty)$ or $(-\infty, -2)$

Q.48 (3)
$f(x) = x^2 \Rightarrow f'(x) = 2x > 0$ (for increasing)

i.e., $0 < x < \infty$. Thus $f(x)$ is increasing in $(0, \infty)$.

Q.49 (1)

$f(x) = x^4 - \frac{x^3}{3} \Rightarrow f'(x) = 4x^3 - x^2$

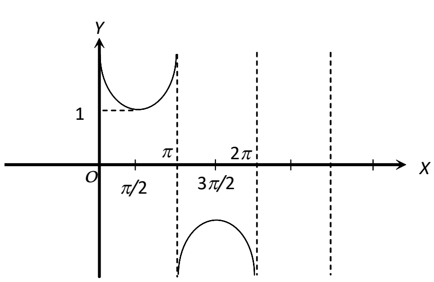
For increasing $4x^3 - x^2 > 0 = x^2(4x - 1) > 0$

Therefore, the function is increasing for $x > \frac{1}{4}$

Similarly decreasing for $x < \frac{1}{4}$.

Q.50 (4)
If the function is monotonic, then its value must change according to its monotonocity.

Q.51 (1) The graph of cosec x is opposite in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



Q.52 (4)
$f(x) = \sin x - \cos x$

$f'(x) = \cos x + \sin x = \sqrt{2}\left[\cos\left(x - \frac{\pi}{4}\right)\right] = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

For $f(x)$ decreasing, $f'(x) < 0$

$\frac{\pi}{2} < \left(x - \frac{\pi}{4}\right) < \frac{3\pi}{2}$, (within $0 \le x \le 2\pi$).

$\Rightarrow \frac{3\pi}{4} < x \le \frac{7\pi}{4}$.

Q.53 (2)
Here $f(x) = |\sin 4x + 3|$
We know that minimum value of $\sin x$ is -1 and maximum is 1.
Hence minimum $|\sin 4x + 3| = |-1 + 3| = 2$ and maximum $|\sin 4x + 3| = |1 + 3| = 4$.

Q.54 (3)
Obviously, it has a maximum at $x = 1$.

Q.55 (1)
At an extreme point of a function f (x), slope is always zero.
Thus, At and extreme point of a finction f(x), the tangent to the curve is parallel to the x-axis.

Q.56 **(1)**
Q.57 (1)

Let $y = x^x \Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$

For $\frac{dy}{dx} > 0$; $x^x(1 + \log x) > 0$

$\Rightarrow 1 + \log x > 0 \Rightarrow \log_e x > \log_e \frac{1}{e}$

For this to be positive, x should be greater than $\frac{1}{e}$.

Q.58 (4)

Here $f(x) = \frac{x^2 - 3x}{x - 1} \Rightarrow f'(x) = \frac{x^2 - 2x + 3}{(x-1)^2}$

Obviously, it is not derivable at $x = 1$ i.e., in $(0, 3)$
Also $f(a) = f(b)$ does not hold for $[-3, 0]$ and $[1.5, 3]$
Hence the answer is (4).

Q.59 (2)

Here $\frac{f(b) - f(a)}{b - a} = f'(c)$

$\Rightarrow \frac{e^b - e^a}{b - a} = f'(c) \Rightarrow \frac{e - 1}{1 - 0} = e^c \Rightarrow c = \log(e - 1)$.

Q.60 (2)

$$f(x)=x(x-1)^2;x\in[0,2]$$

$$f'(c)=\frac{f(b)-f(a)}{b-a};f(2)=2,f(1)=0$$

$$f'(x)=3x^2-4x+1\Rightarrow f'(c)=3c^2=4c+1$$

Thus, $3c^2-4c+1=\dfrac{f(2)-f(0)}{2-0}$

$$=\frac{2-0}{2-0}=1\Rightarrow c=\frac{4}{3}$$

Q.61 (2)

If Rolle's theorem is true for any function $f(x)$ in $[a,b]$.

Then $f(a)=f(b)$, therefore [–2,2].

Q.62 (1)

$f(1)=f(3)\Rightarrow a+b-5=3a+b-27\Rightarrow a=11$

which is given in option (1) only.

Q.63 (2)

$f(b)=f(2)=8-24a+10=18-24a$

$f(a)=f(1)=1-6a+5=6-6a$

$f'(x)=3x^2-12ax+5$

From Lagrange's mean value theorem,

$$f'(x)=\frac{f(b)-f(a)}{b-a}=\frac{18-24a-6+6a}{2-1}$$

$\therefore f'(x)=12-18a$

At $x=\dfrac{7}{4},3\times\dfrac{49}{16}-12a\times\dfrac{7}{4}+5=12-18a$

$$\Rightarrow 3a=\frac{147}{16}-7\Rightarrow 3a=\frac{35}{16}\Rightarrow a=\frac{35}{48}$$

Q.64 (4)

Let $f(x)=x^2\log x\Rightarrow f'(x)=2x\log x+x$

and $f''(x)=2(1+\log x)+1$

Now $f''(1)=3+2\log_e 1$ and

$f''(e)=3+2\log_e e$

$f(x)$ has local minimum at $\dfrac{1}{\sqrt{e}}$, but x lies only in interval $(1,e)$ so that has not extremum in

Hence neither a point of maximum nor minimum.

Q.65 (4)

$x+y=16\Rightarrow y=16-x\Rightarrow x^2+y^2=x^2+(16-x)^2$

Let $z=x^2+(16-x)^2\Rightarrow z'=4x-32$

To be minimum of z, $z''>0$, and it is.

Therefore $4x-32=0\Rightarrow x=8\Rightarrow y=8$

Q.66 (4)

Let a and b are given, then area

$$A=\frac{1}{2}ab\sin C\Rightarrow\frac{dA}{dC}=\frac{1}{2}ab\cos C$$

Hence A is maximum, when

$$\frac{dA}{dC}=0\Rightarrow C=90°$$

Q.67 (1)

Let $y=\sin^3 x\cos x+4\cos^3 x(-\sin x)$

$$\frac{dy}{dx}=4\sin^3 x\cos x+4\cos^3 x(-\sin x)$$

$$4\sin x\cos x(\sin^2 x-\cos^2 x)$$

$$(2\sin 2x)(-\cos 2x)=-\sin 4x$$

$$\therefore\frac{dy}{dx}=0\Rightarrow\sin 4x=0$$

$$\Rightarrow 4x=0,\pi,2\pi,3\pi$$

$$\text{or } x=0,\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\ldots\Rightarrow x=\frac{\pi}{4}$$

Q.68 (1)

$\dfrac{x}{2}+\dfrac{2}{x}$ is of the form $x+\dfrac{1}{x}\geq 2$ and equality holds for x = 1

Q.69 (2)

Let $f(x)=ax^3+bx^2+bx^2+cx+d$

Put x = 0 and x = 1

Then, we get f(0) = –1 and f(1) = 0

$\Rightarrow$ d = –1 and a + b + c + d = 0

$\Rightarrow$ a + b + c = 1 ....(i)

It is given that x = 0 is a stationary point of f(x), but it is not a point of extremum.

Therefore, f'(0) = 0 = f''(0) and f''(0) = 0

Now, $f(x)=3ax^2+2bx+c$,

$$\Rightarrow f''(x)=3ax^2+2bx+c,$$

$f''(x)=6ax+2b$ and $f''(x)=6a$

f'=0, f''(0) = 0 and f''(0) = 0 $\neq$ 0

c = 0, b = 0 and a $\neq$ 0

From Eqs. (i) and (ii), we get

a = 1, b = c = 0 and d = –1

Put these values in f(x)

we get $f(x)=x^3-1$

Hence, $\displaystyle\int\frac{f(x)}{x^3-1}dx=\int\frac{x^3-1}{x^3-1}dx=\int 1dx=x+C$

Q.70 (2)

Q.71 (4)

Let one side of quadrilateral be x and another side be y

So, $2(x + y) = 34$

Or, $(x + y) = 17$

We know from the basic principle that for a given perimeter square has the maximum area, so x = y and putting this value in equation (i)

$$x = y = \frac{17}{2}$$

$$Area = x.y = \frac{17}{2}\times\frac{17}{24} = \frac{289}{4} = 72.25$$

Q.72 (1)

Let = xe^x.

Differentiate both side w.r.t.'x'

$$\Rightarrow \frac{dy}{dx} = e^x + xe^x = e^x(1+x)$$

$$Put \frac{dy}{dx} = 0$$

$$\Rightarrow e^x(1+x) = 0$$

$$\Rightarrow x = -1$$

$$Now, \frac{d^2y}{dx^2} = e^x + e^x(1+x) = e^x(x+2)$$

$$\left(\frac{d^2y}{dx^2}\right)_{(x=-1)} = \frac{1}{e} + 0 > 0$$

Hence, $y = xe^x$ is minimum function and

$$y_{\min} = -\frac{1}{e}$$

Q.73 (3)

$V = \pi r^2 h$ = constant. If k be the thickness of the sides then that of the top will be (5/4)k.

$\therefore$ $S = (2\pi rh)k + (\pi r^2).(5/4)k$

('S' is vol. of material used)

or $S = 2\pi rk \cdot k\left(-\frac{2V}{r^2} + \frac{5}{2}\pi r\right)$

$\therefore$ $r^3 = 4V/5\pi$

$$\frac{d^2S}{dr^2} = k\left(\frac{4V}{r^3} + \frac{5}{2}\pi\right) = \frac{15}{2}k\pi \text{ = positive}$$

When $r^3 = 4V/5\pi$ or $5\pi r^3 = 4\pi r^2 h$

$$\therefore \frac{r}{h} = \frac{4}{5}.$$

Q.74 (2)

Q.75 **(4)**

Q.76 **(3)**

Q.77 (3)

Given function $f : R \to R$ is to be maximum, if $f'(a) = 0$ and $f''(a) < 0$.

Q.78 (4)

$$f(x) = \int_0^x te^{-t^2}dt \Rightarrow f'(x) = xe^{-x^2} = 0 \Rightarrow x = 0$$

$$f''(x) = e^{-x^2}(1-2x^2); \quad f''(0) = 1 > 0$$

$\therefore$ Minimum value $f(0) = 0$

Q.79 (3)

Let $y = \exp(2+\sqrt{3}\cos x + \sin x)$

$\Rightarrow y' = \exp(2+\sqrt{3}\cos x + \sin x)(-\sqrt{3}\sin x + \cos x)$

Now $y' = 0 \Rightarrow -\sqrt{3}\sin x + \cos x = 0$

$$\Rightarrow \sin\left(x - \frac{\pi}{6}\right) = 0 \Rightarrow x = \frac{\pi}{6}$$

Now y'' is –ve at $x = \frac{\pi}{6}$

$\therefore$ Maximum value of

$$y = \exp\left(2 + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\right) = \exp(4).$$

Q.80 **(2)**

EXERCISE-II (JEE MAIN LEVEL)

Q.1 **(1)**

Q.2 **(2)**

Q.3 **(4)**

Q.4 **(3)**

Q.5 (1)

$y = \tan(\tan^{-1} x)$

$\Rightarrow y = x$

$\Rightarrow x = -\sqrt{x} + 2$

$x + \sqrt{x} - 2 = 0$

$\sqrt{x} = 1 \Rightarrow x = 1, \; y = 1$

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = -\frac{1}{2}$$

Slope of normal = 2

Equation of normal is $2x - y = 1$

Q.6 (4)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(-\sin\theta)}{a(1+\cos\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{-\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$$

$$\tan\alpha = -\frac{1}{\sqrt{3}} \Rightarrow \alpha = \pi - \frac{\pi}{6}$$

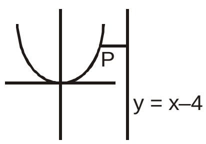
$$\alpha = \frac{5\pi}{6}$$

Q.7 (1)

Let the point on parabola P $(2t, t^2)$

$$y = \frac{x^2}{4}$$

$$\frac{dy}{dx} = \frac{2x}{4} = \left.\frac{x}{2}\right|_{(2t,\, t^2)} = t$$



slope = 1 $\Rightarrow$ t = 1 so P(2, 1)

Q.8 (3)

$$y - e^{xy} + x = 0$$

$$\therefore \frac{dy}{dx} - e^{xy}\left(y + x\frac{dy}{dx}\right) + 1 = 0$$

i.e., $\frac{dy}{dx} - y(x+y) - x(x+y)\frac{dy}{dx} + 1 = 0$

i.e., $\left[1 - x(x+y)\right]\frac{dy}{dx} = y(x+y) - 1$

for the vertical tangents

$$1 - x(x+y) = 0$$

i.e., $y = \frac{1-x^2}{x}$ $\therefore x = 1$ and $y = 0$

Q.9 (3)

Givne equation of a line parallel to X-axis is y = k.

Given eqution of the curve is $y = \sqrt{x}$, On solving equation of line with the equation of curve, we get x = k^2 Thus the intersrecting point is (k^2, k)

It is given that the line y = k intersect the curve $y = \sqrt{x}$ at an angle of $\pi/4$. This means that the slope of the tangent to

$y = \sqrt{x}$ at (k^2, k) is $\tan\left(\pm\frac{\pi}{4}\right) = \pm 1$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(k^2,k)} = \pm 1 \Rightarrow \left(\frac{1}{2\sqrt{x}}\right)_{(k^2,k)} = \pm 1$$

$$\Rightarrow k = \pm\frac{1}{2}$$

Q.10 (3)

Let (x1, y1) be one of the points of contact.

Given curve is y = cos x

$$\Rightarrow \frac{dy}{dx} = -\sin x$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x_1,y_1)} = -\sin x_1$$

Now the equation of the tangent at (x_1, y_1) is

$$y - y1\left(\frac{dy}{dx}\right)_{(x_1,y_1)}(x - x_1)$$

$$\Rightarrow y - y_1 = -\sin x_1 (0 - x_1)$$

Since, it is given that equation of tangent passes through origin.

$\therefore 0 - y_1 = -\sin x_1 (0 - x_1)$

$\Rightarrow y_1 = -x_1 \sin x_1$ ...(i)

Also, point (x_1, y_1) lies on y = cos x.

$\therefore y_1 = \cos x_1$

From Eqs. (i), (ii), we get

$$\sin^2 x_1 + \cos^2 x_1 = \frac{y_1^2}{x_1^2} + y_1^2 = 1$$

$$\Rightarrow x_1^2 = y_1^2 + y_1^2 x_1^2$$

Hence, the locus o (x_1, y_1) is

$$x^2 = y^2 + y^2x^2 \Rightarrow x^2y^2 = x^2 - y^2$$

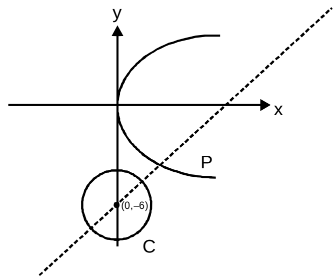
Q.11 **(2)**

Q.12 (1)

$P_1 : y^2 = 8x$

$C_1 : x^2 + (y+6)^2 = 1$

$$2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{4}{y}$$



Equation of normal of parabola
$y = mx - 2am - am^3$
if passes through (0,–6)
$-6 = -2am - am^3$
$\because a = 2$
$\Rightarrow 3 = 2m + m^3$
$m^3 + 2m - 3 = 0 \Rightarrow m = 1.$
Point on parabola $(am^2, -2am) \equiv (2, -4)$.

Q.13 (2)
$x^3 + pxy^2 = -2 ; 3x^2y - y^3 = 2$
$3x^2 + P(y^2 + 2xyy') = 0 ; 6xy + 3x^2y' - 3y^2y' = 0$

$$m_1 = y' = \frac{3x^2 + py^2}{-2pxy} ; \quad m_2 = y' = -\frac{2xy}{3x^2 - 3y^2}$$

$m_1 \times m_2 = -1$

$$\frac{(3x^2 + py^2)}{-2pxy} \times \frac{(-6xy)}{(3x^2 - 3y^2)} = -1$$

$$\frac{3}{p}\frac{(3x^2 + py^2)}{(3x^2 - 3y^2)} = -1$$

p = –3 only possible

Q.14 (2)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} =$$

$$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0 \qquad y + xy' = 0$$

$$m_2 = y' = -\frac{y}{x}$$

$$m_1 = y' = \frac{b^2}{a^2}\frac{x}{y}$$

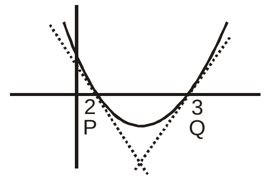
$m_1 \times m_2 = -1$

$$\frac{b^2}{a^2}\frac{x_1}{y_1} \times \left(-\frac{y_1}{x_1}\right) = -1$$

$b^2 = a^2$

Q.15 (1)
$y = x^2 - 5x + 6$
$y = (x - 2)(x - 3)$

$$\frac{dy}{dx} = 2x - 5$$



$$m_1 = \left.\frac{dy}{dx}\right|_P = -1$$

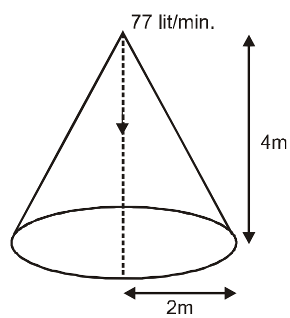
$$m_2 = \left.\frac{dy}{dx}\right|_Q = 1$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-1-1}{1-1}\right| = \infty$$

$$\theta = \frac{\pi}{2}$$

Q.16 (2)

$$V = \frac{1}{3}\pi r^2 h \quad \left(\because \frac{r}{h} = \frac{2}{4} = \frac{1}{2}\right)$$



$$V = \frac{1}{3}\pi\frac{h^3}{4} = \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$$

$$77 \times 10^3 = \frac{22}{7} \times \frac{1}{4} \times 70 \times 70 \times \frac{dh}{dt} \quad (\because 1 \text{ litre} = 10^3 \text{c.c.})$$

$$\therefore \frac{dh}{dt} = 20 \text{ cm/min.}$$

Q.17 (1)
$V = \pi r^2 h$

$$\frac{dv}{dt} = \pi r^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{dv/dt}{\pi r^2} = \frac{1}{9\pi} \text{ m/min.}$$

Q.18 (4)
Q.19 (3)
Q.20 (1)
Q.21 (2)
Q.22 **(1)**
Q.23 (1)

$$\text{L.N.} = y\sqrt{1+y'^2}$$

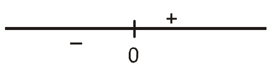
$$\text{L.N.} = \frac{y}{y'}\sqrt{1+y'^2}$$

$$\frac{(\text{L.N.})^2}{(\text{L.T})^2} = y'^2$$

Also $\frac{\text{L.S.N}}{\text{L.S.T}} = \frac{yy'}{\frac{y}{y'}} = y'^2$

Q.24 (3)

$f'(x) = (2^2 + 4^2x^2 + 6^2x^4 + \ldots\ldots + 100^2x^{98})\,x$



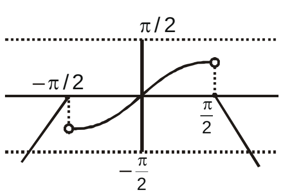
signs of f′(x)

Minimum at $x = 0$

Q.25 (2)

$f(x) = \tan^{-1} x,\ |x| < \frac{\pi}{2}$

$\frac{\pi}{2} - |x|,\ |x| \geq \frac{\pi}{2}$



$x = -\frac{\pi}{2}$ is maxima

Q.26 (3)

$$f'(x) = 3\left(\frac{a^2-1}{a^2+1}\right)x^2 - 3$$

$f''(x) < 0$ for all x if $a^2 - 1 \leq 0 \Rightarrow -1 \leq a \leq 1$

Q.27 (1)

$$f(x) = \tan x - 4x \Rightarrow f'(x) = \sec^2 x - 4$$

When $\frac{-\pi}{3} < x < \frac{\pi}{3}, 1 < \sec x < 2$

Therefore, $1 < \sec^2 x < 4$

$$\Rightarrow -3 < \left(\sec^2 x - 4\right) < 0$$

Thus, for $\frac{-\pi}{3} < x < \frac{\pi}{3}, f'(x) < 0$

Hence, f is strictly decreasing on $\left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$

Q.28 **(1)**
Q.29 (2)

Let

$$f(x) = 2x^3 + 15 \text{ and } g(x) = 9x^2 - 12x \text{ then}$$

$$f'(x) = 6x^2 \forall x \in R$$

$\therefore f(x)$ is increasing function $\forall x \in R$

Also, $g'(x) > 0 \Rightarrow 18x - 12 > 0 \Rightarrow x > \frac{2}{3}$

Thus, f(x) and g(x) both increases for $x > \frac{2}{3}$ Let

$$F(x) = f(x) - g(x), F'(x) < 0$$

(because f(x) increases less rapidly than the function g(x))

$$\Rightarrow 6x^2 - 18x + 12 < 0 \Rightarrow 1 < x < 2$$

Q.30 **(2)**

Given : $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

Differentiating with respect to x, we get

$f'(x) = 12x^3 + 12x^2 - 24x$

For f(x) to be increasing

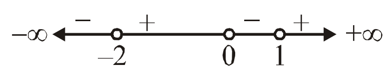
$f'(x) > 0 \Rightarrow 12x^3 + 12x^2 - 24x > 0$

$\Rightarrow 12x(x^2 + x - 2) > 0$

$\Rightarrow 12x(x - 1)(x + 2) > 0$

$\Rightarrow x(x - 1)(x + 2) > 0$

$\Rightarrow -2 < x < 0$ or $x > 1$



It means $x \in (-2, 0) \cup (1, \infty)$.

Hence f(x) is increasing in (–2, 0) and (1, ∞)

Q.31 **(1)**
Q.32 **(4)**
Q.33 (4)

$f(1) = 1 - 1 + 10 - 5 = 5$

for greatest value at x = 1

$f(1^+) \leq f(1)$ $b^2 - 2 > 0$

$-2 + \log_2(b^2 - 2) \leq 5$; $b > \sqrt{2}$ or $b < -\sqrt{2}$

$\log_2(b^2 - 2) \leq 7$

$b^2 - 2 \leq 2^7$

$b^2 \leq 130$

$-\sqrt{130} \leq b \leq \sqrt{130}$

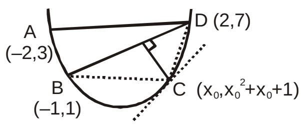
final answer $b \in [-\sqrt{130}, -\sqrt{2}) \cup (\sqrt{2}, \sqrt{130}]$

Q.34 (1)

Let $y = ax^2 + bx + c$

A : $3 = 4a - 2b + c$ ....(1)

B : $1 = a - b + c$ ....(2)



C : $7 = 4a + 2b + c$ ....(3)

$b = 1 = a = c$

$y = x^2 + x + 1$

Method (1) Make determinant using area of ΔBCD then diff with respect to x_0

Method (2) Area will be maximum if tangent at C will be parallel to BD

$$\frac{dy}{dx} = 2x_0 + 1 = \left(\frac{7-1}{2+1}\right)$$

$2x_0 + 1 = 2$

$x_0 = 1/2$

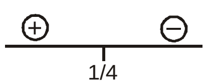
$$y = \frac{1}{4} + \frac{1}{2} + 1 = \frac{1+2+4}{4} = \frac{7}{4}$$

point $\left(\frac{1}{2}, \frac{7}{4}\right)$

Q.35 (4)

$f(x) = x^{25}(1-x)^{75}$

$f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74} = 0$



$\Rightarrow x = 1/4$

$x = 1/4$ maxima

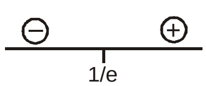
Q.36 (3)

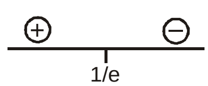
$f(x) = x^x$ $\quad f(x) = x^{-x}$

$f'(x) = x^x(1 + \ell n\, x)$ $\quad f'(x) = -x^{-x}(1 + \ell n\, x)$

$1 + \ell n\, x = 0$ $\;x = 1/e$

$x = 1/e$





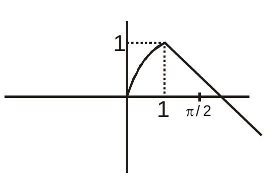
$1/e \rightarrow$ minima $\quad 1/e \rightarrow$ maxima

min. value $= \left(\frac{1}{e}\right)^{1/e}$ $\quad f\left(\frac{1}{e}\right) = e^{1/e}$

product $= (e^{-1/e})(e)^{1/e} = 1$

Q.37 (1)

$x = 1$ local maxima



Q.38 (3)

$$y = \frac{1}{3\sin\theta - 4\cos\theta + 7} \;;\; -5 < 3\sin\theta - 4\cos\theta < 5$$

$$y_{min.} = \frac{1}{(3\sin\theta - 4\cos\theta + 7)_{max}}$$

$$= \frac{1}{5+7} = \frac{1}{12}$$

Q.39 (2)

$$f'(x) = \frac{1}{3(x+1)^{2/3}} - \frac{1}{3(x-1)^{2/3}}$$

$f'(x) = 0 \Rightarrow x = 0$

$f(0) = 1 + 1 = 2$

$f(1) = 2^{1/3}$

max. value = 2

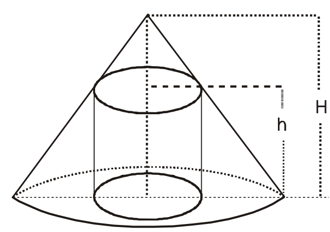
Q.40 (4)

$$\frac{H}{R} = \frac{H-h}{r}$$

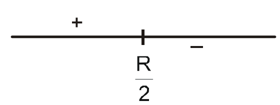
$S = 2\pi rh$

$$= 2\pi H\left(r - \frac{r^2}{R}\right)$$

$$\frac{dS}{dr} = 2\pi H\left(1 - \frac{2r}{R}\right)$$



Figure



sign of $\frac{dS}{dr}$

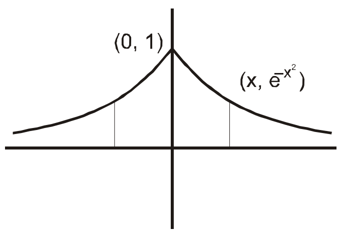
Maximum at $r = \frac{R}{2}$

Q.41 (1)

Let A be area

$A = (2x)(e^{-x^2}), x > 0$

$$\frac{dA}{dx} = -2\left(x + \frac{1}{\sqrt{2}}\right)\left(x - \frac{1}{\sqrt{2}}\right)e^{-x^2}$$



Figure

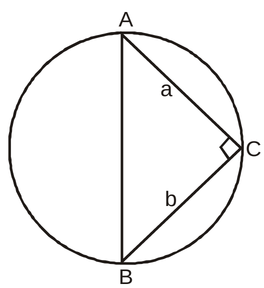
At $x = \frac{1}{\sqrt{2}}$, A is maximum.

Largest area is $2\frac{1}{\sqrt{2}}e^{-1/2}$

Q.42 (1)

$S = \frac{1}{2}ab$

A (circle) = πr^2



$= \pi\frac{(a^2+b^2)}{4}$

$= \frac{\pi}{4}\left[a^2 + \frac{4S^2}{a^2}\right]$

AM ≥ GM

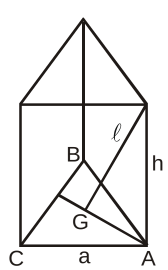
$a^2 + \frac{4S^2}{a^2} \geq 2\sqrt{a^2 \times \frac{4S^2}{a^2}} \Rightarrow a^2 + \frac{4S^2}{a^2} \geq 4S$

Area (max.) = $\frac{\pi}{4}(4S) = \pi S$

Q.43 (2)

$AG = \frac{2}{3}\cdot\frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$

$\ell^2 = \frac{a^2}{3} + h^2$



$v(h) = 3.\frac{\sqrt{3}}{4}h(\ell^2 - h^2)$,

$v'(h) = 0 \Rightarrow h = \frac{\ell}{\sqrt{3}}$

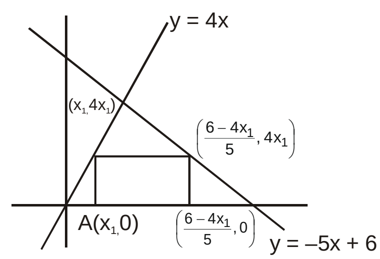
$v_{max} = \frac{\ell^3}{2}$

Q.44 (3)

$A = \left(\frac{6-4x_1}{5} - x_1\right)(4x_1)$

$A = \left(\frac{6-9x_1}{5}\right)(4x_1)$

$\frac{dA}{dx_1} = \frac{4}{5}(6 - 18x_1)$



$\frac{dA}{dx_1} = 0 \Rightarrow x_1 = \frac{1}{3}$

$A = \frac{4}{3}\left(\frac{1}{3}\right)\left(6 - 9\times\frac{1}{3}\right) = \frac{4}{5}$

Q.45 (4)

$\frac{p+q}{2} \geq \sqrt{pq}$

$(p+q)^2 \geq 4pq$

$p^2 + q^2 = 1$

$(p+q)^2 - 2pq = 1$

$2pq = (p+q)^2 - 1$

$4pq = 2(p+q)^2 - 2$

$2(p+q)^2 - 2 \leq (p+q)^2$

$(p+q)^2 \leq 2$

$p + q \leq \sqrt{2}$

Q.46 (1)

$f(x) = \frac{3x^2+9x+17}{3x^2+9x+7}$

$f(x) = 1 + \frac{10}{3x^2+9x+7}$

For f(x) to be maximum the quadratic expression should get its min. value $= -\frac{0}{4a} = \frac{3}{12}$

max. value of $f(x) = 1 + \frac{10}{3/12} = 41$

Q.47 (4)

$x = 1 \Rightarrow 3 = a + b$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 0 \Rightarrow 3a + b = 0$

$a = -\frac{3}{2}, b = \frac{9}{2}$

Q.48 (4)

If the sum of two positive quantities is a constant, then their product is maximum, whenthey are equal.

$\therefore a^2x^4.b^2y^2$ is maximum when

$$a^2x^4 = b^2y^4 = \frac{1}{2}\left(a^2x^4 + b^2y^4\right) = \frac{c^4}{2}$$

$\therefore$ maximum value of $a^2x^4 \cdot b^2y^4 = \frac{c^4}{2} \cdot \frac{c^4}{2} = \frac{c^8}{4}$

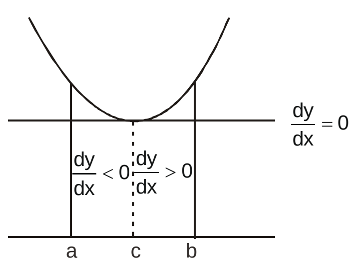
$$\text{Maximum value of } xy = \left(\frac{c^8}{4a^2b^2}\right)^{1/4} = \frac{c^2}{\sqrt{2ab}}$$

Q.49 (3)

Q.50 (3)

For $x \in (a, b)$

$\frac{dy}{dx} \uparrow \Rightarrow \frac{d^2y}{dx^2} > 0$



either $\frac{dy}{dx} < 0 \Rightarrow \frac{d^2y}{dx^2} > 0$

$\frac{dy}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} > 0$

Q.51 (3)

$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ $\quad a > 0$

$f'(x) = 6x^2 - 18ax + 12a^2$

$= 6(x^2 - 3ax + 2a^2) = 0$

$x = 2a, a$

$f''(x) = 6(2x - 3a)|_{x=2a} = 6a > 0$

$f''(x) = 6(2x - 3a)|_{x=a} = -6a < 0$

x = 2a is minima = q

x = a is maxima = p

$p^2 = q$

$a^2 = 2a$

a = 0 (reject), $\quad$ a = 2

Q.52 (4)

Let us assume the functions f(x) and g(x) given by

$f(x) = \tan x - x$ and $g(x) = x - \sin x$, for $0 < x < \frac{\pi}{2}$

Now, $f'(x) = \sec^2 x - 1$ and $g'(x) = 1 - \cos x$

$$\Rightarrow f'(x) > 0 \text{ and } g'(x) > 0, \forall x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow f(x) > f(0) \text{ and } g(x) > g(0) \ \forall x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \tan x - x > 0 \text{ and } x - \sin x > 0, \ \forall x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \tan x > x \text{ and } x > \sin x, \ \forall x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \sin x < x < \tan x \ \forall x \in \left(0, \frac{\pi}{2}\right)$$

Q.53 (2)

$f(x) = x^3 - 3x \quad [0, 2]$

$f'(x) = 3x^2 - 3 = 0 \quad \Rightarrow x = \pm 1$

$f(1) = 1 - 3 = -2$

$f(-1) = -1 + 3 = 2$ (reject)

$f(0) = 0$

$f(2) = 8 - 6 = 2$

max. value = 2

Q.54 (4)

For Rolle's throrem in [a, b]

f(1) = f(b).

In $[0, 1] \Rightarrow f(0) = f(1) = 0$

Q the function has to be continuous in [0, 1]

$\Rightarrow f(0) = \lim_{x \to 0} f(x) = 0$

$\Rightarrow \lim_{x \to 0} x^\alpha \log x = 0 \Rightarrow \lim_{x \to 0} \frac{\log x}{x^{-\alpha}} = 0$

Applying L.H. Rule $\lim_{x \to 0} \frac{1/x}{-\alpha x^{-\alpha-1}} = 0$

$\Rightarrow \lim_{x \to 0} \frac{-x^\alpha}{\alpha} = 0 \Rightarrow \alpha > 0$

Q.55 (1)

Let $f(x) = e^{x-1} + x - 2$

check for x = 1

Then, $f(1) = e^0 + 1 - 2 = 0$

So, x = 1 is a real root of the equation f(x) = 0 Let x = α be the other root such that $\alpha > 1$ or $\alpha < 1$.

Consider the interval $[1, \alpha]$ or $[\alpha, 1]$.

Clearly $f(1) = f(\alpha) = (0)$

By Rolle's theorem f'(x) = 0 has a root in $(1, \alpha)$ or in $(\alpha, 1)$.

But $f'(x) = e^{x-1} + 1 > 0$, for all x. Thus,

$f'(x) \neq 0$, for any $x \in (1,\alpha)$ or $x \in (\alpha, 1)$,

which is a contradiction.

Hence, f(x) = 0 has no real root other then 1.

Q.56 (1)

$$\frac{dy}{dx} \le 0 \quad \forall \ x \in R$$

$\Rightarrow a + 2 < 0, D \le 0$

$\Rightarrow a + 2 < 0, a\,(a + 3) \ge 0$

$\Rightarrow a \le -3$

Q.57 (3)

Let $z = x^3$

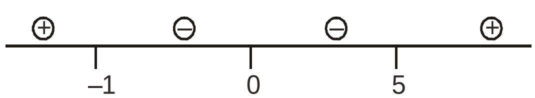
$y = 6x^2 + 15x + 5$

$$\frac{dy}{dx} > 1$$

$$\frac{12x+15}{3x^2} - 1 > 0$$

$$\frac{12x+15-3x^2}{3x^2} > 0$$

$$\frac{x^2-4x-5}{x^2} < 0 \Rightarrow \frac{(x+1)(x-5)}{x^2} < 0$$



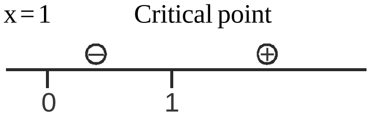
$x \in (-1, 5)$

Q.58 (4)

$f(x) = x\ell nx - x + 1$ $\qquad D_f : x \in R^+$

$f'(x) = \ell nx + 1 - 1$

x = 1 $\qquad$ Critical point



If $x \in (0, 1)$ f is ↓ing

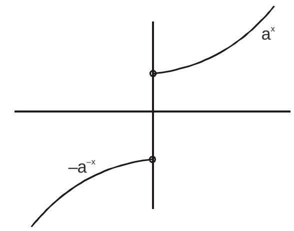
$f(1) > f(x) > f(0) \Rightarrow 0 > f(x) > 1$ positive

If $x \in (1, \infty)$ & is ↑ing

$f(x) > f(1) \Rightarrow$ $\boxed{f(x) > 0}$

Q.59 (4)

$(\ell na)\, h(x) = \ell n\, f(x)\, g(x)$



$= \ell n\ [a^{\{a^{|x|} \text{sgn } x\}} + [a^{|x|} \text{sgn } x]]$

$= \ell n\ a^{a^{|x|} \text{sgn} x}$

$(\ell n\, a)\, h(x) = a^{|x|}$ sgn x $(\ell n\, a)$

$h(x) = a^{|x|}$ sgn x

$h(x) = a^x \qquad x > 0$

$\quad = 0 \qquad x = 0$

$\quad = -a^{-x} \quad x < 0$

h is odd and ↑ing.

Q.60 (1)

$f(x) = x^2 - x \sin x$

$f'(x) = 2x - x\cos x - \sin x$

$= x\,(2 - \cos x) - \sin x$

is $\left[0, \frac{\pi}{2}\right]$ (2 – cos x) is +ve and sin x is +ve. and

(2 – cos x) is greater than sin x so

$f'(x) > 0$

f(x) ↑ing in $\left[0, \frac{\pi}{2}\right]$

Q.61 (3)

$$f(x) = 2x^3 - 3x^2 - 12x + 4$$

$$\Rightarrow f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$$

$$= 6(x-2)(x+1)$$

For maxima and minima f'(x)=0

$$\therefore 6(x-2)(x+1) = 0$$

$$\Rightarrow x = 2, -1$$

Now, $f''(x) = 24 - 6 = 18 > 0$

$\therefore x = 2$, local min. point

At $x = -1$; $f''(x) = 12(-1) - 6 = -18 < 0$

$\therefore x = -1$ local max. point

Q.62 (1)

$$\frac{dy}{dy} = \frac{d}{dx}(x^4 - 2x^2 + 1) = 4x(x^2 - 1)$$

For max or min $\frac{dy}{dy} = 0$

$4x(x^2 - 1)$; either x = 0 or x = ± 1

x=0,and x = -1 does not belong to $\left[\frac{1}{2}, 2\right]$

$\frac{d^2y}{dx^2} = 12x^2 - 4 \therefore \left(\frac{d^2y}{dx^2}\right)_{x=1}$

$= 12(1)^2 - 4 = 8 > 0$

$\therefore$ there is minimum value of function at x = 1

$\therefore$ minimum value is

$y(1) = 1^4 - 2(1)^2 + 1 = 1 - 2 + 1 = 0$

We have : f(x) = sin x – cos x - ax + b

$\Rightarrow f'(x) = \cos x + \sin x - a$

$\Rightarrow f'(x) < 0 \forall x \in R$

$\Rightarrow (\cos x + \sin x) < a \forall x \in R$

As the max value of (cos x + sin x) is $\sqrt{2}$

The above is possible when a $\geq \sqrt{2}$

Q.63 (4)

Let the speed of the train be v and distance to be covered be s so that total time taken is s/v hours.

cost of fuel per hour $= kv^2 (k$ is constant) Also

$48 = k.16^2$ by given condition $\therefore k = \frac{3}{16}$

$\therefore$ cost to fuel per hour $\frac{3}{16}v^2$ Other charges per hour are 300 . Total running cost,

$$C = \left(\frac{3}{16}v^2 + 300\right)\frac{s}{v} = \frac{3s}{16}v + \frac{300s}{v}$$

$$\frac{dC}{dv} = \frac{3s}{16} - \frac{300s}{v^2} = 0 \Rightarrow v = 40$$

20

$\frac{d^2C}{dv^2} = \frac{600s}{v^3} > 0 \therefore v = 40$ results in minimum running cost

Q.64 (3)

Let m be the slope of the tangent to the curve

v = e^x cos x.

Then, $m = \frac{dy}{dx} = e^x(\cos x - \sin x)$

Diff. w.r.t 'x'

$\Rightarrow \frac{dm}{dx} = e^x(\cos x - \sin x) +$

$e^x(-\sin x - \cos x) = -2e^x \sin x$

and $\frac{d^2m}{dx^2} = -2e^x(\sin x + \cos x)$

Put $\frac{dm}{dx} = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

Clearly, $\frac{d^2m}{dx^2} > 0$ for $x = \pi$

Thus, y is minimum at x = π.

Hence the value of of $\alpha = \pi$.

Q.65 (2)

Q.66 (4)

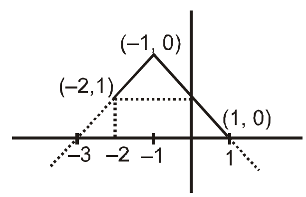
Q.67 **(2)**

Q.68 **(3)**

Q.69 **(3)**

Q.70 (3)

f(x) = 2 – |x + 1|



Figure

From figure it is clear that greatest, least values are respectively 2, 0

Q.71 (2)

$f'(x) = \frac{a}{x} + 2bx + 1$

$f'(-1) = 0$

$-a - 2b + 1 = 0$

$a + 2b = 1$

$f'(2) = 0$

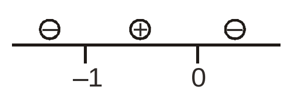
$\frac{a}{2} + 4b + 1 = 0 \Rightarrow a + 8b + 2 = 0$

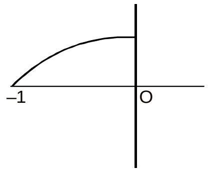
$-6b = 3 \Rightarrow b = \frac{-1}{2}, a = 2$

Q.72 (1)

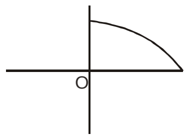
Let $f(x) = \ell n(1 + x) - x$ D_f : $\boxed{x > -1}$

$f'(x) = \frac{1}{1+x} - 1 = \frac{-x}{1+x}$





In $x \in (-1, 0)$ f is ↑ing
$f(x) \le f(0)$
$f(x) \le 0$
In $x \in (0, \infty)$ f is ↓ing.



$f(x) \le f(0)$
$f(x) \le 0$

Q.73 (1)
$f(x) = x^3 - 6x^2 + ax + b$
f(x) satisfies condition in Rolle's theorem on [1, 3]
$f(1) = f(3)$
$\Rightarrow 1 - 6 + a + b = 27 - 54 + 3a + b$
$2a = 22$
$a = 11$
and $b \in R$.

Q.74 (2)
$f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2$
(1) $h(x) = f'(x) - g(x)$
$h(0) = f(0) - g(0) = 2$ wrong
$h(1) = f(1) - g(1) = 6 - 2 = 4$
(2) $h(x) = f(x) - 2g(x)$
$h(0) = f(0) - 2g(0) = 2$ right`
$h(1) = f(1) - 2g(1) = 6 - 4 = 2$

Q.75 (3)
(1) $f(0) = 0$
$f(1) = 0$ Rolle's Thrm. is applicable

(3) $f'(c) = \dfrac{f(3) - f(-3)}{3 + 3}$

$e^c = \dfrac{e^3 - e^{-3}}{6}$

$e^c = \dfrac{e^6 - 1}{6e^3}$

$c = \ell n\left(\dfrac{e^6 - 1}{6e^3}\right) = \ell n(e^6 - 1) - \ell n 6 - 3$

Q.76 (4)
(1) LMVT (2) LMVT
(3) $f(0) = -2$ $f(1) = 4 - 5 + 1 - 2 = -2$ Not applicable

EXERCISE-III

NUMERICAL VALUE BASED

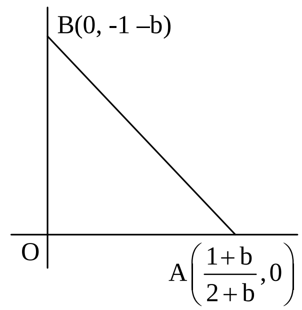
Q.1 0003

$$f'(x) = \begin{cases} 3(2+x)^2 & , \quad -3 < x < -1 \\ \dfrac{2}{3}x^{-1/3}, & -1 < x < 2, \quad x \ne 0 \end{cases}$$

Critical plints are –2, –1, 0

Q.2 0003
$f'(1) = 2 + b$
equation of tangent at (1, 1)
$y - 1 = (2 + b)(x - 1)$

area of triangle $= -\dfrac{1}{2}\dfrac{(1+b)^2}{(2+b)}$



$2 = -\dfrac{1}{2}\dfrac{(1+b)^2}{(2+b)} \Rightarrow b = -3.$

Q.3 0002

$f'\left(\dfrac{\pi}{3}\right) = 0 \Rightarrow \dfrac{a}{2} - 1 = 0 \Rightarrow a = 2$

Q.4 1.5

$S = 2\pi r \Rightarrow \dfrac{ds}{dt} = 2\pi\dfrac{dr}{dt} = .3$

$A = \pi r^2 \Rightarrow \dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 5 \times .3 = 1.5$

Q.5 (0.25)

$$f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74}$$
$$= 25x^{24}(1-x)^{74}[(1-x) - 3x]$$
$$= 25x^{24}(1-x)^{74}(1-4x)$$

$f'(x)$ changes sign about x = 1/4 only.

Q.6 0003
$f(1) = f(2)$
$\Rightarrow 1 + b + c = 8 + 4b + 2c$

$f'(4/3)=0 \Rightarrow 3.\frac{16}{9}+2b\frac{4}{3}+c=0$

Solving both, we get b = –5, c = 8.

Q.7 0004

Let (x_1, y_1) be a point on the curve

$9y_1^2 = x_1^3$

$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{x_1^2}{6y_1}$ $\qquad \frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} = \pm 1$

$x_1 = 0, 4$

but the line making equal intercepts with the axes can not pass through the origin

$x_1 = 4$

Q.8 0001

$\left(\frac{dy}{dx}\right)_{(a,b)} = \frac{-a^2}{b^2}$

$a^3 + b^3 = c^3$ 1

$a_1^3 + b_1^3 = c^3$ 2

$-\frac{a^2}{b^2} = \frac{b_1 - b}{a_1 - a}$ 3

solving $\frac{a_1}{a} + \frac{b_1}{b} = -1$

Q.9 0080

Selling price of each computer = Rs. 330 – x

Selling price of x computer = Rs. 330 – xx

Cost of production of x computers = Rs. x^2 + 10x + 12

Profit = Selling price – Production cost

$f(x) = 330 - xx - x^2 + 10x + 12$

$f(x) = 320x - 2x^2 - 12$

$f'(x) = 320 - 4x$

$f''(x) = -4$

For $f'(x) = 0$

$\Rightarrow x = 80$

$f''(80) = -4 < 0$

Profit is maximum when x = 80

Q.10 0005

$S = \frac{t^3}{3} - \frac{t^2}{2} - 6t + 5$

$\therefore \frac{ds}{dt} = t^2 - t - 6$

$\therefore \frac{d^2s}{dt^2} = 2t - 1$

But $\frac{ds}{dt}$ = velocity = 0

$\therefore t^2 - t - 6 = 0$

$\therefore t = 3, \quad (\because t \neq -2)$

$\therefore$ acceleration = $\frac{d^2s}{dt^2}(t = 3) = 5$

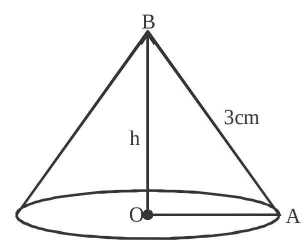
PREVIOUS YEAR'S

MHT CET

Q.1 (3)	**Q.2** (3)	**Q.3** (2)	**Q.4** (1)	**Q.5** (1)
Q.6 (2)	**Q.7** (4)	**Q.8** (2)	**Q.9** (3)	**Q.10** (4)
Q.11 (2)	**Q.12** (1)	**Q.13** (2)	**Q.14** (4)	**Q.15** (4)
Q.16 (2)	**Q.17** (2)	**Q.18** (2)	**Q.19** (4)	**Q.20** (3)
Q.21 (2)	**Q.22** (3)	**Q.23** (1)	**Q.24** (1)	**Q.25** (2)
Q.26 (2)	**Q.27** (3)	**Q.28** (3)	**Q.29** (1)	**Q.30** (2)
Q.31 (1)	**Q.32** (2)	**Q.33** (2)	**Q.34** (4)	**Q.35** (1)
Q.36 (1)	**Q.37** (4)	**Q.38** (4)	**Q.39** (2)	**Q.40** (1)
Q.41 (2)	**Q.42** (1)	**Q.43** (2)	**Q.44** (2)	

Q.45 (2)

Let height of a right circular cone = h cm and OA = rcm



Given , slant height of a right circular cone = 3cm

In ΔOAB, $\angle BOA = 90^o$

$(OB)^2 + (OA)^2 = (AB)^2$

[apply pythogoras theorem]

$(h)^2 + (r)^2 = (3)^2$

$r - = \sqrt{9 - h^2}$ ...(i)

We know that, Volume of cone

$= \frac{\pi}{3}\pi r^2 h$

$V = \frac{\pi}{3}(9 - h^2) \times h$

[from Eq. (i), $r = \sqrt{9 - h^2}$]

$V = \frac{\pi}{3}(9h - h^3)$

$\frac{dV}{dh} = \frac{\pi}{3}(9 - 3h^2)$

$\therefore \frac{dV}{dh} = 0 \Rightarrow \frac{\pi}{3}(9 - 3h^2) = 0$

$\Rightarrow\ h^2\ \frac{9}{3} = 3 \Rightarrow h = \sqrt{3}$

$$\frac{d^2V}{dh^2} = \frac{-6 \times h}{3} < 0$$

Q.46 (2)

Given, function is $y = [x(x-2)]^2 = [x^2 - 2x]^2$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 2\left(x^2 - 2x\right)\frac{d}{dx}\left(x^2 - 2x\right)$$

$= 2(x^2 - 2x)(2x - 2) = 4x(x-2)(x-1)$

On putting $\frac{dy}{dx} = 0$, we get

$4x(x-2)(x-1) = 0 \Rightarrow x = 0, 1$ and 2

Now, we find interval in which f(x) is strictly increasing or strictly decreasing.

Interval	$\frac{dy}{dx} = 4x(x-2)(x-1)$	Sign of f'(x)
$(-\infty, 0)$	(–) (–)(–)	–ve
(0,1)	(+)(–)(–)	+ ve
(1,2)	(+)(–) (+)	–ve
$(2, \infty)$	(+) (+) (+)	+ve

Hence, y is strictly increasing in (0,1) and $(2, \infty)$.

Also, y is a polynomial function, so it continuous at x = 0, 1 and 2.

Hence, y is increasing in $[0, 1] \cup [2, \infty]$

Q.47 (4)

Let r be the radius, V be the volume and S be the surface are of the spherical raindrop at time t.

Then, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

The rate at which the raindrop evaporates is $\frac{dV}{dt}$ which is proportioal to the surface area.

$\therefore \frac{dV}{dt} \propto S \Rightarrow \frac{dV}{dt} = -kS$, where $k > 0$ ...(i)

Now, $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

$$\therefore\ \frac{dv}{dt} = \frac{4\pi}{3} \times 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = -k\left(4\pi r^2\right) \qquad \text{[from Eq. (i)]}$$

$$\therefore\ \frac{dr}{dt} = -k \text{ Þ } dr = -kdt$$

On intergrating, we get

$$\int dr = -k\int dt + C$$

$\therefore\ r = -kt + C$

Initially, i.e. when $t = 0, r = 3$

$\therefore\ 3 = -k \times 0 + C$

$\therefore\ C = 3$

$\therefore\ r = -kt + 3$

When $t = 1, r = 2$

$\therefore\ 2 = -k \times 1 + 3$

$\therefore\ k = 1$

$\therefore\ r = -t + 3$

$\therefore\ r = 3 - t$, where $0 \le t \le 3$

This is the required expression for the radius of the raindrop at any time t.

Q.48 (1)

Let a be the side of an equilateral triangle and A be the area of an equilateral triangle.

Then, $\frac{da}{dt} = 2$cm/s

We know that, area of an equilateral triangel

$$A = \frac{\sqrt{3}}{4}a^2$$

On differentiating both sides w.r.t.t, we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2 \times 20 \times 2$$

[given a = 20]

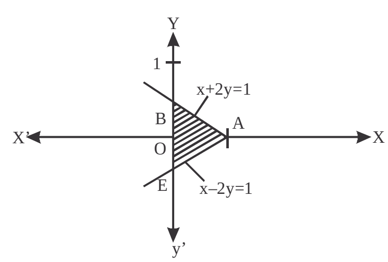
$$\therefore\ \frac{dA}{dt} = 20\sqrt{3}\ \text{cm}^2/\text{s}$$

Q.49 (2)

Given curves are $x = 0$ and $x + 2|y| = 1$

Now, $x + 2|y| = 0$

When $y > 0$, $x + 2y = 1$; when $y < 0$, $x - 2y = 1$



$\therefore$ Area of bounded region ABC

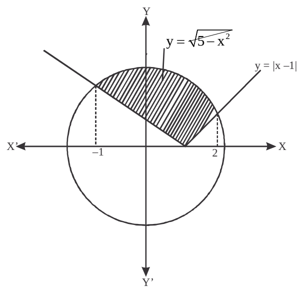
$$= 2AOB = 2\int_0^1 \left(\frac{1-x}{2}\right)dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 = \left[1 - \frac{1}{2} - (0-0)\right] = \frac{1}{2}$$

Q.50 (2)

Given $y = \sqrt{5 - x^2} \Rightarrow y^2 + x^2 = 5$

and $y = |x - 1|$



$\therefore$ Required area

$$= \int_{-1}^{2} \sqrt{5-x^2}\,dx - \int_{-1}^{1} (1-x)\,dx - \int_{1}^{2} (x-1)\,dx$$

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$$

$$= \left[1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right]$$

$$-\left[1 - \frac{1}{2} - \left(-1 - \frac{1}{2}\right)\right] - \left[2 - 2 - \left(\frac{1}{2} - 1\right)\right]$$

$$= 2 + \frac{5}{2}\sin(1) - \frac{1}{2} = \frac{5\pi}{4} - \frac{1}{2} = \left(\frac{5\pi - 2}{4}\right) \text{ sq units}$$

Q.51 (2)

Given curve is $y = x^3 + ax - b$ ...(i)

Passes through the point p(1,5).

$\therefore\ -5 = 1 + a - b$

$\Rightarrow\ b - a = 6$ ...(ii)

and slope of tangent at point p(1,–5) to the curve is,

$$m_1 = \left.\frac{dy}{dx}\right|_{(1,-5)} = [3x^2 + a]_{(1,-5)} = a + 3$$

$\because$ The slope of tangent at point p(1,–5) to the curve is perpendicular to line $-x + y + 4 = 0$, whose slope is $m_2 = 1$.

$\therefore\ a + 3 - 1$

$\Rightarrow\ a = -4$ $[\because m_1 m_2 = -1]$

Now, on substituting a = –4 in Eq. (ii), we get b = 2

On putting a = – 4 and b = 2 in Eq. (i), we get

$$y = x^3 - 4x - 2$$

Now, from option (b), (2,–2) is the required point which lie on it.

Q.52 (3)

Let the equation of drawn line be $\frac{x}{a} + \frac{y}{b} = 1$, where a > 3, b > 4, as the line passes through (3,4) and meets the positive direction of coordinate axes.

We have, $\frac{3}{a} + \frac{4}{b} = 1 \Rightarrow b = \frac{4a}{(a-3)}$

Now, area of ΔAOB, $\Delta = \frac{1}{2}ab = \frac{2a^2}{a-3}$

$$\Rightarrow \frac{d\Delta}{da} = \frac{2a(a-6)}{(a-3)}$$

Clearly, a = 6 is the point of minima for triangle.

Thus, $\Delta_{min} = \frac{2 \times 36}{3} = 24$ sq units

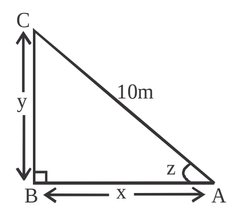
Q.53 (4)

Let AB = xm, BC = y m and AC = 10m

$\therefore\ x^2 + y^2 = 100$ ...(i)

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

So, 2x(3) –2y (4) = 0



Given, $\frac{dx}{dt} = 3\text{m/s}$, $\frac{dy}{dt} = -4\text{ m/s} \Rightarrow x = 4\,y/3$

Putting this value in Eq. (i), we get

$$\frac{16}{9}y^2 + y^2 = 100$$

$\Rightarrow\ 16y^2 + 9y^2 = 900 \Rightarrow 25y^2 = 900$

$\Rightarrow\ y = 30/5 = 6\text{m}$

Q.54 (2)

Given, $f(x) = e^x \sin x$

$f'(x) = e^x \cos x + \sin x\, e^x$

$f''(x) = -e^x \sin x + \cos x\, e^x + e^x \cos x + e^x \sin x$

$= 2e^x \cos x$

For maximum slope,

$f''(x) = 0$

$\Rightarrow\ 2e^x \cos x = 0 \Rightarrow \cos x = 0$

$$\Rightarrow\ x = \frac{\pi}{2}.\frac{3\pi}{2}, \forall x \in [0, 2\pi]$$

$f'''(x) = 2\,[-e^x \sin x + e^x \cos x]$

$f'''(x)|_{x=\frac{\pi}{2}} < 0$ and $f'''(x)|_{x=\frac{3\pi}{2}} > 0$

$\therefore$ Slope is maximum at $x = \pi/2$.

Q.55 (3)
Let v, r, h be the volume, radius adn height of a cylindrical vessel respectively.

$$\therefore \frac{dV}{dt} = 36m^3/s$$

Now, $V = \pi r^2 h$ ...(i)
On differentiating Eq. (i) w.r.t . 't', we get

$$\therefore \frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

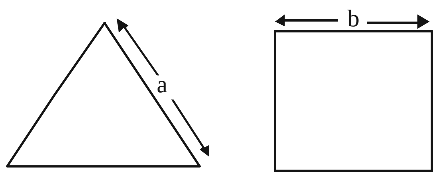
$$\Rightarrow \frac{dh}{dt} = \frac{\left(\frac{dV}{dt}\right)}{\pi r^2} = \frac{36}{\pi(3)^2} \quad [\because r = 3]$$

$$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi} m/s$$

Hence, the water level is rising at the rate of $\frac{4}{\pi}$ m/s.

JEE MAIN
PREVIOUS YEAR'S

Q.1 (2)



$3a = x, \& 4b = 22 - x$
$b = (22 - x)/4$

$$a = \frac{x}{3}$$

$$A_T = \frac{\sqrt{3}}{4} a^2 + b^2$$

$$= \frac{\sqrt{3}}{4}\frac{x^2}{9} + \frac{(22-x)^2}{16}$$

$$= \frac{\sqrt{3}}{4}x^2 + \frac{22^2}{16} - \frac{2}{16}.22.x + \frac{x^2}{16}$$

$$= \frac{dA}{dx} = 0 = \Rightarrow x\left(\frac{\sqrt{3}}{2\times 9} + \frac{1}{8}\right) - \frac{22}{8} = 0$$

$$\Rightarrow x\left(\frac{4\sqrt{3}+9}{36}\right) = \frac{11}{2}$$

$a = x/3$

$$a = \left(\frac{11/2}{\frac{4\sqrt{3}+9}{36}}\right)\left(\frac{1}{3}\right) = \frac{66}{4\sqrt{3}+9}$$

Q.2 (3)

$$f'(x) = (x-3)^{n_1-1}(x-5)^{n_1-1}(n_1+n_2)\left(x - \frac{5n_1+3n_2}{n_1+n_2}\right)$$

Option (3) is incorrect since
for $n_1 = 3, n_2 = 5$

$$f'(x) = 8(x-3)^2(x-5)^4\left(x - \frac{30}{8}\right)$$

minima at $x = \frac{30}{8}$

Q.3 (4)
Let $h(x) = f(x)\,g'(x)$
$h'(x) = f(x)\,g''(x) + f'(x)g'(x)$
Since $f(x)$ is even $\Rightarrow$

$$\therefore f\left(\frac{1}{4}\right) = f\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right) = f\left(-\frac{1}{4}\right) = 0$$

$\therefore f(x) = 0$ has minimum 4 roots

$g(x)$ is even $\Rightarrow g\left(\frac{3}{4}\right) = g\left(-\frac{3}{4}\right) = 0$

$\therefore g'(x) = 0$ has minimum one root
Hence $h'(x)$ has minimum 4 root

Q.4 (1)
Area $\Rightarrow S=4\pi r^2$
D w.r.t. 't'

$$\frac{ds}{dt} = 4\pi(2r)\frac{dr}{dt}$$

$$\left\{\because \frac{ds}{dt} \text{ is constant, Let } \frac{ds}{dt} = k\right\}$$

$$k = 8\pi r.\frac{dr}{dt}$$

$$\int k\,dt = \int 8\pi r.dr$$

$\Rightarrow kt = 4\pi r^2 + C$ .....(1)
$\because$ at $t=0 \Rightarrow r=3$
So $0 = 4\pi(9) + C$
$\Rightarrow C=-36\pi$
Eq. (i) $\Rightarrow kt=4\pi r^2-36\pi$ ....(2)
Now at $t=5; r=7$
$\Rightarrow k(5) =4\pi(7)^2-36\pi$
$\Rightarrow k= 32\pi$
Eq. (2) $32\pi t=4\pi r^2-36\pi$
Now at $t=9$
$\Rightarrow 32\pi(9)=4\pi(r^2)-36\pi$
$\Rightarrow r=9$

Q.5 (1)

Given circle $x^2+y^2+Ax+By+C=0$

$\Downarrow (0\ 6)$

$0+36+0+6B+C=0$

$6B+C=-36$ ....(i)

Given parabola $y=x^2$

Slope of tangent at (2, 4) $=\left(\frac{dy}{dx}\right)(2,4)$

$=(2x)_{(2,4)}=4$

$\therefore$ slope of normal at (2, 4) $=-\frac{1}{4}$

Now equation of normal at (2, 4)

$\Rightarrow (y-4)=-\frac{1}{4}(x-2)$

$\Rightarrow 4y-16=-x+2$

$\Rightarrow 4y+x=18$ .....(2)

Center of circle $\left(\frac{-A}{2},\frac{-B}{2}\right)$

Normal of circle passes through centre of circle

$\Rightarrow 4\left(\frac{-B}{2}\right)-\frac{A}{2}=18$

$\Rightarrow A+4B=-36$ ...(3)

and circle passing through (2, 4)

$\Rightarrow 4+16+2A+4B+C=0$

$\Rightarrow (A+C)+(A+4B)=-20$

$\Rightarrow A+C+(-36)=-20$ (from (3))

$A+C=16$

Q.6 (3)

$f(x)=4\ell_n(x-1)-2x^2+4x+5, x>1$

$f'(x)=\frac{4}{x-1}-4x+4$

$f'(x)=\frac{4}{x-1}-4(x-1)$

for $x\in(1,2)\Rightarrow f'(x)>0$

for $x\in(2,\infty)\Rightarrow f'(x)<0$

$\therefore$ option (A) is correct

$f(x)=-1$ has exactly two solutions

$\therefore$ option (B) exactly is correct

$f(e)>0$ & $f(e+1)<0$

$\therefore f(e).f(e+1)<0$ $\therefore$ option (D) is also correct

$f'(e)-f''(2)<0$ is not correct

$\therefore$ option (c) is incorrect

Q.7 (4)

$y=x^3+3x^2+5$ $\Rightarrow (x_1,y_1)$ lies on curve

$\frac{dy}{dx}=3x^2+6x$ $\therefore y_1=x_1^3+3x_1^2+5$

slope of tangent at (x_1, y_1) is $\frac{dy}{dx}(x_1,y_1)=3x_1^2+6x_1$

also tangent at (x_1, y_1) passes through (0, 0)

$\therefore$ slope of tangent $=\frac{y_1-0}{x_1-0}$

$\therefore \frac{y_1}{x_1}=3x_1^2+6x_1$

$y_1=3x_1^3+6x_1^2$

$x_1^3+3x_1^2+5=3x_1^3+6x_1^2$

$2x_1^3+3x_1^2-5=0$

On $x_1=1$ satisfies above equation

$\therefore y_1=1+3+5$

$y_1=9$ $\therefore (x_1,y_1)=(1,9)$

Now checking option for (1, 9)

(A) put(1, 9) $\Rightarrow 1+\frac{81}{81}=2$ $\Rightarrow$ LHS = RHS

(B) $\frac{81}{9}-1=8$ $\Rightarrow 9-1=8$ $\Rightarrow 8=8$ LHS = RHS

(C) $9=4(1)^2+5$ $\Rightarrow 9=9$ $\Rightarrow$ LHS = RHS

(D) $\frac{1}{3}-(9)^2=2$ $\Rightarrow \frac{1}{3}-81=2$ $\Rightarrow$ LHS = RHS

$\therefore$ option (d) is answer

Q.8 (2)

$f(x)=|2x^2+3x-2|+\sin x\cos x$ in [0, 1]

let $y=2x^2+3x-2=(x+2)(2x-1)$

$\therefore f(x)=|(x+2)(2x-1)|+\frac{1}{2}\sin 2x$

Case - 1 when $x\in\left[0,\frac{1}{2}\right]$

$f(x)=-(2x^2+3x-2)+\frac{1}{2}\sin 2x$

$f'(x)=-(4x+3)+\cos 2x$

when $x\in\left[0,\frac{1}{2}\right]$ then $(4x+3)\in[3,5]$

$\therefore -(4x+3)\in(-5,-3)$

$\therefore f'(x)<0\ \forall x\in\left[0,\frac{1}{2}\right]\Rightarrow f(x)$ is decreasing

$\therefore f_{max}=f(0)=2$ & $f_{min}=f\left(\frac{1}{2}\right)=\frac{\sin 1}{2}$

Case - 2 when $x\in\left[\frac{1}{2},1\right]$

$f(x)=(2x^2+3x-2)+\frac{1}{2}\sin 2x$

$f'(x)=(4x+3)+\cos 2x$

for $x \in \left[\frac{1}{2},1\right]$ then $(4x+3) \in [5,7]$

$\therefore f'(x) > 0 \forall x \in \left[\frac{1}{2},1\right]$

$\Rightarrow$ f(x) is increasing

$f\left(\frac{1}{2}\right)_{min} = \frac{\sin 1}{2}, f(1)_{max} = 3+\frac{1}{2}\sin 2$

sum of max. and min. values

$= \frac{\sin 1}{2} + 3 + \frac{1}{2}\sin 2$

$= 3+\frac{1}{2}[\sin 1 + \sin 2]$

$= 3+\frac{1}{2}[\sin 1 + 2\sin 1.\cos 1]$

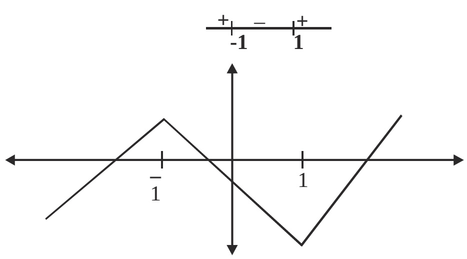
$= 3+\frac{\sin 1}{2}(1+2\cos 1)$

Q.9 (4)

$f(\alpha x) = x^7 - 7x - 2$

$f'(\alpha) = 7x^6 - 7 = 7[x^6 - 1]$

$= 7[(x+1)(x-1)][x^4 + x^2 + 1]$



Q.10 (4)

$f'_{\lambda}(x) \Rightarrow 12\lambda x^2 - 72\lambda x + 36 \geq 0$

$\Rightarrow \lambda x^2 - 6\lambda x + 3 > 0$

$\lambda > 0$ $\quad\quad d \leq 0$

$36\lambda^2 - 12\lambda \leq 0$

$\lambda \in [0, \frac{1}{3}]$

$\lambda = \frac{1}{3}$

$f_{\frac{1}{3}}(1) + f_{\frac{1}{3}}(-1)$

$= \left[\frac{4}{3}(1) - \frac{36}{3}(1) + 36 + 48\right] + \left[\frac{4}{3}(-1) - \frac{36}{3}(1) - 36 + 48\right]$

$= 96 - \frac{72}{3}$

$= 96 - 24$

$= 72$

Q.11 (3)

$x = 12(t + \sin t \cos t)$

$y = 12(1 + \sin t)^2$

$0 < t < \frac{\pi}{2}$

$\frac{dy}{dx} = \frac{12 \times 2(1+\sin t)\cos t}{12\{1 - \sin^2 t + \cos^2 t\}}$

$\frac{dy}{dx} = \frac{2(1+\sin t)\cos t}{(1+\cos 2t)} = \frac{2(1+\sin t)\cos t}{2\cos^2 t}$

Now, given $\tan\theta = \tan\frac{\pi}{3} = \sqrt{3}$

$\Rightarrow \sqrt{3} = \frac{1+\sin t}{\cos t}$

$\Rightarrow \sqrt{3}\cos t - \sin t = 1$

$\Rightarrow \frac{\sqrt{3}}{2}\cos t - \frac{\sin t}{2} = \frac{1}{2} \Rightarrow \cos\left(t+\frac{\pi}{6}\right) = \cos\frac{\pi}{3}$

$\Rightarrow t + \frac{\pi}{6} = \frac{\pi}{3} \Rightarrow t = \frac{\pi}{6}$

Now, $y_0 = 12\left(1+\sin\frac{\pi}{6}\right)^2$

$= 12\left\{1+\frac{1}{2}\right\}^2 \Rightarrow 9 \times 3 = 27$

Q.12 [3]

$$f(x) = \begin{cases} (x^2-1)(x-3)+(x-3), & x \in (0,1] \cup [3,4) \\ -(x^2-1)(x-3)+(x-3), & x \in [1, 3] \end{cases}$$

$$\Rightarrow f'(x) \begin{cases} 3x^2 - 6x, & x \in (0, 1) \cup (3,4) \\ -3x^2 + 6x + 2, & x \in (1,3) \end{cases}$$

f(x) is non-derivable at x = 1 and x = 3

also $f'(x) = 0$ at $x = 1 + \sqrt{\frac{5}{3}} \Rightarrow m + M = 3$

Q.13 (2)

Total surface area = $76x^2 + 3\pi r^2 = k$

$\Rightarrow r = \left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}}$

total volume (v) = $40x^3 + \frac{2}{3}\pi r^3 = 40x^3 + \frac{2}{3}\pi\left(\frac{k-76x^2}{3\pi}\right)^{3/2}$

$\frac{dv}{dx} = 120x^2 + \frac{2\pi}{3}\left(\frac{3}{2}\right)\left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}\left(\frac{-152x}{3\pi}\right)$

Put $\frac{dv}{dx} = 0 \Rightarrow \frac{x}{r} = \frac{19}{45}$

Q.14 (2)

Let $f(x) = x^4 - 4x + 1$

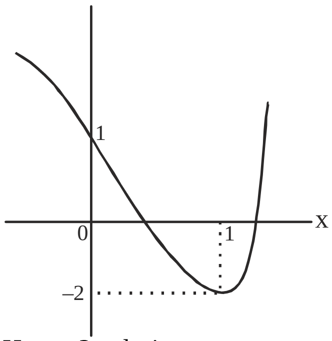
$f'(x) = 4x^3 - 4$

$f'(x) = 0 \Rightarrow x = 1$

x = 1 is point of minima

$f(1) = -2$

$f(0) = 1$



Hence 2 solutions

Q.15 (3)

$a = 20 - 2x^2,\ b = 10 + x^2,\ c = 10 + x^2$

$= \dfrac{a+b+c}{2}$

$= 20$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{20(2x^2)(10-x^2)(10-x^2)}$

$= 2\sqrt{10}\ \sqrt{x^2(10-x^2)^2}$

$= 2\sqrt{10}\ |10x - x^3|$

$S = 10x - x^3$

$\dfrac{ds}{dx} = 10 - 3x^2$

$\dfrac{ds}{dx} = 0$

$\Rightarrow 3x^2 = 10$

Q.16 **(2)**

$m = L \cdot \max$

$N = L \cdot \min$

$$f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}\,dt$$

$$f'(x) = \frac{(x^4 - 5x^2 + 4)2x}{2 + e^{x^2}} = \frac{2x(x^2-1)(x^2-4)}{2+e^{x^2}}$$

$$f'(x) = \frac{2x(x-1)(x+1)(x-2)(x+2)}{2+e^{x^2}}$$

–	+	–	+	–	+
	–2	–1	0	1	2
	L.min	L.max	min	max	min

So, m = 2 and n = 3

Q.17 (4)

$$\frac{x(\cos x - \sin x)}{e^x + 1} + \left\{\frac{g(x)(e^x+1) - xe^x}{(e^x+1)^2}\right\}$$

$$= \frac{(e^x+1)(g(x) + xg'(x)) - e^x xg(x)}{(e^x+1)^2}$$

$\Rightarrow (e^x+1)(x\cos x - x\sin x) + g(x)(e^x+1) - xe^x g(x)$

$= (e^x+1)g(x) + (e^x+1)xg'(x) - e^x.xg(x)$

$\Rightarrow (e^x+1)(x\cos x - x\sin x) = (e^x+1).xg'(x)$

$\Rightarrow x(\cos x - x\sin x) = x.g'(x)$

$\therefore g'(x) = \cos x - \sin x$

$\Rightarrow g'(x) = \sqrt{2}\cos x\left(x + \dfrac{\pi}{4}\right) \downarrow \text{ in } \left(0, \dfrac{\pi}{4}\right)$

$g(x) = \sin x + \cos x + \lambda$

$\Rightarrow g(x) = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right) + \lambda \uparrow \text{in } \left(0, \dfrac{\pi}{4}\right)$

$g(x) + g'(x) = 2\text{cox} + \lambda$ is decreasing in $\left(0, \dfrac{\pi}{2}\right)$

$g(x) - g'(x) = 2\sin x + \lambda$ is Increasing in $\left(0, \dfrac{\pi}{2}\right)$

Q.18 (1)

$f(x) = \log_e(x^2+1) - e^{-x} + 1$	$g(x) = \dfrac{1 - 2e^{2x}}{e^x}$
$f'(x) = \dfrac{2x}{1+x^2} + e^{-x}$	$g'(x) = -e^{-x} - 2e^x$
$f'(x) > 0$	$g'(x) = -\left\{2ex + \dfrac{1}{e^x}\right\} < 0$
$\therefore$ f(x) $\uparrow$ function	g(x) $\downarrow$ function

$f\left(g\left(\dfrac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \dfrac{5}{3}\right)\right)$

$\Rightarrow g\left(\dfrac{(\alpha-1)^2}{3}\right) > g\left(\alpha - \dfrac{5}{3}\right)$

$\Rightarrow \dfrac{(\alpha-1)^2}{3} < \alpha - \dfrac{5}{3}$

$\Rightarrow \alpha^2 + 1 - 2\alpha < 3\alpha - 5$

$\alpha^2 - 5\alpha + 6 < 0$

$\alpha \in (2, 3)$ Ans.

Q. 19 (1)

$$f(x) = \begin{cases} x^2 - 4x - 2, & \forall x \in \left(-1, \dfrac{3-\sqrt{17}}{2}\right) \\ -x^2 + 2x + 2, & \forall x \in \left(\dfrac{3-\sqrt{17}}{2}, 2\right) \end{cases}$$

$f'(x)$ when $x \in \left(-1, \frac{3-\sqrt{17}}{2}\right)$

f'(x) = 2x–4=0 ⇒ x=2
f'(x) = 2(x–2) ⇒ f'(x) is always ↓
f(2) = 2
f(–1) = 3

$$f\left(\frac{3-\sqrt{17}}{2}\right) = \frac{\sqrt{17}-3}{2}$$

f'(x) when $x \in \left(\frac{3-\sqrt{17}}{2}, 2\right)$

f'(x) = –2x+2
f'(x) = –2(x-1)
f'(x) = 0 when x=1
f(1) = 3

absolute minimum value = $\frac{\sqrt{17}-3}{2}$

absolute maximum value = 3

Sum = $\frac{\sqrt{17}-3}{2} + 3 = \frac{\sqrt{17}+3}{2}$

Q.20 (2)

$$f'(x) = \frac{-2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2$$

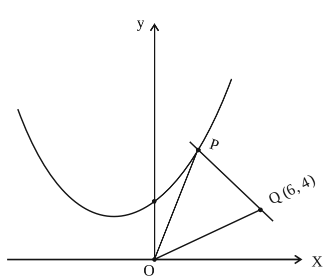
$$= -2\left[\frac{1}{\sqrt{1-x^2}} + \frac{2}{1+x^2} + 3x + 1\right]$$

f'(x)<0 ⇒ f(x) is a dec. function
f(1) = π+5
f(–1) = 5π+5
Range : [a,b] ≡ [π+5, 5π+5]
a = π + 5, b = 5π + 5 ⇒ 4a–b = 11– π

Q.21 (2)

$f(x) = x^7 + 5x^3 + 3x + 1$
$f'(x) = 7x^6 + 15x^2 + 3 > 0$
∴ f(x) is strictly increasing function
$x \to -\infty \Rightarrow y \to -\infty$
$x \to \infty \Rightarrow y \to \infty$
∴ no. of real solution = 1

Q.22 (13)



$y = 2x^2 + x + 2$

$$\frac{dy}{dx} = 4x + 1$$

Let P be (h, k), then normal at P is

$$y - k = -\frac{1}{4h+1}(x-h)$$

This passes through Q(6, 4)

$$\therefore\ 4 - k = -\frac{1}{4h+1}(6-h)$$

$\Rightarrow (4h + 1)(4 – k) + 6 – h = 0$
Also $k = 2h^2 + h + 2$
$\therefore (4h + 1)(4 – 2h^2 – h – 2) + 6 – h = 0$
$\Rightarrow 4h^3 + 3h^2 – 3h – 4 = 0$
⇒ h = 1, k = 5

Now area of ΔOPQ will be $= \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 5 \\ 1 & 6 & 4 \end{vmatrix} = 13$

Q.23 (1)

$f(x) = xe^{x(1-x)}$
$f'(x) = -e^{x(1-x)}(2x + 1)(x – 1)$

f(x) is increasing in $\left(-\frac{1}{2}, 1\right)$

Q.24 (2)

$$f(x) = \left(x^2 - 2x + 7\right)e^{\left(4x^3 - 12x^2 - 180x + 31\right)}$$

$$f'(x) = (2x-2)e^{\left(4x^3 - 12x^2 - 180x - 31\right)} +$$

$$\left(x^2 - 2x + 7\right)e^{\left(4x^3 - 12x^2 - 180x + 31\right)}\left(12x^2 - 24x - 180\right)$$

$$f'(x) = e^{\left(4x^3 - 12x^2 - 180x + 31\right)}$$

$$\left[(2x-2) + \left(x^2 - 2x + 7\right)12.\left(x^2 - 2x - 15\right)\right]$$

$$f'(x) = e^{\left(4x^3 - 12x^2 - 180x + 31\right)}$$

$$\left[2x - 2 + 12\left(x^2 - 2x + 7\right)(x-5)(x+3)\right]$$

Now $f'(x) < 0 \forall \in [-3, 0]$

$\Rightarrow f'(x) < 0 \forall x \in [-3, 0]$

$\Rightarrow f'(x)$ dec. $\forall x \in [-3, 0]$

f(x) max. at $\boxed{x = -3}$

Q.25 (1)

$f(x) = ax^3 + bx^2 + cx + 5$

$$\left.\begin{aligned} f'(-2) = 0 &\Rightarrow 12a - 4b + c = 0 \\ f'(-2) = 0 &\Rightarrow -8a + 4b - 2c + 5 = 0 \end{aligned}\right] 4a - c + 5 = 0$$

$$\boxed{a = -\frac{1}{2}}$$

Q: (0,5) $\Rightarrow f'(0) = 3$ & $b = -\dfrac{3}{4}$

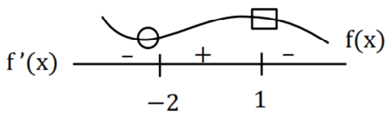
$\Rightarrow \boxed{c = 3}$

Hence $f(x) = -\dfrac{x^3}{2} - \dfrac{3x^2}{4} + 3x + 5$

$f'(x) = -\dfrac{3x^3}{2} - \dfrac{3x}{2} + 3$

$f'(x) = -\dfrac{3}{2}[x^2 + x - 2]$

$f'(x) = -\dfrac{3}{2}[x+2][x-1]$



$f(x)_{max}$ at x = 1

$f_{max} = f(1) = -\dfrac{1}{2} - \dfrac{3}{4} + 3 + 5 = \dfrac{32-3-2}{4} = \boxed{\dfrac{27}{4}}$

Q.26 (1)

$I(x) = \int \underset{II}{\sec^2 x} \cdot \underset{I}{\sin^{-2022} x}\, dx - 2022 \int \sin^{-2022} x\, dx$

$= \tan x \cdot (\sin x)^{-2022} + \int (2022) \tan x . (\sin x)^{-2023} \cos x dx$

$-2022 \int (\sin x)^{-2022} dx$

$I(x) = (\tan x)(\sin x)^{-2022} + C$

At $X = \pi/4, 2^{1011} = \left(\dfrac{1}{\sqrt{2}}\right)^{-2022} + C$

$\therefore C = 0$

Hence, $I(x) = \dfrac{\tan x}{(\sin x)^{2022}}$

$I\left(\dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{3}\left(\dfrac{1}{2}\right)^{2022}} = \dfrac{2^{2022}}{\sqrt{3}}$

$I(\pi/3) = \dfrac{\sqrt{3}}{\left(\dfrac{\sqrt{3}}{2}\right)^{2022}} = \dfrac{2^{2022}}{(\sqrt{3})^{2021}} = \dfrac{1}{3^{1010}} I\left(\dfrac{\pi}{6}\right)$

$3^{1010} I(\pi/3) = I(\pi/6)$

Q.27 (195)

$y = 5x^2 + 2x - 25$ P(2, –1)

$y' = 10x + 2$

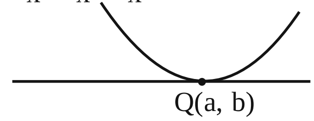
$y_p' = 22$

$\therefore$ tangent to curve at P

$y + 1 = 22(x - 2)$

$y = 22x - 45$

$y = x^3 - x^2 + x$



$\left.\dfrac{dy}{dx}\right|_Q = 3a^2 - 2a + 1$

Hence, $3a^2 - 2a + 1 = 22$

$\therefore 3a^2 - 2a - 21 = 0$

$3a^2 - 9a + 7a - 21 = 0$

$(3a + 7)(a - 3) = 0$ ⟨ $a = 3$, $a = -7/3$

from curve $b = a^3 - a^2 + a$

at a = 3

b = 21 $|2a + 9b| = 195$

at a = –7/3 tangent will be parallel

Hence, it is rejected

Q.28 (4)

$f(x) = 81.3^{(x^2-2)^3}$

$f'(x) = 81.3^{(x^2-2)^3} \cdot \ell n3.3(x^2-2)^2.2x$

$= (81 \times 6)3^{(x^2-2)^3} x(x^2-2)^2 \ell n3$

+	–	+	+
$-\sqrt{2}$	0	$\sqrt{2}$	

x = 6 is point of local min

$f'(x) = \underbrace{(486.\ell n3)}_{k} \underbrace{3^{(x^2-2)^3} x(x^2-2)^2}_{g(x)}$

$g'(x) = 3^{(x^2-2)^3}(x^2-2)^2 + x.3^{(x^2-2)^3}.4x.(x^2-2)$

$+ x.(x^2-2)^2.3^{(x^2-2)^3} \ell n3.3(x^2-2)^2.2x$

$= 3^{(x^2-2)^3}(x^2-2)[x^2 - 2 + 4x^2 + 6x^2 \ell n3(x^2-2)^3]$

$g'(x) = 3^{(x^2-2)^3}(x^2-2)[5x^2 - 2 + 6x^2 \ell n3(x^2-2)^3]$

$f''(x) = k.g'(x)$

$f''(\sqrt{2}) = 0,\ f''(\sqrt{2}^+) > 0,\ f''(\sqrt{2}^-) < 0$

$x = \sqrt{2}$ is point of inflection

$f''(x) > 0$ for $x > \sqrt{2}$

So, $f'(x)$ is increasing

Q.29 (3)

$\dfrac{x^2}{2} + \dfrac{x^2}{2} + \dfrac{x^2}{2} + \dfrac{x^2}{2} + \dfrac{x^2}{2} + \dfrac{\alpha}{2x^5} + \dfrac{\alpha}{2x^5}$

$\geq 7\left(\frac{\alpha^2}{2^7}\right)^{\frac{1}{7}}$

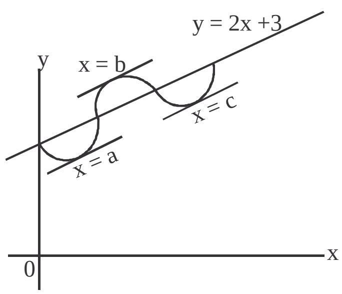
$\frac{7(\alpha)^{2/7}}{2} = 14$

$(\alpha^2)^{1/7} = 2^2$

$\alpha = (2^2)^{7/2} = 2^7$

$\alpha = 128$

Q.30 (2)



f'(a) = f'(b) = f'(c) = 2

$\Rightarrow$ f''(x) is zero

for at least $x_1 \in (a,b)$ & $x_2 \in (b,c)$

Q.31 (3)

Given $f(x) = \begin{cases} \int_0^x (5-|t-3|)dt & x > 4 \\ x^2 + bx & x \leq 4 \end{cases}$

f(x) is continuous at x =4

so $\lim_{x\to4} f(x) = \lim_{x\to4^+} f(x) = f(4)$

So $16 + 4b = \int_0^3 (2+t)dt + \int_3^4 (8-t)dt \Rightarrow 16 + 4b = 15$

So $b = \frac{-1}{4}$

At x = 4

LHD = $2x + b = \frac{31}{4}$

RHD = $5 - |x-3| = 4$

LHD $\neq$ RHD

Option (A) is true

and $f'(3) + f'(5) = \frac{23}{4} + 3 = \frac{35}{4}$

Option (B) is true

$\because f(x) = x^2 - \frac{x}{4}$ at $x \leq 4$

$f'(x) = 2x - \frac{1}{4}$

This function is not increasing.

In the interval in $x \in \left(-\infty, \frac{1}{8}\right)$

Option (C) is NOT TRUE

This function f (x) is also local minima at x = $\frac{1}{8}$

Q.32 [2]

$y^5 - 9xy + 2x = 0$

$5y^4\frac{dy}{dx} - 9x\frac{dy}{dx} - 9y + 2 = 0$

$\frac{dy}{dx}(5y^4 - 9x) = 9y - 2$

$\frac{dy}{dx} = \frac{9y-2}{5y^4 - 9x} = 0$ (for horizontal tangent)

$y = \frac{2}{9} \Rightarrow$ Which does not satisfy the origial equation

$\Rightarrow$ M = 0.

Now $5y^4 - 9x = 0$ (For vertical tangent)

$\therefore 5y^4 = 9x$

Putting value of 9x in the equation of curve

$y^5 - 5y^5 + 2x = 0 \Rightarrow x = y^5$

So, $5y^4 = 9y^5$

$\Rightarrow y = 0$ & $y = \frac{5}{9}$

y = 0 gives x = 0

$y = \frac{5}{9}$ gives $x = \left(\frac{5}{9}\right)^5$

So, N = 2 $\Rightarrow$ M + N = 2

Q.33 (Bonus)

$f_a(X) = \tan^{-1} 2x - 3ax + 7$

$f_a'(x) = \frac{2}{1+4x^2} - 3a \geq 0$

$a \leq \left(\frac{2}{3(1+4x^2)}\right)_{\min.}$ at $x = \pm\frac{\pi}{6}$

$a_{\max} = \bar{a} = \frac{6}{9+\pi^2}$

$f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1}\frac{\pi}{4} - 3\frac{6}{9+\pi^2}\frac{\pi}{8} + 7$

$= \tan^{-1}\frac{\pi}{4} - \frac{9\pi}{4(\pi^2+9)} + 7$

Q.34 (3)

$f(1) \geq f(1^+)$

$1 - 1 + 10 - 7 \geq -2 + \log_2(b^2 - 4)$

$5 \geq \log_2(b^2 - 4)$

$36 \geq b^2$

$b \in [-6, 6]$

$\because b^2 - 4 > 0 \quad \Rightarrow b \in (-\infty, -2) \cup (2, \infty)$

$\therefore b \in [-6, -2] \cup [2, 6]$

Q.35 [45]

$$\frac{dy}{dx} = 4x - \frac{1}{x}$$

$$= \frac{(2x+1)(2x-1)}{x}$$

$\downarrow$ in (0, 1/2)

$\uparrow$ (1/2, ∞)

$y^2 = 4ax$ $\quad$ $P(at^2, 2at)$

tangent $\quad$ $yt = x + at^2$

pass (4, 3)

$$3t = 4 + \frac{1}{2}t^2$$

$t^2 - 6t + 8 = 0 \Rightarrow t = 2, 4$

$t = 2$ $\quad$ $2y = x + 2$

pass (–2, 0)

$$\left(-\frac{1}{a}, 0\right) = (-2, 0)$$

$\therefore$ t = 2 not possible

$t = 4$ $\quad$ $4y = x + 8$

equation of normal at (8, 4)

p(8, 4)

$y - 4 = -4(x - 8)$

$4x + y = 36$

$$\frac{x}{9} + \frac{y}{36} - 1 \quad \alpha + \beta = 45$$

Q.36 **[5]**

$$V = \frac{\pi r^2 h}{3}$$

$$V = \frac{\pi r^3}{3}.\frac{4r}{3}$$

$$= \frac{4}{9}\pi r^3 \qquad \frac{r}{h} = \frac{3}{4}$$

$$\Rightarrow \frac{dv}{dt} = \frac{4}{9}\pi.3r^2\frac{dr}{dt} \qquad h = \frac{4r}{3}$$

$$\Rightarrow 6 = \frac{4}{9}\pi.3r^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{9}{2\pi r^2}$$

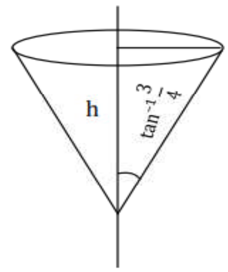
$$A = \pi r\sqrt{r^2 + h^2}$$

$$= \pi r\sqrt{\frac{16r^2}{9} + r^2}$$

$$A = \frac{5\pi r^2}{3} = \frac{dA}{dt} = \frac{5\pi}{3}2r.\frac{dr}{dt} = \frac{5\pi}{3}2r.\frac{9}{2r^2} = \frac{15}{r}$$

At $h = 4 \Rightarrow r = 3$

$$\frac{dA}{dt} = \frac{15}{3} = m^2 / hr$$



Q.37 (2)

$$\frac{dy}{dx} = -\frac{ky}{x}$$

$\ln y = -k\ln x + \ln c$

$y \cdot x^k = c$

passes through (1, 2) and (8, 1)

$\therefore$ c = 2 and $1.8^k = 2$

$$k = \frac{1}{3}$$

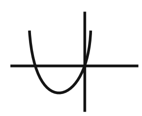
$$\therefore \; y = \frac{2}{x^{\frac{1}{3}}}$$

$$\therefore \; y\left(\frac{1}{8}\right) = \frac{2}{\left(\frac{1}{8}\right)^{\frac{1}{3}}} = 4$$

Q.38 [15]

$f(x) = |5x - 7| + [x^2 + 2x]$

For $(x^2 + 2x)$, $x \in \left[\frac{5}{4}, 2\right]$

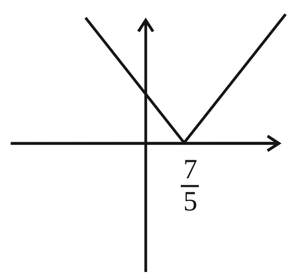


$$\text{at } x = \frac{5}{4} \Rightarrow x^2 + 2x = \frac{25}{16} + \frac{5}{2} = \frac{25+40}{16} = \frac{65}{16}$$

at $x = 2 \Rightarrow x^2 + 2x = 4 + 4 = 8$

For $|5x - 7|$,

minimum at $x = \frac{7}{5}$



f_{max} will occur at x = 2 and f(2) = 11

f_{min} at either $x = \frac{7}{5}$ or $x = \frac{5}{4}$

$$f\left(\frac{7}{5}\right)=0+\left[\frac{49}{25}+\frac{14}{5}\right]=\left[\frac{49+70}{25}\right]=4$$

$$f\left(\frac{5}{4}\right)=\left|\frac{25}{4}-7\right|+4>f\left(\frac{7}{5}\right)$$

$\therefore$ m + M = 4 + 11 = 15

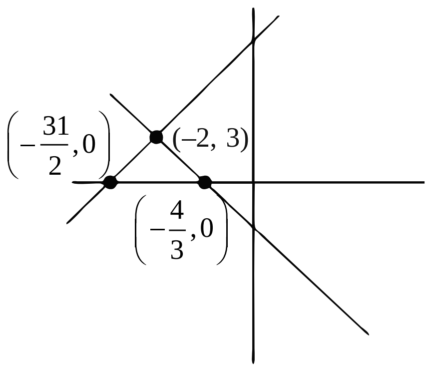
Q.39 [170]

$$12x^2-3y^2-6xy\frac{dy}{dx}+12x-5y-5x\frac{dy}{dx}-16y\frac{dy}{dx}+9=0$$

x = –2, y = 3

$$48-27+36\frac{dy}{dx}-24-15+10\frac{dy}{dx}-48\frac{dy}{dx}+9=0$$

$$-2\frac{dy}{dx}=9\Rightarrow m_t=\frac{-9}{2},m_n=\frac{2}{9}$$



Tangent $y-3=\frac{-9}{2}(x+2):9x+2y=-12$

Normal $y-3=\frac{2}{9}(x+2):2x-9y=-31$

Area $=\frac{1}{2}\left(\frac{31}{2}-\frac{4}{3}\right)\cdot 3$

$$=\frac{1}{2}\left(\frac{93-8}{6}\right)3=\frac{85}{4}$$

$\therefore$ 8A = 170

INDEFINITE INTEGRATION

EXERCISE-I (MHT CET LEVEL)

Q.1 (2)

$\int 5\sin x\, dx = -5\cos x + c$

Q.2 (4)

$\int \frac{3x^3 - 2\sqrt{x}}{x} dx = \int 3x^2 dx - 2\int x^{-1/2} dx = x^3 - 4\sqrt{x} + c$

Q.3 (2)

$\int \text{cosec}^2 x\, dx = -\cot x + c$

Q.4 (2)

$\frac{d}{dx}(A in|\cos x + \sin x - 2| + Bx + C)$

$= A\frac{\cos x - \sin x}{\cos x + \sin x - 2} + B$

$= \frac{A\cos x - \sin x + B\cos x + B\sin x + -2B}{\cos x + \sin x - 2}$

$\therefore 2 = A + B\, or\, -1 = -A + B; \lambda$

$\therefore 2 = 3/2, B = 1/2, \lambda = -1$

Q.5 (3)
Q.6 (4)
Q.7 **(2)**
Q.8 (4)

Put x=2a–t

so t hat dx=–dt

when x=a, t=a and when x= 2a, t = 0

$\int_0^2 f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a - t)\, dt = n + m$

Q.9 (2)

$\int e^{\log(\sin x)} dx = \int \sin x\, dx = -\cos x + c.$

Q.10 (2)

$\int e^{x\log a} e^x dx = \int e^{\log a^x}.e^x dx = \int a^x e^x dx$

$= \int (ae)^x dx = \frac{(ae)^x}{\log(ae)} + C.$

Q.11 (2)

$I = \int \frac{1}{(x-5)^2} dx = \frac{(x-5)^{-2+1}}{-2+1} + c = \frac{(x-5)^{-1}}{-1} + c$

$= -\frac{1}{(x-5)} + c$

Q.12 (2) $\int \frac{dx}{1+e^x} = \int \frac{e^{-x}}{1+e^{-x}} dx$

Put $1 + e^{-x} = t \Rightarrow e^{-x} dx = -dt$, then it reduces to

$-\int \frac{dt}{t} = -\log t = -\log(1 + e^{-x})$

Q.13 (1)

$\int \frac{dx}{x + x\log x} = \int \frac{dx}{x(1 + \log x)}$

Now putting $1 + \log x = t \Rightarrow \frac{1}{x} dx = dt,$ it reduces to

$\int \frac{dt}{t} = \log(t) = \log(1 + \log x)$

Q.14 **(3)**
Q.15 **(2)**
Q.16 **(2)**
Q.17 **(3)**
Q.18 **(3)**

$I = \int_1^2 [x^2]\, dx - \int_1^2 [x]^2\, dx$

$= \int_1^{\sqrt{2}} dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^2 3dx - \int_1^2 1dx$

$= 4 - \sqrt{2} - \sqrt{3}$

Q.19 **(1)**

$I = \int_0^{\frac{\pi}{2}} \log(\tan x)\, dx = \int_0^{\frac{\pi}{2}} \log\left\{\tan\left(\frac{\pi}{2} - x\right)\right\} dx$

$= \int_0^{\frac{\pi}{2}} \log(\cot x)\, dx$

$\therefore 2I = \int_0^{\frac{\pi}{2}} \log(\tan x)\, dx + \int_0^{\frac{\pi}{2}} \log(\cot x)\, dx$

$= \int_0^{\frac{\pi}{2}} [\log\tan x + \log\cot x]\, dx$

$= \int_0^{\frac{\pi}{2}} \log(\tan x.\cot x)\, dx$

$$\int_0^{\frac{\pi}{2}} \log(1)\,dx = \int_0^{\frac{\pi}{2}} 0\,dx = 0 \qquad \therefore I = 0$$

Q.20 (1)

$$\int \frac{\sec x\,dx}{\sqrt{\cos 2x}} = \int \frac{\sec x}{\sqrt{\cos^2 x - \sin^2 x}}dx$$

$= \int \frac{\sec^2 x\,dx}{\sqrt{1-\tan^2 x}}$ {Multiplying N'r and D'r by $\sec x$}

Now putting $\tan x = t \Rightarrow \sec^2 x\,dx = dt,$ we get the integral $= \sin^{-1} t = \sin^{-1}(\tan x).$

Trick : Since $\frac{d}{dx}\{\sin^{-1}(\tan x)\} = \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$

$$= \frac{\sec^2 x.\cos x}{\sqrt{\cos^2 x - \sin^2 x}} = \frac{\sec x}{\sqrt{\cos 2x}}.$$

Q.21 (1)

Put $\log \sin x = t$.

Q.22 (3)

Put $(1+\log x) = t \Rightarrow \frac{1}{x}dx = dt$

$$\int \frac{(1+\log x)^2}{x}dx = \int t^2 dt = \frac{t^2}{3} + c = \frac{(1+\log x)^3}{3} + c.$$

Q.23 (2)

$$\int \frac{x-2}{x(2\log x - x)}dx = -\int \frac{\left(\frac{2}{x}-1\right)}{(2\log x - x)}dx$$

Now put $(2\log x - x) = t \Rightarrow \left(\frac{2}{x}-1\right)dx = dt,$ then it reduces to $-\int \frac{1}{t}dt = -\log t = -\log(2\log x - x)$

$$= \log\left(\frac{1}{2\log x - x}\right) + c$$

Q.24 (2)

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$.

Q.25 (3)

Putting $t = \tan^{-1} x \Rightarrow dt = \frac{1}{1+x^2}dx,$ we get

$$\int \frac{e^{\tan^{-1} x}}{1+x^2}dx = \int e^t dt = e^t + c = e^{\tan^{-1} x} + c.$$

Q.26 (1)

$$\int \sec x \tan^3 x\,dx = \int \sec x(\sec^2 x - 1)\tan x\,dx$$

$$= \int \sec x \tan x \sec^2 x\,dx - \int \sec x \tan x\,dx$$

$$= \frac{\sec^3 x}{3} - \sec x + c,$$

(Putting $\sec x = t$ in first part).

Q.27 (1)

Put $t = \tan x \Rightarrow dt = \sec^2 x\,dx,$ then

$$\int \frac{\sec^2 x\,dx}{\sqrt{\tan^2 x + 4}} = \int \frac{1}{\sqrt{t^2 + 2^2}}dt$$

$$= \log[\tan x + \sqrt{\tan^2 x + 4}] + c.$$

Q.28 (4)

$$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$$

$$= x\tan x + \log(\cos x) + c.$$

Q.29 (3)

Let $I = \int \sin(\log x)\,dx$

Put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt,$ then

$$I = \int \sin t.e^t\,dt = \sin t.e^t - \int e^t.\cos t\,dt$$

$$= \sin t.e^t - [\cos t.e^t + \int e^t.\sin t\,dt]$$

$$\Rightarrow 2I = \sin t.e^t - \cos t.e^t$$

$$\Rightarrow I = \int \sin(\log x)\,dx = \frac{1}{2}x[\sin(\log x) - \cos(\log x)].$$

Q.30 (1)

$$\int x\tan^{-1} x\,dx = \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$$

$$\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int\left[1 - \frac{1}{1+x^2}\right]dx$$

$$= \frac{1}{2}(x^2+1)\tan^{-1} x - \frac{1}{2}x + c$$

Q.31 (4)

$$\int x\log\left(1+\frac{1}{x}\right)dx$$

$$=\log\left(1+\frac{1}{x}\right)\cdot\frac{x^2}{2}-\int\frac{x}{x+1}\cdot\left(-\frac{1}{x^2}\right)\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2}{2}\log\left(\frac{x+1}{x}\right)\cdot\frac{x^2}{2}+\frac{1}{2}\int\frac{x+1-1}{x+1}dx$$

$$=\frac{x^2}{2}\log\left(\frac{x+1}{x}\right)+\frac{1}{2}x-\frac{1}{2}\log(x+1)+c$$

$$=\left(\frac{x^2-1}{x}\right)\log(x+1)-\frac{x^2}{2}\log x+\frac{1}{2}+c$$

Q.32 (2)
Q.33 (2)
Q.34 (1)
Q.35 (3)
Q.36 **(3)**

Q.37 (1) Since, $\int x\sin x\,dx=-x\cos x+A$

$\Rightarrow -x\cos x+\sin x+\text{constant}=-x\cos x+A$

Equating it, we get $A=\sin x+\text{constant}$.

Q.38 (2)

$$\int x\log x\,dx=\frac{x^2}{2}\log x-\int\frac{1}{x}.\frac{x^2}{2}dx+c=\frac{x^2\log x}{2}-\frac{x^2}{4}+c.$$

Q.39 (1)

$$\int x\cos x\,dx=x\sin x-\int\sin x\,dx+c=x\sin x+\cos x+c$$

Q.40 (1)

$$I=\int\log x(\log x+2)dx$$

Put $\log x=t\Rightarrow e^t=x\Rightarrow e^t dt=dx$, then

$$I=\int t(t+2)e^t dt=t^2.e^t+c=x(\log x)^2+c.$$

Q.41 (4)

$$\int e^{2x}(-\sin x+2\cos x)dx$$

$$=-\int e^{2x}\sin x\,dx+2\int e^{2x}\cos x\,dx$$

$$=e^{2x}\cos x-2\int e^{2x}\cos x\,dx+2\int e^{2x}\cos x\,dx+c$$

$$=e^{2x}\cos x+c.$$

Aliter : $\int e^{2x}(2\cos x-\sin x)dx=e^{2x}\cos x+c$

$$\left\{\because \int e^{kx}\{kf(x)+f'(x)\}dx=e^{kx}f(x)+c\right\}$$

Q.42 (3)

Put $x^2=t\Rightarrow 2x\,dx=dt$, then

$$\int x^3e^{x^2}dx=\frac{1}{2}\int te^t dt$$

$$=\frac{1}{2}\left[te^t-e^t\right]+c=\frac{1}{2}e^{x^2}(x^2-1)+c.$$

Q.43 (1)

Putting $\tan^{-1}x=t$ and $\frac{dx}{1+x^2}=dt$, we get

$$\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx=\int e^t(\tan t+\sec^2 t)\,dt$$

$$=e^t\tan t+c=x\,e^{\tan^{-1}x}+c$$

$$\left[\text{Using}\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+C\right].$$

Q.44 (1)

$$\int\frac{dx}{(x-x^2)}=\int\left(\frac{1}{x}+\frac{1}{1-x}\right)dx=\log x-\log(1-x)+c$$

Q.45 (3)

Given integral $I=\int\left(1+\frac{1}{x^2-1}\right)dx$

$$=\int dx+\int\frac{dx}{(x-1)(x+1)}$$

$$=x+\frac{1}{2}\int\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx$$

$$=x+\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)+c$$

Q.46 (3)
Q.47 (2)

$$\int\frac{x-1}{(x-3)(x-2)}dx$$

$$=\int\frac{x-3}{(x-3)(x-2)}dx+\int\frac{2}{(x-3)(x-2)}dx$$

$$= \log\left[\frac{(x-2)(x-3)^2}{(x-2)^2}\right] + c = \log\left[\frac{(x-3)^2}{(x-2)}\right] + c.$$

Trick : By inspection, $\frac{d}{dx}\{\log(x-3) - \log(x-2)\}$

$$= \frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{(x-3)(x-2)}$$

$$\Rightarrow \frac{d}{dx}\{2\log(x-3) - \log(x-2)\}$$

$$= \frac{2}{x-3} - \frac{1}{x-2} = \frac{x-1}{(x-3)(x-2)}$$

Q.48 (1) $\int \frac{x}{(x-2)(x-1)}dx = -\int \frac{1}{x-1}dx + \int \frac{2}{x-2}dx$

$$= -\log_e(x-1) + 2\log_e(x-2) + c = \log_e \frac{(x-2)^2}{(x-1)} + c.$$

Q.49 (3)

$$\int \frac{dx}{(x^2+1)(x^2+4)} = \frac{1}{3}\left[\int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+4}\right]$$

$$= \frac{1}{3}\left[\tan^{-1}x - \frac{1}{2}\tan^{-1}\frac{x}{2}\right] + c = \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2} + c.$$

Q.50 (4)

$$\int \frac{1}{x-x^3}dx = \int \frac{1}{x(1+x)(1-x)}dx$$

$$= \frac{1}{2}\int \left(\frac{2}{x} - \frac{1}{1+x} + \frac{1}{1-x}\right)dx$$

$$= \frac{1}{2}[2\log x - \log(1+x) - \log(1-x)] = \frac{1}{2}\log\frac{x^2}{(1-x^2)} + c$$

Q.51 (2)

$$\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} = \int \frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}dx$$

$$= \frac{1}{(a-b)}\int (x+a)^{1/2}dx - \frac{1}{(a-b)}\int (x+b)^{1/2}dx$$

$$= \frac{2}{3(a-b)}[(x+a)^{3/2} - (x+b)^{3/2}] + c.$$

Q.52 (4)

$$\int \frac{1}{\cos x(1+\cos x)}dx = \int \frac{dx}{\cos x} - \int \frac{dx}{1+\cos x}$$

$$= \int \sec x\, dx - \frac{1}{2}\int \sec^2\frac{x}{2}dx$$

$$= \log(\sec x + \tan x) - \tan\frac{x}{2} + c.$$

Q.53 (1)

$$\int 4\cos\left(x+\frac{\pi}{6}\right)\cos 2x.\cos\left(\frac{5\pi}{6}+x\right)dx$$

$$= 2\int\left(\cos(2x+\pi)\cos\frac{2\pi}{3}\right)\cos 2x\, dx$$

$$= 2\int\left(-\cos 2x - \frac{1}{2}\right)\cos 2x\, dx$$

$$= \int\left(-2\cos^2 2x - \cos 2x\right)dx$$

$$= \int\left(1+\cos\ 4x + \cos 2x\right)dx$$

$$= -x - \frac{\sin 4x}{x} - \frac{\sin 2x}{2} + c$$

Q.54 (4)

Q.55 (1)

Q.56 (1)

$$\int \sin^3 x \cos^2 x\, dx = \int (1-\cos^2 x)\cos^2 x.\sin x\, dx$$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$, then it reduces to

$$-\int (t^2 - t^4)dt = \frac{t^5}{5} - \frac{t^3}{3} + c = \frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + c$$

Q.57 (2)

$$\int \sin 2x \cos 3x\, dx = \frac{1}{2}\int 2(\sin 2x \cos 3x)\, dx$$

$$= \frac{1}{2}\int (\sin 5x - \sin x)\, dx = \frac{1}{2}\left[-\frac{\cos 5x}{5} + \cos x\right] + c$$

$$= \frac{1}{2}\left[\cos x - \frac{\cos 5x}{5}\right] + c.$$

Q.58 (2)

$$\int \frac{dx}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a).\cos(x-b)}dx$$

$$= \frac{1}{\sin(a-b)}\int \left\{\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)}\right\}dx$$

$$= \operatorname{cosec}(a-b)\log\frac{\cos(x-a)}{\cos(x-b)}+c$$

Q.59 (1)

$$\int (\sin 2x+\cos 2x)\,dx = -\frac{\cos 2x}{2}+\frac{\sin 2x}{2}+k$$

$$= \frac{1}{\sqrt{2}}\left(\sin 2x\cos\frac{\pi}{4}-\cos 2x\sin\frac{\pi}{4}\right)+k$$

$$= \frac{1}{\sqrt{2}}\sin\left(2x-\frac{\pi}{4}\right)+k$$

$\Rightarrow c=\frac{\pi}{4}$ and $a=k,$ an arbitrary constant.

Q.60 (2)

$$\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$$

$$= \int \frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x}dx$$

$$= \int (\sin^4 x-\cos^4 x)\,dx$$

$$= \int (\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)\,dx$$

$$= \int (\sin^2 x-\cos^2 x)\,dx \int -\cos 2x\,dx = -\frac{\sin 2x}{2}+c.$$

Q.61 (4)
We know that

$$\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)=\log\left(\frac{1+\tan\theta}{1-\tan\theta}\right)=\log\tan\left(\frac{\pi}{4}+\theta\right)$$

$$\int \sec\theta\,d\theta=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

$$\therefore \int \sec 2\theta\,d\theta=\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)$$

$$\therefore 2\sec 2\theta=\frac{d}{d\theta}\log\tan\left(\frac{\pi}{4}+\theta\right) \quad \text{.....(i)}$$

Integrating the given expression by parts, we get

$I=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\int \sin 2\theta.2\sec 2\theta\,d\theta$ by (i)

$$=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\int \tan 2\theta\,d\theta$$

$$=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta$$

Q.62 (4)

$$\int \frac{x^2+1}{x^4+1}dx=\int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}\right)}dx=\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$$

Put $x-\frac{1}{x}=t\Rightarrow\left(1+\frac{1}{x^2}\right)dx=dt,$ then the required integral is $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$.

Q.63 (3)

Put $1+x^3=t^2\Rightarrow 3x^2dx=2t\,dt$ and $x^3=t^2-1$

So, $\int \frac{x^5}{\sqrt{1+x^3}}dx=\int \frac{x^2.x^3}{\sqrt{1+x^3}}dx$

$$=\frac{2}{3}\int \frac{(t^2-1).t\,dt}{t}=\frac{2}{3}\int (t^2-1)\,dt=\frac{2}{3}\left[\frac{t^3}{3}-t\right]+c$$

$$=\frac{2}{3}\left[\frac{(1+x^3)^{3/2}}{3}-(1+x^3)^{1/2}\right]+c$$

Q.64 (3)

$$\int_0^1 e^{2\log x}dx=\int_0^1 e^{\log x^2}dx=\int_0^1 x^2dx=\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3}$$

Q.65 (1)

$$\int\frac{dx}{\sqrt{2-x^2}}=\sin^{-1}\frac{x}{\sqrt{2}}+c$$

So $\int_0^1\frac{dx}{\sqrt{2-x^2}}=\sin^{-1}\frac{x}{\sqrt{2}}\Big|_0^1$

$$=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+c-\sin^{-1}(0)-c=\frac{\pi}{4}-0=\frac{\pi}{4}$$

Q.66 (3)

$$I_{10}=\int_1^e 1.(\ln x)^{10}dx=\left[(\ln x)^{10}x\right]_1^e$$

$$-\int_1^e 10(\ln x)^9.\frac{1}{x}.x\,dx$$

$$= e - 0 - 10\int_1^e (\ln x)^9 dx$$

Q.67 (2)

If $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}, I_2 = \int_1^2 \frac{dx}{x}$

$$I_1 = \ell n\left(\frac{2+\sqrt{5}}{1+\sqrt{2}}\right), I_2 = \ell n 2 \Rightarrow I_1 < I_2$$

Q.68 (1)
Q.69 (2)
Q.70 (2)
Q.71 (4)
Q.72 **(4)**
Q.73 (1)

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\ d\theta$

Also as $x = 0, \theta = 0$ and $x = 1, \theta = \frac{\pi}{4}$

Therefore, $\int_0^1 \tan^{-1} x\, dx = \int_0^{\pi/4} \theta \sec^2\theta\, d\theta$

$$= \frac{\pi}{4} - \log\sqrt{2} = \frac{\pi}{4} - \frac{1}{2}\log 2$$

Q.74 (1)

$$\int_{-\pi/4}^{\pi/4} e^{-x}\sin x\, dx = \left[\frac{e^{-x}}{2}(-\sin x - \cos x)\right]_{-\pi/4}^{\pi/2}$$

$$= \frac{1}{2}[e^{-x}(-\sin x - \cos x)]_{-\pi/4}^{\pi/2}$$

$$= \frac{1}{2}\left[e^{-\pi/2}(-1-0) - \left\{e^{\pi/4}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)\right\}\right] = -\frac{e^{-\pi/2}}{2}.$$

Q.75 (3)

$$\int_0^{\pi/2} \frac{(1+2\cos x)}{(2+\cos x)^2} dx = \int_0^{\pi/2} \frac{2(\cos x + 2) - 3}{(2+\cos x)^2} dx$$

$$= 2\int_0^{\pi/2} \frac{dx}{2+\cos x} - 3\int_0^{\pi/2} \frac{dx}{(2+\cos x)^2}$$

$$= 4\int_0^1 \frac{dt}{3+t^2} - 6\int_0^1 \frac{1+t^2}{(3+t^2)^2} dt,$$

$$\left[\text{Put } \tan\frac{x}{2} = t\right]$$

$$= -2\int_0^1 \frac{dt}{3+t^2} + 12\int_0^1 \frac{dt}{(3+t^2)^2}$$

$$= -2\int_0^1 \frac{dt}{3+t^2} + 12\left[\frac{1}{6}.\frac{t}{t^2+3}\right]_0^1 + \frac{1}{6}\int_0^1 \frac{dt}{3+t^2}$$

$$= 2\left[\frac{t}{t^2+3}\right]_0^1 = \frac{1}{2}$$

Q.76 (2)

$$\int_0^1 \frac{dx}{e^x + e^{-x}} = \int_0^1 \frac{e^x}{1+e^{2x}} dx$$

Now put $e^x = t \Rightarrow e^x dx = dt$

Also as $x = 0$ to 1, $t = 1$ to *e*, then reduced form is

$$\int_1^e \frac{dt}{1+t^2} = [\tan^{-1} t]_1^e = \tan^{-1}\left(\frac{e-1}{e+1}\right)$$

$$\left[\because \tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

Q.77 (1)

Let $I = \int_0^{\pi/4} \frac{\cos x}{\cos^2 x(1+2\sin^2 x)} dx$

$$= \int_0^{\pi/4} \frac{\cos x\, dx}{(1-\sin^2 x)(1+2\sin^2 x)}$$

$$= \frac{1}{3}\int_0^{1/\sqrt{2}} \left(\frac{1}{1-t^2} + \frac{2}{1+2t^2}\right) dt$$

By partial fractions, where $t = \sin x$

$$= \frac{1}{3}\left[\frac{1}{2.1}\log\frac{1+t}{1-t} + \frac{2}{\sqrt{2}}\tan^{-1} t\sqrt{2}\right]_0^{1/\sqrt{2}}$$

$$= \frac{1}{3}\left[\frac{1}{2}\log\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)} + \sqrt{2}\tan^{-1} 1\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}\log(\sqrt{2}+1)^2 + \sqrt{2}.\frac{\pi}{4}\right] = \frac{1}{3}\left[\log(\sqrt{2}+1) + \frac{\pi}{2\sqrt{2}}\right]$$

Q.78 (1)

We have $\int_3^8 \frac{2-3x}{x\sqrt{1+x}} dx = I$

Put $1 + x = t^2 \Rightarrow dx = 2t\, dt$

When $x = 3 \to 8$, then $t = 2 \to 3$

$\therefore\ I = 2\int_2^3 \frac{5-3t^2}{t^2-1}dt$; $I = 2\int_2^3\left(\frac{2}{t^2-1}-3\right)dt$

$I = 2\left[\frac{2}{2.1}\log\frac{t-1}{t+1}-3t\right]_2^3$; $I = 2\log\left(\frac{3}{2e^3}\right)$.

Q.79 (2)

$\int_0^\pi xf(\sin x)dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx$

Since $\int_0^a xf(x)dx = \frac{1}{2}a\int_0^a f(x)dx,$

if $f(a-x) = f(x)$.

Q.80 (3)

$I = \int_0^{\pi/2}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx$ .....(i)

$= \int_0^{\pi/2}\frac{\sqrt{\cot\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cot\left(\frac{\pi}{2}-x\right)}+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}dx$

$= \int_0^{\pi/2}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$ .....(ii)

Now adding (i) and (ii), we get

$2I = \int_0^{\pi/2}\frac{\sqrt{\cot x}+\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = [x]_0^{\pi/2} \Rightarrow I = \frac{\pi}{4}$.

Q.81 (2)

$f(x) = \int_a^x t^3e^tdt = \int_a^0 t^3.e^tdt + \int_0^x t^3e^t\ dt$

$\Rightarrow \frac{df(x)}{dx} = \frac{d}{dx}\left(\int_a^0 t^3.e^tdt\right) + \frac{d}{dx}\left(\int_0^x t^3.e^t\ dt\right) = x^3e^x$

Q.82 (2)

$I = \int_0^\pi x\log\sin x\,dx$ …..(i)

$= \int_0^\pi(\pi-x)\log\sin(\pi-x)\,dx$ …..(ii)

By adding (i) and (ii), we get

$2I = \int_0^\pi \pi\log\sin x\,dx \Rightarrow I = \frac{2\pi}{2}\int_0^{\pi/2}\log\sin x\,dx$

$= \pi\left(\frac{\pi}{2}\log\frac{1}{2}\right) = \frac{\pi^2}{2}\log\frac{1}{2}$

Q.83 (4)

$\int_0^{\pi/2}\log\tan x\,dx = \int_0^{\pi/2}\log\left(\frac{\sin x}{\cos x}\right)dx$

$= \int_0^{\pi/2}\log\sin x\,dx - \int_0^{\pi/2}\log\cos x\,dx = 0$,

$\left\{\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$.

Q.84 (3)

Let $I = \int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ .....(i)

and $I = \int_0^{\pi/2}\frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$

$I = \int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ …..(ii)

Adding (i) and (ii), we get

$2I = \int_0^{\pi/2}(1)dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$.

Q.85 (2)

$\int_0^{\pi/2}|\sin x - \cos x|\,dx$

$= \int_0^{\pi/4}-(\sin x - \cos x)dx + \int_{\pi/4}^{\pi/2}(\sin x - \cos x)dx = 2(\sqrt{2}-1)$

Q.86 (4)

$I = \int_0^\pi\frac{x\tan x}{\sec x+\tan x}dx = \int_0^\pi\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx$

$\Rightarrow 2I = \frac{\pi}{2}\int_0^\pi\frac{\tan x}{\sec x+\tan x}dx = \frac{\pi}{2}\int_0^\pi\frac{\sin x}{1+\sin x}dx$

$= \frac{\pi}{2}\left[\int_0^\pi 1dx - \int_0^\pi\frac{dx}{1+\sin x}\right]$

On solving, we get

$I = \frac{\pi^2}{2} - \pi = \pi\left(\frac{\pi}{2}-1\right)$

Q.87 (2)

Let $f(x)=\int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x.dx$

Since $\cos(2n+1)(\pi-x)=\cos[(2n+1)\pi-(2n+1)x]$

$=-\cos(2n+1)x$ and $\sin^2(\pi-x)=\sin^2 x$

Hence by the property of definite integral,

$$\int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x\,dx=0,$$

$[f(2a-x)=-f(x)]$

Q.88 (3)

$\int_{-3}^{3}\frac{x^2\sin x}{1+x^6}dx=0$. By the property of definite integral, $\int_{-a}^{a}f(x)dx=0$, when $f(x)=-f(-x)$.

Q.89 (4)

$I=\int_0^{\pi/2}\frac{dx}{1+\tan^3 x}=\int_0^{\pi/2}\frac{\cos^3 x}{\sin^3 x+\cos^3 x}dx$ .....(i)

$=\int_0^{\pi/2}\frac{\sin^3 x}{\cos^3 x+\sin^3 x}dx$ .....(ii)

Adding (i) and (ii), we get $2I=\int_0^{\pi/2}dx\Rightarrow I=\frac{\pi}{4}$.

Q.90 (1)

$I=\int_0^{\pi}x\sin x dx=\int_0^{\pi}(\pi-x)\sin x\,dx$

$\Rightarrow 2I=\pi\int_0^{\pi}\sin x dx=\pi[-\cos x]_0^{\pi}\Rightarrow I=\pi$.

Q.91 (3)

Let $I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$.

Then, $I=\int_0^{\pi/2}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$,

$\left[\because\int_0^{\pi/2}f(x)dx=\int_0^{\pi/2}f\left(\frac{\pi}{2}-x\right)dx\right]$

$\Rightarrow I=-\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx=-I$

$\Rightarrow 2I=0\Rightarrow I=0$

Q.92 (3) $f(\cos x)$ is an even function.

$\because f(\cos(-x))=f(\cos x)$

$\therefore\int_{-\pi/2}^{\pi/2}f(\cos x)dx=2\int_0^{\pi/2}f(\cos x)dx=2\int_0^{\pi/2}f(\sin x)dx$.

Q.93 (1)

$I=\int_0^{\pi/2}\frac{e^{x^2}\,dx}{e^{x^2}+e^{\left(\frac{\pi}{2}-x\right)^2}}$ and

$I=\int_0^{\pi/2}\frac{e^{\left(\frac{\pi}{2}-x\right)^2}dx}{e^{\left(\frac{\pi}{2}-x\right)^2}+e^{x^2}}$

$\left[\because\int_0^{a}f(x)dx=\int_0^{a}f(a-x)dx\right]$

$\Rightarrow 2I=\int_0^{\pi/2}1dx=(x)_0^{\pi/2}\Rightarrow I=\frac{\pi}{4}$

Q.94 (1)

$\int_0^{9}[\sqrt{x}+2]dx=\int_0^{1}2\,dx+\int_1^{4}3dx+\int_4^{9}4dx$

$=2+(12-3)+(36-16)=2+9+20=31$

Q.95 (3)

$I=\int_0^{\pi/2}\sin 2x\log\tan x\,dx$,

$I=\int_0^{\pi/2}\sin 2\left(\frac{\pi}{2}-x\right)\log\tan\left(\frac{\pi}{2}-x\right)dx$,

$[\because\int_0^{a}f(x)dx=\int_0^{a}f(a-x)dx]$

$=\int_0^{\pi/2}\sin 2x\log\cot x\,dx=-\int_0^{\pi/2}\sin 2x\log\tan x\,dx$

$\therefore I=-I\Rightarrow 2I=0\Rightarrow I=0$.

Q.96 (4)

$\int_{-2}^{2}|[x]|dx=\int_{-2}^{-1}|[x]|dx+\int_{-1}^{0}|[x]|dx+\int_0^{1}|[x]|dx+\int_1^{2}|[x]|dx$

$=\int_{-2}^{-1}2dx+\int_{-1}^{0}1dx+\int_0^{1}0dx+\int_1^{2}1dx$

$=2[x]_{-2}^{-1}+[x]_{-1}^{0}+0+[x]_1^{2}$

$=2(-1+2)+(0+1)+(2-1)=2+1+1=4$.

Q.97 (3)

Since $\log\left(\frac{1+x}{1-x}\right)$ is an odd function

$$\therefore \int_{-2}^{2}\left\{p\log\left(\frac{1+x}{1-x}\right)+q\log\left(\frac{1-x}{1+x}\right)^{-2}+r\right\}dx$$

$= r\int_{-2}^{2} dx = 4r.$ Hence depends on the value of r.

Q.98 (3)

$$\int_o^n [x]dx = \int_0^1 0dx + \int_1^2 1dx + \int_2^3 2dx + ... +$$

$$\int_{n-1}^{n}(n-1)dx$$

$$=1+2+3+.....+(n-1) = \frac{(n-1)n}{2} = 66$$

$$\Rightarrow n(n-1) = 132 \Rightarrow n = 12$$

Q.99 (1)

We know that $|\sin x|$ is a periodic function of π

Hence

$$\int_0^{4\pi}|\sin x|\,dx = 4\int_0^{\pi}|\sin x|\,dx = 4\int_0^{\pi}\sin x dx$$

Q.100 $I = \int_0^{\pi/2}\frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx$

$$I = \int_0^{\pi/2}\frac{2^{\sin(\pi/2-x)}}{2^{\sin(\pi/2-x)}+2^{\cos(\pi/2-x)}}dx$$

=

$$\int\frac{2^{\cos x}}{2^{\cos x}+2^{\sin x}}dx \Rightarrow 2I = \int_0^{\pi/2}dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

Q.101 (2)

Let $I = \int_0^{10}\frac{x^{10}}{(10-x)^{10}+x^{10}}dx$

$$I = \int_0^{10}\frac{(10-x)^{10}}{(10-x)^{10}+x^{10}}dx$$

Adding (1) and (2), we get

$$2I = \int_0^{10}dx \Rightarrow 2I = 10 \Rightarrow I = 5$$

Q.102 (2) **Q.103** (1) **Q.104** (2) **Q.105 (4)** **Q.106 (2)**
Q.107 (3) **Q.108 (1)** **Q.109 (1)** **Q.110 (4)** **Q.111 (4)**
Q.112 (3) **Q.113 (2)** **Q.114 (2)** **Q.115 (2)** **Q.116 (3)**

Q.117 (2)

We have, $\lim_{n\to\infty}\left[\frac{n}{1+n^2}+\frac{n}{4+n^2}+......+\frac{1}{2n}\right]$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{r^2+n^2} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n}{n^2\left(1+\frac{r^2}{n^2}\right)}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)} = \int_0^1\frac{dx}{1+x^2},$$

$$\left\{\text{Applying formula, } \lim_{n\to\infty}\sum_{r=0}^{n-1}\left\{f\left(\frac{r}{n}\right)\right\}.\frac{1}{n} = \int_0^1 f(x)dx\right\}$$

$$= [\tan^{-1}x]_0^1 = \tan^{-1}1 - \tan^{-1}0 = \frac{\pi}{4}.$$

Q.118 (2)

$$\lim_{n\to\infty}\frac{1^{99}+2^{99}+.....+n^{99}}{n^{100}} = \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r^{99}}{n^{100}}\right)$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{99} = \int_0^1 x^{99}dx = \left[\frac{x^{100}}{100}\right]_0^1 = \frac{1}{100}.$$

Q.119 (2)

$$L = \lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{n}.\frac{r/n}{\sqrt{1+(r/n)^2}} = \int_0^2\frac{x}{\sqrt{1+x^2}}dx = \sqrt{5}-1$$

Q.120 (4)

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{2n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{n+n}\right]$$

$$= \frac{1}{n}\lim_{n\to\infty}\left[1+\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+....+\frac{1}{1+\frac{n}{n}}\right]$$

$$= \frac{1}{n}\lim_{n\to\infty}\sum_{r=0}^{n}\left[\frac{1}{1+\frac{r}{n}}\right] = \int_0^1\frac{1}{1+x}dx$$

$$= [\log_e(1+x)]_0^1 = \log_e 2 - \log_e 1 = \log_e 2.$$

Q.121 (1)

Let $I = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2} = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\frac{\left(\frac{k}{n}\right)}{1+\left(\frac{k}{n}\right)^2}$

$$I = \int_0^1 \frac{x}{1+x^2}dx = \frac{1}{2}[\log(1+x^2)]_0^1 = \frac{1}{2}[\log 2]$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (1)

$$\int \frac{dx}{\sin x.\sin(x+\alpha)}$$

$$= \frac{1}{\sin\alpha}\int \frac{\sin(\alpha+x-x)}{\sin x\,\sin(x+\alpha)}dx$$

$$= \text{cosec}\,\alpha \int \frac{\sin(x+\alpha)\cos x - \cos(x+\alpha)\sin x}{\sin x \sin(x+\alpha)}$$

$$= \text{cosec}\,\alpha \left[\int \cot x\,dx - \int \cot(x+\alpha)\right] + C$$

$= \text{cosec}\,\alpha\ [\log|\sin x| - \log|\sin(x+\alpha)|] + C$

$$= \text{cosec}\,\alpha \log\left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$$

Q.2 (1)

Q.3 (2)

$$I = -\frac{1}{2}\cos 2x - \frac{\sin 2x}{2} + b = -\frac{1}{\sqrt{2}}\sin\left(2x+\frac{\pi}{4}\right)+b$$

$$= \frac{1}{\sqrt{2}}\sin\left(2x+\frac{5\pi}{4}\right)+b \qquad \therefore\ a = -\frac{5\pi}{4},$$

$b \in R$

Q.4 (1)

$I = \int \frac{\cos 2x}{\cos x}dx = \int \frac{2\cos^2 x - 1}{\cos x}dx = 2\sin x - \int \sec x\,dx = 2\sin x - \ell n|\sec x + \tan x| + C$

Q.5 (3)

$$\int (f(x)\,g''(x) - f''(x)\,g(x))\,dx$$

$$= f(x)\int g''(x).dx - \int f'(x)\,g'(x)\,dx - g(x)\int f''(x)\,dx + \int g'(x)\,f'(x)\,dx$$

$= f(x)\,g'(x) - f'(x)\,g(x) + c$

Q.6 (2)

$$\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x}.\ (4)x$$

$$\int \frac{(\sin^4+\cos^4 x)(\sin^2 x+\cos^2 x)(\sin^2 x - \cos^2 x)}{(\sin^4 x+\cos^4 x)}dx$$

$$= -\int \cos 2x\,dx = -\frac{\sin 2x}{2} + c$$

Q.7 (2)

$$\int \frac{a^{\sqrt{x}}}{\sqrt{x}}dx$$

put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$ Þ $\frac{dx}{\sqrt{x}} = 2dt = 2\int a^t\,dt$

$$= \frac{2a^t}{\ell n\,a} + c = 2\frac{a^{\sqrt{x}}}{\ell n\,a} + c$$

Q.8 (1)

$$\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx$$

$$\int \frac{\sqrt{\tan x}\,\sec^2 x}{\tan x}dx$$

$\tan x = t^2 \Rightarrow \sec^2 x\,dx = 2t\,dt$

$$\int \frac{t.2t\,dt}{t^2} = 2t + c = 2\sqrt{\tan x} + c$$

Q.9 (1)

$$I = \int\sqrt{\frac{x}{4-x^3}}dx = \int\frac{\sqrt{x}dx}{\sqrt{4-x^3}}$$

Here integral of $\sqrt{x} = \frac{2}{3}x^{3/2}$ and

$4 - x^3 = 4 - (x^{3/2})^2$

Put $x^{3/2} = t \Rightarrow \sqrt{x}d = \frac{2}{3}dt$

$$\text{So } I = \frac{2}{3}\int\frac{dt}{\sqrt{4-t^2}} = \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{2}\right)+c$$

Q.10 (2)

$$I = \int \frac{x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)^{1/2}}$$

$$\text{Let } x+\frac{1}{x} = p \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dp$$

$$I=\int\frac{dp}{p\sqrt{p^2-2}}=\frac{1}{\sqrt{2}}\sec^{-1}\frac{p}{\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+c$$

Q.11 (4)

Put $x^2=t\Rightarrow 2x\,dx=dt$

$$I=\int\frac{e^{x^2}\left(2+x^2\right)xdx}{\left(3+x^2\right)^2}=\frac{1}{2}\int e^t\,\frac{(2+t)}{(3+t)^2}dt$$

$$=\frac{1}{2}\int\frac{e^t(3+t-1)xdx}{(3+t)^2}=\frac{1}{2}\int e^t\left[\frac{1}{3+t}-\frac{1}{(3+t)^2}\right]dt$$

$$=\frac{1}{2}e^t.\frac{1}{3=t}+k\left[\because\frac{d}{dt}\left(\frac{1}{3+t}\right)=\frac{-1}{(3+t)^2}\right]$$

$$=\frac{1}{2}\frac{e^{x^2}}{3+x^2}+k$$

Q.12 (2)
Q.13 **(2)**
Q.14 **(3)**
Q.15 **(3)**
Q.16 **(2)**
Q.17 (4)

$$\int\frac{2^x}{\sqrt{1-4^x}}dx$$

$2^x=t\Rightarrow 2^x\ln 2\,dx=dt\Rightarrow 2^x\,dx=\frac{dt}{\ell n\,2}$

$$\frac{1}{\ell n\,2}\int\frac{dt}{\sqrt{1-t^2}}=\frac{1}{\ell n\,2}\sin^{-1}(2^x)+c$$

Q.18 (3)

$$\int\tan^3 2x\sec 2x\,dx$$

$$\int\tan 2x\,(\sec^2 2x-1)\sec 2x\,dx$$

$$=\int\frac{\sin 2x}{\cos^4 2x}dx-\int\frac{\sin 2x}{\cos^2 2x}dx$$

put $\cos 2x=t$

$\sin 2x\,dx=-\frac{dt}{2}$

$$=-\frac{1}{2}\int\frac{dt}{t^4}+\frac{1}{2}\int\frac{dt}{t^2}$$

$$=-\frac{1}{2}\left[\frac{t^{-3}}{-3}\right]-\frac{1}{2}\,\frac{1}{t}+c$$

$$=\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+c$$

Q.19 (1)

$$\int\sqrt{\frac{e^x-1}{e^x+1}}dx=\int\frac{e^x-1}{\sqrt{e^{2x}-1}}dx$$

$$=\int\frac{e^x}{\sqrt{e^{2x}-1}}dx-\int\frac{dx}{\sqrt{e^{2x}-1}}$$

$$=\int\frac{dt}{\sqrt{t^2-1}}-\int\frac{e^x}{e^x\sqrt{e^{2x}-1}}dx$$

$$=\int\frac{dt}{\sqrt{t^2-1}}-\int\frac{du}{u\sqrt{u^2-1}}$$

$$=\ell n\left(e^x+\sqrt{e^{2x}-1}\right)-\sec^{-1}(e^x)+c$$

Q.20 (3)

$$I=\int\sqrt{\frac{1-\cos x}{\cos x}}dx=\int\sqrt{\frac{2\sin^2\frac{x}{2}}{\cos x}}dx=\int\frac{\sqrt{2}\sin\frac{x}{2}}{\sqrt{2\cos^2\frac{x}{2}-1}}dx$$

Let $\cos\frac{x}{2}=t\Rightarrow-\frac{1}{2}\sin\frac{x}{2}dx=dt$

$$I=-2\sqrt{2}\int\frac{dt}{\sqrt{2t^2-1}}=\frac{-2\sqrt{2}}{\sqrt{2}}\int\frac{dt}{\sqrt{t^2-\frac{1}{2}}}=-$$

$$2\int\frac{dt}{\sqrt{t-\left(\frac{1}{\sqrt{2}}\right)^2}}$$

$$\Rightarrow I=-2\log\left|t+\sqrt{t^2-\frac{1}{2}}\right|+c$$

$$=-2\log\left|\cos\frac{x}{2}+\sqrt{\cos^2\frac{x}{2}-\frac{1}{2}}\right|+c$$

Q.21 (1)

$$\int\frac{\sin^2 x}{1+\sin^2 x}dx=\int\frac{\sin^2 x+1-1}{1+\sin^2 x}dx$$

$= \int dx - \int \frac{dx}{1+\sin^2 x}$

$= \int dx - \int \frac{\sec^2 x}{\sec^2 x + \tan^2 x} dx$

$= \int dx - \int \frac{\sec^2 x}{1+2\tan^2 x} dx$

$= \int dx - \int \frac{dt}{1+2t^2} = x - \left(\frac{1}{\sqrt{2}}\right) \tan^{-1} \sqrt{2}\, t + c$

$= x - \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2} \tan x) + c$

Q.22 (1)

$\int \left\{\frac{(\log x - 1)}{1+(\log x)^2}\right\}^2 dx$

$\ell n\, x = t \Rightarrow x = e^t \Rightarrow dx = e^t\, dt$

$= \int e^t \left(\frac{t-1}{t^2+1}\right)^2 dt = \int e^t \left(\frac{t^2+1-2t}{(t^2+1)^2}\right) dt$

$= \int e^t \left\{\underset{\substack{\uparrow \\ f(t)}}{\frac{1}{(t^2+1)}} - \underset{\substack{\uparrow \\ f'(t)}}{\frac{2t}{(1+t^2)^2}}\right\} dt$

$= \frac{e^t}{1+t^2} = \frac{x}{1+\log^2 x} + c$

Q.23 (1)

$\int \frac{\ell n|x|}{x\sqrt{1+\ell n\, x}} dx$

$1 + \ell n\, x = t^2 \Rightarrow \frac{1}{x} dx = 2t\, dt$

$\int \frac{(t^2-1)\, 2t\, dt}{t} = 2\int (t^2-1)\, dt$

$= 2\left[\frac{t^3}{3} - t\right] + c = \frac{2}{3} t\, [t^2 - 3] + c$

$= \frac{2}{3}\sqrt{1+\ell n\, x}\, [1 + \ell n\, x - 3] + c$

$= \frac{2}{3}\sqrt{1+\ell n\, x}\, [\ell n\, x - 2] + c$

Q.24 (3)

$\int \{1 + 2\tan x\, (\tan x + \sec x)\}^{1/2} dx$

$\int \{1 + 2\tan^2 x + 2\tan x \sec x\}^{1/2} dx$

$\int \{\sec^2 x - \tan^2 x + 2\tan^2 x + 2\tan x \sec x\}^{1/2} dx$

$= \int (\sec x + \tan x)\, dx$

$= \ell n \sec x + \ell n (\sec x + \tan x) + c$

$= \ell n \sec x\, (\sec x + \tan x) + c$

Q.25 (3)

$\int (x-1)\, e^{-x} dx$

$= \int x\, e^{-x} dx - \int e^{-x} dx$

$= -xe^{-x} + \int e^{-x} dx - \int e^{-x} dx$

$= -xe^{-x} + c$

Q.26 Let, $I = \int \frac{\cos x - 1}{\sin x - 1} e^x dx$

$= \int \left[\frac{\cos x}{\sin x - 1} - \frac{1}{\sin x + 1}\right] e^x dx$

$= \int \frac{\cos x}{1+\sin x} e^x dx - \int \frac{1}{\sin x + 1} e^x dx$

$= \frac{e^x \cdot \cos x}{1+\sin x} - \int \frac{-(1+\sin x)\sin x - \cos^2 x}{(1+\sin x)^2} e^x dx$

$- \int \frac{e^x}{\sin x + 1} dx$

$= \frac{e^x \cos x}{1+\sin x} + \int \frac{e^x}{1+\sin x} dx - \int \frac{e^x dx}{1+\sin x}$

$= \frac{e^x \cos x}{1+\sin x} + C$

$\left[\text{Using } \int e^x \{f(x) + f(x)\} dx = e^x f(x) + C\right]$

Q.27 (2)
Q.28 **(1)**
Q.29 **(1)**
Q.30 **(1)**
Q.31 (3)

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} (x + \sqrt{x})\, dx$

$\sqrt{x} = t \Rightarrow \frac{1}{\sqrt{x}} dx = 2\, dt$

$2\int e^t (t^2 + t)\, dt$

$= 2\int e^t (t^2 + 2t) - 2\int e^t\, t\, dt$

$= 2\, e^t (t^2) - 2[t\, e^t - e^t] + c$

$= 2e^{\sqrt{x}}.x - 2[\sqrt{x}\, e^{\sqrt{x}} - e^{\sqrt{x}}] + c$

$= 2e^{\sqrt{x}}[x - \sqrt{x} + 1] + c$

Q.32 (4)

$\int e^{\tan\theta}(\sec\theta - \sin\theta)\, d\theta$

$\tan\theta = t \Rightarrow d\theta = \frac{dt}{1+t^2}$

$I = \int \frac{e^t}{1+t^2}\left(\sqrt{1+t^2} - \frac{t}{\sqrt{1+t^2}}\right) dt$

$= \int e^t \left(\frac{1}{\sqrt{1+t^2}} - \frac{t}{\sqrt{(t^2+1)^3}}\right) dt$

$= e^t \frac{1}{\sqrt{(t^2+1)}} + c = e^{\tan\theta} \frac{1}{\sec\theta} + c = e^{\tan\theta}\cos\theta + c$

Q.33 (2)

$y = \int \frac{dx}{x^2+x+1} = \int \frac{dx}{x^2 + x + \frac{1}{4} + 1 - \frac{1}{4}} =$

$\int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

Q.34 **(3)**
Q,35 **(4)**
Q.36 (3)

$I = \int e^{3x}\cos 4x\, dx = e^{3x}(A\sin 4x + B\cos 4x) + C$

....(i)

$I = \frac{1}{4}e^{3x}\sin 4x - \int \frac{3}{4}e^{3x}\sin 4x\, dx$

$= \frac{1}{4}e^{3x}\sin 4x + \frac{3}{16}e^{3x}\cos 4x -$

$\int \frac{9}{16}e^{3x}\cos 4x\, dx$

$\frac{25}{16} I = \frac{1}{16}\left(4e^{3x}\sin 4x + 3e^{3x}\cos 4x\right)$

comparing with equation (i)

$\Rightarrow A = \frac{4}{25}, B = \frac{3}{25}$

$\Rightarrow \frac{A}{B} = \frac{4}{3} \Rightarrow 3A = 4B$

Q.37 (3)

$\int \frac{dx}{x^3(1+x)} = \frac{A}{x^2} + \frac{B}{x} + \ell n\left(\frac{x}{x+1}\right) + c$

$\frac{1}{x^3(1+x)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \frac{d}{(x+1)}$

$1 = ax^2(x+1) + bx(x+1) + c(x+1) + d\,x^3$

put $x = 0 \Rightarrow c = 1$

put $x = -1 \Rightarrow d = -1$

put $x = 1$

$1 = 2a + 2b + 2c + d$

$1 = 2a + 2b + 2 - 1 \Rightarrow a + b = 0$

put $x = 2$

$1 = 12a + 6b + 3c + 8d$

$1 = 12a + 6b + 3 - 8$

$12a + 6b = 6 \Rightarrow 2a + b = 1 \Rightarrow a = 1$

$\int \frac{dx}{x^3(1+x)} = \int \left(\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} - \frac{1}{x+1}\right) dx$

$= \ell n\, x + \frac{1}{x} - \frac{1}{2x^2} - \ell n\,(x+1) + c$

$= -\frac{1}{2x^2} + \frac{1}{x} + \ell n\left(\frac{x}{x+1}\right) + c$

$A = -1/2, B = 1$

Aliter : $\int \frac{1+x^3-x^3}{x^3(1+x)} = \int \frac{1+x^3}{x^3(1+x)} - \int \frac{dx}{1+x}$

Q.38 (2)

$\int \frac{x^3-1}{x(x^2+1)} dx = \int \frac{x^2}{x^2+1} dx - \int \frac{1}{x(x^2+1)} dx$

$= \int dx - \int \frac{dx}{x^2+1} - \int \frac{1}{x^3(1+x^{-2})} dx$

Let $1 + x^{-2} = t \Rightarrow \frac{dx}{x^3} = -\frac{dt}{2}$

$= x - \tan^{-1} x + \frac{1}{2}\int \frac{dt}{t}$

$= x - \tan^{-1} x + \frac{1}{2}\,\ell n\,|1 + x^{-2}| + c$

$= x - \tan^{-1} x + \frac{1}{2}\,\ell n\,(x^2 + 1) - \ell n\, x + c$

Q.39 (4)

$y = \int \frac{dx}{x^3\left(1+\frac{1}{x^2}\right)^{3/2}}$ put $1 + \frac{1}{x^2} = t^2 \Rightarrow -\frac{2}{x^3}\,dx$

$= 2t\,dt$

$\Rightarrow y = \int \frac{-t\,dt}{t^3} = -\int \frac{dt}{t^2} = \frac{1}{t} + C = \frac{x}{\sqrt{1+x^2}} + C$

$\because y(0) = 0 \Rightarrow C = 0$

$\therefore y(1) = \frac{1}{\sqrt{2}}$

Q.40 (2)

$I = \int \frac{\cos 2x\,dx}{(\sin x + \cos x)^2} = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2}dx$

$= \int \frac{\cos x - \sin x}{\sin x + \cos x}dx$ Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\,dx = dt$

$\Rightarrow I = \int \frac{dt}{t} = \ell n\,|t| + C = \ell n|\sin x + \cos x| + C$

Q.41 (2)

$\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$

$\int \frac{dx}{\cos^3 x\sqrt{2\sin x \cos x}}$

$= \frac{1}{\sqrt{2}}\int \frac{dx}{\cos^4 x\sqrt{\tan x}} = \frac{1}{\sqrt{2}}\int \frac{\sec^4 x}{\sqrt{\tan x}}dx$

$= \frac{1}{\sqrt{2}}\int \frac{(1+\tan^2 x)}{\sqrt{\tan x}}\sec^2 x\,dx$

put $\tan x = t^2 \Rightarrow \sec^2 x\,dx = 2t\,dt$

$= \frac{1}{\sqrt{2}}\int \left(\frac{1+t^4}{t}\right)2t\,dt = \sqrt{2}\int (1+t^4)\,dt$

$= \sqrt{2}\left[t + \frac{t^5}{5}\right] + c$

$= \sqrt{2}\left[\tan^{1/2} x + \frac{\tan^{5/2} x}{5}\right] + c$

Q.42 (2)

$\int \frac{\cos 4x + 1}{\cot x - \tan x} = A\cos 4x + B$

$I = \int \frac{(\cos 4x + 1)}{(\cos^2 x - \sin^2 x)}\cos x\,\sin x\,dx$

$= \int \left(\frac{2\cos^2 2x}{\cos 2x}\right)(\cos x \sin x)\,dx$

$= \int \cos 2x\,\sin 2x\,dx$

$= \frac{1}{2}\int \sin 4x\,dx = -\frac{\cos 4x}{8} + B$

Q.43 (3)

$$I = \int \frac{1}{1+3\sin^2 x + 8\cos^2 x}dx$$

Dividing the numerator and denominator by $\cos^2 x,$ we get

$$I = \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x + 8}dx = \int \frac{\sec^2 x}{4\tan^2 x + 9}dx$$

Putting tan $x = t \Rightarrow \sec^2 x dx = dt,$ we get

$$I = \int \frac{dt}{4t^2+9} = \frac{1}{4}\int \frac{dt}{t^2 + (3/2)^2}$$

$$= \frac{1}{4}\times\frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right) + C$$

$$\Rightarrow I = \frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right) + C = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$$

Q.44 **(2)**

Put $x = \cos 2\theta$

$\therefore I = \int \cos\{2\tan^{-1}\tan\theta\} - (-2\sin 2\theta)\,d\theta$

$\int \sin 4\theta\,d\theta = \frac{1}{4}\cos 4\theta + c$

$= \frac{1}{4}(2x^2 - 1) + c = \frac{1}{2}x^2 + k$

Q.45 (1)
Q.46 (4)
Q.47 **(3)**
Q.48 **(4)**
Q.49 **(2)**
Q.50 **(1)**

Q.51 (2)

$$\int 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}\, dx$$

$$2\int \left(2 \sin x \cos \frac{x}{2}\right) \cos \frac{3x}{2}\, dx$$

$$2\int \left(\sin\frac{3x}{2} + \sin\frac{x}{2}\right)\cos \frac{3x}{2}\, dx$$

$$= \int 2 \sin \frac{3x}{2} \cos \frac{3x}{2}\, dx + \int 2 \sin \frac{x}{2} \cos \frac{3x}{2}\, dx$$

$$= \int \sin 3x\, dx + \int ((\sin 2x) - \sin x)\, dx$$

$$= -\frac{\cos 3x}{3} - \frac{\cos 2x}{2} + \cos x + c$$

Q.52 (2)

$$\int \sin x.\cos x.\cos 2x \cos 4x.\cos 8x.\cos 16x\, dx$$

$$= \frac{1}{2}\int (\sin 2x.\cos 2x) \cos 4x.\cos 8x.\cos 16x\, dx$$

$$= \frac{1}{4}\int (\sin 4x.\cos 4x) \cos 8x.\cos 16x\, dx$$

$$= \frac{1}{8}\int (\sin 8x.\cos 8x) \cos 16x\, dx$$

$$= \frac{1}{16}\int \sin 16x \cos 16x\, dx$$

$$= \frac{1}{32}\int \sin 32x\, dx = -\frac{1}{32} \times \frac{\cos 32x}{32}$$

$$= -\frac{1}{1024}\cos 32x + c$$

Q.53 (1)

$$\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c$$

$$= \int \frac{\sec^4 x\, dx}{\sqrt{\tan^3 x}}$$

$\tan x = t^2 \Rightarrow \sec^2 x\, dx = 2t\, dt$

$$\int \frac{(1+\tan^2 x)}{\tan^{3/2} x}\sec^2 x\, dx$$

$$= \int \left(\frac{1+t^4}{t^3}\right) 2t\, dt = 2\int \left(\frac{1}{t^2} + t^2\right) dt$$

$$= -\frac{2}{t} + \frac{2}{3}t^3 + c = -2\sqrt{\cot x} + \frac{2}{3}\sqrt{\tan^3 x} + c$$

Q.54 (1)

$$\int \frac{dx}{\cos x - \sin x} = \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{1-\tan^2\frac{x}{2} - 2\tan\frac{x}{2}}$$

$$= \int \frac{\sec^2\frac{x}{2}\, dx}{1-\tan^2\frac{x}{2} - 2\tan\frac{x}{2}}$$

Put $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\, dx = 2dt$

$$= 2\int \frac{dt}{1-t^2-2t} = 2\int \frac{-dt}{(t^2+2t-1)}$$

$$= 2\int \frac{-dt}{(t^2+2t+1-2)} = 2\int \frac{dt}{2-(t+1)^2}$$

$$= \frac{2}{2\sqrt{2}}\,\ell n\, \frac{\sqrt{2}+t+1}{\sqrt{2}-(t+1)} + c$$

$$= \frac{1}{\sqrt{2}}\,\ell n\, \frac{\sqrt{2}+\tan\frac{x}{2}+1}{\sqrt{2}-\left(\tan\frac{x}{2}+1\right)} + c$$

Q.55 (1)

$$\int \frac{dx}{\sqrt{\sin^3 x \cos x}}$$

$$= \int \frac{dx}{\sqrt{\tan^3 x.\cos^4 x}}$$

$$= \int \frac{\sec^2 x\, dx}{\sqrt{\tan^3 x}}$$ [$\tan x = t \Rightarrow \sec^2 x\, dx = dt$]

$$= \int \frac{dt}{t^{3/2}} = \frac{t^{-3/2+1}}{1-3/2} + c = \frac{-2}{\sqrt{\tan x}} + c$$

Q.56 (1)

Sol. $\displaystyle\int \frac{1}{x(x^n+1)}dx = \int \frac{1}{x^{1+n}(1+x^{-n})}dx$

$1 + x^{-n} = t \Rightarrow -nx^{-n-1}\, dx = dt \Rightarrow \dfrac{dx}{x^{n+1}} = \dfrac{-1}{n}dt$

$$= -\frac{1}{n}\int \frac{dt}{t} = -\frac{1}{n}\,\ell n\,(1+x^{-n}) + c = -$$

$\frac{1}{n}\ell n\left(\frac{x^n+1}{x^n}\right)+c = \frac{1}{n}\ell n\left(\frac{x^n}{1+x^n}\right)+c$

Q.57 (4)

$I = \int \frac{1}{x^5(1+\frac{1}{x^4})^{3/4}} dx$ put $1+\frac{1}{x^4}=t$

$\Rightarrow -\frac{4}{x^5}dx = dt$

$\Rightarrow I = -\frac{1}{4}\int\frac{dt}{t^{3/4}} = -\frac{1}{4}\cdot\frac{t^{1/4}}{1/4}+C = -\left(1+\frac{1}{x^4}\right)^{1/4} + C$

Q.58 (3)

$\int \frac{1-x^7}{x(1+x^7)}dx = \int \frac{1}{x(1+x^7)}dx - \int \frac{x^6}{(1+x^7)}dx$

$= \int \frac{1}{x^8(x^{-7}+1)}dx - \int \frac{x^6}{(1+x^7)}dx$

$1 + x^{-7} = t$ $1 + x^7 = u$

$\frac{-7}{x^8}dx = dt$ $x^6\,dx = \frac{du}{7}$

$\frac{dx}{x^8} = \frac{-1}{7}dt$

$= -\frac{1}{7}\int\frac{dt}{t} - \frac{1}{7}\int\frac{du}{u} = -\frac{1}{7}\ell n\, t - \frac{1}{7}\ell n\, u$

$= -\frac{1}{7}\ell n\,(1+x^{-7}) - \frac{1}{7}\ell n\,(1+x^7) + c = -\frac{1}{7}\ell n\left(\frac{x^7+1}{x^7}\right) - \frac{1}{7}\ell n\,(1+x^7)+c$

$= -\frac{2}{7}\ell n\,(1+x^7) + \ell n\, x + c$

Q.59 (2)

$\int \frac{3x^4-1}{(x^4+x+1)^2}dx = \int \frac{3x^4-1}{x^2\left(x^3+1+\frac{1}{x}\right)^2}dx$

$= \int \frac{3x^2-1/x^2}{\left(x^3+1+\frac{1}{x}\right)^2}dx$

$x^3+1+\frac{1}{x} = t \Rightarrow \left(3x^2-\frac{1}{x^2}\right)dx = dt$

$= \int\frac{dt}{t^2} = -\frac{1}{t}+c$

$= -\frac{1}{\left(x^3+1+1/x\right)}+c = -\frac{x}{x^4+x+1}+c$

Q.60 (1)

$\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \pi \Rightarrow \left[\sec^{-1}\right]_1^x = \frac{\pi}{6} = \sec^{-1}x - \sec^{-1}1$

$= \frac{\pi}{6} = \sec^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \sec^{-1}\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

Q.61 (3)

$\int_0^2 x^2 f(x)dx = \int_0^1 x^3\,dx + \int_1^2 (x^3-x^2)dx =$

$\left[\frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4}-\frac{x^3}{3}\right]_1^2$

$= \frac{1}{4}+\left[4-\frac{8}{3}-\left(\frac{1}{4}-\frac{1}{3}\right)\right] = 4-\frac{8}{3}+\frac{1}{3} = 4-\frac{1}{3} = \frac{5}{3}$

Q.62 (3)

$\int_n^{n+1} f(x)dx = n^2$

$\int_{-2}^{4} f(x)dx = \int_{-2}^{-1} f(x)dx + \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{4} f(x)dx$

$= 4 + 1 + 0 + 1 + 4 + 9 = 19$

Q.63 (2)

Let $I = \int_0^{\pi/2}\frac{\sin x}{1+\cos^2 x}dx$

Let $\cos x = t$ and $-\sin x dx = dt$

Now, $x = 0 \Rightarrow t = \cos 0 = 1$ and

$x = \frac{\pi}{2} \Rightarrow t = \cos\frac{\pi}{2} = 0$

$\therefore\ I = \int_1^0 \frac{\sin x}{1+t^2}\left(\frac{-dt}{\sin x}\right) = -\int_1^0\frac{dt}{1+t^2}$

$= \left[\tan^{-1}t\right]_1^0 = -\left[0-\frac{\pi}{4}\right] = \frac{\pi}{4}$

Q.64 (1)
Q.65 (1)
Q.66 **(2)**
Q.67 **(1)**
Q.68 **(4)**
Q.69 (1)

$$I = \int_{-1}^{3} (|x-2|+[x])dx$$

$$= \int_{-1}^{2} |x-2|+dx + \int_{2}^{3} |x-2|+dx + \int_{-1}^{0} (-1)dx +$$

$$\int_{0}^{1} 0.dx + \int_{1}^{2} 1.dx + \int_{2}^{3} 2.dx$$

$$= \int_{-1}^{2} (2-x)\,dx + \int_{2}^{3} (x-2)dx \; -1 + 0 + 1 + 2$$

$$= 2x - \frac{x^2}{2}\Big|_{-1}^{2} + \frac{x^2-2x}{2}\Big|_{2}^{3} + 2 = 7$$

Q.70 (3)

$$I = \int_{0}^{1} \underset{I}{x} \underset{II}{f''(2x)} dx = \left[\frac{x f'(2x)}{2} - \frac{f(2x)}{4}\right]_0^1$$

$$= \frac{f'(2)}{2} - \frac{f(2)}{4} + \frac{f(0)}{4} = \frac{5}{2} - \frac{3}{4} + \frac{1}{4} = 2$$

Q.71 (1)

$$\int_{-1}^{3/2} |x\sin\pi x|\,dx$$

$$= \int_{-1}^{1} |x\sin\pi x|\,dx + \int_{1}^{3/2} |x\sin\pi x|\,dx$$

$$= 2\int_{0}^{1} |x\sin\pi x|\,dx + \int_{1}^{3/2} |x\sin\pi x|\,dx$$

$$= 2\int_{0}^{1} x\sin\pi x\,dx - \int_{1}^{3/2} x\sin\pi x\,dx$$

$$\int x\cos\pi x dx = -\frac{x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}$$

$$\int_{0}^{1} x\sin\pi x dx = \frac{1}{\pi}$$

$$\int_{1}^{3/2} x\sin\pi x\,dx = \frac{-1}{\pi^2} - \frac{1}{\pi}$$

$$I = \frac{2}{\pi} + \frac{2}{\pi^2} + \frac{1}{\pi} = \frac{3\pi+1}{\pi^2}$$

$k = 3\pi + 1$

Q.72 (1)

$$I = \int_{\log\pi - \log 2}^{\log\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)} dx$$

Put $\frac{e^x}{3} = t \Rightarrow e^x\,dx = 3dt$

$$= 3\int_{\pi/6}^{\pi/3} \frac{dt}{1-\cos 2t} = \frac{3}{2}\int_{\pi/4}^{\pi/3} \frac{dt}{\sin^2 t} = \frac{3}{2}\int \operatorname{cosec}^2 t\,dt$$

$$= -\frac{3}{2}\,[\cot t]_{\pi/6}^{\pi/3} = -\frac{3}{2}\left[\frac{1}{\sqrt{3}} - \sqrt{3}\right] = \sqrt{3}$$

Q.73 (1)

$$\int_{0}^{\pi/2} \ell n|\tan x + \cot x|\,dx$$

$$= \int_{0}^{\pi/2} \ell n\left(\frac{2}{\sin 2x}\right)dx = \int_{0}^{\pi/2} \ell n 2dx - \int_{0}^{\pi/2} \ell n(\sin 2x)dx$$

Q.74 (2)

$$I = \int_{0}^{\infty} [2e^{-x}]dx$$

Let $2e^{-x} = t$

$$-2e^{-x}\,dx = dt \Rightarrow dx = \frac{-dt}{t}$$

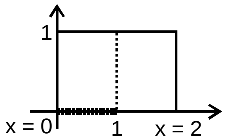
$$= -\int_{2}^{0} [t]\,\frac{dt}{t} = \int_{0}^{2} [t]\frac{dt}{t} = \int_{0}^{1} \frac{0}{t}dt + \int_{1}^{2} \frac{1}{t}dt = \ell n2$$

Q.75 (3)

$f(x) = 0$ where $\pi = \frac{n}{n+1}$, $n = 1, 2, 3, ...$

$= 1$ else where

$$\text{Find} = \int_{0}^{2} f(x)dx = 2 \times 1 = 2$$



Q.76 (2)

$$A = \int_0^1 \frac{e^t dt}{1+t}$$

$$I = \int_{a-1}^{a} \frac{e^{-t}}{t-a-1} dt = -\int_{a-1}^{a} \frac{e^{-t}}{1+a-t} dt$$

Put a – t = z
dt = – dz

$$= \int_1^0 \frac{e^{a(z-a)}dz}{1+z} = -\int_0^1 \frac{e^z.dz}{1+z} = -Ae^{-a}$$

Q.77 (1)

$$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$$

$$\sec^{-1}x - \sec^{-1}\sqrt{2} = \frac{\pi}{2}$$

$$\sec^{-1}x = \frac{\pi}{2} = \frac{\pi}{2}$$

$$\sec^{-1}x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\sec^{-1}x = \frac{3\pi}{4}$$

$$x = \sec\frac{3\pi}{4}$$

$$x = -\sqrt{2}$$

Q.78 (3)

$$I = \int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{|\sin x+\cos x|} dx = \int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{(\sin x+\cos x)} dx$$

$$= \int_0^{\pi/2} (\sin x+\cos x)dx \quad [\sin x - \cos x]_0^{\pi/2} = 1 + 1 = 2$$

Q.79 (3)

Let $I_1 = \int_a^b f(x)g(x)dx$

[given $\frac{d}{dx}f(x) = g(x)$]

$$\Rightarrow I_1 = \left[f(x)\int g(x)dx\right]_a^b - \int_a^b \left(\frac{d}{dx}f(x)\int g(x)dx\right)dx$$

$$= \left[f^2(x)\right]_a^b - \int_a^b f(x)g(x)dx \Rightarrow 2I_1 = [f(2)]^2 - [f(1)]^2$$

$$\Rightarrow I_1 = \frac{[f(b)]^2 - [f(a)]^2}{2}$$

Q.80 (1)

$$\int_1^2 (x - \log_2 a)\, dx = 2\log_2\left(\frac{2}{a}\right)$$

$$\left.\frac{x^2}{2} - (\log_2 a)x\right|_a^2 = 2\log_2\left(\frac{2}{a}\right)$$

$$2 - 2\log_2 a = 2\log_2 \frac{2}{a}$$

$2 - 2\log_2 a = 2\log_2 2 - 2\log_2 a$
1 = 1
a > 0 because of log properties.

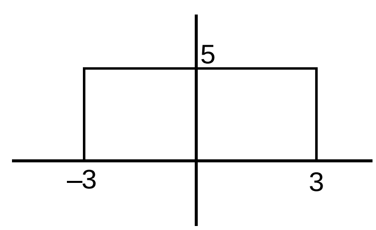
Q.81 (2)

$$I = \frac{1}{C}\cdot\int_{ac}^{bc} f\left(\frac{x}{c}\right)dx$$

Put $\frac{x}{c} = t \Rightarrow \frac{dx}{c} = dt$ $= \int_a^b f(t)dt$

Q.82 (4)

$$\int_{-3}^{3} f(x)dx = 5 \times (3 + 3) = 30$$



Q.83 (1)

$$I = \int_{5/2}^{5} \frac{\sqrt{(25-x^2)^3}}{x^4}dx$$

Put x = 5 sinθ
⇒ dx = 5 cosθ dθ

$$= \int_{\pi/6}^{\pi/2} \cot^4\theta\, d\theta = \int_{\pi/6}^{\pi/2} \cot^2\theta\,(\text{cosec}^2\theta - 1)\, d\theta$$

$$= \int \cot^2\theta\,\text{cosec}^2\theta d\theta - \int \cot^2\theta\, d\theta$$

$$= \frac{\cot^2 3\theta}{3}\Bigg|_{\pi/6}^{\pi/2} - \int \text{cosec}^2\theta + \theta$$

$$= -\frac{\cot^3\theta}{3} + \cot\theta + \theta|_{\pi/6}^{\pi/2}$$

$$= \frac{\pi}{2} + \frac{(\sqrt{3})^3}{3} - \sqrt{3} - \frac{\pi}{6} = \frac{\pi}{3}$$

Q.84 (3)

$$I = \int_{\ell n\lambda}^{-\ell n\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x)-f(-x)]}{f\left(\frac{x^2}{4}\right)[g(x)+g(-x)]}dx$$

odd function by P – 5
I = 0

Q.85 (3)

Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$

.....(i)
Then,

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2-x)} + \sqrt{\cos(\pi/2-x)}}\,dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$$

....(ii)
Adding (i) and (ii), we get

$$2I \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$$

$$+ \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$$

$$\int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$+ \int_0^{\pi/2} 1.dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$\Rightarrow I = \frac{\pi}{4} \Rightarrow \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx = \frac{\pi}{4}$$

Q.86 (2)

Given, $I = \int_a^b \frac{\sqrt{x}dx}{\sqrt{x} + \sqrt{a+b-x}} ...(i)$

Note : $\int_a^b \frac{\sqrt{a+b-x}}{\sqrt{a+b-x} + \sqrt{x}}dx \quad ...(i)$

Add. (i) and (ii),

$$2I = \int_a^b \frac{\sqrt{x}dx}{\sqrt{x} + \sqrt{a+b-x}} + \int_a^b \frac{\sqrt{a+b-x}dx}{\sqrt{a+b-x} + \sqrt{x}}$$

$$= \int_a^b \frac{\sqrt{x} + \sqrt{a+b-x}dx}{\sqrt{a+b-x} + \sqrt{x}} = \int_a^b 1.dx = [x]_a^b$$

$$2I = b - a \qquad \therefore I = \frac{b-a}{2}$$

Q.87 **(1)**

Let $I = \int_0^{\frac{\pi}{2}} x\sin^2 x\cos^2 x dx$ …(i)

From the definite integral property

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

we have

$$I = \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)\sin^2 x\cos^2 x dx \quad …(ii)$$

$$\left(\because \cos^2 x = \sin^2\left(\frac{\pi}{2} - x\right) \,\&\, \sin^2 x = \cos^2\left(\frac{\pi}{2} - x\right)\right)$$

By adding (i) and (ii)

$$2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \sin^2 x\cos^2 x dx$$

or $2I = \frac{\pi}{8}\int_0^{\frac{\pi}{2}} \sin^2 2x\,dx$

[$\because$ sin2x = 2sin x cos x]

$$= \frac{\pi}{8}\int_0^{\frac{\pi}{2}} (1 - \cos 4x)dx \left(\therefore \cos 2\theta = 1 - 2\sin^2\theta\right)$$

$$\Rightarrow 2I = \frac{\pi}{8}\left[x - \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{8}\left[\frac{\pi}{2} - 0\right] \Rightarrow I = \frac{\pi^2}{32}$$

Q.88 (4)
Q.89 (4)
Q.90 (3)
Q.91 **(2)**
Q.92 **(4)**
Q.93 **(4)**
Q.94 **(1)**
Q.95 **(1)**
Q.96 (3)

$I_1 = \int_0^{3\pi} f(\cos^2 x)dx$; period is π

$= 3\int_0^{\pi} f(\cos^2 x)dx$

$I_1 = 3I_3$
Similalry $I_2 = 2I_3$
$I_2 + I_3 \;\; 3I_3$
$I_2 + I_3 = I_1$

Q.97 (2)
$f(x) = f(a - x)$
$g(x) + g(a - x) = 2$

$I = \int_0^a f(x)\,g(x)\,dx = \int_0^a f(a-x)g(a-x)dx$

$I = \int_0^a f(x)\;(2 - g(x))\,dx = 2\int_0^a f(x)dx - I$

$2I = 2\int_0^a f(x)dx \;\Rightarrow\; I = \int_0^a f(x)dx$

Q.98 (2)

$I = \sum_{r=1}^{100}\left(\int_0^1 f(r-1+x)\,dx\right)$

$= \int_0^1 f(x)\,dx + \int_0^1 f(1+x)\,dx +$

$\int_0^1 f(2+x)\,dx + ... + \int_0^1 f(99+x)\,dx$

=

$\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + ... + \int_{99}^{100} f(x)\,dx$

$= \int_0^{100} f(x)\,dx = 1 = a$

Q.99 (4)

$\int_0^{\pi/3} f(x)dx = 0\int_0^{\pi/4} \tan x dx + \int_{\pi/4}^{\pi/3} \cot x\, dx$

$= \ell n \sec x|_0^{\pi/4} + \ell n \sec x|_{\pi/4}^{\pi/3}$

$= \ell n\sqrt{2} + \ell n\frac{\sqrt{3}}{2} - \ell n\frac{1}{\sqrt{2}} = \ell n\sqrt{3}$

Q.100 (2)

Q.101 (3)

$\because\; f'(x) = f(x) \;\Rightarrow\; \frac{f'(x)}{f(x)} = 1$

$\int \frac{f'(x)}{f(x)}dx = \int dx \;\Rightarrow\; \ell n\, f(x) = x + c$

$f(x) = e^{x+c}$ .....(1)
$f(0) = 1 \;\Rightarrow\; e^c = 1 \;\Rightarrow\; c = 0$
Now $f(x) = e^x$
$f(x) + g(x) = x^2 \;\Rightarrow\; g(x) = x^2 - e^x$

$I = \int_0^1 f(x)\,g(x)\,dx = \int_0^1 e^x\,\{x^2 - e^x\}\,dx$

$= e - \frac{e^2}{2} - \frac{3}{2}$

Q.102 **(4)**

$I = \int_0^{\pi/2}\frac{dx}{1+\tan^3 x}$

Applying King

$I = \int_0^{\pi/2}\frac{dx}{1+\cot^3 x}$

Add

$2I = \int_0^{\pi/2} dx =$ $\qquad I = \frac{\pi}{4}$

Q.103 **(1)**

$I = \int_{-1}^{3}\left(\tan^{-1}\frac{x}{1+x^2} + \tan^{-1}\frac{x^2+1}{x}\right)dx$

$I = \int_{-1}^{1}\left(\tan^{-1}\frac{x}{1+x^2} + \tan^{-1}\frac{x^2+1}{x}\right)dx$

O is odd function

$+\int_1^3\left(\tan^{-1}\frac{x}{1+x^2} + \tan^{-1}\frac{x^2+1}{x}\right)dx$

$= \int_1^3\left(\tan^{-1}\frac{x}{1+x^2} + \cot^{-1}\frac{x}{1+x^2}\right)dx$

$= \int_1^3 \frac{\pi}{2}.dx = \frac{\pi}{2}(z) = \pi$

Q.104 (3)

$$I = \int_{-1}^{1} \frac{x^4}{1+e^{x^7}}\,dx$$

King Replace $x \to -x$

$$I = \int_{-1}^{1} \frac{x^4}{1+e^{-x^7}}\,dx$$

$$2I = \int_{-1}^{1} \frac{x^4(1+e^{x^7})}{(1+e^{x^7})}\,dx$$

$$I = \frac{1}{5}$$

Q.105 (4)

$$I = \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{r^3}{r^4+n^4}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{r^3}{n^4\left(1+\frac{r}{n}\right)^4}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{\left(\frac{r}{n}\right)^3}{1+\left(\frac{r}{n}\right)^4}\frac{1}{n}\right) = \int_0^1 \frac{x^3}{1+x^4}\,dx$$

$$= \frac{1}{4}\ \ell n\ (1+x^4)\Big|_0^1 = \frac{1}{4}\ \ell n\ 2$$

Q.106 (4)

Consider $\lim\limits_{x\to\infty} \dfrac{\int_0^{2x} xe^{x^2}dx}{e^{4x^2}}$

$$= \lim_{x\to\infty} \frac{2\int_0^{2x} xe^{x^2}dx}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \frac{2\int_0^{2x} xe^{x^2}d\left(x^2\right)}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \frac{[e^{x^2}]_0^{2x}}{2e^{4x^2}} = \lim_{x\to\infty} \frac{e^{4x^2}-1}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \left(\frac{1}{2} - \frac{1}{2e^{4x^2}}\right) = \frac{1}{2}$$

Q.107 (3)

$$L = \lim_{h\to\infty} \left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{h^2}\right)\ldots\left(1+\frac{n^2}{h^2}\right)\right]^{1/n}$$

$$\ell nL = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \ell n\left(1+\left(\frac{r}{n}\right)^2\right)$$

$$= \int_0^1 \ell n(1+x^2)dx$$

$$= x\ell n\ (1+x^2) - 2x + 2\tan^{-1}x\Big|_0^1$$

$$\ell nL = \ell n2 - 2 + \frac{\pi}{2} \Rightarrow L = \frac{2}{e^2}\ e^{\pi/2}$$

Q.108 (1)

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2 = \int_0^1 x\sec^2 x^2\,dx$$

Put $x^2 = t$

$$x\ dx = \frac{dt}{2} = \frac{1}{2}\int_0^1 \sec^2 t = \frac{1}{2}\ [\tan t]_0^1 = \frac{1}{2}\ \tan 1$$

Q.109 (3)

$$I = \lim_{n\to\infty} \sum_{r=1}^{n-1} \frac{\pi}{n}\sin\frac{r\pi}{n} = \pi \int_0^1 \sin \pi x\,dx = \pi$$

$$\left[-\frac{\cos \pi x}{\pi}\right]_0^1 = [-\cos\pi + 1] = 2$$

Q.110 (4)

$$f(x) = 1 + x + \int_1^x (\ell n^2 t + 2\ell nt)dt$$

$f'(x) = 1 + \ell n^2x + 2\ell nx$

$f'(x) = 0$

$(\ell nx + 1)^2 = 0 \Rightarrow x = e^{-1}$

$$f\left(\frac{1}{e}\right) = 1 + \frac{1}{e} + \int_1^{1/e}\Big[\underset{\substack{\uparrow\\ f(t)}}{\ell n^2 t} + \underset{\substack{\uparrow\\ f'(t)}}{\left(\frac{2}{t}\ell nt\right)}\Big]dt$$

$$= 1 + \frac{1}{e} + t\ \ell n^2 t\Big|_1^{1/e} = 1 + \frac{2}{e} = 1 + 2e^{-1}$$

Q.111 (1)

$f(x) = e^{g(x)}$

$$g(x) = \int_2^x \frac{t\,dt}{1+t^4}$$

$$g'(x) = \frac{x}{1+x^4}$$

$$g'(2) = \frac{2}{17}$$

$f'(x) = e^{g(x)} . g'(x)$

$f'(2) = e^{g(x)} . g'(2)$

$$= e^0 . \frac{2}{17} = \frac{2}{17}$$

Q.112 (3)

$$L = \lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 + dt}{x\sin x} = \lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x^2}$$

$$= \lim_{x\to 0} \frac{\sec^2 x^2 .2x}{2x} = \sec^2 (0) = 1$$

Q.113 **(2)**

$$I = \lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2\,dt}{x\sin x} = \lim_{x\to 0} \frac{\int_0^{x^2} \cot^2 dt}{x^2}$$

$$= \lim_{x\to 0} \frac{\cos x^4 .2x}{2x} = 1$$

Q.114 **(2)**

$$\int_a^y \cos t^2 \; dt = \int_a^{x^2} \frac{\sin t}{t} dt$$

differentiating both sides w.r.t x we get

$$\frac{d}{dx}\int_a^y \cot\, t^2\,dt = \frac{d}{dx}\int_a^{x^2} \frac{\sin t}{t} dt$$

$$\text{RHS} = \frac{\sin[x^2]}{x^2}\frac{dx^2}{dx} = 2\times\frac{\sin x^2}{x^2}$$

$$\text{L.H.S.} = \frac{d}{dy}$$

$$\left(\int_a^y \cos t^2\,dt\right)\frac{dy}{dx} = \cos\, y^2 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{2\sin x^2}{x\cos y^2}$$

Q.115 (2)

$$I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}} = \ell n \left(x+\sqrt{x^2+1}\right)_1^2 = \ell n \left(\frac{2+\sqrt{5}}{1+\sqrt{2}}\right)$$

$$I_2 = \int_1^2 \frac{dx}{x}$$

$= \ell n\ 2$

$I_2 > I_1$

Q.116 **(1)**

$$I = \int_0^1 c_2x^2 + c_1x + c_0 = \frac{c_2x^2}{3} + \frac{c_1x^2}{2} + c_0x\Big|_0^1$$

$$= \frac{c_2}{3} + \frac{c_1}{2} + c_0$$

$I = 0$ than definately one root will lie in (0, 1)

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 0008

$$I = \int \frac{3x^2+2x}{(x^3+x^2)^2+2(x^3+x^2)+5}dx$$

Put $x^3 + x^2 = t,$ we get

$$I = \frac{1}{2}\tan^{-1}\left(\frac{x^3+x^2+1}{2}\right)+K$$

$\therefore A + B + C + D = 8$

Q.2 0005

$$\cos^4 x = \frac{1}{4}(1+\cos 2x)^2 = \frac{1}{8}(3+4\cos 2x + \cos 4x)$$

$$\therefore \int \cos^4 x\,dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + D$$

$\therefore$ 8A + 4B + 32C = 5

Q.3 0009

$$I = \int x^{13/2}(1+x^{5/2})^{1/2}dx = \int (x^{5/2})^{1/2}x^{3/2}\cdot dx$$

put $1 + x^{5/2} = t$

$$\therefore\ I = \frac{2}{5}\int (t-1)^2 t^{1/2}dt$$

$$\therefore\ I = \frac{2}{5}\left[\frac{2}{7}(1+x^{5/2})^{7/2} - \frac{4}{5}(1+x^{5/2})^{5/2} + \frac{2}{3}(1+x^{5/2})^{3/2}\right]+C$$

$\therefore$ A + B + C = 9

Q.4 0.66

Use $\sin x = \dfrac{2\tan x/2}{1+\tan^2 x/2}$

Put $\tan\dfrac{x}{2} = t$

Q.5 0.125

$$\int \frac{\cos 4x + 1}{\cot x - \tan x}\,dx = \int \frac{2\cos^2 2x}{\left(\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}\right)}$$

$$= \int \frac{2\cos^2 2x.\sin x \cos x}{\cos 2x}\,dx \Rightarrow \int \cos 2x \sin 2x\,dx$$

$$\Rightarrow \int \frac{\sin 4x}{2}\,dx = -\frac{1}{8}\cos 4x + c$$

Q.6 0002

$$I = \int \frac{dx}{\left(\sqrt{x}\right)^2 + \left(\sqrt{x}\right)^7}$$

$$I = \int \frac{1 \quad dx}{\left(\sqrt{x}\right)^7 \left(\dfrac{1}{\left(\sqrt{x}\right)^5} + 1\right)}$$

Put $1 + \dfrac{1}{\left(\sqrt{x}\right)^5} = t$

Q.7 0002

$$I = \int \frac{\cos x + \sin 2x}{(2 - \cos^2 x)(\sin x)}\,dx$$

$$I = \int \frac{(1 + 2\sin x)\cos x}{(1 + \sin^2 x)(\sin x)}\,dx$$

$$I = \int \frac{1 + 2t}{(1 + t^2)t}\,dt, \text{ (Put } \sin x = t)$$

Use parial fractions

$$\frac{1 + 2t}{t(1 + t^2)} = \frac{1}{t} + \frac{(-t + 2)}{1 + t^2}$$

Q.8 0.33

$$\int \tan^4 x\,dx$$

$$= \int \tan^2 x(\sec^2 x - 1)\,dx$$

$$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$$

$$= \int \tan^2 x \sec^2 x\,dx - \int (\sec^2 x - 1)\,dx$$

$$= \frac{\tan^3 x}{3} - \tan x + x + c$$

Q.9 0001

$$I = \int \frac{dx}{x^2 (x^4 + 1)^{3/4}} = \int \frac{dx}{x^2.x^3\left(1 + \dfrac{1}{x^4}\right)^{3/4}},$$

Put $1 + x^{-4} = t$

$$\Rightarrow \frac{-4}{x^5}\,dx = dt \quad = -\frac{1}{4}\int \frac{dt}{t^{3/4}}$$

$$= \frac{1}{4}\int t^{-3/4}\,dt = -\frac{1}{4}.\frac{t^{1/4}}{1/4} + c$$

$$= -\left(1 + \frac{1}{x^4}\right)^{1/4} + c = -\left(\frac{1 + x^4}{x^4}\right)^{1/4} + C$$

Hence $A = -1, B = \dfrac{1}{4}$

Q.10 0.5

$$\int \frac{\log_x e.\log_{ex} e.\log_{e^2x} e}{x}\,dx$$

$$\int \frac{dx}{x\log_e x\,(1 + \log_e x)\,(2 + \log_e x)}$$

Put $\log x = t$

$$\int \frac{dt}{t(1 + t)\,(2 + t)}$$

Now $\dfrac{1}{t(1 + t)\,(2 + t)} = \dfrac{A}{t} + \dfrac{B}{1 + t} + \dfrac{C}{2 + t}.$

$$A = \frac{1}{2}, B = -1, C = \frac{1}{2}$$

Q.11 0050

$$100\int_0^1 \{x\}\,dx = 100\int_0^1 x\,dx$$

$$\frac{100}{2}\left[x^2\right]_0^1 = 50$$

Q.12 0.67

Apply Newton Liebnitz rule and L'Hospital rule.

$$= \lim_{x \to 0^+} \frac{2x \sin x}{3x^2} = \frac{2}{3}.1$$

Q.13 0.277

Put $\sin x = t$,

$$I = \int_0^1 t^{1/2}\left(1 - t^2\right)^2\,dt$$

$$\int_0^1 t^{1/2}(1 - 2t^2 + t^4)\,dt = \int \left[t^{1/2} - 2.t^{5/2} + t^{9/2}\right]dt$$

$$= \frac{2}{3}t^{3/2} - 2.\frac{2}{7}.t^{7/2} + \frac{2}{11}.t^{11/2} = \frac{2}{3} - \frac{4}{7} + \frac{2}{11} = \frac{64}{231}$$

Q.14 0002

$$\lim_{x\to 0}\frac{1}{n}\sum_{r=0}^{3n-3}\sqrt{\frac{n}{n+r}}$$

$$=\int_0^3 \frac{1}{\sqrt{1+x}}dx$$

$$=2\left[\sqrt{1+x}\right]_0^3=2[2-1]=2$$

Q.15 (0000)

$$I=\int_0^{\pi/2}\frac{\cos x-\sin x}{1+\sin x\cos x}dx$$

$$I=\int_0^{\pi/2}\frac{\cos\left(\frac{\pi}{2}-x\right)-\sin\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\pi/2}\frac{\sin x-\cos x}{1+\sin x\cos x}dx \quad =\int_0^{\pi/2}\frac{\cos x-\sin x}{1+\sin x\cos x}dx$$

$I=-I \Rightarrow 2I=0 \Rightarrow I=0$

Q.16 0003

$$\int_0^{\pi} \underset{I}{f(x)}\underset{II}{\sin x}dx+\int_0^{\pi} \underset{II}{f''(x)}\underset{I}{\sin x}dx=$$

$$=\left(\Big|_0^{\pi} f(x)(-\cos x)-\int_0^{\pi} f'(x)(-\cos x)dx\right)$$

$$+\left(\Big|_0^{\pi}\sin x f'(x)-\int_0^{\pi} f'(x)\cos x dx\right)$$

$$=\left(f(\pi)+f(0)+\int_0^{\pi} f'(x)\cos x dx\right)+\left(0-\int_0^{\pi} f'(x)\cos x dx\right)$$

$$=f(\pi)+f(0)=5$$

$$\therefore\ f(0)=3$$

Q.17 0031

$$\int_0^9\left[\sqrt{x}+2\right]dx$$

$$=\int_0^1\left[\sqrt{x}+2\right]dx+\int_1^4\left[\sqrt{x}+2\right]dx+\int_4^9\left[\sqrt{x}+2\right]dx$$

$$=\int_0^1 2dx+\int_1^4(1+2)dx+\int_4^9(2+2)dx=\ 2+9+20=31$$

Q.18 11.33

$$\sqrt{x+\left(2\sqrt{x-1}\right)}=\left|\sqrt{x-1}+1\right|$$

and $\sqrt{x-2\sqrt{x-1}}=\left|\sqrt{x-1}-1\right|$

$$I=\int_1^2 2\,dx+\int_2^5 2\sqrt{x-1}\,dx$$

$$=2.1+2.(x-1)^{3/2}.\frac{2}{3}\Big|_2^5\quad =2+\frac{28}{3}=\frac{34}{3}$$

Q.19 (0.5)

$$I=\int_2^3\frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$$

$$I+I=\int_2^3\frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$$

$$2I=1\Rightarrow I=\frac{1}{2}$$

Q.20 (0.5)

$$I_1=\int_{1-k}^{k} xf\left[x(1-x)\right]dx$$

$$=\int_{1-k}^{k}(1-x)f\left[(1-x)\left[1-(1-x)\right]\right]dx$$

$$=\int_{1-k}^{k} f\left[x(1-x)\right]dx-\int_{1-k}^{k} xf\left[x(1-x)\right]dx$$

$$I_1=I_2-I_1\Rightarrow\frac{I_1}{I_2}=\frac{1}{2}$$

PREVIOUS YEAR'S

MHT CET

Q.1 (1)	**Q.2** (1)	**Q.3** (2)	**Q.4** (3)	**Q.5** (3)
Q.6 (2)	**Q.7** (3)	**Q.8** (3)	**Q.9** (1)	**Q.10** (2)
Q.11 (2)	**Q.12** (2)	**Q.13** (3)	**Q.14** (1)	**Q.15** (2)
Q.16 (4)	**Q.17** (3)	**Q.18** (2)	**Q.19** (2)	**Q.20** (1)
Q.21 (2)	**Q.22** (3)	**Q.23** (1)	**Q.24** (1)	**Q.25** (2)
Q.26 (2)	**Q.27** (2)	**Q.28** (1)	**Q.29** (1)	**Q.30** (1)
Q.31 (4)	**Q.32** (2)	**Q.33** (4)	**Q.34** (1)	**Q.35** (1)
Q.36 (1)	**Q.37** (2)	**Q.38** (2)	**Q.39** (4)	**Q.40** (2)
Q.41 (1)	**Q.42** (1)	**Q.43** (1)	**Q.44** (3)	
Q.45 (4)				

Let $I=\int\frac{(\log x)^2}{x}dx$

Put $\log x=t\Rightarrow\frac{1}{x}dx=dt$

$\therefore\ I = \int \frac{(\log x)^2}{x} dx = \int t^2 dt$

$= \frac{t^3}{3} + C = \frac{(\log x)^3}{3} + c \quad [\text{put } t = \log x]$

Q.46 (3)

Let $I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx$

Put $\tan \sqrt{x} = t$

$\Rightarrow \frac{\sec^2 \sqrt{x}}{2\sqrt{x}} dx = dt$

$\therefore\ I = \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx = \int 2t^4 dt$

$= \frac{2t^5}{5} + C = \frac{2(\tan\sqrt{x})^5}{5} + C$

Q.47 (2)

$I = \int \frac{dx}{\sin^2 x \cos^2 x}$

$= \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cos^2 x}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \int (\sec^2 x + \operatorname{cosec}^2 x) dx$

$= \int (\sec^2 x dx + \int \operatorname{cosec}^2 x dx$

$= \tan x - \cot x + C$

Q.48 (3)

Let $I = \int \frac{e^{\tan^{-1} x}}{1+x^2} dx$

Put $\tan^{-1} x = t$

$\Rightarrow \frac{1}{1+x^2} dx = dt$

$\therefore\ I = \int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^t dt$

$= e^t + C \qquad \left[\because \int e^x dx = e^x\right]$

$= e^{\tan^{-1} x} + C \qquad [\text{put } t = \tan^{-1} x]$

Q.49 (4)

Let $I = \sqrt{2} \int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)} dx$

Put $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$\therefore\ I = \sqrt{2} \int \frac{\sin\left(\frac{\pi}{4} + t\right)}{\sin t} dt$

$= \sqrt{2} \int \frac{\sin\frac{\pi}{4}\cos t + \cos\frac{\pi}{4}\sin t}{\sin t} dt$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\frac{\cos t}{\sin t} + \frac{1}{\sqrt{2}}\right) dt$

$\int (\cot t + 1)\, dt = \log|\sin t| + t + C_1|$

$= x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + C \left[\because C_1 - \frac{\pi}{4} = C\right]$

Q.50 (3)

Let $I = \int \frac{(x^2 - 1)dx}{x^3\sqrt{2x^4 - 2x^2 + 1}}$

Dividing numerator and denominator by x^5,

$I = \int \frac{\left(\frac{1}{x^3} - \frac{1}{x^5}\right)dx}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}$

Put $2 - \frac{2}{x^2} + \frac{1}{x^4} = t \Rightarrow \left(\frac{4}{x^3} - \frac{4}{x^5}\right)dx = dt$

$J. = \frac{1}{4}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\sqrt{t} + C = \frac{1}{2}\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + C$

Q.51 (1)

Given,

$\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx = x\,[f(x) - g(x)] + C$

$\text{LHS} = \int \underset{\text{II}}{1}.\log(\underset{\text{I}}{\log} x)dx + \int \frac{1}{(\log x)^2} dx$

$= x\log(\log x) - \int \frac{1}{\log x} dx + \int \frac{1}{(\log x)^2} dx$

$= x\log(\log x) - \frac{x}{\log x}$

$- \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx + C$

$= x\left[\log(\log x) - \frac{1}{\log x}\right] + C$

$\therefore\ f(x) = \log(\log x);\ g(x) = \frac{1}{\log x}$

Q.52 (4)

Let $I = \int \frac{x\,dx}{\sqrt{1-2x^4}} = \int \frac{x\,dx}{\sqrt{1-\left(\sqrt{2}x^2\right)^2}}$

Let $\sqrt{2}x^2 = t$

$\Rightarrow 2\sqrt{2}\,x\,dx = dt$ Þ $x\,dx = dt/2\sqrt{2}$

$\therefore\ I = \frac{1}{2\sqrt{2}}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{2\sqrt{2}}\sin^{-1}(t) + C$

$I = \frac{1}{2\sqrt{2}}\sin^{-1}\left(\sqrt{2}x^2\right) + C$

Q.53 (2)

Let $I = \int \frac{dx}{2+\cos x}$

$I = \int \frac{dx}{2 + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} = \int \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{2+2\tan^2\frac{x}{2}+1-\tan^2\frac{x}{2}}$

$I = \int \frac{\sec^2\frac{x}{2}dx}{3+\tan^2\frac{x}{2}}$

Let $\tan\frac{x}{2} = t$ $\Rightarrow \sec^2 x/2dx = 2dt$

$\therefore\ I = \int \frac{2dt}{3+t^2} = 2\int \frac{dt}{\left(\sqrt{3}\right)^2 + t^2}$

$I = 2\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + C$

$\Rightarrow I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + C$

Q.54 (2)	**Q.55** (2)	**Q.56** (3)	**Q.57** (2)	**Q.58** (3)
Q.59 (2)	**Q.60** (1)	**Q.61** (2)	**Q.62** (1)	**Q.63** (3)
Q.64 (1)	**Q.65** (1)	**Q.66** (4)	**Q.67** (2)	**Q.68** (2)
Q.69 (4)	**Q.70** (2)	**Q.71** (4)	**Q.72** (3)	**Q.73** (4)
Q.74 (1)	**Q.75** (4)	**Q.76** (3)	**Q.77** (2)	**Q.78** (3)
Q.79 (4)	**Q.80** (2)	**Q.81** (2)	**Q.82** (2)	**Q.83** (4)
Q.84 (3)	**Q.85** (4)	**Q.86** (3)	**Q.87** (3)	**Q.88** (3)
Q.89 (4)				

Q.90 (3)

$\int_{-1}^{1} \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2+1}dx$

$= \int_{-1}^{1} \underbrace{\frac{17x^5 + 29x^3 - 31x + 1}{x^2+1}}_{\text{Odd function}}dx - \int_{-1}^{1} \underbrace{\frac{x^4 - 1}{x^2+1}}_{\text{Even function}}dx$

$= 0 - 2\int_0^1 \frac{\left(x^2-1\right)\left(x^2+1\right)}{\left(x^2+1\right)}dx$

$= -2\left[\left(\frac{x^3}{3} - x\right)\right]_0^1 = \frac{4}{3}$

Q.91 (2)

Let $I = \int_{\pi/6}^{\pi/2} \frac{\operatorname{cosec} x.\cot x}{1+\operatorname{cosec}^2 x}dx$

Let cosec x = t

Þ – cosec x cot x dx = dt

When $x = \frac{\pi}{6}$, then $t = \operatorname{cosec}\frac{\pi}{6} = 2$

and when $x = \frac{\pi}{2}$, then $t = \operatorname{cosec}\frac{\pi}{2} = 1$

$\therefore\ I = \int_2^1 -\frac{dt}{1+t^2} = -\int_2^1 \frac{dt}{1+t^2}$

$= \int_1^2 \frac{dt}{1+t^2} = \left[\tan^{-1}(t)\right]_1^2$

$= \tan^{-1}(2) - \tan^{-1}(1)$

$= \tan^{-1}\left(\frac{2-1}{1+2\times 1}\right) = \tan^{-1}\left(\frac{1}{1+2}\right)$

$= \tan^{-1}\left(\frac{1}{3}\right)$

Q.92 (2)

$\int_3^5 \frac{x^2}{x^2-4}dx = \int_3^5 \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4}\right)dx$

$= \int_3^5 \left(1 + \frac{4}{x^2-4}\right)dx = \left[x + \frac{4}{2\times 2}\log_e\left(\frac{x-2}{x+2}\right)\right]_3^5$

$= \left[5 + \log_e\left(\frac{5-2}{5+2}\right) - 3 - \log_e\left(\frac{3-2}{3+2}\right)\right]$

$= 2 + \log_e\left(\frac{3}{7}\right) - \log_e\left(\frac{1}{5}\right)$

$= 2 + \log_e\left(\frac{3}{7}\times\frac{5}{1}\right) = 2 + \log_e\left(\frac{15}{7}\right)$

Q.93 (4)

$\int_{\pi/6}^{\pi/2} \left(\frac{1+\sin 2x + \cos 2x}{\sin x + \cos x}\right)dx$

$$=\int_{\pi/6}^{\pi/2}\left(\frac{1+\sin x\cos x+2\cos^2 x-1}{(\sin x+\cos x)}\right)dx$$

$$=\int_{\pi/6}^{\pi/2}\left(\frac{2\cos x(\sin x+\cos x)}{(\sin x+\cos x)}\right)dx$$

$$=\int_{\pi/6}^{\pi/2}2\cos x dx=2[\sin x]_{\pi/6}^{\pi/2}$$

$$=2\left(\sin\frac{\pi}{2}-\sin\frac{\pi}{6}\right)=2\left(1-\frac{1}{2}\right)=2\times\frac{1}{2}=1$$

Q.94 (1)

Let $I=\int_0^1\frac{x^4(1-x)^4}{1+x^2}dx$

$$=\int_0^1\frac{(x^4-1)(1-x)^4+(1-x)}{(1+x^2)}dx$$

$$=\int_0^1(x^2-1)(1-x)^4\,dx+\int_0^1\frac{(1+x^2-2x)^2}{(1+x^2)}$$

$$=\int_0^1\left\{(x^2-1)(1-x)^4+(1+x^2)-4x\frac{4x^2}{(1+x^2)}\right\}dx$$

$$=\int_0^1\left\{(x^2-1)(1-x)^4+(1+x^2)+4x+4\frac{4}{(1+x^2)}\right\}dx$$

$$=\int_0^1\left(x^6-4x^5+5x^4-4x^2+4-\frac{4}{1+x^2}\right)dx$$

$$=\left[\frac{x^7}{7}-\frac{4x^6}{6}+\frac{5x^5}{5}-\frac{4x^3}{3}+4x-4\tan^{-1}x\right]_0^1$$

$$=\frac{1}{7}-\frac{4}{6}+\frac{5}{5}-\frac{4}{3}+4-4\left(\frac{\pi}{0}-0\right)$$

$$=\frac{22}{7}-\pi$$

Q.95 (1)

$\because$ |z| = real and positive and imaginary part is zero.

$\therefore$ arg | z| = 0

$\Rightarrow$ [arg|z|] = 0

$\therefore \int_{x=0}^{100}[\arg|z|]dx=\int_{x=0}^{100}0dx=0$

Q.96 (4)

Given $\int_{-1}^{3}(|x-2|+[x])dx$

$$=\int_{-1}^{0}\{-(x-2)-1\}dx+\int_0^1-(x-2)+0dx$$

$$+\int_1^2\{-(x-2)+1\}dx+\int_2^3(x-2)+2dx$$

$$=\int_{-1}^{0}(-x+1)dx+\int_0^1(-x+2)dx$$

$$+\int_1^2(-x+3)dx+\int_2^3(x)dx$$

$$=\left[\frac{-x^2}{2}+x\right]_{-1}^{0}+\left[\frac{-x^2}{2}+2x\right]_0^1+\left[\frac{-x^2}{2}+3x\right]+\left[\frac{x^2}{2}\right]_2^3$$

$$=\frac{3}{2}+\frac{3}{2}+(-2+6)-\left(\frac{-1}{2}+3\right)+\left(\frac{9}{2}-\frac{4}{2}\right)$$

$$=3+4-\frac{5}{2}+\frac{5}{2}=7$$

Q.97 (2)

Given $\lim_{x\to0}\frac{1}{x^3}\int_0^x\frac{t\log(1+t)}{t^4+4}dt$

$$\Rightarrow \lim_{x\to0}\frac{\int_0^x\frac{t\log(1+t)}{t^4+4}dt}{x^3}$$

Using L' Hospital's rule, we get

$$\lim_{x\to0}\frac{\frac{x\log(1+x)}{x^4+4}}{3x^2}=\lim_{x\to0}\frac{\log(1+x)}{3x}.\frac{1}{x^4+4}$$

$$=\frac{1}{3}.\frac{1}{4}=\frac{1}{12}$$

Q.98 (4)

Let $I=\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx$ ...(i)

$$\therefore\ I=\int_{-1}^{1}\log\left[\frac{2-(1-1-x)}{2+(1-1-x)}\right]dx$$

$$\left[\because\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

$I=\int_{-1}^{1}\log\left(\frac{2+x}{2-x}\right)dx$ ...(ii)

Adding Eqs. (i) and (ii), we get

$$2I=\int_{-1}^{1}\left[\log\left(\frac{2-x}{2+x}\right)+\log\left(\frac{2+x}{2-x}\right)\right]dx$$

$$2I=\int_{-1}^{1}\log\left[\left(\frac{2-x}{2+x}\right)\left(\frac{2+x}{2-x}\right)\right]dx$$

$$[\because \log m+\log n=\log mn]$$

$$2I=\int_{-1}^{1}\log 1\,dx=0\Rightarrow I=0$$

JEE MAIN
PREVIOUS YEAR'S

Q.1 **(1)**

Let $I = \int \frac{1-x}{x\sqrt{1-x^2}}dx$

$= \int \frac{1}{x\sqrt{1-x^2}}dx - \int \frac{1}{\sqrt{1-x^2}}dx$

Put $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}$

$= \int \frac{\frac{-1}{t^2}}{\frac{1}{t}\sqrt{1-\frac{1}{t^2}}} dt - \sin^{-1}(x) + C_1$

$= \int \frac{-dt}{\sqrt{t^2-1}}$

$= -\ell n|t + \sqrt{t^2-1}\,|$

$= -\ell n\left|\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right| - \sin^{-1}(x) + C_1$

$= -\ell n\left|\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right| - \left(\frac{\pi}{2} - \cos^{-1}x\right) + C_1$

$= -\ell n\left|\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right| + \cos^{-1}x - \frac{\pi}{2} + C_1$

$= g(x) = \cos^{-1}(x) - \ell n\left|\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right|$

$= g(1) = \cos^{-1}(1) - \ell n|1| = 0$

$= g\left(\frac{1}{2}\right) = \cos^{-1}\left(\frac{1}{2}\right) - \ell n|2 + \sqrt{3}\,|$

$= \frac{\pi}{3} - \ell n\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

$= \frac{\pi}{3} + \ell n\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$

Q.2 (2)

$\int (\frac{x^2+1}{(x+1)^2})e^x \cdot dx$

$\int (\frac{x^2-1+2}{(x+1)^2})e^x \cdot dx$

$\int (\frac{x-1}{x+1} + \frac{2}{(x+1)^2})e^x \cdot dx$

$\int (f(x) + f'(x))e^x dx$

$= f(x)\, e^x + c$

Where $f(x) = \frac{x-1}{x+1}$

$f'(x) = \frac{2}{(x+1)^2}$

$f''(x) = \frac{-4}{(x+1)^3}$

$f'''(x) = \frac{12}{(x+1)^4}$

$f'''(1) = \frac{12}{16}$

$= \frac{3}{4}$

Q.3 (1)

$I = \int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$

$\frac{\sqrt{3}}{2}\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(\frac{\sqrt{3}}{2} + \sin 2x\right)}dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos x - \sin x)}{\sin 60^\circ + \sin 2x}dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\sin x\right)}{2\sin\left(X+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$

$\int \frac{\left(\cos\left(x-\frac{\pi}{6}\right) - \sin\left(x+\frac{\pi}{6}\right)\right)}{2\sin\left(X+\frac{\pi}{6}\right)\cos\left(X-\frac{\pi}{6}\right)}dx$

$\frac{1}{2}\left(\int \frac{dx}{\sin\left(X+\frac{\pi}{6}\right)} - \int \frac{dx}{\cos\left(x-\frac{\pi}{6}\right)}\right)$

$$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$$

Q.4 (2)

$$g(x) = \int_x^{\frac{\pi}{4}} d(f(t).\sec t)$$

$$= f(t)\sec t\Big|_x^{\frac{\pi}{4}}$$

$$g(x) = f\left(\frac{\pi}{4}\right)\sec\frac{\pi}{4} - f(x)\sec x$$

$$g(x) = 2 - \frac{f(x)}{\cos x}$$

$$\lim_{x\to\left(\frac{\pi}{2}\right)^-} g(x) = 2 - \lim_{x\to\left(\frac{\pi}{2}\right)^-}\left(\frac{f(x)}{\cos x}\right)$$

Using L' Hopital Rule

$$= 2 - \lim_{x\to\left(\frac{\pi}{2}\right)^-}\left(\frac{f'(x)}{-\sin x}\right) \qquad = 2 + \left(\frac{f'\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)}\right) = 3$$

Q.5 (3)

$f(x) + f(x+k) = n$

$f(x+k) + f(x+2k) = n$

$\Rightarrow f(x+2k) - f(x) = 0$

$\Rightarrow f(x+2k) = f(x)$

$\Rightarrow f(x)$ is periodic with period 2k

$$I_1 = \int_0^{4nk} f(x)\,dx = 2n\int_0^{2k} f(x)dx$$

$$I_2 = \int_{-k}^{3k} f(x)\,dx = 2\int_0^{2k} f(x)dx$$

Now,

$f(x) + f(x+k) = n$

$$\Rightarrow \int_0^k f(x)\,dx + \int_0^k (x+k)dx = nk$$

$$\Rightarrow \int_0^k f(x)dx + \int_k^{2k} f(x)dx = nk$$

$$\Rightarrow \int_0^{2k} f(x)dx = nk$$

$\Rightarrow I_1 = 2n^2 k,\ I_2 = 2nk$

$\Rightarrow I_1 + nI_2 = 4n^2k$

Q.6 (3)

$$f(x) = \begin{cases} x^3 - 3x, & x \le -1 \\ 2, & -1 < x \le 2 \\ x^2 + 2x - 6 & 2 < x \le 3 \\ 9 & 3 < x \le 4 \\ 10 & 4 < x < 5 \\ 11 & x = 5 \\ 2x+1 & x > 5 \end{cases}$$

Clearly f(x) is not differentiable at

$x = 2, 3, 4, 5 \Rightarrow m = 4$

$$I = \int_{-2}^{-1}(x^3 - 3x)dx + \int_{-1}^{2} 2\cdot dx = \frac{27}{4}$$

$$I = \frac{27}{4}$$

Q.7 (4)

$$I = \int_0^5 \cos\left(\pi x - \pi\left[\frac{x}{2}\right]\right)dx$$

$$I = \int_0^2 \cos(\pi x)dx + \int_2^4 \cos(\pi x - \pi)dx + \int_4^5 \cos(\pi x - 2\pi)dx$$

$$I = \left[\frac{\sin \pi x}{\pi}\right]_0^2 + \left[\frac{\sin(\pi x - \pi)}{\pi}\right]_2^4 + \left[\frac{\sin(\pi x - 2\pi)}{\pi}\right]_4^5$$

$I = 0$

Q.8 (4)

$$f(x) = x + \int_0^1 (x-t)f(t)\,dt$$

$$f(x) = x(1 + \int_0^1 f(t)\,dt) - \int_0^1 tf(t)\,dt$$

$f(x) = Ax - B$ ... (1)

$\Rightarrow f(t) = At - B$

Now, $A = 1 + \int_0^1 f(t)\,dt = 1 + \int_0^1 (At - B)\,dt$

$\Rightarrow A = 2(1 - B)$ ... (2)

Also $B = \int_0^1 tf(t)\,dt = \int_0^1 (At^2 - Bt)\,dt$

$A = \frac{9}{2}B$ ... (3)

From (2), (3)

$A = \frac{18}{13},\ B = \frac{4}{13}$

So $f(6) = 8$

Q.9 (3)

$$\text{LHS} = \int_0^2 (\sqrt{2x} - \sqrt{2x - x^2})\,dx = \frac{8}{3} - \frac{\pi}{2}$$

$$\text{RHS} = \int_0^1\left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right)dy + \int_1^2\left(2 - \frac{y^2}{2}\right)dy + I$$

$$= I + \frac{5}{3} - \frac{\pi}{4}$$

$$\text{So, } I = 1 - \frac{\pi}{4} = \int_0^1\left(1 - \sqrt{1-y^2}\right)dy$$

Q.10 **[1]**

$$f(\theta)=\sin\theta+\int_{-\pi/2}^{\pi/2}(\sin\theta+t\cos\theta)f(t).dt$$

$$f(\theta)=\sin\theta+\int_{-\pi/2}^{\pi/2}\sin\theta.f(t).dt+\int_{-\pi/2}^{\pi/2}\cos\theta.t.f(t).dt$$

$$f(\theta)=\left(1+\int_{-\pi/2}^{\pi/2}f(t).dt\right)\sin\theta+\left(\int_{-\pi/2}^{\pi/2}t.f(t).dt\right)\cos\theta$$

$$\Rightarrow f(\theta)=A.\sin\theta+B.\cos\theta$$

$$A=1+\int_{-\pi/2}^{\pi/2}f(t).dt \qquad B=\int_{-\pi/2}^{\pi/2}t.f(t).dt$$

$$A=1+\int_{-\pi/2}^{\pi/2}(A.\sin t+B.\cos t).dt$$

$$B=\int_{-\pi/2}^{\pi/2}t.(A.\sin t+B\cos t).dt$$

$$A=1+0+2B\int_0^{\pi/2}\cos t.dt$$

$$B=2A\int_0^{\pi/2}t.\sin t.dt+0$$

$$A=1+2B(1)$$

$$B=2A\left(-t\cos t+\int_0^{\pi/2}\cos t.dt\right)$$

$$A=1+2B...(1) \qquad B=2A(-t\cos t+\sin t)_0^{\pi/2}$$

from (1) & (2) B=2A.... (2)

A=1+4A $\Rightarrow$ A=–1/3

$\Rightarrow$ B=–2/3

$$\Rightarrow f(\theta)=\frac{-\sin\theta}{3}-\frac{2\cos\theta}{3}$$

Now

$$\left|\int_0^{\pi/2}f(\theta)\cdot d\theta\right|=\left|-\frac{1}{3}\int_0^{\pi/2}\sin\theta+2\cos\theta.d\theta\right|=\frac{1}{3}\left|(-\cos\theta+2\sin\theta)_0^{\pi/2}\right|$$

$$=\frac{1}{3}\left|(0+2)-(-1+0)\right|=1$$

Q.11 **[34]**

Let $y=\frac{9-x^2}{5-x}$

$$\frac{dy}{dx}=\frac{(5-x)(-2x)-(9-x^2)(-1)}{(5-x^2)}$$

$$\frac{dy}{dx}=\frac{x^2-10x+9}{(5-x)^2}=\frac{(x-1)(x-9)}{(5-x)^2}$$

So critical points x=1, 9

$1\in[0,2]$

$$y(0)=\frac{9}{5};\ y(1)=2;\ y(2)=\frac{5}{3}$$

$$\Rightarrow \alpha=2\ \&\ \beta=\frac{5}{3}$$

Now $I=\int_{\beta-\frac{8}{3}}^{2\alpha-1}\text{Max}\left\{\frac{9-x^2}{5-x},x\right\}\cdot dx$

$$=\int_{-1}^{3}\text{Max}\left\{\frac{9-x^2}{5-x},x\right\}\cdot dx=\int_{-1}^{9/5}\frac{9-x^2}{5-x}\cdot dx+\int_{9/5}^{3}x.dx$$

$$=\int_{-1}^{9/5}\frac{9}{5-x}dx+=\int_{-1}^{9/5}\frac{9-x^2}{5-x}\cdot dx+\int_{-1}^{9/5}\frac{25-x^2-25}{5-x}dx+\left(\frac{x^2}{2}\right)_{9/5}^{3}$$

$$-16\int_{-1}^{9/5}\frac{dx}{5-x}-\int_{-1}^{9/5}(5+x)dx+\left(\frac{x^2}{2}\right)_{9/5}^{3}$$

$$=16\left[\ell n|5-x|\right]_{-1}^{9/5}+\left[5x+\frac{x^2}{2}\right]_{-1}^{9/5}+\left(\frac{x^2}{2}\right)_{9/5}^{3}$$

$$=16\left[\ell n[16/5]-\ell n6\right]+\left[\left(9+\frac{81}{50}\right)-(-5+1/2)\right]+\left[\frac{9}{2}-\frac{81}{50}\right]$$

$$=16\ell n\left(\frac{16}{30}\right)+14+4$$

$$=18+16\ell n(8/15)$$

$$\alpha_1=18,\alpha_2=16$$

$$\alpha_1+\alpha_2=34$$

Q.12 (3)

Let $I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{(1+e^x)(\sin^6x)+\cos^6x)}$ ...(1)

Applying king

$$I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{(1+e^{-x})[\sin^6(-x)+\cos^6(-x)]}$$

$I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{e^x}{(e^x+1)(\sin^6x)+\cos^6x)}dx$ ...(2)

(1) + (2)

$$2I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{(1+e^x)}{(1+e^x)(\sin^6x+\cos^6x)}dx$$

Applying NANO Prop.

$$2I=2\int_0^{\frac{\pi}{2}}\frac{dx}{\sin^6x+\cos^6x}$$

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{(1)[\sin^4x+\cos^4x-\sin^2x\cos^2x]}$$

divide by $\cos^4x$

$$I=\int_0^{\frac{\pi}{2}}\frac{\sec^4 x dx}{\tan^4 x+1-\tan^2 x}$$

put tanx = t

$$I=\int_0^{\infty}\frac{(1+t^2)}{t^4+1-t^2}dt$$

$$=\int_0^{\infty}\frac{t^2(1+\frac{1}{t^2})dt}{t^2(t^2+\frac{1}{t^2}-1)}$$

$$I=\int_0^{\infty}\frac{1+\frac{1}{t^2}}{(t-\frac{1}{t})^2+1}dt$$

put $t-\frac{1}{t}=y$

$$I=\int_{-\infty}^{\infty}\frac{dy}{y^2+1}$$

$$I=[\tan^{-1}y]_{-\infty}^{\infty}$$

$$I=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\pi$$

Q.13 (1)

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{n^2[1+\frac{r^2}{n^2}]n(1+\frac{r}{n})}$$

$$I=\int_0^1\frac{dx}{(1+x^2)(1+x)}$$

put x = tan θ

$$I=\int_0^{\frac{\pi}{4}}\frac{\sec^2\theta d\theta}{(\sec^2\theta)[1+\tan\theta]}$$

$$I=\int_0^{\frac{\pi}{4}}\frac{\cos\theta d\theta}{(\cos\theta+\sin\theta)}$$

$$I=\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)}d\theta$$

$$I=\frac{1}{2}[\theta]_0^{\frac{\pi}{4}}+\frac{1}{2}(\ell n|\cos\theta+\sin\theta|)_0^{\frac{\pi}{4}}$$

$$I=\frac{\pi}{8}+\frac{1}{2}(\ell n\sqrt{2})$$

$$=\frac{\pi}{8}+\frac{1}{4}\ell n2$$

Q.14 (4)

$$b_n=\int_0^{\frac{\pi}{2}}\frac{\cos^2(n\pi)}{\sin x}dx,\ n\in N$$

$$b_n-b_{n-1}=\int_0^{\frac{\pi}{2}}\frac{\cos^2 n\pi-\cos^2(n-1)x}{\sin x}$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin^2(n-1)x-\sin^2 nx}{\sin x}$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin(2nx-x)(-\sin x)}{\sin x}$$

$$=-\int_0^{\frac{\pi}{2}}\sin(2n-1)x$$

$$=\frac{\cos(2n-1)x}{2n-1}\Big|_0^{\pi/2}$$

$$=\frac{-1}{2n-1}$$

Now, $b_3-b_2=-\frac{1}{5}$

$b_4-b_3=\frac{-1}{7}\Rightarrow\frac{1}{b_3-b_2},\frac{1}{b_4-b_3},\frac{1}{b_5-b_4}$ are in A.P. with common difference –2

$$b_5-b_4=\frac{-1}{9}$$

Q.15 [6]

$$12\int_3^b\frac{dx}{(x^2-1)(x^2-4)}=\ln\left(\frac{49}{40}\right)$$

$$\Rightarrow\frac{12}{3}\int_3^b\frac{(x^2-1)-(x^2-4)}{(x^2-1)(x^2-4)}dx=\ln\left(\frac{49}{40}\right)$$

$$\Rightarrow 4\left[\int_3^b\frac{dx}{x^2-4}-\int_3^b\frac{dx}{x^2-1}\right]=\ln\left(\frac{49}{40}\right)$$

$$\Rightarrow 4\left[\frac{1}{4}\left(\ln\left|\frac{x-2}{x+2}\right|\right)_3^b-\frac{1}{2}\left(\ln\left|\frac{x-1}{x+1}\right|\right)_3^b\right]=\ln\left(\frac{49}{40}\right)$$

$$\Rightarrow\ln\left|\frac{b-2}{b+2}\right|-\ln\left(\frac{1}{5}\right)-2\ln\left|\frac{b-1}{b+1}\right|+2\ln\left(\frac{1}{2}\right)=\ln\left(\frac{49}{40}\right)$$

$$\Rightarrow\ln\left[\left(\frac{b-2}{b+2}\right)\times\frac{(b+1)^2}{(b-1)^2}\times\frac{5}{4}\right]=\ln\left(\frac{49}{40}\right)$$

$$\Rightarrow \left(\frac{b-2}{b+2}\right)\times\frac{(b+1)^2}{(b-1)^2}\times\frac{5}{4}=\frac{49}{40}$$

$$\Rightarrow \frac{(b-2)(b^2+2b+1)}{(b+2)(b^2-2b+1)}=\frac{49}{50}$$

$$\Rightarrow \frac{b^3+2b^2+b-2b^2-4b-2}{b^3-2b^2+b+2b^2-4b+2}=\frac{49}{50}$$

$(b^3-3b-2)50 = 49(b^3-3b+2)$
$b^3-150b+147b-100-98=0$
$b^3-3b-198=0$
$\Rightarrow b=6$

Q.16 **(3)**

$$\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$$

$$\text{Let } I=\int_0^{\sqrt{2}}\frac{2-x^2}{(2+x^2)\sqrt{4+x^4}}dx$$

$$=\int_0^{\sqrt{2}}\frac{(2-x^2)}{x(\frac{2}{x}+x)x\sqrt{\frac{4}{x^2}+x^2}}dx$$

$$=\int_0^{\sqrt{2}}\frac{(\frac{2}{x^2}-1)}{(\frac{2}{x}+x)\sqrt{(x+\frac{2}{x})^2-2^2}}dx$$

$$\text{Put } x+\frac{2}{x}=t$$

$$\left(1-\frac{2}{x^2}\right)dx=dt$$

$$I=-\int_{\infty}^{2\sqrt{2}}\frac{dt}{t.\sqrt{t^2-2^2}}$$

$$=\frac{1}{2}\left[\sec^{-1}\frac{t}{2}\right]_{2\sqrt{2}}^{\infty}$$

$$=\frac{1}{2}\left[\sec^{-1}(\infty)-\sec^{-1}\left(\sqrt{2}\right)\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{2}-\frac{\pi}{4}\right]$$

$$=\frac{\pi}{8}$$

$$\text{Answer}=\frac{24}{\pi}\times\frac{\pi}{8}=3$$

Q.17 (4)

$$f(x)=\frac{|x^3+x|}{(e^{x|x|}+1)}dx$$

$$\int_{-2}^{2}f(x)\,dx=\int_0^2(f(x)+f(-x))dx$$

$$=\int_0^2\left(\frac{|x^3+x|}{(e^{x|x|}+1)}+\frac{|-x^3-x|}{(e^{-x|-x|}+1)}\right)dx$$

$$=\int_0^2\left(\frac{|x^3+x|}{(e^{x|x|}+1)}+\frac{|x^3+x|}{(e^{-x|x|}+1)}\right)dx$$

$$=\int_0^2\left(\frac{x^3+x}{(e^{x^2}+1)}+\frac{x^3+x}{(e^{-x^2}+1)}\right)dx$$

$$I=\int_0^2\left(\frac{x^3+x}{(1+ex^2)}+\frac{ex^2(x^3+x)}{(1+e^{x^2})}\right)dx$$

$$\int_0^2(x^3+x)dx$$

$$=\left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2$$

$=4+2=6$

Q.18 **(2)**

At right hand vicinity of $x=0$ given equation does not satisfy

$$\because \text{LHS}=\int_{1^-}^{1}t^2f(t)\,dt=0,\ \text{RHS}=\lim_{x\to0^+}(\sin^3x+\cos x)=1$$

LHS $\neq$ RHS hence data given in question is wrong hence BONUS

Correct data should have been

$$=\int_{1^-}^{1}t^2f(t)\,dt=\sin^3x+\cos x-1$$

Calculation for option

differentiating both sides

$-\cos^2x\, f(\cos x)\cdot(-\sin x)=3\sin^2x\cdot\cos x-\sin x$

$\Rightarrow f(\cos x)=3\tan x-\sec^2x$

$\Rightarrow f'(\cos x)(-\sin x)=3\sec^2x-2\sec^2x\tan x$

$$\Rightarrow f'(\cos x)(-\sin x)=\frac{3}{\cos^2x}-\frac{2\sin x}{\cos^3x}$$

$$\Rightarrow f'(\cos x)\cos x=\frac{2}{\cos^2x}-\frac{3}{\sin x\cdot\cos x}$$

When $\cos x=\frac{1}{\sqrt{3}};\ \sin x=\frac{\sqrt{2}}{\sqrt{3}}$

$$f'\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{3}}=6-\frac{9}{\sqrt{2}}$$

Q.19 **(1)**

$$\int_0^1\frac{1}{7^{\left[\frac{1}{x}\right]}}dx=-\int_1^0\frac{1}{7^{\left[\frac{1}{x}\right]}}dx$$

$$= (-1)\left[\int_{1}^{\frac{1}{2}} \frac{1}{7}\,dx + \int_{\frac{1}{2}}^{\frac{1}{3}} \frac{1}{7^2}\,dx + \int_{\frac{1}{3}}^{\frac{1}{4}} \frac{1}{7^3}\,dx + \ldots\infty\right]$$

$$= \left(\frac{1}{7} + \frac{1}{2.7^2} + \frac{1}{3.7^3} + \infty\right) - \left(\frac{1}{7^2} + \frac{1}{7^2.3} + \frac{1}{7^3.4} + \ldots\infty\right)$$

$$= -\ln\left(1-\frac{1}{7}\right) - 7\left(\frac{1}{7^2.2} + \frac{1}{7^3.3} + \frac{1}{7^4.4} + \ldots\infty\right)$$

$$\left[\text{as } \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\infty\right]$$

$$\left[\text{as } \ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\infty\right]$$

$$= -\ln\frac{6}{7} - 7\left(-\ln\left(1-\frac{1}{7}\right) - \frac{1}{7}\right)$$

$$= 6\ln\frac{6}{7} + 1$$

Q.20 (3)

$$I = \int_0^{\pi} \frac{e^{\cos x}\sin x dx}{(1+\cos^2 x)(e^{\cos x} + e^{-\cos x})} \quad .....(A)$$

Replace $x \to \pi - x$

$$I = \int_0^{\pi} \frac{e^{-\cos x}.\sin x dx}{(1+\cos^2 x)(e^{\cos x} + e^{-cox})} \quad .....(B)$$

Add (A) and (B)

$$2I = \int_0^{\pi} \frac{\sin x dx}{1+\cos^2 x}$$

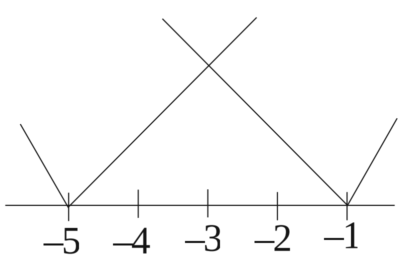
$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{1+\cos^2 x}$$

Put
cosx = t
-sinx dx =dt

$$I = -\int_1^0 \frac{dt}{1+t^2} = [\tan^{-1} t]_0^1 = \frac{\pi}{4}\, Ans.$$

Q. 21 (21)

f(x) = max{|x+1|, |x + 2|, |x + 3|, |x + 4|, x + 5|}



$$\int_{-6}^{0} f(x)dx = \int_{-6}^{-3} |x+1|\,dx + \int_{-3}^{0} |x+5|dx$$

$$= -\int_{-6}^{-3} x+1dx + \int_{-3}^{0} x+5dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{9}{2} - 3\right) - (18-6)\right] + \left[0 - \left(\frac{9}{2} - 15\right)\right]$$

$$= -\left[\frac{3}{2} - 12\right] + \frac{21}{2} = \frac{21}{2} + \frac{21}{2} = 21$$

Q. 22 (6)

$$I = \frac{48}{\pi^4}\int_0^{\pi} x^2\left(\frac{3\pi}{2} - x\right)\frac{\sin x}{1+\cos^2 x}dx \quad ...(1)$$

Apply king property

$$I = \frac{48}{\pi^4}\int_0^{\pi} (\pi - x)^2\left(\frac{\pi}{2} + x\right)\frac{\sin x}{1+\cos^2 x}dx \quad ...(2)$$

(1) + (2)

$$I = \frac{12}{\pi^3}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\left[\pi^2 + (\pi-2).x.(\pi-2x)\right]dx \quad ...(3)$$

Apply king again

$$I = \frac{12}{\pi^3}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\left[\pi^2 + (\pi-2)(\pi-x)(2x-\pi)\right]dx \quad (4)$$

(3)+(4)

$$I = \frac{6}{\pi^2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}[2\pi + (\pi-2)(\pi-2x)]dx \quad ...(5)$$

Apply king

$$I = \frac{6}{\pi^2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}[2\pi + (\pi-2)(2x-\pi)]dx$$

....(6)

(5)+(6)

$$I = \frac{12}{\pi}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

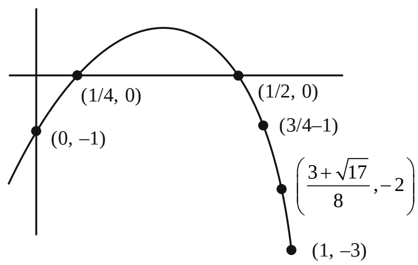
Let cox=t $\Rightarrow$ sinxdx = –dt

$$I = \frac{12}{\pi}\int_{1}^{-1}\frac{-dt}{1+t^2} = 6$$

Q.23 (3)

$$\int_0^1 [-8x^2 + 6x - 1]\,dx$$

$$= \int_0^{1/4}(-1)\,dx + \int_{1/4}^{1/2}(0)\,dx + \int_{1/2}^{3/4}(-1)\,dx$$



$$+ \int_{3/4}^{\frac{3+\sqrt{17}}{8}}(-2)\,dx + \int_{\frac{3+\sqrt{17}}{8}}^{1}(-3)\,dx$$

$$= -[x]_0^{1/4} + 0 - [x]_{1/2}^{3/4} + -2[x]_{3/4}^{\frac{3+\sqrt{17}}{8}} - 3[x]_{\frac{3+\sqrt{17}}{8}}^{1}$$

$$= -\left(\frac{1}{4} - 0\right) - \left(\frac{3}{4} - \frac{1}{2}\right) - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)$$

$$= -\frac{1}{4} - \frac{1}{4} + \frac{-6-2\sqrt{17}}{8} + \frac{3}{2} - 3 + \frac{9+3\sqrt{17}}{8}$$

$$= \frac{\sqrt{17}-13}{8}$$

Q.24 **(385)**

if $f(x) = \min\{[x-1], [x-2], ... [x-10]\}$

So $f(x) = [x-10] = [x] - 10$

$$= \int_0^{10}([x]-10)\,dx + \int_0^{10}([x]-10)^2\,dx + \int_0^{10}(|[x]-10)\,dx$$

$$= \int_0^{10}[x]\,dx - 10\int_0^{10}1\,dx + (10^2 + 9^2 + 1^2) + (10 + 9 + ... + 1)$$

$$= \frac{10\times 9}{2} - 100 + \frac{1}{6}10\times 11\times 21 + \frac{10\times 11}{2}$$

$$= -55 + 385 + 55 = 385$$

Q.25 **(12)**

$$f(x) = \frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right)dx$$

Let $\lambda^2 = \frac{3}{x}$ $\qquad \lambda = 0\,\frac{3}{x} = 0 \to x \to \infty$

$$2\lambda\,d\lambda = -\frac{3}{x^2}dx$$

$$\lambda = \sqrt{3}, 3 = \frac{3}{x} \Rightarrow x = 1$$

$$d\lambda = -\frac{3}{\lambda^2 x^2}dx$$

$$f(x) = \frac{2}{\sqrt{3}}.\int_\infty^1 f(1)\frac{(-3)}{2\lambda x^2}\,dx$$

$$f(x) = \frac{2}{\sqrt{3}}.f(1).\frac{(-3)}{2\lambda}\int_\infty^1 \frac{1}{x^2}dx$$

$$f(x) = \frac{2}{\sqrt{3}}.\sqrt{3}.\frac{(-3)}{2\lambda}\left(-\frac{1}{x}\right)_\infty^1 \qquad (\because f(1) = \sqrt{3})$$

$$f(x) = \frac{+3}{\lambda} \qquad \lambda^2 = \frac{3}{x}$$

$$f(x) = \frac{3}{\frac{\sqrt{3}}{\sqrt{x}}} = \sqrt{3x} \qquad \lambda = \frac{\sqrt{3}}{\sqrt{x}}$$

$$\lambda = \frac{\sqrt{3}}{x}$$

$f(\alpha) = \sqrt{3\alpha} = 6 \Rightarrow 3\alpha = 36$

$\alpha = 12$

Q.26 (1)

$$I_n(x) = \int_0^x \frac{dt}{(t^2+5)^n}$$

Applying integral by parts

$$I_n(x) = \left[\frac{t}{(t^2+5)^n}\right]_0^x - \int_0^x n(t^2+5)^{-n-1}\cdot 2t^2$$

$$I_n(x) = \frac{x}{(x^2+5)^n} + 2n\int_0^x \frac{t^2}{(t^2+5)^{n+1}}dt$$

$$I_n(x) = \frac{x}{(x^2+5)^n} + 2n\int_0^x \frac{(t^2+5)-5}{(t^2+5)^{n+1}}dt$$

$$I_n(x) = \frac{x}{(x^2+5)^n} + 2nI_n(x) - 10n\ I_{n+1}(x)$$

$$10_n I_{n+1}(x) + (1-2)I_n(x) = \frac{x}{(x^2+5)^n}$$

Put n = 5

Q.27 (104)

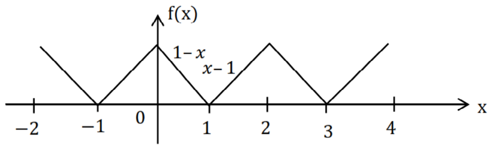
$$I = 60\int_0^{\frac{\pi}{2}}\left(\frac{\sin 6x - \sin 4x}{\sin x} + \frac{\sin 4x - \sin 2x}{\sin x} + \frac{\sin 2x}{\sin x}\right)dx$$

$$I = 60\int_0^{\frac{\pi}{2}}\left(2\cos 5x + 2\cos 3x + 2\cos x\right)dx$$

$$I = 60\left(\frac{2}{5}\sin 5x + \frac{2}{3}\sin 3x + 2\sin x\right)\Bigg|_0^{\pi/2} = 104$$

Q.28 (1)

$$f(x) = \begin{cases} \{x\} & [x] \text{ odd} \\ 1-\{x\} & [x] \text{ even} \end{cases}$$



f(x) is periodic with period 2 and even ins

Hence $I = \frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x$

↓ Jack property (P–7) & (P–4)

$$I = \frac{\pi^2}{20}2.5\int_0^2 f(x)\cos \pi x$$

$$I = \pi\left[\int_0^1 (1-x)\cos \pi x dx + \int_1^2 (x-1)\cos \pi x dx\right]$$

Using by prop.

$$I = \pi^2 . \frac{4}{\pi^2} \Rightarrow \boxed{I = 4}$$

Q.29 (2)

$$\frac{dy}{dx} = \frac{2e^{2x} - 6x^{-x} + 9}{2 + 9e^{-2x}}$$

$$\int_{\frac{1}{2}+\frac{\pi}{2\sqrt{2}}}^{\frac{1}{2}e^{2\alpha}} dy = \int_0^{\alpha} \frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}} dx$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_0^{\alpha} \frac{\left(2e^{2x} - 6e^{-x} + 9\right)}{\left(2 + 9e^{-2x}\right)} \frac{e^{2x}}{e^{2x}} dx$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_0^{\alpha} \frac{2e^{4x} + 9e^{2\pi} - 6e^{x}}{2e^{2x} + 9} dx$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_1^{e^{\alpha}} \frac{2t^3 + 9t - 6}{2t^2 + 9} dt$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \int_1^{e^{\alpha}} \left(t - \frac{6}{2t^2 + 9}\right) dt$$

$$\frac{1}{2}e^{2\alpha} - \frac{1}{2} - \frac{\pi}{2\sqrt{2}} = \frac{t^2}{2}\int_1^{e^{\alpha}} - 3\int_1^{e^{\alpha}} \frac{dt}{t^2 + \frac{9}{2}}$$

$$\frac{e^{2\alpha} - 1}{2} - \frac{\pi}{2\sqrt{2}} = \frac{e^{2\alpha} - 1}{2} - 3\left(\frac{\sqrt{2}}{3}\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)\right)$$

$$\frac{\pi}{2\sqrt{2}} = \sqrt{2}\left[\tan^{-1}\left(\frac{\sqrt{2}e^{\alpha}}{3}\right) - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)\right]$$

$$\frac{\pi}{4} = \tan^{-1}\left(\frac{\left(\sqrt{2}e^{\alpha} - \sqrt{2}\right)3}{9 + 2e^{\alpha}}\right)$$

$$\frac{3\sqrt{2}\left(e^{\alpha} - 1\right)}{2e^{\alpha} + 9} = 1 \Rightarrow \left(e^{\alpha} - 1\right)3\sqrt{2} = 2e^{\alpha} + 9$$

$$e^{\alpha}\left(3\sqrt{2} - 2\right) = 9 + 3\sqrt{2}$$

$$e^{\alpha} = \frac{3\left(3 + \sqrt{2}\right)}{\sqrt{2}\left(3 - \sqrt{2}\right)}$$

Q.30 (5)

$$\lim_{n\to\infty} \frac{1}{n^2}\left(\frac{n+1}{n}\right)^{k-1} . [(nK+1) + (nk+2) + ... + (nk+n)]$$

$$= \lim_{n\to\infty} \frac{1}{n} . \left[1 + \frac{1}{n}\right]^{k-1} \left[\Sigma\left(K + \frac{r}{n}\right)\right]$$

$$=33 \lim_{n\to\infty} \frac{1}{n}\left[\Sigma\left(\frac{r}{n}\right)^K\right]$$

$$\int_0^1 (K + x)dx = 33\int_0^1 x^k dx$$

$$K + \frac{1}{2} = 33\left(\frac{1}{k+1}\right)$$

$(2k+1)(K+1) = 66$

$2K^2 + 3K - 65 = 0$

$2k^2 + 13K - 10K - 65 = 0$

$(K - 5)(2K + 13) = 0$

$\Rightarrow K = 5$ Ans.

Q.31 (1)

$$I = \int_0^1 [2x - |3x^2 - 3x - 2x + 2| + 1]dx$$

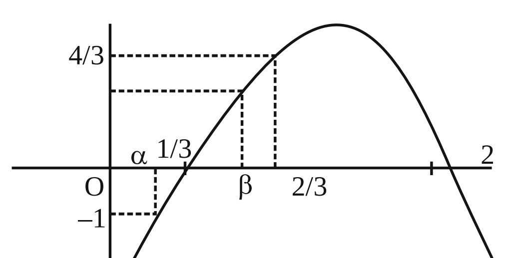
$$I = \int_0^1 [2x - |(3x-2)(x-1)|]dx + \int_0^1 1 dx$$

$$I = \int_0^{2/3} [(2x - (3x^2 - 5x + 2))]dx$$

$$+ \int_{2/3}^1 (2x + (3x^2 - 5x + 2))dx + 1$$

$$I = \int_0^{2/3} [-3x^2 + 7x + 2]dx + \int_{2/3}^1 (3x^2 - 3x + 2))dx + 1$$

$y = -3x^2 + 7x - 2$

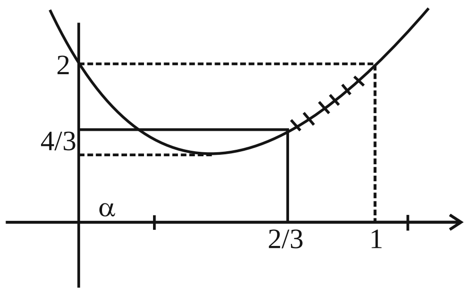


$$\alpha = \frac{7 - \sqrt{37}}{6}, \beta = \frac{7 - \sqrt{13}}{6}$$

$$\int_0^{\alpha}(-2)dx+\int_0^{1/3}(-1)dx+\int_{1/3}^{\beta}0\,dx+\int_{\beta}^{1/3}1.dx$$

$$=-2\alpha-\left(\frac{1}{3}-\alpha\right)+\frac{2}{3}-\beta=-\alpha-\beta+\frac{1}{3}$$

$y=3x^2-3x+2$



When $x\in\left(\frac{2}{3},1\right)$

$$3x^2-3x+2\in\left(\frac{4}{3},2\right)$$

$[3x^2-3x+2]=1$

$$\therefore\int_{2/3}^{1}[3x^2-3x+2]\,dx=1\left(1-\frac{2}{3}\right)=\frac{1}{3}$$

Hence, $I=\left(\frac{1}{3}-(\alpha+\beta)\right)+\frac{1}{3}+1$

$$=\frac{5}{3}-\left(\frac{7-\sqrt{37}+7+\sqrt{13}}{6}\right)$$

$$=\frac{-2}{3}+\frac{\sqrt{37}+\sqrt{13}}{6}$$

$$=\frac{\sqrt{37}+\sqrt{13}-4}{6}$$

Q.32 (2)

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}\cdot dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Put $\tan\frac{x}{2}=t,$ so

$$I=\int_0^{1}\frac{dt}{(t+1)^2+1}=\tan^{-1}(x+1)\Big|_0^1=\tan^{-1}2-\frac{\pi}{4}$$

Q.33 (4)

$$f(e^3)=\int_1^{e^3}\frac{\ell nt}{\ell n10(1+t)}dt \qquad ...(1)$$

$$f(\alpha)=\int_1^{\alpha}\frac{\ell nt}{(\ell n10)(1+t)}dt$$

$$t=\frac{1}{x}\Rightarrow x=\frac{1}{t}$$

$$dt=\frac{-1}{x^2}dx$$

$$=\int_1^{\frac{1}{\alpha}}\frac{-\ell nx}{(\ell n10)\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)dx$$

$$f(\alpha)=\frac{1}{\ell n10}\int_1^{\frac{1}{\alpha}}\frac{\ell nx}{x(x+1)}dx$$

$$f(e^{-3})=\frac{1}{\ell n10}\int_1^{e^3}\frac{\ell nx}{t(t+1)}dt \qquad ...(2)$$

Add (1) & (2)

$f(e^3)+f(e^{-3})$

$$=\left(\frac{1}{\ell n10}\right)\int_1^{e^3}\frac{\ell nt}{(1+t)}\left[1+\frac{1}{t}\right]dt$$

$$=\left(\frac{1}{\ell n10}\right)\int_1^{3}\frac{\ell nt}{t}dt$$

$\ell nt=r$

$$\frac{dt}{t}=dr$$

$$=\frac{1}{\ell n10}\int_0^{3}rdr$$

$$=\left(\frac{1}{\ell n10}\right)\left(\frac{r^2}{2}\right)\Bigg|_0^3$$

$$=\left(\frac{1}{\log 10}\right)\left(\frac{9}{2}\right)$$

$$=\frac{9}{2\log_e 10}$$

Q.34 (1)

$$f(x)=e^x.\int_0^{x}\frac{f'(t)}{e^t}dt$$

$f'(x)=e^{x.}$

$$\int_0^{x}\frac{f'(t)}{e^t}dt+e^x.\frac{f'(x)}{e^x}-[2x-1].e^x+(x^2-x+1).e^x]$$

$$\int_0^{x}\frac{f'(t)}{e^t}dt=x^2+x$$

$$\frac{f'(x)}{e^x} = 2x+1$$

$f'(x) = (2x + 1).e^x$

$f'(x) = 0 \quad \Rightarrow \quad x = -\frac{1}{2}$

$f(x) = (2x+1).e^x - 2e^x + C$

$f(0) = -1$

$-1=1-2+C$

$C = 0$

$f(x) = e^x(2x-1)$

$$f\left(-\frac{1}{2}\right) = \frac{-2}{\sqrt{e}}$$

Q.35 (10)

Put $1 + x^2 = t^2$

$2x\,dx = 2t\,dt$

$x\,dx = t\,dt$

$$\therefore \int_1^2 \frac{15(t^2-1)t\,dt}{\sqrt{t^2+t^3}}$$

$$15\int_1^2 \frac{t(t^2-1)}{t\sqrt{1+t}}dt$$

Put $1 + t = u^2$

$dt = 2u\,du$

$$15\int_{\sqrt{2}}^{\sqrt{3}} \frac{(u^2-1)^2}{u}\times 2u\,du$$

$$30\int_{\sqrt{2}}^{\sqrt{3}} (u^4-2u^2)du$$

$$30\left(\frac{u^5}{5}-\frac{2u^3}{3}\right)_{\sqrt{2}}^{\sqrt{3}}$$

$$30\left[\frac{1}{5}\left((\sqrt{3})^5-(\sqrt{2})^5\right)-\frac{2}{3}\left((\sqrt{3})^3-(\sqrt{2})^3\right)\right]^3$$

$$30\left[\frac{1}{5}(9\sqrt{3}-4\sqrt{2})-\frac{2}{3}(3\sqrt{3}-2\sqrt{2})\right]$$

$$30\left[-\frac{1}{5}\times\sqrt{3}+\frac{8}{15}\sqrt{2}\right]$$

$$-6\sqrt{3}+16\sqrt{2} = \alpha\sqrt{2}+\beta\sqrt{3}$$

$\alpha = 16, \beta = -6$

$\therefore \alpha + \beta = 10$

Q.36 (2)

$$\lim_{x\to -1^+} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x] = -a+2$$

$$\lim_{x\to -1^-} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x] = 0+3=3$$

$\lim_{x\to -1} f(x)$ exist when $a = -1$

Now,

$$\int_0^4 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^3 f(x)dx + \int_3^4 f(x)dx$$

$$= \int_0^1 (0+1)dx + \int_1^2 (-1+0)dx + \int_2^3 (0-1)dx + \int_3^4 (1-2)dx$$

$= 1 - 1 - 1 - 1 = -2$

Q.37 (3)

Consider

$f(x) = 8\sin x - \sin 2x$

$f'(x) = 8\cos x - 2\cos 2x$

$f''(x) = -8\sin x + 4\sin 2x$

$$\therefore f''(x) < 0 \qquad x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$$

$\therefore f'(x)$ is $\downarrow$ function

$$f'\left(\frac{\pi}{3}\right) < f'(x) < f'\left(\frac{\pi}{4}\right)$$

$$5 < f'(x) < \frac{8}{\sqrt{2}}$$

$$5 < f'(x) < 4\sqrt{2}$$

$$5x < f(x) < 4\sqrt{2}x$$

$$5 < \frac{f(x)}{x} < 4\sqrt{2}$$

$$\int_{\pi/4}^{\pi/3} 5 < \int \frac{f(x)}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$\int_{\pi/4}^{\pi/3} 5 < \int \frac{8\sin x - \sin 2x}{x} < \int_{\pi/4}^{\pi/3} 4\sqrt{2}$$

$$\frac{5\pi}{12} < I < \frac{\sqrt{2}\pi}{3}$$

Q.38 (4)

$$\int_0^{20\pi} (|\sin x|+|\cos x|)^2 dx$$

$$\int_0^{20\pi} (1+|\sin 2x|)dx$$

$$20\pi + 40\int_0^{\frac{\pi}{2}} |\sin 2x|dx$$

$$20\pi + 40\int_0^{\frac{\pi}{2}} \left(-\frac{\cos 2x}{2}\right)_0^{\frac{\pi}{2}}$$

$20\pi - 20\{\cos\pi - \cos 0\}$

$20\pi + 40 \Rightarrow 20(\pi + 2)$

Q.39 (3)

$$a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n\left[1+\left(\frac{k}{n}\right)^2\right]}$$

$$a = \int_0^1 \frac{2dx}{1+x^2} \qquad = (2\tan^{-1}x)_0^1$$

$$a = \frac{\pi}{2}$$

$$f(x) = \left|\tan\frac{x}{2}\right|$$

$$f(x) = \tan\left(\frac{x}{2}\right)$$

$$f\left(\frac{a}{2}\right) = \tan\left(\frac{a}{2}\right) \Rightarrow \frac{x}{2}\in\left(0, \frac{1}{2}\right)$$

$$f'(x) = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

$$f'\left(\frac{a}{2}\right) = \frac{1}{2}\sec^2\left(\frac{a}{4}\right)$$

$$f'\left(\frac{\pi}{4}\right) = \frac{1}{2}\sec^2\left(\frac{\pi}{8}\right) = \frac{1}{2}\left[1+\tan^2\frac{\pi}{8}\right]$$

$$= \frac{1}{2}\left[1+\left(\sqrt{2}-1\right)^2\right] = \frac{1}{2}\left[4-2\sqrt{2}\right] \quad = 2-\sqrt{2}$$

$$f'\left(\frac{\pi}{4}\right) = \sqrt{2}f\left(\frac{\pi}{4}\right)$$

$$f'\left(\frac{a}{2}\right) = \sqrt{2}f\left(\frac{a}{2}\right)$$

Q.40 (24)

$$I_1 = \int_0^1 (1-x^n)^{2n+1}\cdot I dx$$

$$\text{Let } I_2 = \int_0^1 (1-x^n)^{2n} dx$$

$$= [(1-x^n)^{2n+1}\cdot x]_0^1 - \int_0^1 (2n+1)(1-x^4)^{2n}(-nx^{n-1}\cdot n)\,dx$$

$$= (1-x^n)^{2n+1}dx = n(2n+1)\int_0^1 [(1-x^n)^{2n}]x^n\,dx$$

$$= -(n\,|\,2n+1\,|)\int_0^1 [(1-x^n)^{2n}][1-x^n-1]\,dx$$

$$I_1 = -n(2n+1)\left(\int_0^1 (1-x^n)^{2n+1}dx - \int_0^1 (1-x^n)^{2n}dx\right)$$

$I_1 = -n(2n + 1)\, I_1 + n(2n + 1)I_2$

$= (1 + n(2n + 1)]I_1 = n(2n + 1)I_2$

$\therefore\ 1 + n(2n + 1) = 1177 \Rightarrow 2n^2 + n + 1176 = 0$

$n = 24$

Q.41 (4)

Let $f(x) = 2 + |x| - |x-1| + |x+1|$

$$f(x) = \begin{cases} -x & x < -1 \\ x+2 & -1 \le x < 0 \\ 3x+2 & 0 \le x < 1 \\ x+4 & x \ge 1 \end{cases}$$

$$f'(x) = \begin{cases} -1 & x < -1 \\ 1 & -1 \le x < 0 \\ 3 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

S (1) : $f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right) = -1 + 1 + 3 + 1 = 4$

S(2): $\int_{-2}^{2} f(x)\,dx = \int_{-2}^{-1} -x\,dx + \int_{-1}^{0}(x+2)\,dx + \int_0^1 (3x+2)\,dx + \int_1^2 (x+4)\,dx = \frac{3}{2}+\frac{3}{2}+\frac{7}{2}+\frac{11}{2} = \frac{24}{12} = 12$

Q.42 (2)

$$\int_0^2 \left|2x^2-3x\right|dx + \left[x-\frac{1}{2}\right]dx$$

$$= \int_0^2 \left|2x^2-3x\right|dx + \int_0^2\left[x-\frac{1}{2}\right]dx$$

$$= \int_0^{3/2}(3x-2x^2)\,dx + \int_{3/2}^2 (2x^2-3x)\,dx + \int_0^{1/2} -1dx + \int_{1/2}^{3/2} 0\,dx + \int_{3/2}^2 1\,dx$$

$$= \left(\frac{3x^2}{2}-\frac{2x^3}{3}\right)\Bigg|_0^{3/2} + \left(\frac{2x^3}{3}-\frac{3x^2}{2}\right)\Bigg|_{3/2}^2 + \left(-\frac{1}{2}\right) + \left(2-\frac{3}{2}\right)$$

$$= \frac{27}{8}-\frac{9}{8}+\frac{16}{3}-6-\frac{9}{4}-\frac{1}{2}-+\frac{1}{2}$$

$$= \frac{9}{2}+\frac{64-72-27}{72} = \frac{9}{2}+\frac{64-99}{12} = \frac{19}{12}$$

Q.43 (3)

$$\text{Let } \lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+\ldots+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$$

Let $2^n = t$

$n \to \infty \Rightarrow t \to \infty$

$$S = \lim_{t\to\infty}\frac{1}{t}\left(\frac{1}{\sqrt{1-\frac{1}{t}}}+\frac{1}{\sqrt{1-\frac{2}{t}}}+\ldots+\frac{1}{\sqrt{1-\frac{t-1}{t}}}\right)$$

$$S = \lim_{t\to\infty}\frac{1}{t}\left(\sum_{k=1}^{t-1}\frac{1}{\sqrt{1-\frac{k}{t}}}\right)$$

$$S = \int_0^1 \frac{1}{\sqrt{1-x}}dx = (-2)[\sqrt{1-x}]_0^1$$

$= (-2)[0-1]$

$= 2$

Q.44 (2)

Let $I = \int_{-3}^{101}([\sin\pi x] + e^{[\cos 2\pi x]})\,dx$

By using Jack property

$\therefore\ I = 52\int_0^2 [\sin\pi x]\,dx + 104\int_0^1 e^{[\cos 2\pi x]}dx$

$$= 52\int_1^2(-1)\,dx + 104\left[\int_0^{\frac{1}{4}} e^0 dx + \int_{\frac{1}{4}}^{\frac{3}{4}} e^{-1}dx + \int_{\frac{3}{4}}^{1} e^0 dx\right]$$

$$= \frac{52}{e}$$

Q.45 [8]

$$x\int_0^x f'(t)dt - \int_0^x t f'(t)dt = (e^{2x} + e^{-2x})\cos 2x + \frac{2x}{a}$$

$$f'(x) + \int_0^x f'(t)dt + x\cdot f'(x) - xf'(x)$$

$$= 2(e^{2x} - e^{-2x})\cos 2x - 2(e^{2x} + e^{-2x})\sin 2x + \frac{2}{a}$$

Put $x = 0$

$4 + 0 = 2(e^0 - e^0).\cos 2x$

$$-2(e^0 - e^0).0 + \frac{2}{a}$$

$$4 = \frac{2}{a} \Rightarrow a = \frac{1}{2}$$

$2a = 1$

$$\therefore\ (2a+1)^5\cdot a^2 = 2^5 \times \frac{1}{4} = 8$$

Q.46 [5]

$$a_n = \int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{3} + \ldots + \frac{x^{n-1}}{n}\right)dx \ \Rightarrow a_n \text{ is increasing}$$

$$a_1 = \int_{-1}^{1} dx = 2$$

$$a_2 = \int_{-1}^{2}\left(1 + \frac{x}{2}\right)dx = \left[x + \frac{x^2}{4}\right]_{-1}^{2}$$

$$= (3) - ((-1) + \frac{1}{4})$$

$$= 4 - \frac{1}{4} = \frac{15}{4}$$

$$a_3 = \int_{-1}^{3}\left(1 + \frac{x}{2} + \frac{x^2}{3}\right)dx = \left(x + \frac{x^2}{4} + \frac{x^3}{9}\right)_{-1}^{3}$$

$$= \left(3 + \frac{9}{4} + \frac{27}{9}\right) - \left(-1 + \frac{1}{4} - \frac{1}{9}\right)$$

$$= 4 + 2 + \frac{28}{9} = 6 + \frac{28}{9} = \frac{82}{9}$$

$$a_4 = \int_{-1}^{4}\left(1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4}\right)dx$$

$$= \left[x + \frac{x^2}{4} + \frac{x^2}{9} + \frac{x^4}{16}\right]_{-1}^{4}$$

$$= \left(4 + \frac{16}{4} + \frac{64}{9} + \frac{256}{16}\right) - \left(-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16}\right)$$

$$= 24 + 7 + \frac{1}{9} + 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} > 30$$

So, $\{n \in N : a_n \in (2, 30)\} = \{2, 3\}$

$\therefore$ Sum $= 2 + 3 = 5$

AREA UNDER CURVE

EXERCISE-I (MHT CET LEVEL)

Q.1 (3)

Given curve $y = \log x$ and $x = 1$, $x = 2$.

Hence required area = $\int_1^2 \log x\,dx = (x\log x - x)_1^2 =$

$2\log 2 - 1 = (\log 4 - 1)$ sq. unit.

Q.2 (2)

Required area is $\int_0^a y\,dx = \int_0^a xe^{x^2}\,dx$

We put $x^2 = t \Rightarrow dx = \frac{dt}{2x}$ as $x = 0 \Rightarrow t = 0$ and

$x = a \Rightarrow t = a^2$, then it reduces to

$\frac{1}{2}\int_0^{a^2} e^t dt = \frac{1}{2}[e^t]_0^{a^2} = \frac{e^{a^2} - 1}{2}$ sq. unit.

Q.3 (b)

Required area $= \int_0^{\frac{\pi}{4}} \tan x\,dx$

$= In\left|\sec 1\right|_0^{\frac{\pi}{4}} = In\sqrt{2} = \frac{In2}{2}$

Q.4 (c)

Area = $\int_0^{\pi/2} y dx = \int_0^{\pi/2} \sin x dx = [-\cos x]_0^{\pi/2} = 1$

Q.5 (d)

Given $\int_1^b f(x)dx = \sqrt{b^2+1} - \sqrt{2}$

Differentiate with respect to b

$f(b) = \frac{b}{\sqrt{b^2+1}} \Rightarrow f(x) = \frac{x}{\sqrt{x^2+1}}$

Q.6 (2)

Q.7 **(4)**

Q.8 **(2)**

Q.9 **(3)**

Q.10 (3)

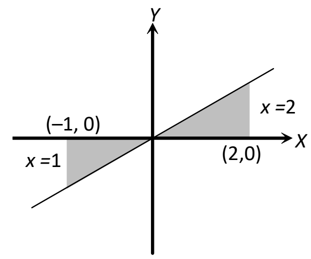
Required area = $\int_1^4 x\,dy = \int_1^4 \frac{\sqrt{y}}{2}dy$

$= \frac{1}{2}.\frac{2}{3}|y^{3/2}|_1^4 = \frac{7}{3}$ sq. unit.

Q.11 (1)

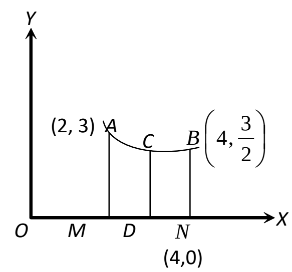
Required area

$\int_{-1}^2 y\,dx = \int_{-1}^0 y.dx + \int_0^2 y.dx = \frac{5}{2}$ sq. unit.



Q.12 (2)

Let the ordinate at $x = a$ divide the area into two equal parts



Area of AMNB $= \int_2^4 \left(1 + \frac{8}{x^2}\right) dx = \left[x - \frac{8}{x}\right]_2^4 = 4$

Area of ACDM $= \int_2^a \left(1 + \frac{8}{x^2}\right) dx = 2$

On solving, we get $a = \pm 2\sqrt{2}$;Since $a > 0 \Rightarrow$

$a = 2\sqrt{2}$

Q.13 (1)

Required area

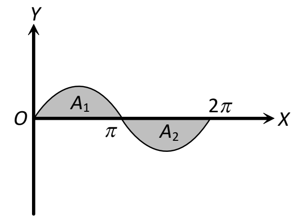
$= k\int_\pi^{2\pi} \sin x\,dx = k[-\cos x]_\pi^{2\pi} = -2k$

Hence, area = 2k sq. unit.

Q.14 (4)

Required area is

$A_1 + A_2 = \int_0^\pi y\,dx + \left|\int_\pi^{2\pi} y\,dx\right| = 4\pi$ sq.unit



Q.15 (2)

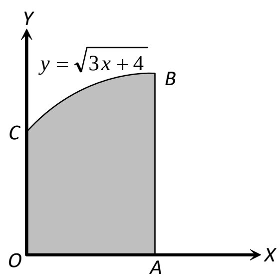
Required area $= \int_0^{\pi/4} (\sin 2x + \cos 2x)dx$

$$= \left[-\frac{\cos 2x}{2} + \frac{\sin 2x}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[-\cos\frac{\pi}{2} + \sin\frac{\pi}{2} + \cos 0 - \sin 0\right] = 1 \text{sq. unit.}$$

Q.16 (4)

Area $= \int_0^4 \sqrt{3x+4}\,dx = \left|\frac{(3x+4)^{3/2}}{3.(3/2)}\right|_0^4$



$= \frac{2}{9} \times 56 = \frac{112}{9}$ sq. unit.

Q.17 (3)
Q.18 **(1)**
Q.19 **(4)**
Q.20 (1)

$y^2 = x$ and $2y = x \Rightarrow y^2 = 2y \Rightarrow y = 0, 2$

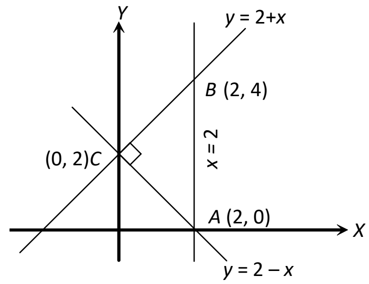
$\therefore$ Required

area $= \int_0^2 (y^2 - 2y)dy = \left(\frac{y^3}{3} - y^2\right)_0^2 = \frac{4}{3}$ sq. unit.

Q.21 (2)
Obviously, triangle ACB is right angled at C.

$\therefore$ Required area $= \frac{1}{2} \times AC \times BC$

$= \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} = 4$ sq. unit.



Q.22 (4)

$A_1 = \int_0^{\pi/3} \cos x\,dx = \frac{\sqrt{3}}{2}$,

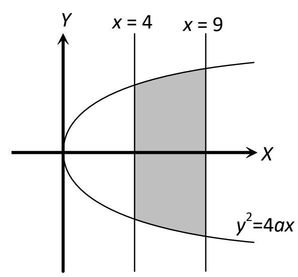
$A_2 = \int_0^{\pi/3} \cos 2x\,dx = \frac{\sqrt{3}}{4}$

$\therefore\ A_1 : A_2 = 2 : 1$

Q.23 (4)

Shaded area $A = 2\int_4^9 \sqrt{4ax}\,dx$

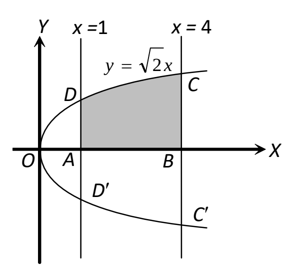
$A_2 = \int_0^{\pi/3} \cos 2x\,dx = \frac{\sqrt{3}}{4}$



Q.24 (2)

$\int_0^2 2^{kx}dx = \frac{3}{\log 2} \Rightarrow 2^{2k} - 1 = 3k$. Now check from options, only (2) satisfies the above condition.

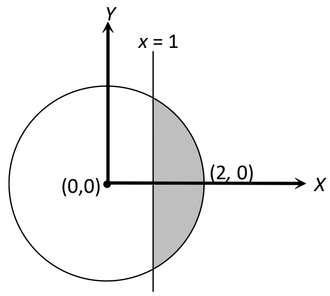
Q.25 (2)
Required area = CDD'C' = 2 × ABCD



$= 2\int_1^4 \sqrt{2}x^{1/2}\,dx = \frac{28\sqrt{2}}{3}$ sq. unit.

Q.26 (2)

Area of smaller part $= 2\int_1^2 \sqrt{4-x^2}\,dx$



$$= 2\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_1^2 = 2\left[2.\frac{\pi}{2} - \left[\frac{\sqrt{3}}{2} - 2.\frac{\pi}{6}\right]\right]$$

$$= 2\left[\pi - \left[\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]\right] = \frac{8\pi}{3} - \sqrt{3}$$

Q.27 (2)
Required area

$$A = \int_0^{\pi/2} \sin^2 x.dx = \int_0^{\pi/2}\left(\frac{1-\cos 2x}{2}\right)dxa$$

$$= \frac{1}{2}[x]_0^{\pi/2} - \frac{1}{4}[\sin 2x]_0^{\pi/2} = \frac{\pi}{4}$$

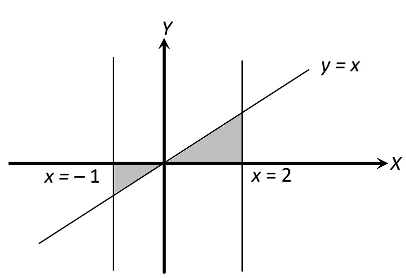
Q.28 (1)

Solving $y = 0$ and $y = 4 + 3x - x^2$, we get $x = -1, 4$. Curve does not intersect x-axis between $x = -1$ and $x = 4$.

$\therefore$ Area $= \int_{-1}^{4}(4+3x-x^2)dx = \frac{125}{6}$.

Q.29 (4)

Bounded area $= \left|\int_{-1}^{0} x\,dx\right| + \left|\int_0^2 x\,dx\right|$



$= \left|-\frac{1}{2}\right| + |2| = 2 + \frac{1}{2} = \frac{5}{2}$

Q.30 (3)

We have $y^2 = 4ax \Rightarrow y = 2\sqrt{ax}$

We know the equations of lines $x = a$ and $x = 4a$

$\therefore$ The area inside the parabola between the lines

$$A = \int_a^{4a} y\,dx = \int_a^{4a} 2\sqrt{ax}\,dx = 2\sqrt{a}\int_a^{4a} x^{\frac{1}{2}}dx = 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^{4a}$$

$$= \frac{4}{3}a^{\frac{1}{2}}\left[(4a)^{\frac{3}{2}} - (a)^{\frac{3}{2}}\right] = \frac{4}{3}a^{\frac{1}{2}}a^{\frac{3}{2}}[8-1] = \frac{28}{3}a^2$$

Q.31 (2)

Given, $y = -x^2 + 2x + 3$ and $y = 0$

Therefore, $x = -1$ and $x = 3$

$\therefore$ Required area $= \int_{-1}^{3}(-x^2 + 2x + 3)dx$

$$= \left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^{3} = \frac{32}{3}$$

Q.32 (3)

Given curves are, $y = x^3$ and $y = \sqrt{x}$

On solving, we get $x = 0, x = 1$

Therefore, required area $= \int_0^1 (x^3 - \sqrt{x})\,dx$

$= \left[\frac{x^4}{4} - \frac{2x\sqrt{x}}{3}\right]_0^1 = \left[\frac{1}{4} - \frac{2}{3}\right] = \frac{5}{12}$, (Area can't be negative).

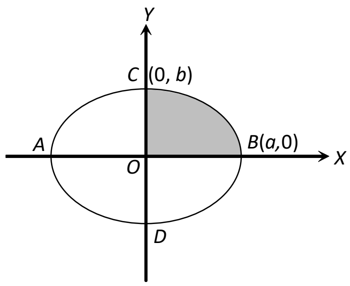
Q.33 (1)
The parabola meets x-axis at the points, where

$\frac{3}{a}(a^2 - x^2) = 0 \Rightarrow x = \pm a.$ So the required area

$= \int_{-a}^{a}\frac{3}{a}(a^2 - x^2)dx = \frac{6}{a}\int_0^a (a^2 - x^2)dx = 4a^2$ sq. unit.

Q.34 (1)

Since the given equation contains only even powers of x and only even powers of y, the curve is symmetrical
about y-axis as well as x-axis.



$\therefore$ Whole area of given ellipse

$$= 4(\text{area of BCO}) = 4\times\int_0^a y\,dx = 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

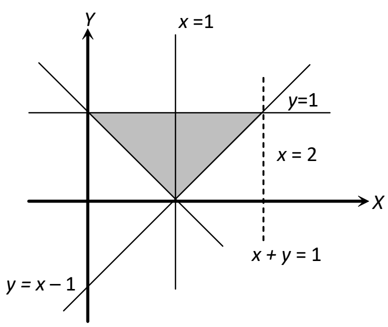
$$= 4ab\int_0^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta\ ,\ \{\text{Putting } x = a\sin\theta\}$$

$$= 2ab\left(\int_0^{\pi/2} d\theta + \int_0^{\pi/2}\cos 2\theta\,d\theta\right)$$

$$= [\theta]_0^{\pi/2} + \left[\frac{\sin 2\theta}{2}\right]_0^{\pi/2} = \pi ab \text{ sq. unit.}$$

Q.35 (2)

$y = x-1$, if $x>1$ and $y = -(x-1)$, if $x<1$

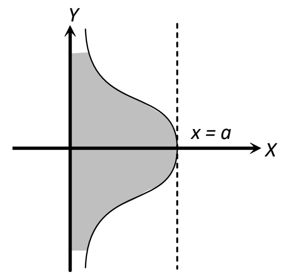


Area

$$= \int_0^1 (1-x)dx + \int_1^2 (x-1)dx = \left[x-\frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^2$$

$$= \left[1-\frac{1}{2}\right] + \left[-\left(\frac{1}{2}-1\right)\right] = \frac{1}{2}+\frac{1}{2} = 1$$

Q.36 (1)



Since the curve is symmetrical about x-axis, therefore

Required area $A = 2\int_0^a a\sqrt{\frac{a-x}{x}}\,dx$

Put $x = a\sin^2\theta$

$\Rightarrow dx = 2a\sin\theta.\cos\theta\,d\theta$

$$A = 2\int_0^{\pi/2} a\sqrt{\frac{a\cos^2\theta}{a\sin^2\theta}}\,a\sin 2\theta\,d\theta$$

$$= 2a^2\int_0^{\pi/.2}\frac{\cos\theta}{\sin\theta}2\sin\theta\cos\theta\,d\theta$$

$$A = 4a^2\int_0^{\pi/2}\cos^2\theta\,d\theta \Rightarrow A = 4a^2.\frac{1}{2}.\frac{\pi}{2} = \pi a^2.$$

Q.37 (2)

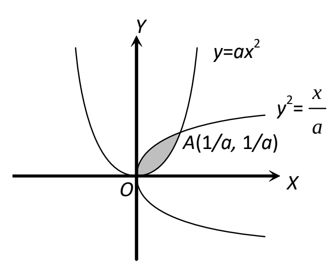
The x-coordinate of A is $\frac{1}{a}$

According to the given condition,

$$1 = \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx$$

$$\Rightarrow 1 = \frac{1}{\sqrt{a}}.\frac{2}{3}[x^{3/2}]_0^{1/a} - \frac{a}{3}[x^3]_0^{1/a} \Rightarrow$$

$$a^2 = \frac{1}{3} \Rightarrow a = \frac{1}{\sqrt{3}}$$



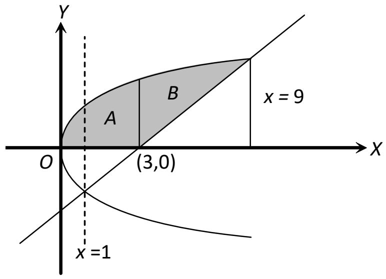
Q.38 (1)

Solving $y^2 = x$ and $x = 2y+3$

$$4y^2 = (x-3)^2,\ 4x = x^2 - 6x + 9$$

$\Rightarrow x^2 - 10x + 9 = 0 \Rightarrow (x-1)(x-9) = 0 \Rightarrow$

$x = 1, 9$



$= -4[x \log x - x]_0^1 = -4(-1) = 4$ sq. unit,

$(\because \lim_{x\to 0} x \log x = 0)$.

Required area =

$$A+B = \int_0^3 \sqrt{x}dx + \int_3^9 \left[\sqrt{x} - \left(\frac{x-3}{2}\right)\right]dx$$

$$= \frac{2}{3}[x^{3/2}]_0^3 + \frac{2}{3}[x^{3/2}]_3^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9$$

$$= \frac{2}{3}3\sqrt{3} + \frac{2}{3}[9\times 3 - 3\sqrt{3}] - \frac{1}{2}\left[\left(\frac{81}{2} - 27\right) - \left(\frac{9}{2} - 9\right)\right]$$

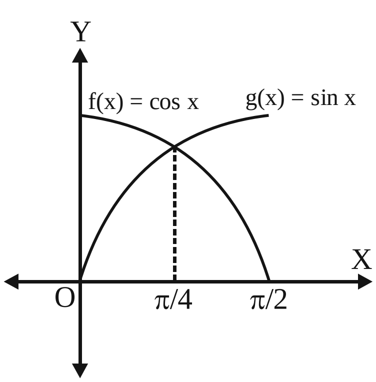
$$= 18 - \frac{1}{2}[36 - 18] = 18 - 9 = 9 \text{ sq. unit.}$$

Q.39 (4) $9x^2 + 4y^2 = 36 \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$, which is equation of an ellipse. Remember area enclosed by ellipse is πab i.e.

$\pi\, 2.3 = 6\pi$.

Q.40 (b)

$y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4} (\cos x - \sin x)\, dx$

$= 2[\sin x + \cos x]_0^{\pi/4}$

$= 2\left[\frac{2}{\sqrt{2}} - 1\right](2\sqrt{2} - 2$sq.units

Q.41 (3)
Q.42 **(2)**
Q.43 **(1)**
Q.44 **(3)**
Q.45 **(2)**
Q.46 **(2)**
Q.47 (3)

Given equations of curves $y = \cos x$ and $y = \sin x$

and ordinates $x = 0$ to $x = \frac{\pi}{4}$. We know that area bounded
by the curves

$$= \int_{x_1}^{x_2} ydx = \int_0^{\pi/4} \cos x dx - \int_0^{\pi/4} \sin x\, dx$$

$$= [\sin x]_0^{\pi/4} - [-\cos x]_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4} - \sin 0\right) + \left(\cos\frac{\pi}{4} - \cos 0\right) = \left(\frac{1}{\sqrt{2}} - 0\right) + \left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= \sqrt{2} - 1$$

Q.48 (1)

Area of the circle in first quadrant is $\frac{\pi(\pi^2)}{4}$ i.e., $\frac{\pi^3}{4}$.

Also area bounded by curve $y = \sin x$ and x-axis is 2 sq.

unit. Hence required area is $\frac{\pi^3}{4} - 2 = \frac{\pi^3 - 8}{4}$.

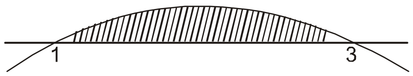
Q.49 (2)

$$\int_0^1 (\sqrt{x} - x^2)dx = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)_0^1 = \frac{1}{3}$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 **(3)**

$y = 0 \Rightarrow x = 1, 3$



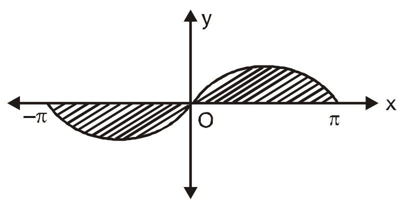
Graph of $y = 4x - x^2 - 3$

$$\text{Area} = \int_1^3 (4x - x^2 - 3)dx = \frac{4}{3}$$

Q.2 **(1)**

$$\text{Area of bounded region} = 2\int_0^{\pi} \sin x \, dx = 2[-\cos x]_0^{\pi}$$

$= 2 \; [1- \; (-1)] = 4$



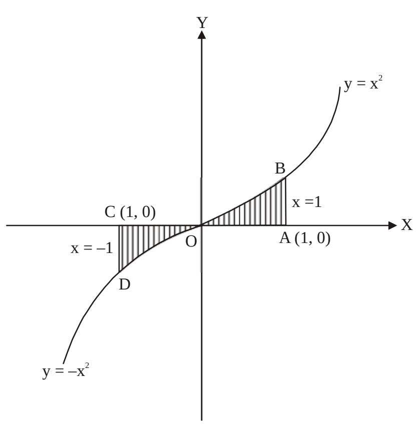
Q.3 (c)

The area of the region bounded by the curve y = f (x) and the ordinatess x = a, x = b is given by

$$\cdot \text{Area} = \left| \int_a^b y \, dx \right|$$

According to the question,

$$y = x|x| = \in \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$



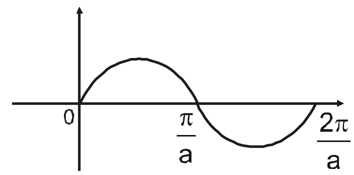
Required area
= area of region OAB + area of region OCD
= 2 × Area of region OAB

$$= 2\int_0^1 x^2 dx = \frac{2}{3} \text{sq.units}$$

Q.4 **(c)**

Q.5 **(2)**

$x = 0, x = \frac{\pi}{a}$ are successive points of inflection

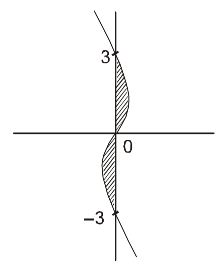


Graph of y = sin ax

$$\text{Area} = \int_0^{\pi/a} \sin ax \, dx = \frac{2}{a}$$

Q.6 **(3)**

$x = 0 \Rightarrow y = 0, -3, 3$



Figure

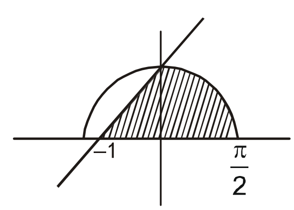
$$\text{Required area} = 2\int_0^3 (9y - y^3)\,dy = \frac{81}{2}$$

Q.7 **(4)**

$$\text{Area} = 2\int_0^2 2\sqrt{x}\,dx = \frac{16\sqrt{2}}{3}$$

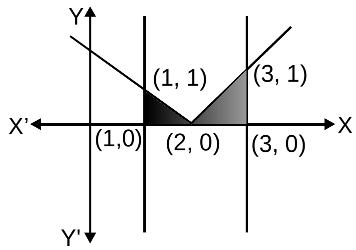
Q.8 **(4)**

From figure it is clear that required



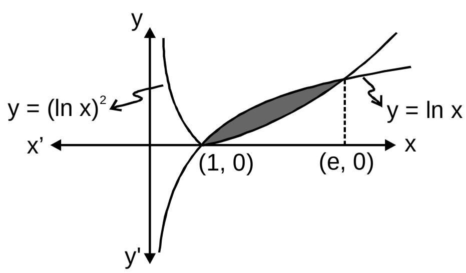
$$\text{area} = \frac{1}{2} + \int_0^{\pi/2} \cos x \, dx = \frac{3}{2}$$

Q.9 **(3)**



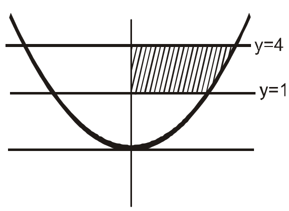
Area A = $\frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 1 \times 1 = 1$

Q.10 **(3)**



$A = \int_1^e (\ln^2 x - \ln x)\, dx = 3 - e$

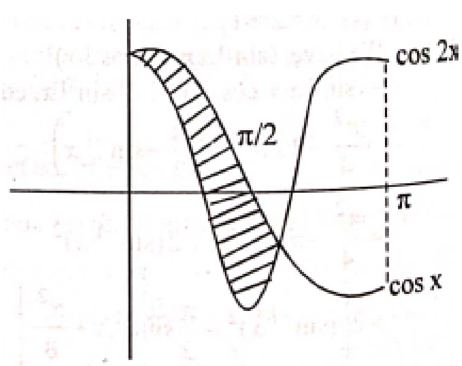
Q.11 **(3)**



Graph of $y = 4x^2$

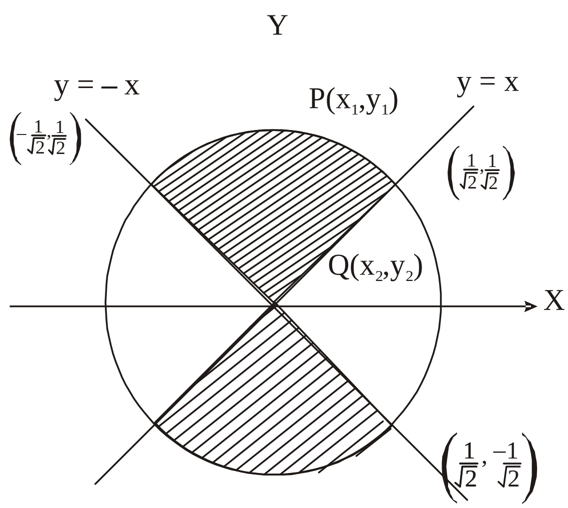
Area = $\int_1^4 \frac{1}{2}\sqrt{y}dy = \frac{7}{3}$

Q.12 (d)



$$\text{Area} = \int_0^{2\pi/3} (\cos x - \cos 2x)dx = \frac{3\sqrt{3}}{4}$$

Q.13 (c)



Required area = 44 (Area of the shaded region in first quadrant)

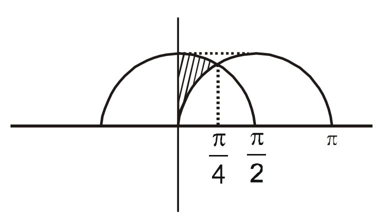
$$= 4\int_0^{1\sqrt{2}} (y_1 - y_2)\, dx = 4\int_0^{1\sqrt{2}} \left(\sqrt{1-x^2} - x\right) dx$$

$$= 4\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x - \frac{x^2}{2}\right]_0^{1/\sqrt{2}}$$

$$= 4\left[\frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{\pi}{4} - \frac{1}{4}\right]$$

$$= 4\left[\frac{1}{4} + \frac{\pi}{8} - \frac{1}{4}\right] = \frac{4\pi}{8} = \frac{\pi}{2} \text{ sq units}$$

Q.14 (c)
Q.15 (b)
Q.16 **(c)**
Q.17 **(3)**



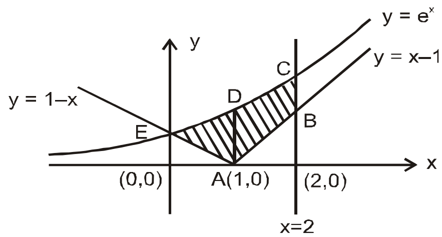
From figure

Area = $\int_0^{\pi/4} (\cos x - \sin x)dx = \sqrt{2} - 1$

Q.18 **(3)**

$$\text{Area} = \int_0^1 \left(e^x - (1-x)\right) dx + \int_1^2 \left(e^x - (x-1)\right) dx$$

$$= \left(e^x - x + \frac{x^2}{2}\right)_0^1 + \left(e^x - \frac{x^2}{2} + x\right)_1^2$$



Figure

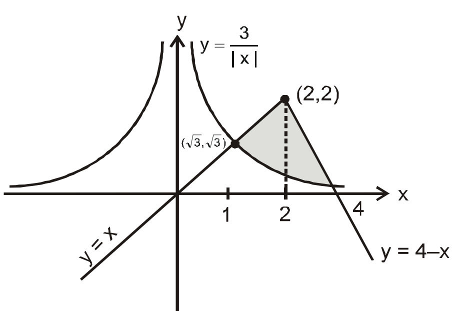
$$= \left(e^1 - 1 + \frac{1}{2}\right) - 1 + \left(e^2 - 2 + 2\right) - \left(e^1 - \frac{1}{2} + 1\right)$$

$= e^2 - 2$

Q.19 **(2)**

$$A = \int_{\sqrt{3}}^{2} \left(x - \frac{3}{x}\right) dx + \int_2^3 \left(4 - x - \frac{3}{x}\right) dx$$

$$= \left(\frac{x^2}{2} - 3\ell nx\right)_{\sqrt{3}}^{2} + \left(4x - \frac{x^2}{2} - 3\ell nx\right)_2^3$$



Figure

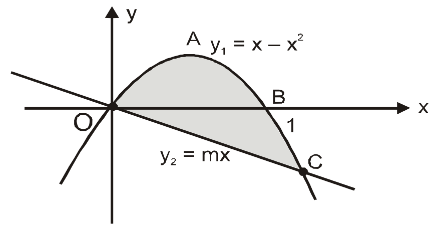
$$= \frac{4 - 3\ell n3}{2}$$

Q.20 $y = x - x^2$: $y = mx$

first find point of intersection : $x - x^2 = mx$

$x^2 + (m-1)x = 0 \Rightarrow x = 0, 1-m$

Case - I



Figure

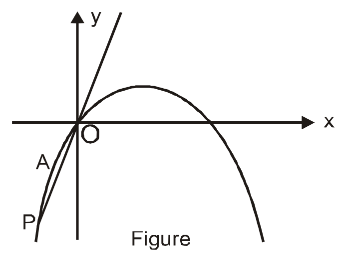
Area of OABCO

$$A = \int_0^{1-m} (y_1 - y_2)\, dx$$

$$= \int_0^{1-m} (x - x^2 - mx)\, dx = 9/2$$

$= \mathbf{m = -2}$

Case - II



Figure

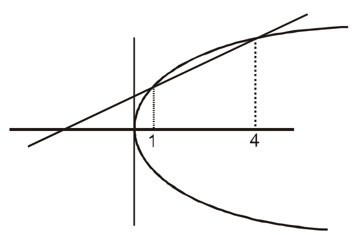
Area of PAOP

$$\int_{1-m}^{0} (x - x^2 - mx)\, dx = 9/2$$

$\Rightarrow m = 4$

Q.21 **(1)**

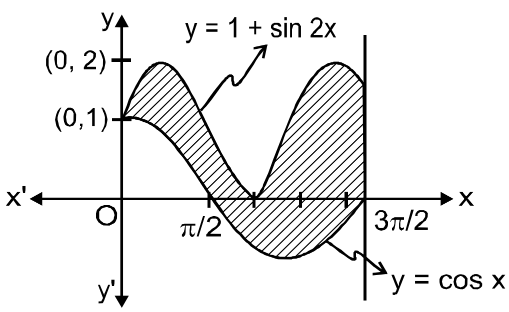
Solving $x = 1, 4$



From graph it is clear that required

$$\text{area} = \int_1^4 \left(2\sqrt{x} - \frac{1}{3}(2x+4)\right) dx = \frac{1}{3}$$

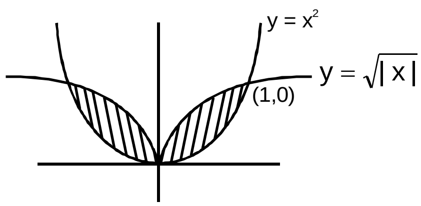
Q.22 **(3)**



$$A = \int_{0}^{3\pi/2} (1 + \sin 2x - \cos x)dx$$

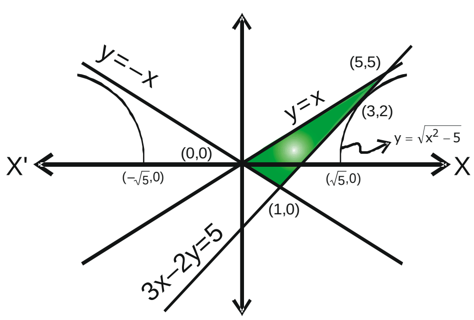
$$A = \int_0^{3\pi/2} (1 + \sin 2x - \cos x)\, dx = 2 + \frac{3\pi}{2}$$

Q.23 **(2)**



$$A = 2\int_0^1 \left(\sqrt{|x|} - x^2\right)dx = \frac{2}{3}$$

Q.24 **(1)**

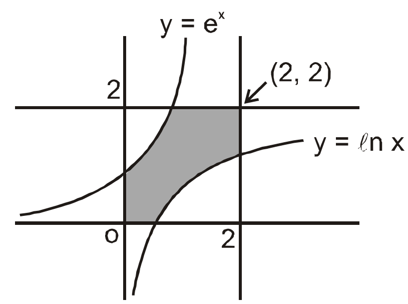


Equation of tangent area of shaded region

$$= \frac{1}{2}|5(-1) - 5(1)| = 5$$

Q.25 **(1)**

$$A = \int_1^2 \ell n\, x\, dx = 2\,\ell n\, 2 - 1$$



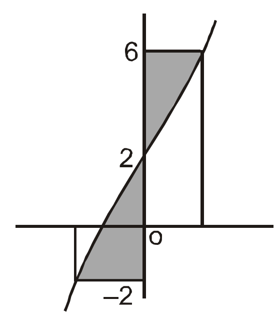
$\Rightarrow$ Required area = $4 - 2\,(\ell n\, 2 - 1) = 6 - 4\,\ell n\, 2$

Q.26 **(3)**

The required area will be equal to the area enclosed by y = f(x),
y-axis between the abscissa at y =–2 and y = 6

$$\text{Hence, Area} = \int_0^1 (6 - f(x))\, dx + \int_{-1}^0 (f(x) - (-2))\, dx$$

$$= \frac{9}{2}$$

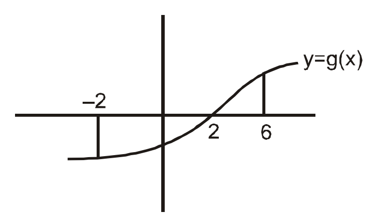


Graph of y = f(x)

Alternative

Clearly $g(x) < 0$ for $x < 2$ and $g(x) > 0$ for $x > 2$

$$\text{Area} = -\int_{-2}^{2} g(x)\,dx + \int_{2}^{6} g(x)\,dx$$



Figure

put $x = f(t)$

$$= -\int_{-1}^{0} t f'(t)\,dt + \int_{0}^{1} t f'(t)\,dt = \frac{9}{2}$$

Q.27 **(1)**

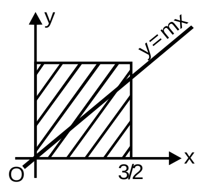
$\frac{d^2y}{dx^2} = 0$ at x = 2 so A

$= \int_0^2 xe^{-x}\,dx = 1 - 3e^{-2}$

Q.28 **(1)**

$A = \int_0^{3/2} y\,dx = \frac{39}{8}$

and $\left(\frac{39}{8}\right) \times \frac{1}{2} = \int_0^{3/2} mx\,dx$



$\Rightarrow m = 13/6$

Q.29 **(1)**

$\sin 2x - \sqrt{3}\sin x = 0 \Rightarrow \sin x\left(\cos x - \frac{\sqrt{3}}{2}\right) = 0$

x = 0 on $\pi/6$

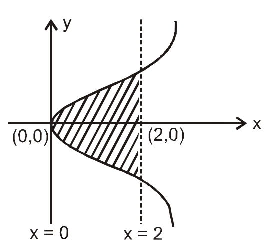
so $A = \int_0^a (\sin 2x - \sqrt{3}\sin x)\,dx$

$\Rightarrow 4A + 8\cos a = 7.$

Q.30 **(2)**

$\text{Area} = \int_1^3 \left(\frac{1}{x^2}\right) dx = \left(-\frac{1}{x}\right)_1^3 = -\frac{1}{3} - (-1) = \frac{2}{3}$

Q.31 **(3)**



$A = 2\int_0^2 x^3 dx = 8$

Q.32 **(2)**

from at point (1, 3)

A + B + C = 3 ....(i)

equation of tangent at (2, 0)

$y = 4Ax + Bx + 2B + 2\ell$ ....(ii)

comparing with 4x + y = 8 (given tangent)

get A, B, C & area.

Q.33 **(4)**

Let s be side, r be radius

$4s = 2\pi r$

$s = \frac{\pi}{2} r$

$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{x^2} = \frac{4}{\pi} > 1$

Q.34 **(3)**

$\int_1^b f(x)\,dx = (b-1)\sin(3b+4)$

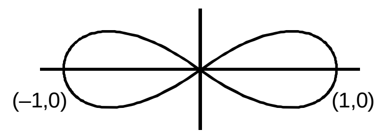
differentiate w.r.t. 'b'

$f(b)\cdot 1 = 3(b-1)\cos(3b+4) + \sin(3b+4)$

so $f(x) = 3(x-1)\cos(3x+4) + \sin(3x+4)$

Q.35 **(2)**

curve is symmetric about both the axes & cuts x-axis at (–1, 0) (0, 0) & (1, 0)



$\text{Area of loop} = 2\int_0^1 x\sqrt{1-x^2}\,dx = 2.\frac{2}{3} = \frac{4}{3}$

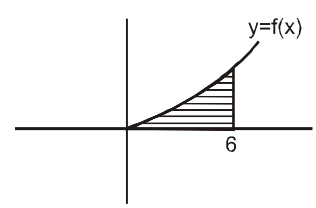
Q.36 **(2)**

$\overline{OA} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\overline{OB} = t\hat{i} + \hat{j} + (t+1)\hat{k}$

$s(t) = \frac{1}{2}\left|\overline{OA} \times \overline{OB}\right| = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ t & 1 & t+1 \end{vmatrix}\right|$

$= \frac{1}{2}\sqrt{(2t+1)^2 + (t+2)^2 + (2-2t)^2}$

$= \frac{3}{2}\sqrt{t^2+1}$



Figure

$f(x) = \int_0^x \frac{9}{4}(t^2+1)\,dt$

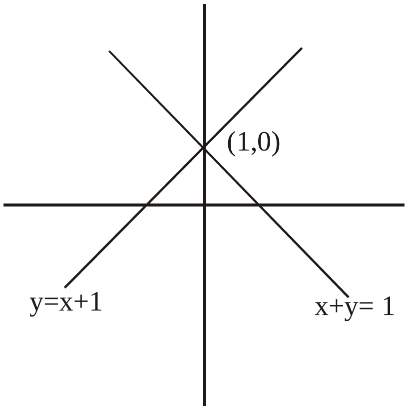
$= \frac{3x^3}{4} + \frac{9x}{4}.$

$\text{Area} = \frac{3}{16}6^4 + \frac{9}{8}6^2 = \frac{567}{2}$

Q.37(a)

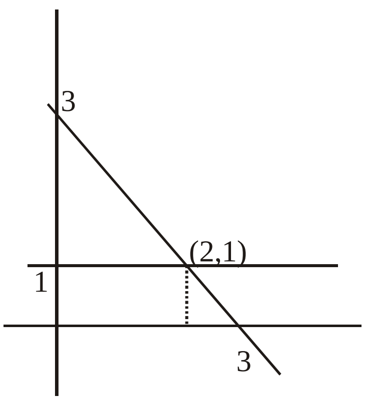
$x^2 - y^2 + 2y = 1 \Rightarrow x = \pm(y-1)$

Bisectors of above line are x = 0 & y = 1



So area between x = 0, y = 1 & x + y = 3 is shaded

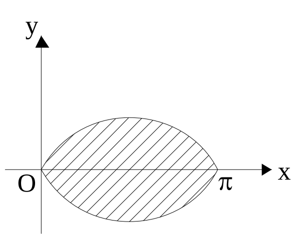
Region shown in figure.



EXERCISE-III

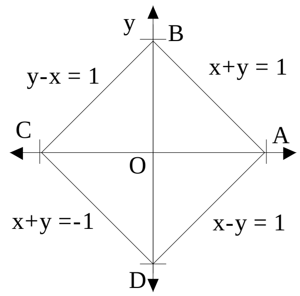
Q.1 0004

Area = $2\int_0^{\pi} \sin x dx = 2\left|-\cos\right|_0^{\pi}$ = 4 sq. units



Q.2 0002

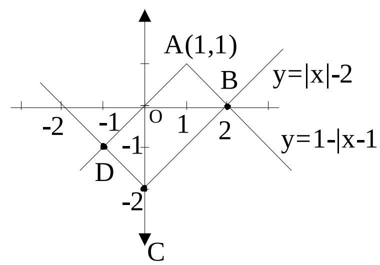
After shifting the origin at the point (2, −1) the equation of curve becomes, |x| + |y| = 1 This curve will represent a square as shown in the adjacent figure.



Area of this square is clearly equal to 4 times the area of triangle OAB. Thus required area = 2sq. units.

Q.3 0004

Bounded fig ABCD is rectangle.



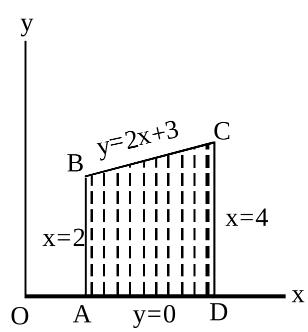
$AB = \sqrt{1+1} = \sqrt{2}$

$BC = \sqrt{4+4} = 2\sqrt{2}$

This, bounded area = $\left(\sqrt{2}\right)\left(2\sqrt{2}\right)$ = 4sq. units .

Q.4 0018

Required area ABCDA $\int_2^4 y dx = \int_2^4 (2x+3) dx$A



$= \left[x^2 + 3x\right]_2^4$

$= (16+12) - (4+6)$

= 18 sq. units.

Q.5 0004

Curves can also be written as

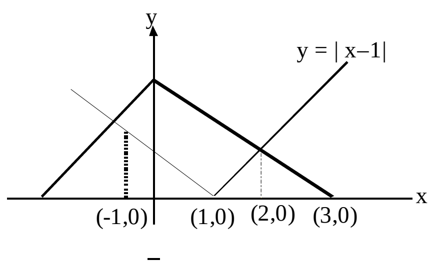
$$y_1 = |x-1| = \begin{cases} x-1 : x \ge 1 \\ 1-x : x < 1 \end{cases} \quad \ldots(i)$$

$$y_2 = 3-|x| = \begin{cases} 3-x : x \ge 0 \\ 3+x : x < 0 \end{cases}$$

….(ii)

These two curves meet at (–1, 2) and (2, 1)

Now, the graph of these function is



Required area

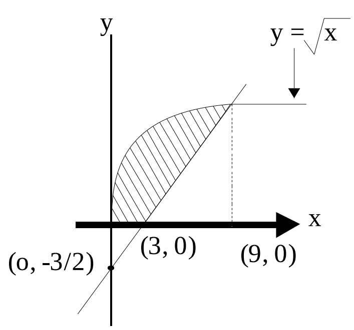
$$= \int_{-1}^{0} (y_2 - y_1)\,dx + \int_{0}^{1} (y_2 - y_1)\,dx + \int_{1}^{2} (y_2 - y_1)\,dx$$

$$= \int_{-1}^{0} (2+2x)\,dx + \int_{0}^{1} 2dx + \int_{1}^{2} (4-2x)\,dx$$

$$= 1 + 2 + (4-3) = 4 \text{ sq. units.}$$

Q.6 0009

Graph of the function is



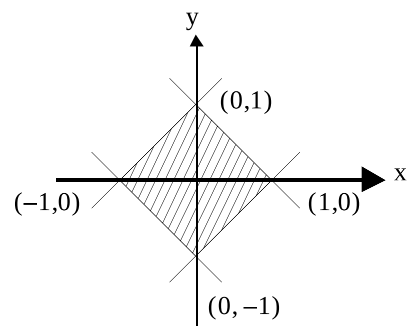
Required area

$$\int_0^9 \sqrt{x}\,dx - \int_3^9 \frac{x-3}{2}\,dx \;=\; \frac{2}{3}x^{3/2}\Big|_0^9 - \frac{1}{2}\left(\frac{x^2}{2} - 3x\right)\Big|_3^9$$

$$= 18 - \frac{1}{2}(18) = 9 \text{ sq.units}$$

Q.7 0002

Graph of these function is



This is obviously a square and area $= \sqrt{2} \times \sqrt{2} = 2$ sq.units

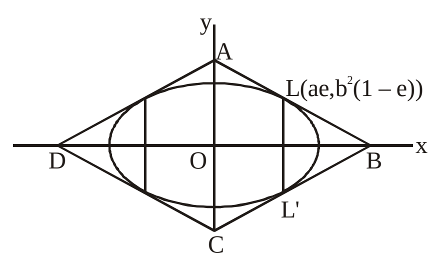
Q.8 0027

Given : $\frac{x^2}{9} + \frac{y^2}{5} = 1$

to find tangents at the points of latus rectum, we find ae,

i.e. $ae = \sqrt{a^2 - b^2} = \sqrt{4} = 2$

By symmetry the quadrilateral is rhombus.



So area is four times the area of the right angled Δ formed by the tangent and axes in the I quadrant.

$\Rightarrow$ Equation of tanegent at $\left(ae, b^2\left(1-e^2\right)\right) = \left(2, \frac{5}{3}\right)$ is

$$\frac{2}{9}x + \frac{5}{3}\cdot\frac{y}{5} = 1 \qquad \Rightarrow \frac{x}{9/2} + \frac{y}{3} = 1$$

$\therefore$ Area of quadrilateral ABCD = 4(area of ΔAOB)

$$= 4.\left\{\frac{1}{2}.\frac{9}{2}.3\right\} = 27 \text{ sq.units}$$

Q.9 0007

Equation of AB

$$y = \frac{5}{2}(x-2)$$

Equation of BC $\qquad y - 5 = \frac{3-5}{6-4}(x-4)$

$$y = -x + 9$$

Equation of CA

$$y-3=\frac{0-3}{2-6}(x-6)$$

$$y=\frac{3}{4}(x-2)$$

Required area

$$=\frac{5}{2}\int_2^4(x-2)\,dx+\int_4^6-(x-9)dx-\frac{3}{4}\int_2^6(x-2)\,dx$$

$$=\frac{5}{2}\left[\frac{(x-2)^2}{2}\right]_2^4-\left[\frac{(x-9)^2}{2}\right]_4^6-\frac{3}{4}\left[\frac{(x-2)^2}{2}\right]_2^6$$

$$=\frac{5}{2}\left[2^2-0\right]-\frac{1}{2}\left[(-3)^2-(-5)^2\right]-\frac{3}{8}\left[4^2-0\right]$$

$$=\frac{5}{4}\times4-\frac{1}{2}[9-25]-\frac{3}{8}[16-0]$$

$$=5-\frac{1}{2}[-16]-\frac{3}{8}\times16$$

$= 5+8-6=7$ sq.unit.

Q.10 0011

$y=2-x^2,\ x+y=0$

$\Rightarrow x^2=2-y=-(y-2)$

$x=0,\ y=2\quad y=0,\ x=\pm\sqrt{2}$

Point of intersection

$y=2-x^2$ (put $y=-x$)

$$\Rightarrow -x=2-x^2$$

$$\Rightarrow\left(x-\frac{1}{2}\right)^2=2+\frac{1}{4}=\frac{9}{4}$$

$$\Rightarrow x=\pm\frac{3}{2}+\frac{1}{2}$$

$$\Rightarrow x=2 \quad \text{or} \quad x=-1$$

Required area

$$\int_{-1}^2(2-x^2)\,dx+\int_{-1}^2 x.dx$$

$$=\left[2x-\frac{x^3}{3}\right]_{-1}^2+\left[\frac{x^2}{2}\right]_{-1}^2$$

$$=\left[2x-\frac{x^3}{3}\right]_{-1}^2+\left[\frac{x^2}{2}\right]_{-1}^2$$

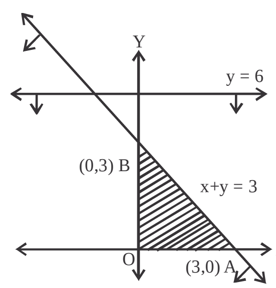
$$=\left[4-\frac{8}{3}+2-\frac{1}{3}\right]+\left[\frac{4}{2}-\frac{1}{2}\right]=3+\frac{3}{3}=\frac{9}{2}\text{sq.unit}$$

PREVIOUS YEAR'S

MHT CET

Q.1 (1)
Q.2 (1)
Q.3 (1)
Q.4 (3)
Q.5 (2)
Q.6 (2)
Q.7 (1)
Q.8 (3)
Q.9 (1)
Q.10 (3)
Q.11 (3)
Q.12 (3)

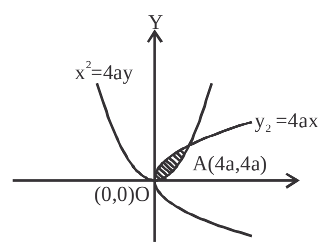
The given region is bounded in first quadrant.



Q.13 (1)

The equations of given curves are

$y^2 = 4ax$ and $x^2 = 4ay$



On solving these equations, we get the intersection points, i.e. (0,0) and (4a, 4a).

$$\therefore \text{Required area}=\int_0^{4a}\left(2\sqrt{a}\sqrt{x}-\frac{x^2}{4a}\right)dx$$

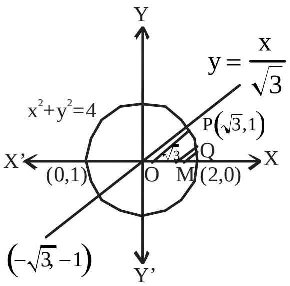
$$=2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^{4a}-\left[\frac{x^3}{12a}\right]_0^{4a}$$

$$=\frac{32a^2}{3}-\frac{16a^2}{3}\qquad=\frac{16a^2}{3}$$

Q.14 (3)

The intersection points of curves $x^2 + y^2 = 4$ and

$y = \frac{x}{\sqrt{3}}$ are (0,0) and $P\left(\sqrt{3},1\right)$



$\therefore$ Area of DOPM = $\frac{1}{2}\times\sqrt{3}\times 1 = \frac{\sqrt{3}}{2}$

and area of curve MPQ = $\int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$

$$= \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$

$$= \left[0+2\left(\frac{\pi}{2}\right)-\left(\frac{\sqrt{3}}{2}+2\times\frac{\pi}{3}\right)\right]$$

$$= \left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$$

$\therefore$ Required area = $\frac{\sqrt{3}}{2}+\frac{\pi}{3}-\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

Q.15 (2)

Given equuation of curve = $y^2 = 2x$ ...(i)

Which is a parabola with vertex (0,0) and its axis parallel to X-axis

and another cux²ex $y^2 = 4x$ ...(ii)

which is a circel with centre (2, 0) and radius is 2. On substituting $y^2 = 2x$ in Eq. (ii), we get

$x^2 + 2x = 4x$

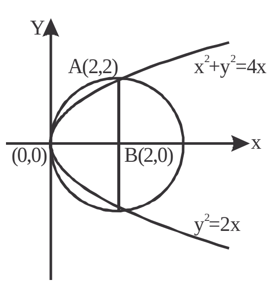
$\Rightarrow x^2 = 2x$

$\Rightarrow x(x-2) = 0$

$\Rightarrow x = 0$ or $x = 2$

$\Rightarrow y = 0$ or $y = \pm 2$ [using Eq. (i)]

Now, the required area is the area of shaded region



$\therefore$ Required area = $\frac{\text{Area of a circle}}{4} - \int_0^2\sqrt{2x}\,dx$

$$= \frac{\pi(2)^2}{4} - \sqrt{2}\int_0^2 x^{1/2}dx = \pi - \sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^2$$

$$= \pi - \frac{2\sqrt{2}}{3}[2\sqrt{2}-0] = \left(\pi-\frac{8}{3}\right)\text{sq. units}$$

Q.16 (2)

We have, $x > 0$

$\therefore \sin x < x$

$\Rightarrow \frac{\sin x}{x} < 1$

$\Rightarrow \int_0^{\pi/4}\frac{\sin x}{x}dx < \int_0^{\pi/4}1dx = \frac{\pi}{4}$

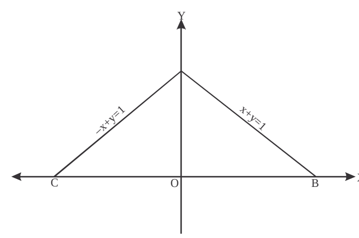
or $\int_0^{\pi/4}\frac{\sin x}{x}dx < \frac{\pi}{4}$

Q.17 (1)

Given curve is $|x| + y = 1$

$\therefore$ Curve is $x + y = 1$, when $x \geq 0$ and $-x + y = 1$, when $x \leq 0$.

The graph of the given curve is as shown below,



$\therefore$ Required area = Area CAOC + Area OABO

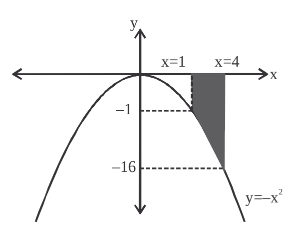
$$\int_{-1}^{0}ydx+\int_0^1 ydx = \int_{-1}^{0}(x+1)dx+\int_0^1(1-x)dx$$

$$= \left[\frac{x^2}{2}+x\right]_{-1}^{0}+\left[x-\frac{x^2}{2}\right]_0^1$$

$$= \left[0-\left(\frac{1}{2}-1\right)\right]+\left[\left(1-\frac{1}{2}\right)-0\right]$$

$$= \frac{1}{2}+\frac{1}{2} = 1\text{sq. unit}$$

Q.18 (1)



$\therefore$ Required area = Area of shaded region

$$= \int_1^4 (-y)\,dx = \int_1^4 x^2 dx = \left[\frac{x^3}{3}\right]_1^4$$

$$= \frac{64}{3} - \frac{1}{3} = \frac{63}{3}$$

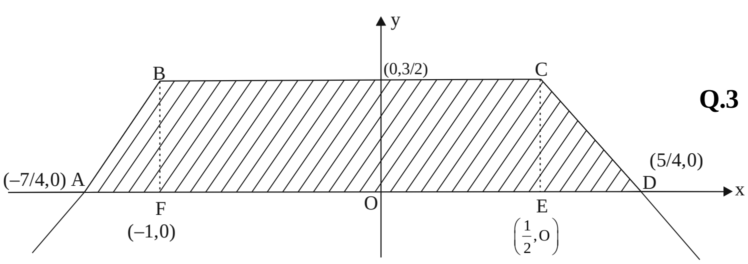
= 21 sq units

JEE MAIN

Q.1 (3)

$$y = \begin{cases} 3 + (x+1) + \left(x - \frac{1}{2}\right), & x < -1 \\ 3 - (x-1) + \left(x - \frac{1}{2}\right), & -1 \le x < \frac{1}{2} \\ 3 - (x+1) - \left(x - \frac{1}{2}\right), & \frac{1}{2} \le x \end{cases}$$

$$y = \begin{cases} \frac{7}{2} + 2x, & x < -1 \\ \frac{3}{2}, & -1 \le x < \frac{1}{2} \\ \frac{5}{2} - 2x, & \frac{1}{2} \le x \end{cases}$$

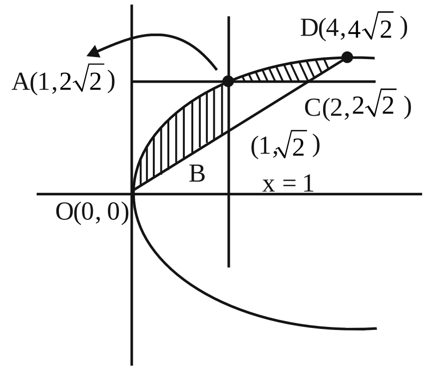


Area bounded = ar (ABF) + ar (BCEF) + ar (CDE)

$$= \frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)\left(\frac{3}{2}\right) + \frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)$$

$$= \frac{27}{8}\text{sq. units}$$

Q.2 (3)



Area of ΔABC $\frac{1}{2}(\sqrt{2})\,.1 = \frac{\sqrt{2}}{2}$

Area between two curves = $\int_0^4 (\sqrt{8x} - \sqrt{2x})dx - \frac{\sqrt{2}}{2}$

$$= \left[\frac{2\sqrt{2}x^{3/2}}{3/2} - \frac{x^2}{\sqrt{2}}\right]_0^4$$

$$= \frac{32\sqrt{2}}{3} - 8\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3}$$

Required Area = $\frac{8\sqrt{2}}{3} - \frac{1}{\sqrt{2}} = \frac{8\sqrt{2}}{3} - \frac{\sqrt{2}}{2} = \frac{13\sqrt{2}}{6}$

Q.3 (12)

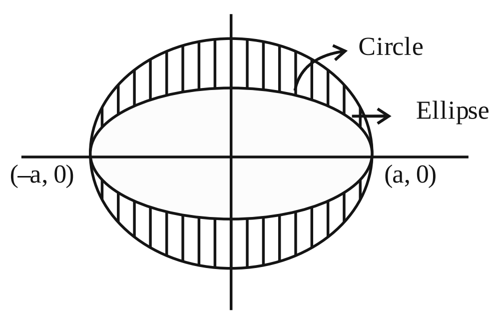
Case-I : $x^2 + y^2 \le a^2 \Rightarrow$ Circle

$\frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1 \Rightarrow$ ellipse

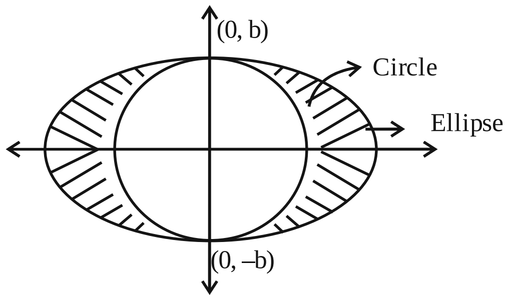
area of circle – area of ellipse = 30π

$\pi a^2 - \pi ab = 30\pi \Rightarrow a^2 - ab = 30$ ... (i)

Case II : $x^2 + y^2 \ge b^2 \Rightarrow$ circle



$\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1 \Rightarrow$ ellipse



Area of (ellipse) – Area of (circle) = 18π

$\pi ab - \pi b^2 = 18\pi \Rightarrow b^2 = ab - 18$ ... (ii)

(i) + (ii)

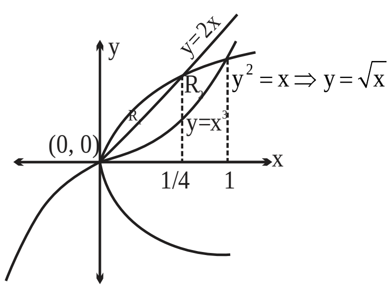
$a^2 + b^2 - 2ab = 12$

$(a-b)^2 = 12$

Q.4 **[19]**

Given curve $\Rightarrow y=x^3$

$\& \Rightarrow y^2=x$



Now $R_1 = \int_0^{1/4}\left(\sqrt{x} - 2x\right).dx$

$= \left(\frac{x^{3/2}}{3/2} - x^2\right)_{10}^{1/4}$

$= \frac{1}{12} - \frac{1}{16} - 0 = \frac{1}{48}$

Now $R_2 = \int_0^{1/4}\left(2x - x^3\right)dx + \int_{1/4}^{1}\sqrt{x} - x^3 \cdot dx$

$\left(x^2 - \frac{x^4}{4}\right)_0^{1/4} + \left(\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right)_{1/4}^{1}$

$= \frac{1}{16} + \frac{2}{3} - \frac{1}{4} - \frac{1}{12} = \frac{19}{48}$

So, $\frac{R_2}{R_1} = \frac{19/48}{1/48} = 19$

Q.5 (18)

$y^2 = 2x$

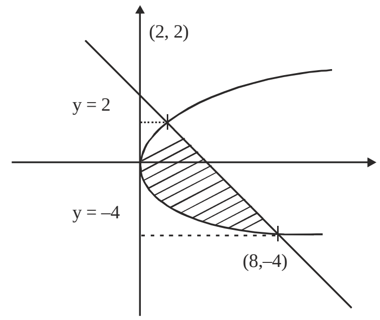
$x + y = 4$

$(4-x^2) = 2x$

$x^2 - 8x + 16 = 2x$

$x^2 - 10x + 16 = 0$

$x = 2, 8$



$A = \int_{-4}^{2}[(4-y) - \frac{y^2}{2}]dy$

$= [4y - \frac{y^2}{2} - \frac{y^3}{6}]_{-4}^{2}$

$= [\left(8 - 2 - \frac{4}{3}\right) - \left(-16 - 8 + \frac{64}{6}\right)$

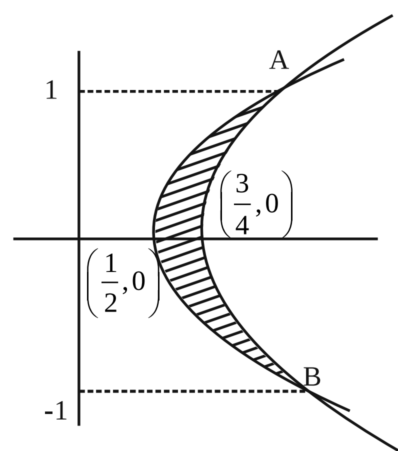
$= 30 - \frac{4}{3} - \frac{64}{6}$

$= \frac{180 - 8 - 64}{6} = \frac{108}{6} = 18$

Q.6 **(1)**

$2x - 1 = 4x - 3$

$x = 1$



So, A(1, 1), B(1, –1)

Area $= \int_{-1}^{1}\left(\frac{3+y^2}{4} - \frac{1+y^2}{2}\right)dy$

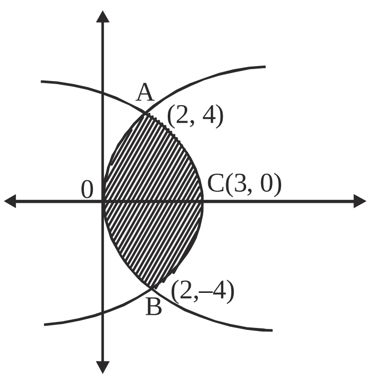
$= 2\int_0^1\left\{\frac{y^2+3}{4} - \frac{y^2+1}{2}\right\}$

$$= 2\left[\frac{1}{4}\left(\frac{y^3}{3}+3y\right)_0^1 - \frac{1}{2}\left(\frac{y^3}{3}+y\right)_0^1\right]$$

$$= 2\left[\frac{1}{4}\left(\frac{1}{3}+3\right)-\frac{1}{2}\left(\frac{1}{3}+1\right)\right]$$

$$= \frac{10}{6}-\frac{4}{3} \Rightarrow \frac{2}{5} = \frac{1}{3}$$

Q.7 **(3)**



Given curves

$y^2 = 16(3–x)$ and $y^2 = 8x$

$8x = 16(3–x)$

$\Rightarrow x = 6 - 2x$

$\Rightarrow x = 2$

$\Rightarrow y = \pm 4$

Area bounded between curves

A = 2(area OAC)

$$= 2\int_0^4 [(3-\frac{y^2}{16}) - \frac{y^2}{8}]\, dy = 2\int_0^4 \left(3-\frac{3y^2}{16}\right) dy$$

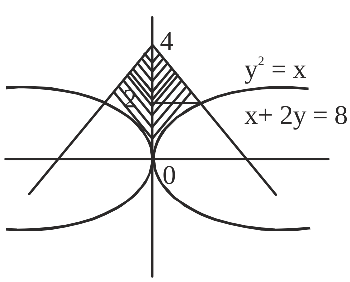
$$= 2\left[3y-\frac{y^3}{16}\right]_0^4$$

$2 = [12–4] = 16$

Q.8 (6)

$A_1 = \{(x,y): |x| \le y^2, |x| + 2y \le 8\}$ and

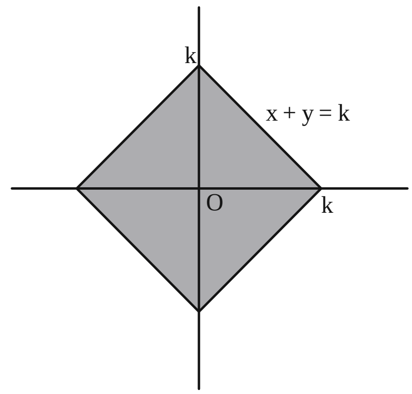
$A_2 = \{(x,y): |x| + |y| \le k\}$.



$$\text{area } (A_1) = 2\left[\int_0^2 y^2 dy + \int_2^4 (8-2y)dy\right]$$

$$= 2\left[\left(\frac{y^3}{3}\right)_0^2 + (8y-y^2)_2^4\right]$$

$$\text{area } (A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$$



$$\text{Area}(A_2) = 4 \times \frac{1}{2} k^2$$

$\text{Area}(A_2) = 2k^2$

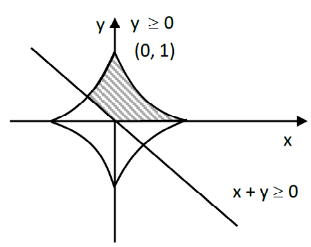
Now

$27 (\text{Area } A_1) = 5 (\text{Area } A_2)$

$9 \times 4 = k^2$

$k = 6$

Q.9 **(36)**



$$A = \frac{3}{2}\int_0^1 (1-x^{2/3})^{3/2}\, dx$$

Let $x = \sin^3\theta$

$dx = 3\sin^2\theta\cos\theta d\theta$

$$A = \frac{3}{2}\int_0^{\pi/2} (1-\sin^2\theta)^{3/2} \cdot 3\sin^2\theta\cos\theta\, d\theta$$

$$= \frac{3}{2}\int_0^{\pi/2} 3\sin^2\theta\cos^4\theta\, d\theta$$

$$= \frac{9}{2}\int_0^{\pi/2} \sin^2\theta\cos^4\theta\, d\theta$$

$$A=\frac{9}{2}\times\frac{1.3.1}{(2+4)(4)(2)}\cdot\frac{\pi}{4}$$

$$\Rightarrow A=\frac{9\pi}{64}\Rightarrow\frac{64A}{\pi}=9$$

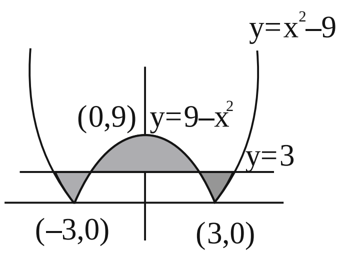
$$\Rightarrow\frac{256A}{\pi}=36$$

Q. 10 (4)

Area of shaded region

$$=2\int_0^3(\sqrt{9+y}-\sqrt{9-y})dy+2\int_3^9(\sqrt{9-y})dy$$

$$=2\left[\int_0^3(9+y)^{1/2}dy-\int_0^3(9-y)^{1/2}dy+\int_0^9(9-y)^{1/2}dy\right]$$



$$=2\left[\frac{2}{3}\left[(9+y)^{3/2}\right]_0^3+\frac{2}{3}\left[(9-y)^{3/2}\right]_0^3-\frac{2}{3}\left[(9-y)^{3/2}\right]_3^9\right]$$

$$=\frac{4}{3}\left[12\sqrt{12}-27+6\sqrt{6}-27-(0-6\sqrt{6})\right]$$

$$=\frac{4}{3}\left[24\sqrt{3}+12\sqrt{6}-54\right]$$

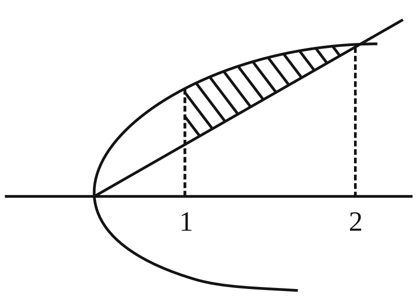
$$=8(4\sqrt{3}+2\sqrt{6}-9)$$

Q.11 (2)

$y^2=8x$ ... (1)

$y=\sqrt{2}x$ ... (2)

$y^2=2x^2$



$\Rightarrow 8x=2x^2$

$\Rightarrow x=0\ \&\ 4$

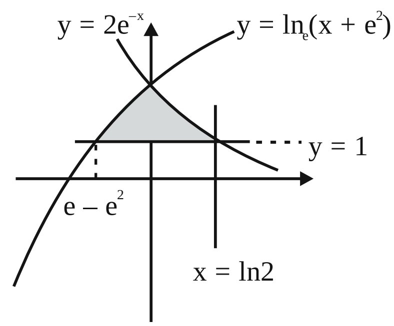
$$\text{Area : }=\int_1^4 2\sqrt{2}\sqrt{x}-\sqrt{2}x\,dx$$

$$=2\sqrt{2}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^4-\sqrt{2}\left(\frac{x^2}{2}\right)_1^4$$

$$=\frac{4\sqrt{2}}{3}(8-1)-\frac{\sqrt{2}}{2}(16-1)$$

$$=\frac{28\sqrt{2}}{3}-\frac{15\sqrt{2}}{2}=\frac{11\sqrt{2}}{6}$$

Q.12 (2)

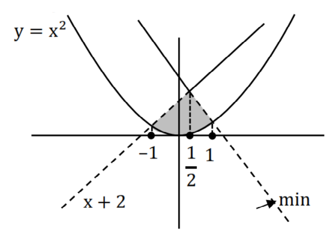


Required area is

$$=\int_{e-e^2}^{0}\ell n(x+e^2)-1dx+\int_0^{\ell n2}2e^{-x}-1dx=1-e-\ell n2$$

Q.13 (2)

$x^2\le y\le\min\{x+2,4-3x\}$



$$\text{Area = }\int_{-1}^{1/2}\left(x+2-x^2\right)dx+\int_{1/2}^{1}\left(4-3x-x^2\right)dx$$

$$\text{Area = }\left.\frac{x^2}{2}+2x\right|_{-1}^{\frac{1}{2}}+\left.4x\frac{-3x^2}{2}\right|_{\frac{1}{2}}^{1}-\left.\frac{x^3}{3}\right|_{-1}^{1}$$

$$=\left(\frac{1}{8}+1\right)-\left(\frac{1}{2}-2\right)+\left(4-\frac{3}{2}\right)-\left(2-\frac{3}{8}\right)-\left(\frac{1}{3}-\left(-\frac{1}{3}\right)\right)$$

$$=\frac{1}{8}+1+\frac{3}{2}+2-\frac{3}{2}+\frac{3}{8}-\frac{2}{3}$$

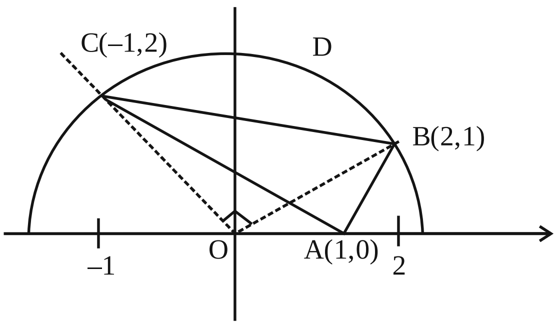
$$=\frac{1}{8}+3+\frac{3}{8}-\frac{2}{3}$$

$$=\frac{1}{2}+3-\frac{2}{3}$$

$$=\frac{7}{2}-\frac{2}{3}$$

$$\frac{(21-4)}{6}=\frac{17}{6}$$

Q.14 (4)



$$|x-1|<y<\sqrt{5-x^2}$$

When $|x-1|=\sqrt{5-x^2}$

$\Rightarrow (x-1)^2=5-x^2$

$\Rightarrow x^2-x-2=0$

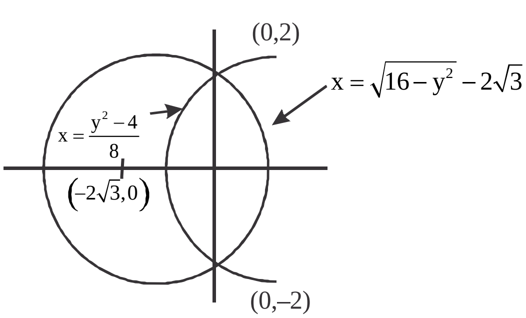
$\Rightarrow x=2,-1$

Required Area = Area of ΔABC + Area of region BCD

$$=\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ 2 & 1 & 1\\ -1 & 2 & 1\end{vmatrix}+\frac{\pi}{4}(\sqrt{5})^2-\frac{1}{2}(\sqrt{5})^2$$

$$=\frac{5\pi}{4}-\frac{1}{2}$$

Q.15 (3)



$y^2=8x+4$ ....(1)

& $x^2+y^2+4\sqrt{3}\,x-4=0$ ....(2)

Points of intersection of (1) & (2)

$x^2+8x+4+4\sqrt{3}\,x-4=0$

$x^2+8x+4\sqrt{3}\,x=0$

$x(x+8+4\sqrt{3})=0$

$x=0,\ x=-(4\sqrt{3}+8)$

at $x=0$, $y\pm 2$

$\Rightarrow$ (0,2) and (0,–2) are points of intersection

$$A=2\int_0^2\left(\sqrt{16-y^2}-2\sqrt{3}\right)-\left(\frac{y^2-4}{8}\right)dy$$

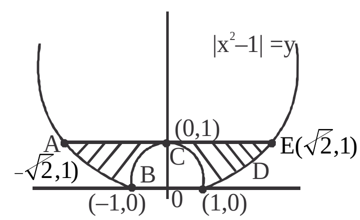
$$=2\left[\frac{y}{2}\sqrt{16-y^2}+8\sin^{-1}\frac{y}{4}-2\sqrt{3}y-\frac{y^2}{24}+\frac{y}{2}\right]_0^2$$

$$=2\left[\sqrt{16-4}+\frac{8\pi}{6}-4\sqrt{3}-\frac{8}{24}+1\right]$$

$$=\frac{1}{3}\left[4-12\sqrt{3}+8\pi\right]$$

Q.16 (4)

$Y=|X^2-1|$



Area = ABCDEA

$$=2\left(\int_0^1\left(1-\left(1-x^2\right)\right)dx+\int_1^{\sqrt{2}}\left(1-\left(x^2-1\right)\right)dx\right)$$

$$=\frac{8}{3}\left(\sqrt{2}-1\right)$$

Q.17 (2)

$$\int_1^3 x\,dy=\frac{364}{3}$$

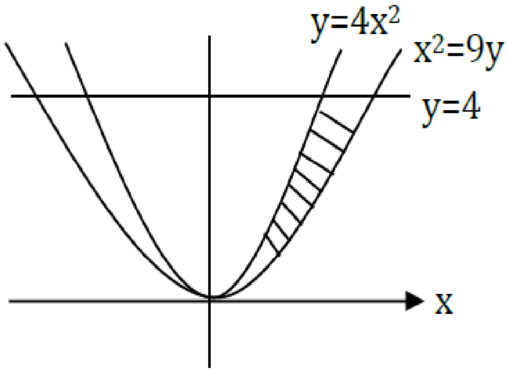
$$\int_1^3 y^a\,dy=\frac{364}{3}$$

$$\left[\frac{y^{a+1}}{a+1}\right]_1^3=\frac{364}{3}$$

$$\frac{3^{a+1}-1}{a+1}=\frac{364}{3}$$

$a=5$

Q.18 **[4]**



Area of shaded region =

$$\mathbf{2}\int_0^4 (x_1 - x_2)dx = 2\int_0^4 (3\sqrt{y} - \frac{\sqrt{y}}{2})dy$$

$$= 2\,x\,\frac{5}{2}\int_0^4 (\sqrt{y})\,d\,y = \frac{15}{2}$$

$$= 5\,\frac{2}{3}\,y^{\frac{3}{2}}\Big|_0^4 = \frac{10}{3}.(4)^{\frac{3}{2}} = \frac{10}{3}\,x\,8 = \frac{80}{3}$$

Q.19 **(3)**

Total area = xy - 8 = $A_1 + A_2$

$$xy - 8 = \frac{3A_1}{2} \;....\;(1)\;(\because A_1 = 2A_2)$$

$A_1 = \int_4^x f(x)\,dx$ value of A_1 put in eq (1)

$$xy - 8 = \frac{3}{2}.\int_4^x f(x)\,dx .....(i)$$

Now, differentate both side the equation (ii)

$$x\frac{dy}{dx} + y = \frac{3}{2}f(x) \Rightarrow x\frac{dy}{dx} + y = \frac{3}{2}\,y$$

$$x\,\frac{d\,y}{d\,x} = \frac{1}{2}\,y$$

Solve the differential equation get, y = $\sqrt{xC}$ ... (iii)

Now from equation (ii) , put x = 4 both side we get, 4y – 8 = 0 ⇒ y = 2

Now put the Value of x & y in equation (iii) we get C = 1

Now equation of curve is y = $\sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

Slope at $(x_1 y_1)$ = $\frac{1}{\sqrt[2]{x_1}} = \frac{1}{6} \Rightarrow$ $x_1 = 9$ & $y_1 = 3$

Ponit on curve is (9,3)

Now equation of normal = y - 3 = – 6 (x-9), 6x+y-57 = 0

ponit [10, – 4] is not satisfy the equation.

DIFFERENTIAL EQUATION

EXERCISE-I (MHT CET LEVEL)

Q.1 (1)

Given differential equation can be written as

$$y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy.\frac{dy}{dx} = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$

Hence it is of 1^{st} order and 2^{nd} degree differential equation.

Q.2 (2)

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$$

$$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 = \left[3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right]^3$$

It is a differential equation of degree 2.

Q.3 (4)

Q.4 (3)

Q.5 **(4)**

Q.6 **(1)**

Q.7 **(4)**

Q.8 **(4)**

Q.9 **(2)**

Q.10 **(2)**

Q.11 (1)

Order is 2 and degree is 2.

Q.12 (4)

Making fourth power both the sides, we get the differential equation $\left(\frac{d^2y}{dx^2}\right)^4 = y + \left(\frac{dy}{dx}\right)^2$

Obviously, order is 2 and degree is 4.

Q.13 (4)

Clearly, order = 2, degree = 3

Q.14 (2)

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$$

$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^9 = \left(\frac{d^2y}{dx^2}\right)^4$$

Clearly, degree is 4.

Q.15 (3)

Let $y = 4\sin 3x \Rightarrow \frac{dy}{dx} = 12\cos 3x$

$$\Rightarrow \frac{d^2y}{dx^2} = -36\sin 3x = -9\times 4\sin 3x = -9y$$

$$\Rightarrow \frac{d^2y}{dx^2} + 9y = 0$$

Q.16 (1)

Q.17 (1)

Q.18 (2)

$$y = \frac{x}{x+1} \Rightarrow \frac{1}{y} = 1 + \frac{1}{x}$$

$$-\frac{1}{y^2}\frac{dy}{dx} = 0 - \frac{1}{x^2} \Rightarrow x^2\frac{dy}{dx} = y^2$$

Q.19 (1)

$$y = A\sin x + B\cos x \Rightarrow \frac{dy}{dx} = A\cos x - B\sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -A\sin x - B\cos x = -(A\sin x + B\cos x) = -y$$

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$ is the required differential equation.

Q.20 (2)

Given equation $y = a\cos(x+b)$

Differentiate it *w.r.t. x* we get $\frac{dy}{dx} = -a\sin(x+b)$

Again $\frac{d^2y}{dx^2} = -a\cos(x+b) = -y$ or $\frac{d^2y}{dx^2} + y = 0$.

Q.21 (1)

$y = ce^{\sin^{-1}x}$ Differentiate it *w.r.t. x*, we get

$$\frac{dy}{dx} = ce^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}} = \frac{y}{\sqrt{1-x^2}} \text{ or } \frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}$$

Q.22 (1)

$x^2y = a$ (On differentiating)

$$x^2\frac{dy}{dx}+y\frac{d}{dx}(x^2)=0 \Rightarrow x^2\frac{dy}{dx}+2xy=0$$

$$\Rightarrow \frac{dy}{dx}+\frac{2y}{x}=0$$

Q.23 (4)
Since the equation of line passing through (1,–1) is $y+1=m(x-1)$

$$\Rightarrow y+1=\frac{dy}{dx}(x-1) \Rightarrow y=(x-1)\frac{dy}{dx}-1$$

Q.24 (2)

Given $y^2=4a(x+a)$. Differentiating, $2y\left(\frac{dy}{dx}\right)=4a$

Eliminating *a* from (i) and (ii), required equation is

$$y\left[1-\left(\frac{dy}{dx}\right)^2\right]=2x\frac{dy}{dx}$$

Q.25 (2)
The displacement of *x* for all S.H.M.is given by

$$x=a\cos(nt+b)\Rightarrow\frac{dx}{dt}=-na\sin(nt+b)$$

$$\Rightarrow\frac{d^2x}{dt^2}=-n^2a\cos(nt+b) \Rightarrow \frac{d^2x}{dt^2}=-n^2x$$

$$\Rightarrow\frac{d^2x}{dt^2}+n^2x=0.$$

Q.26 (1)
The equation of a member of the family of parabolas having axis parallel to *y*-axis is

$y=Ax^2+Bx+C$ .....(i) where *A*, *B*, *C* are arbitrary constants.
Differentiating (i) *w.r.t. x*, we get

$\frac{dy}{dx}=2Ax+B$ .....(ii)
Which on differentiating *w.r.t. x*

gives $\frac{d^2y}{dx^2}=2A$ .....(iii)

Differentiating *w.r.t. x* again, we get $\frac{d^3y}{dx^3}=0$.

Q.27 (1)
It can be written in the form of

$$\frac{\sec^2 y}{\tan y}dy=-3\frac{e^x}{1-e^x}dx$$

$$\int\frac{\sec^2 y}{\tan y}dy=-3\int\frac{e^x}{1-e^x}dx$$

$$\Rightarrow \log(\tan y)=3\log(1-e^x)+\log c \Rightarrow$$

$$\tan y=c(1-e^x)^3$$

Q.28 (1)

$$\frac{dy}{dx}=e^{x-y}+x^2e^{-y}=e^{-y}(e^x+x^2)$$

$$\Rightarrow e^y dy=(x^2+e^x)dx$$

Now integrating both sides, we get $e^y=\frac{x^3}{3}+e^x+c$

Q.29 (4)
We have,

$$2x\frac{dy}{dx}=y+3\Rightarrow\frac{2}{y+3}dy=\frac{dx}{x}$$

intergrating, 2 ln (y + 3) = ln x + ln c = ln cx

$$\Rightarrow \ln(y+3)^2=\ln cx\Rightarrow(y+3)^2=cx$$

which is a family of parabolas.

Q.30 (2)
The equation is,

$$\frac{dy}{dx}=\sin(x-y)-\sin(x+y)=2\cos x\sin(-y)$$

$$\Rightarrow\frac{dy}{\sin y}+2\cos x dx=0$$

$$\Rightarrow\int \text{cosec}y dy+2\int \cos x dx=C$$

$$\Rightarrow \log\tan\frac{y}{2}+2\sin x=C$$

Q.31 (4)

Q.32 (2)

Q.33 **(3)**

Q.34 **(1)**

Q.35 (2)

$$\frac{dy}{dx}+\frac{1+x^2}{x}=0 \Rightarrow dy+\left(\frac{1}{x}+x\right)dx=0$$

On integrating, we get $y+\log x+\frac{x^2}{2}+c=0$

Q.36 (2)

$$\frac{dy}{dx}=-\frac{\cos x-\sin x}{\sin x+\cos x}\Rightarrow$$

$$dy=-\left(\frac{\cos x-\sin x}{\sin x+\cos x}\right)dx$$

On integrating both sides, we get

$$\Rightarrow y=-\log(\sin x+\cos x)+\log c$$

$$\Rightarrow y=\log\left(\frac{c}{\sin x+\cos x}\right)\Rightarrow$$

$$e^{y}(\sin x+\cos x)=c.$$

Q.37 (4)

$$\frac{dy}{dx}=(1+x)(1+y^2)\Rightarrow\frac{dy}{1+y^2}=(1+x)dx$$

On integrating both sides, we get

$$\tan^{-1}y=\frac{x^2}{2}+x+c\Rightarrow y=\tan\left(\frac{x^2}{2}+x+c\right)$$

Q.38 (2)

$\frac{dy}{dx}=\frac{2}{x^2}\Rightarrow dy=\frac{2}{x^2}dx$, Now integrate it.

Q.39 (2)

$$x\sec y\frac{dy}{dx}=1\Rightarrow\sec y\,dy=\frac{dx}{x}$$

On integrating both sides, we get

$$\log(\sec y+\tan y)=\log x+\log c\Rightarrow$$

$$\sec y+\tan y=cx.$$

Q.40 (1)

$$\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}\Rightarrow dy=-\frac{1}{\sqrt{1-x^2}}dx$$

On integrating, we get $y=\cos^{-1}x+c$

$$\Rightarrow y=\frac{\pi}{2}-\sin^{-1}x+c\Rightarrow y+\sin^{-1}x=c$$

Q.41 (4)

$$(e^x+1)y\,dy=(y+1)e^x dx$$

$$\Rightarrow\left(\frac{y}{y+1}\right)dy=\left(\frac{e^x}{e^x+1}\right)dx\Rightarrow$$

$$\left[1-\frac{1}{y+1}\right]dy=\left(\frac{e^x}{e^x+1}\right)dx$$

$$\Rightarrow\int\left\{1-\frac{1}{y+1}\right\}dy=\int\frac{e^x}{e^x+1}dx$$

$\Rightarrow y=\log(y+1)+\log(e^x+1)+\log c$ or

$$e^y=c(y+1)(e^x+1)$$

Q.42 (3)

Let $x-y=v$ and $\frac{dy}{dx}=1-\frac{dv}{dx}$, thus the equation reduces to $\frac{dv}{dx}=\frac{v+2}{2v+5}\Rightarrow\int\frac{2v+5}{v+2}dv=\int dx$

$$\Rightarrow\int\left[2+\frac{1}{(v+2)}\right]dv=\int dx\Rightarrow$$

$$2v+\log(v+2)=x+c$$

$$2(x-y)+\log(x-y+2)=x+c$$

Q.43 (4)

$$(1-x^2)(1-y)dx=xy(1+y)dy$$

$$\Rightarrow\int\frac{y(1+y)}{(1-y)}dy=\int\frac{(1-x^2)}{x}dx;$$

Now integrate it.

Q.44 (1)

We have $y^2dy=x^2dx$

Integrating, we get $y^3-x^3=c\Rightarrow x^3-y^3=c$

Q.45 (2)

Given $\sin\frac{dy}{dx}=a$; $dy=\sin^{-1}a\,dx$

Integrating both sides, $\int dy=\int\sin^{-1}a\,dx$

$y=x\sin^{-1}a+c$ and $y(0)=0+c=1$, $\therefore c=1$

$\therefore y=x\sin^{-1}a+1\Rightarrow a=\sin\frac{y-1}{x}$.

Q.46 (4)

Put $x+y=v\Rightarrow 1+\frac{dy}{dx}=\frac{dv}{dx}\Rightarrow\frac{dy}{dx}=\frac{dv}{dx}-1$

$$\therefore v^2\left(\frac{dv}{dx}-1\right)=a^2$$

$$\Rightarrow\frac{dv}{dx}=\frac{a^2}{v^2}+1=\frac{a^2+v^2}{v^2}\Rightarrow\frac{v^2}{a^2+v^2}dv=dx$$

$$\Rightarrow\left(1-\frac{a^2}{a^2+v^2}\right)dv=dx\Rightarrow v-a\tan^{-1}\frac{v}{a}=x+c$$

$\Rightarrow y = a\tan^{-1}\left(\frac{x+y}{a}\right) + c.$

Q.47 (3)

$\frac{dy}{2y-1} = \frac{dx}{2x+3}$

$\Rightarrow \frac{1}{2}\log(2y-1) = \frac{1}{2}\log(2x+3) + \log c \Rightarrow \frac{2x+3}{2y-1} = c$

Q.48 (1)

$\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}.$

Separating the variables and integrating

$\int(\sin y + y\cos y)dy = \int(x\log x^2 + x)dx$

$\Rightarrow -\cos y + y\sin y + \cos y$

$= \frac{x^2}{2}\log x^2 - \int \frac{x^2}{2}.\frac{1}{x^2}.2xdx + \int x\,dx + c$

$\Rightarrow y\sin y = \frac{x^2}{2}2\log x - \int x\,dx + \int xdx + c$

$\Rightarrow y\sin y = x^2\log x + c$

Q.49 (1)

$\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$

$\frac{dy}{dx}(\tan y) = 2\sin x\cos y \Rightarrow \frac{\sin y}{\cos^2 y}dy = 2\sin xdx$

$\Rightarrow \int\frac{\sin y}{\cos^2 y}dy = 2\int\sin xdx \Rightarrow \frac{1}{\cos y} = -2\cos x + c$

$\therefore \sec y + 2\cos x = c$

Q.50 (4)

It is homogeneous equation

$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So, we get $x\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\Rightarrow \frac{2vdv}{1+v^2} = \frac{dx}{x}$

On integrating, we get $x^2 + y^2 = px^3$

Q.51 (1)

Given $\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x.\frac{dv}{dx}$

$\therefore v + x.\frac{dv}{dx} = v(\log v + 1)$

$v + x\frac{dv}{dx} = v\log v + v \Rightarrow x\frac{dv}{dx} = v\log v \Rightarrow$

$\frac{dv}{v\log v} = \frac{dx}{x}$

Integrating both sides, $\int\frac{dv}{v\log v} = \int\frac{dx}{x}$

$\log\log v = \log x + \log c \Rightarrow \log v = xc \Rightarrow \log(y/x) = xc.$

Q.52 (1)

$ydx - xdy + 3x^2y^2e^{x^3}dx = 0$

$\frac{ydx - xdy}{y^2} + 3x^2e^{x^3}dx = 0 \Rightarrow$

$d\left(\frac{x}{y}\right) + de^{x^3} = 0$

On integrating, we get $\frac{x}{y} + e^{x^3} = c$

Q.53 (2)

$ydx + xdy + xy^2dx - x^2ydy = 0$

$\frac{ydx + xdy}{x^2y^2} + \frac{dx}{x} - \frac{dy}{y} = 0$

On integrating, we get

$-\frac{1}{xy} + \log x - \log y = k \Rightarrow \log\frac{x}{y} = \frac{1}{xy} + k.$

Q.54 (2)

$xdx - y^3dx + 3xy^2dy = 0$

Put $y^3 = t \Rightarrow dt = 3y^2dy$

$x\,dx - tdx + xdt = 0 \Rightarrow xdx + xdt - tdx = 0$

$\Rightarrow \frac{dx}{x} + d\left(\frac{t}{x}\right) = 0$

On integration, we get $\log x + \frac{t}{x} = k$

$\Rightarrow \log x + \frac{y^3}{x} = k$

Q.55 (1)

$$y e^{-x/y}dx-(xe^{-x/y}+y^3)dy=0$$

$$e^{-x/y}(ydx-xdy)=y^3dy \Rightarrow$$

$$e^{-x/y}\frac{(ydx-xdy)}{y^2}=ydy$$

$e^{-x/y}d\left(\frac{x}{y}\right)=ydy$. Integrating both sides, we get

$$k-e^{-x/y}=\frac{y^2}{2} \Rightarrow \frac{y^2}{2}+e^{-x/y}=k$$

Q.56 (1)

$$\frac{ydx-xdy}{y^2}=-xdx \Rightarrow d\left(\frac{x}{y}\right)=-xdx$$

Integrating both side, we get

$$\frac{x}{y}=-\frac{x^2}{2}+c$$

$$\Rightarrow 2x+x^2y=2cy \Rightarrow 2x+x^2y=\lambda y \quad [\lambda=2c]$$

Q.57 (1)

$$x^2dy+y^2dy=xydx \Rightarrow$$

$$x(xdy-ydx)=-y^2dy$$

$$\Rightarrow x\frac{(ydx-xdy)}{y^2}=dy \Rightarrow \frac{x}{y}d\left(\frac{x}{y}\right)=\frac{dy}{y}$$

Integrating, $\frac{x^2}{2y^2}=\log_e y+c$

Given $y(1)=1 \Rightarrow c=\frac{1}{2} \Rightarrow$

$$\frac{x^2}{2y^2}=\log_e y+\frac{1}{2}$$

Now $y(x_0)=e \Rightarrow \frac{x_0^2}{2e^2}-\log_e e-\frac{1}{2}=0$

$$\Rightarrow x_0^2=3e^2$$

$$\Rightarrow x_0=\pm\sqrt{3}e$$

Q.58 (2)

$$(1+y^2)dx-(\tan^{-1}y-x)dy=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{1+y^2}{\tan^{-1}y-x} \Rightarrow \frac{dx}{dy}=\frac{\tan^{-1}y}{1+y^2}-\frac{x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$$

This is equation of the form

$$\frac{dx}{dy}+Px=Q$$

So, I.F. $=e^{\int P dy}=e^{\int \frac{1}{1+y^2}.dy}=e^{\tan^{-1}y}$.

Q.59 (3)

$$\frac{dy}{dx}+\frac{1}{x}y=\sin x\left[type\frac{dy}{dx}+py=Q\right]$$

$$e^{\int pdx}=e^{\int \frac{1}{x}dx}=e^{\log x}=x$$

$$\therefore Sol.is\ y\,x=\int x\sin x dx+C$$

$$=x(-\cos x)-\int 1(-\cos x)dx+C$$

$$=-x\cos x+\sin x+C$$

$$\Rightarrow x(y+\cos x)=\sin x+C$$

Q.60 (3)

Q.61 (1)

Given differential equation as follows:

$\frac{dy}{dx}+\frac{2x}{x^2-1}y=\frac{1}{x^2-1}$, which is a linear form

The integrating factor I.F.

$$=e^{\int \frac{2x}{x^2-1}dx}=e^{\ell n\left(x^2-1\right)}=x^2-1$$

Thus multiplying the given equation by $\left(x^2-1\right)$,

we get $\left(x^2-1\right)\frac{dy}{dx}+2xy=1$

$$\Rightarrow \frac{d}{dx}\left[y\left(x^2-1\right)\right]=1$$

On integrating we get $y\left(x^2-1\right)=x+c$

Q.62 (c,d)

Q.63 (3)

Q.64 (3)

Q.65 (3)

Q.66 (3)

Q.67 (3)

Q.68 (4)

$$x\frac{dy}{dx}+\frac{3}{(dy/dx)}=y^2 \Rightarrow$$

$$x\left(\frac{dy}{dx}\right)^2-y^2\frac{dy}{dx}+3=0$$

Hence it is a non-linear differential equation.

Q.69 (1)

The given equation $\frac{dy}{dx}+\frac{y}{x}=x^2$ is of the form

$\frac{dy}{dx}+Py=Q$. So, I.F.= $e^{\int\frac{1}{x}dx}=e^{\log x}=x$

Hence required solution $xy=\int x.x^2dx+c$

$\Rightarrow xy=\frac{x^4}{4}+c \Rightarrow 4xy=x^4+c$.

Q.70 (1)

$$x\frac{dy}{dx}+y=x^2+3x+2 \Rightarrow$$

$$\frac{dy}{dx}+\frac{y}{x}=x+3+\frac{2}{x}$$

Here $P=\frac{1}{x}, Q=x+3+\frac{2}{x}$, therefore I.F. $e^{\int\frac{1}{x}dx}=x$

Now solve it.

Q.71 (2)

$x^2\frac{dy}{dx}+y=e^x$ can be written as $\frac{dy}{dx}+\frac{y}{x^2}=\frac{e^x}{x^2}$, which is a linear equation.

Q.72 (2)

$$x\frac{dy}{dx}+3y=x \Rightarrow \frac{dy}{dx}+\frac{3y}{x}=1$$

It is in the form of $\frac{dy}{dx}+Py=Q$

So, I.F. $=e^{\int Pdx}=e^{3\int\frac{1}{x}dx}=e^{3\log x}=x^3$

Hence required solution is

$$y+x^2+2x+2=ce^x \Rightarrow yx^3=\frac{x^4}{4}+c .$$

Q.73 (1)

$$y+x^2=\frac{dy}{dx} \Rightarrow \frac{dy}{dx}-y=x^2$$

This is the linear differential equation in *y*, where

$P=-1, Q=x^2$

I.F. $=e^{\int P.dx}=e^{\int -dx}=e^{-x}$

Hence solution, $y.(I.F).=\int Q.(I.F)dx+c$

$\Rightarrow ye^{-x}=-x^2e^{-x}-2xe^{-x}-2e^{-x}+c$

$\Rightarrow y+x^2+2x+2=ce^x$.

Q.74 (1)

$$\frac{dy}{dx}=\frac{-2xy}{(x^2+1)} \Rightarrow \frac{dy}{y}=-\frac{2x}{x^2+1}dx$$

On integrating, we get

$\log y=-\log(1+x^2)+\log c \Rightarrow y(1+x^2)=c$

Since curve passes through (1, 2), we have

$c=2(1+1^2) \Rightarrow c=4$

Hence solution is $y(x^2+1)=4$.

Q.75 (2)

We have $\frac{dy}{dx}=1-\frac{1}{x^2} \Rightarrow y=x+\frac{1}{x}+c$

This passes through $\left(2,\frac{7}{2}\right)$, $\therefore \frac{7}{2}=2+\frac{1}{2}+c$

$\Rightarrow c=1$

Thus the equation of the curve is

$y=x+\frac{1}{x}+1$ or $xy=x^2+x+1$.

Q.76 (3)

$\frac{dy}{dx}=\frac{x^2+y^2}{x^2-y^2}$ and $\frac{dy}{dx}$ is the slope of the curve,

$$\left(\frac{dy}{dx}\right)_{(1,0)}=\frac{1+0}{1-0}=1$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 **(4)**

$y = k_1 \sin(x + C_3) - k_2 e^x$

$k_1 : C_1 + C_2 ; k_2 = c_4 e^{C_5}$

order : 3

Q.2 **(1)**

tangent to $x^2 = 4y$

$x = my + \frac{1}{m}$

$m = \frac{dy}{dx} \Rightarrow x = y\left(\frac{dy}{dx}\right) + \frac{1}{(dy/dx)}$

$$\Rightarrow x\left(\frac{dy}{dx}\right) = y\left(\frac{dy}{dx}\right)^2 + 1$$

$\therefore$ order = 1
degree = 2

Q.3 (4)

$$Ax^2 + By^2 = 1 \qquad ......(1)$$

$$Ax + By\frac{dy}{dx} = 0 \qquad ......(2)$$

$$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0 \qquad .....(3)$$

From (2) and (3)

$$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$$

Dividing both sides by –B, we get

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

Which is a DE of order 2 and degree 1

Q.4 (1)

Put x = sin θ and y = sin ϕ

$$\Rightarrow \cos\theta + \cos\theta = a(\sin\theta - \sin\phi)$$

$$\Rightarrow 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = 2a\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$$\Rightarrow \cot\frac{\theta-\phi}{2} = a \Rightarrow \theta - \phi = 2\cot^{-1} a \; 7$$

$$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$$

Differentiate $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

so the degree is one

Q.5 (1)
Q.6 **(1)**
Q.7 **(1)**

Q.8 **(4)**

$$y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x = -3x\left(\frac{dy}{dx}\right)^{1/3}$$

$$\left(y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x\right)^3 = -9x^3\left(\frac{dy}{dx}\right)$$

here order = 2 = p
Degree = 6 = q
$\therefore$ p < q

Q.9 (1)

$$y = Ax + A^3 \Rightarrow \frac{dy}{dx} = A$$

$$\therefore y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$$

Degree = 3

Q.10 **(4)**

$$\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$$

$$\left(1 + \frac{3dy}{dx}\right)^2 = \left(4\frac{d^3y}{dx^3}\right)^3$$

order = 3
Degree = 3

Q.11 **(2)**

$y^2 = 4ax + k$

$$2y\frac{dy}{dx} = 4a$$

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$

degree = 1
order = 2

Q.12 **(3)**
3

$$\left(\frac{dy}{dx}\right)^{1/3} = 4\frac{d^2y}{dx^2} + 7x$$

$$\left(\frac{dy}{dx}\right) = \left(4\frac{d^2y}{dx^2} + 7x\right)^3$$

order =2 = a
degree = 3 = b
a + b = 5

Q.13 **(1)**

$ax^2 + 2hxy + by^2 = 1$
order : 3

Q.14 **(3)**

$y = e^{mx}$

$D^3y - 3D^2y - 4\,Dy + 12\,y = 0$
$m^3 e^{mx} - 3m^2 e^{mx} - 4me^{mx} + 12\,e^{mx} = 0$
$m^3 - 3m^2 - 4m + 12 = 0$
$m^2(m - 3) - 4(m - 3) = 0$
$m = 3, 2, -2$
Two Natural numnber of m possible

Q.15 **(4)**
$y = mx + c$
$y' = m$
$D^2y - 3Dy - 4y = -4x$
$0 - 3m - 4(mx + c) = -4x$
$-3m - 4mx - 4C = -4x$
$-4m = -4 \Rightarrow m = 1$

$-3m - 4C = 0 \Rightarrow 4C = -3m \Rightarrow C = -\frac{3}{4}$

Q.16 **(3)**

$\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{e^{2y}}{2} = x + c$

$y = 0,\ x = 5 \Rightarrow c = -\frac{9}{2}$

$y(x_0) = 3$

$\Rightarrow \frac{e^6}{2} = x_0 - \frac{9}{2} \Rightarrow x_0 = \frac{e^6 + 9}{2}$

Q.17 **(3)**
Let equation of St. Line
$Y - y = m(X - x)$

Distance from origin $\Rightarrow \left|\frac{mx - y}{\sqrt{1+m^2}}\right| = 1$

$\therefore\ (mx - y)^2 = 1 + m^2$

$\left(y - \frac{dy}{dx}x\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

Q.18 **(1)**

$y' = \frac{x^2 + y^2}{x^2 - y^2}$

$y'_{(1,2)} = \frac{1+4}{1-4} = \frac{-5}{3}$

Q.19 **(2)**
$y = a + bx + ce^{-x}$
$y' = b - ce^{-x}$
$y'' = ce^{-x}$
$y''' = -ce^{-x}$
$y''' = -y'' \Rightarrow y''' + y'' = 0$

Q.20 **(1)**
$(x - h)^2 + (y - k)^2 = a^2$

$(x - h) + (y - k)\,y' = 0 \Rightarrow y' = \frac{-(x-h)}{(y-k)}$

$1 + (y - k)\,y'' + (y)^2 = 0 \Rightarrow y'' = \frac{-a^2}{(y+k)^3}$

(1) option satisfy the given conditions

Q.21 **(3)**
$y = e^{(k+1)x}$
$y' = (k+1)e^{(k+1)x}$
$y'' = (k+1)^2 e^{(k+1)x}$

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

$(k+1)^2 - 4(k+1) + 4 = 0$
$k^2 + 2k + 1 - 4k = 0$
$(k-1)^2 = 0$
$k = 1$

Q.22 **(1)**

$x^2 + y^2 - 2ay = 0 \Rightarrow a = \frac{(x^2 + y^2)}{2y}$

$2x + 2yy' - 2ay' = 0$

$x + yy' - \left(\frac{x^2 + y^2}{2y}\right)y' = 0$

$x + y'\left(\frac{y^2 - x^2}{2y}\right) = 0$

$2xy + y'(y^2 - x^2) = 0$
$y'(x^2 - y^2) = 2xy$

Q.23 **(3)**

$\frac{dy}{dx} - ky = 0, \qquad \frac{dy}{y} = k\,dx$

$\ell ny = kx + c$
at $x = 0,\ y = 1\ \therefore\ C = 0$
Now $\ell ny = kx$
$y = e^{kx}$

$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{kx} = 0$

$\therefore\ k < 0$

Q.24 **(2)**

$\frac{dy}{dx} = 1 + x + y + xy = (1 + x)(1 + y)$

$\Rightarrow \int \frac{dy}{1+y} = \int (1+x)\,dx$

$\Rightarrow \ell n(1 + y) = x + \frac{x^2}{2} + c$

$y(-1) = 0 \Rightarrow c = \frac{1}{2}$

$\ell n(1 + y) = x + \frac{x^2}{2} + \frac{1}{2} = \frac{(1+x)^2}{2}$

$\Rightarrow y = e^{\frac{(1+x)^2}{2}} - 1$

Q.25 (1)

$$\frac{dy}{dx} + 2y = 1 \Rightarrow \frac{dy}{dx} = 1 - 2y$$

$$\int \frac{dy}{1-2y} = \int dx - \frac{1}{2}\log|1-2y| = x + C$$

at $x = 0, y = 0; -\frac{1}{2}\log 1 = 0 + C \Rightarrow C = 0$

$$1 - 2y = e^{-2x} \Rightarrow y = \frac{1-e^{-2x}}{2}$$

Q.26 (1)

$$x^2 = e^{\left(\frac{x}{y}\right)^{-1}\left(\frac{dy}{dx}\right)} \Rightarrow x^2 = e^{\left(\frac{y}{x}\right)\left(\frac{dy}{dx}\right)}$$

$$\Rightarrow \ln x^2 = \frac{y}{x}\frac{dy}{dx} \; or \int x \ln x^2 dx = \int y dy$$

Put $x^2 = t \Rightarrow 2xdx = dt \therefore \frac{1}{2}\int \ln t dt = \frac{y^2}{2}$

$$C + t\ln t - t = y^2 \; or \; y^2 = x^2\left(\ln x^2 - 1\right) + C$$

Q.27 **(3)**

We have $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$\Rightarrow ydx - xdy = ay^2dx + ady$

$\Rightarrow y(1 - ay)dx = (x + a)dy$

$$\Rightarrow \frac{dx}{x+a} - \frac{dy}{y(1-ay)} = 0$$

Intergrating, we ge

$\log(x + a) - \log y + \log(1 - ay) = \log C$ or

$$\log\frac{(a+x)(1-ay)}{y} = \log C \text{ i.e. } (x+a)(1-ay) = Cy$$

Since the curve passes through $\left(a, -\frac{1}{a}\right)$

$$\therefore 2a \times (1+1) = -\frac{C}{a} \text{ i.e } C = -4a^2$$

So, $(x + a)(1 - ay) = -4a^2y$

Q.28 **(3)**

Since, $(e^x + 1)ydy = (y + 1)e^x dx$

$$\Rightarrow \frac{dx}{dy} = \frac{y}{1+y} + \frac{y}{(1+y)e^x}$$

$$\Rightarrow \frac{dx}{dy} = \left(\frac{y}{1+y}\right)\left(\frac{e^x+1}{e^x}\right)$$

$$\Rightarrow \left(\frac{y}{1+y}\right)dy = \left(\frac{e^x+1}{e^x}\right)dx$$

After integrating on both sides, we have

$$\int \frac{y}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow \int 1 dy - \int \frac{y}{1+y}dy = \int \frac{e^x}{1+e^x}dx$$

$$\Rightarrow y - \log|(1+y) = \log|(1+e^x) + \log k$$

Hence $y = \log[k(1 + y)(1 + e^x)]$

Q.29 (1)

Q.30 (4)

Q.31 **(1)**

Q.32 **(4)**

Q.33 **(2)**

x dy = y dx

$\frac{dy}{y} \Rightarrow \frac{dx}{x} \Rightarrow \ell n\, y - \ell n\, x = c$

y = kx

$\therefore$ straight line passing through origin

Q.34 **(2)**

ydy = (1 – x) dx

$\frac{y^2}{2} = x - \frac{x^2}{2} + C$

$x^2 + y^2 - 2x - C = 0$

Q.35 **(1)**

$y\ell ny + xy' = 0$

$y\ell ny + x\frac{dy}{dx} = 0 \Rightarrow \frac{dx}{x} + \frac{dy}{y\ell ny} = 0$

$\ell nx + \ell n(\ell ny) = \ell nC$

$x(\ell ny) = C$

$y(1) = e$

$\ell ne = C \Rightarrow C = 1 \qquad x(\ell ny) = 1$

Q.36 **(2)**

$\frac{dy}{dx} = 100 - y$

$-\ell n(100 - y) = x + C$; $y(0) = 50$

$-\ell n\ (100 - y) = x - \ell n\ 50 \Rightarrow C = -\ \ell n\ 50$

$\ell n \left(\frac{100 - y}{50}\right) = -x$

$100 - y = 50\ e^{-x}$

$y = 100 - 50\ e^{-x}$

Q.37 **(4)**

$ydx + xdy + x\ (xy)\ dy = 0$

Let $xy = t \Rightarrow x = \frac{t}{y}$

$xdy + ydx = dt$

$dt + \left(\frac{t}{y}\right) t\ dy = 0$

$\frac{dt}{t^2} + \frac{dy}{y} = 0 \Rightarrow -\frac{1}{t} + \ell ny = C$

$\frac{-1}{xy} + \ell ny = C$

Q.38 **(1)**

$\frac{dv}{dt} + \frac{K}{m} v = -g$

Integrating factor (I.F.) = $e^{\int \frac{k}{m} dt} = e^{\frac{K}{m}t}$

$\therefore\ V\ e^{\frac{K}{m}t} = -\int g\ e^{K \cdot t/m}$

$V\ e^{\frac{K}{m}t} = \frac{-gm}{K} e^{\frac{K}{m}t} + c$

$V = C \cdot e^{\frac{-K}{m}t} - \frac{mg}{K}$

Q.39 (1)

We have $\frac{dy}{dx} = \frac{f'(x)}{f(x)} y - \frac{y^2}{f(x)}$

$\Rightarrow \frac{dy}{dx} - \frac{f'(x)}{f(x)} y = -\frac{y^2}{f(x)}$

Divide by y^2 : $y^{-2}\frac{dy}{dx} - y^{-1}\frac{f'(x)}{f(x)} = -\frac{1}{f(x)}$

Put $y^{-1} = z \Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$

$-\frac{dz}{dx} - \frac{f'(x)}{f(x)}(z) = -\frac{1}{f(x)}$

$\frac{dz}{dx} + \frac{f'(x)}{f(x)} z = \frac{1}{f(x)}$

I.F. $= e^{\int \frac{f'(x)}{f(x)}dx} = e^{\log f(x)} = f(x)$

$\therefore$ The solution is

$z(f(x)) = \int \frac{1}{f(x)}(f(x))dx + c$

$\Rightarrow y^{-1}(f(x)) = x + c \Rightarrow f(x) = y(x + c)$

Q.40 (1)

Given differential equation is

$dy + \{y\phi'(x) - \phi(x)\phi'(x)\}dx = 0$

$\Rightarrow \frac{dy}{dx} + \phi'(x)y = \phi(x)\phi'(x)$

which is a linear differential equation with

$P = \phi'(x), Q = \phi(x).\phi'(x)$ and

$\text{I.F} = e^{\int \phi'(x)dx} = e^{\phi(x)}$

$\therefore$ Solution is $y.e^{\phi(x)} = \int \phi(x).\phi'(x)e^{\phi(x)}dx + C$

$\Rightarrow y.e^{\phi(x)} = \int \underset{I}{\phi(x)}.e^{\phi(x)}\underset{II}{\phi'(x)}dx + C$

$\Rightarrow y.e^{\phi(x)} = \phi(x)e^{\phi(x)} - \int \phi'(x)e^{\phi(x)}dx + C$

$\Rightarrow y.e^{\phi(x)} = \phi(x)e^{\phi(x)} - e^{\phi(x)} + C$

$\Rightarrow y = [\phi(x) - 1] + Ce^{-\phi(x)}$

Q.41 **(2)**

Given differential equation is :

$x \cos x\ dy/dx + y(x \sin x + \cos x) = 1$

Dividing both the sides by $x \cos x$,

$\Rightarrow \frac{dy}{dx} + \frac{xy\sin x}{x\cos x} + \frac{y\cos x}{x\cos x} = \frac{1}{x\cos x}$

$\Rightarrow \frac{dy}{dx} + y\tan x + \frac{y}{x} = \frac{1}{x\cos x}$

$\Rightarrow \frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right)y = \frac{\sec x}{x}$

Which is of the form $\frac{dy}{dx} + Py = Q$

Here, $P = \tan x + \frac{1}{x}$ and $Q = \frac{\sec x}{x}$

Integrating factor $e^{\int Pdx} = e^{\int \tan x + \frac{1}{x}dx}$

$= e^{(\log \sec x + \log x)} = e^{\log(\sec x.x)} = s \sec x$

Q.42 **(3)**

$(2x - 10y^3)\ \frac{dy}{dx} + y = 0$

$$\frac{dy}{dx} = \frac{y}{10y^3 - 2x} \Rightarrow \frac{dx}{dy} = \frac{10y^3 - 2x}{y}$$

$$\frac{dx}{dy} + \frac{2x}{y} = 10y^2$$

$$\text{I.F.} = e^{\int \frac{2}{y} dy} = y^2$$

$$x\,(y^2) = \int 10y^4 dy$$

$$y^2 x = 2y^5 + C$$

Q.43 **(1)**

$y' + y\phi' - \phi\phi' = 0$
$y' + \phi'(y - \phi) = 0$
$dy + \phi'(y - \phi)\,dx = 0$
Let $\phi = t \Rightarrow \phi'\,dx = dt$
$dy + (y - t)\,dt = 0$

$$\frac{dy}{dt} + y = t$$

I.F. $= e^t$

$$ye^t = \int te^t\,dt$$

$ye^t = te^t - e^t + C$
$y = t - 1 + ce^{-t}$
$y = \phi(x) - 1 + ce^{-\phi(x)}$

Q.44 **(3)**

$$\frac{xdy}{x^2+y^2} = \frac{ydx}{x^2+y^2} - dx$$

$$\frac{xdy - ydx}{x^2+y^2} = -\,dx$$

$$\frac{\frac{xdy - ydx}{x^2}}{1+\left(\frac{y}{x}\right)^2} = -\,dx \Rightarrow \frac{d(y/x)}{1+(y/x)^2} = -\,dx$$

$$d\left(\tan^{-1}\frac{y}{x}\right) = -\,dx \Rightarrow \tan^{-1}\frac{y}{x} = -x + C$$

$$\frac{y}{x} = \tan(C - x) \Rightarrow y = x\tan(C - x)$$

Q.45 **(3)**

EXERCISE-III

Q.1 0005

We have

$$y = c_1\cos(2x + c_2) - (c_3 + c_4)a^{x+c_5} + c_6\sin(x - c_7)$$

$$= c_1\cos(2x + c_2) - c_8.a^{c_5}.a^x + c_6\sin(x - c_7)$$

where $c_3 + c_4 = c_8$.

Since the above relation contains five arbitrary constants, so the order of the differential equation satisfying it, is 5.

Q.2 1.5

Hint: $y = u^m \Rightarrow \frac{dy}{dx} = mu^{m-1}\frac{du}{dx}$,

Hence $2x^4.u^m.m\,u^{m-1}.\frac{du}{dx} = u^{4m} = 4x^6$.

$$\frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4u^{2m-1}} \Rightarrow 4m = 6 \Rightarrow m = \frac{3}{2}$$

Q.3 0002

The equation of any tangent to the parabola

$y^2 = 4ax$ is $y = mx + \frac{a}{m}$, where m is any arbitrary constant.

On differentiating w.r.t. x, we get $\frac{dy}{dx} = m$

On substituting the alue of m in (1), we get

$$y = x\frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$$

$$\Rightarrow \quad x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + a = 0,$$

which is a differential equation of degree 2.

Q.4 0.5

Let $m_1 = \frac{dy}{dx}$ for required family of curves at (x, y)

Let $m_2 = \frac{dy}{dx}$ for the hyperbola $xy = 2$.

Then $m_2 = \frac{dy}{dx} = -\frac{2}{x^2}$.

Since the required family of curves is orthogonal to the hyperbola.

$\therefore \quad m_1 \times m_2 = -1$

$\Rightarrow \quad \frac{dy}{dx} \times \left(-\frac{2}{x^2}\right) = -1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2}{2}$

$\Rightarrow \quad dy = \frac{x^2}{2} dx$

Integrating, we get $y = \frac{x^3}{6} + c$ which is the required family.

Q.5 0012

Since, $y = e^{4x} + 2e^{-x}$

$\Rightarrow \quad y_1 = 4e^{4x} - 2e^{-x}$

$\Rightarrow \quad y_2 = 16e^{4x} + 2e^{-x}$

$\Rightarrow \quad y_3 = 64e^{4x} - 2e^{-x}$

Now,

$y_3 - 13y_1 = (64e^{4x} - 2e^{-x}) - 13(4e^{4x} - 2e^{-x})$

$= 12e^{4x} + 24e^{-x} = 12(e^{4x} + 2e^{-x}) = 12y$

$\therefore \quad \frac{y_3 - 13y_1}{y} = 12.$

Q.6 0.25

$\frac{dy}{dx} = 2c_1e^{2x} + c_2e^{x} - c_3e^{-x}$ $\frac{d^2y}{dx^2} = 4c_1e^{2x} + c_2e^{x} + c_3e^{-x}$

$\frac{d^3y}{dx^3} = 8c_1e^{2x} + c_2e^{x} - c_3e^{-x}$, Putting into the given differential equatiopn.

We get, $8 + 4a + 2b + c = 0, 1 + a + b + c = 0,$
$-1 + a - b + c = 0$

$\Rightarrow a = -2, b = -1, c = 2.$

Thus $\frac{a^3 + b^3 + c^3}{abc} = -\frac{1}{4}.$

Q.7 0004

let m_1 is the slope of y=f(x) and m_2 is the slope of xy=4

$m_1m_2 = -1$

$\frac{4}{x^2} \cdot \frac{dy}{dx} = -1 \quad 4dy = x^2dx$ then integrating both side

$y = \frac{x^3}{12} + \frac{c}{4}$

Q.8 0000

$\frac{x\,dy - y\,dx}{y^2} = \frac{dy}{y}$

$\Rightarrow -d\left(\frac{x}{y}\right) = \frac{dy}{y} \qquad \Rightarrow -\frac{x}{y} = \log y$

$\Rightarrow e^{-x/y} = cy \qquad \Rightarrow ye^{x/y} = c$ at $x = 0, y = 1, c = 1 \quad y \cdot e^{x/y} = 1$

At $y = e \quad e \cdot e^{x/e} = 1 \quad e^{x/e} = e^{-1} \Rightarrow x = -e \quad a = -b, b = e$

$\therefore a + b = 0$

Q.9 0003

The differential equation is

$\frac{d}{dx}\left(y\frac{dy}{dx}\right) = x$

$\Rightarrow y\frac{dy}{dx} = \frac{x^2}{2} + c_1 \qquad \Rightarrow ydy = \left(\frac{x^2}{2} + c_1\right)dx$

$\Rightarrow \frac{y^2}{2} = \frac{x^3}{6} + c_1x + c_2 \quad \Rightarrow y^2 = \frac{x^3}{3} + c_1x + c_2$

$\begin{pmatrix} 2c_1 \to c_1 \\ 2c_2 \to c_2 \end{pmatrix}$

$\Rightarrow \lambda = 3$

Q.10 0.5

Given $\frac{dx}{dt} = \cos^2 \pi x$. Differentiate w.r.t,

$\frac{d^2x}{dt^2} = -2\pi \sin 2\pi x = -ve$

$\because \frac{d^2x}{dt^2} = 0 \Rightarrow -2\pi \sin 2\pi x = 0 \Rightarrow \sin 2\pi x = \sin \pi$

$\Rightarrow 2\pi x = \pi \Rightarrow x = 1/2.$

PREVIOUS YEAR'S

MHT CET

Q.1 (4)	**Q.2** (4)	**Q.3** (3)	**Q.4** (1)	**Q.5** (4)
Q.6 (3)	**Q.7** (1)	**Q.8** (2)	**Q.9** (4)	**Q.10** (2)
Q.11 (4)	**Q.12** (1)	**Q.13** (2)	**Q.14** (2)	**Q.15** (2)
Q.16 (2)	**Q.17** (2)	**Q.18** (1)	**Q.19** (2)	**Q.20** (1)
Q.21 (4)	**Q.22** (2)	**Q.23** (3)	**Q.24** (1)	**Q.25** (4)
Q.26 (4)	**Q.27** (4)	**Q.28** (4)	**Q.29** (3)	**Q.30** (2)
Q.31 (4)	**Q.32** (4)	**Q.33** (2)	**Q.34** (2)	**Q.35** (2)
Q.36 (2)	**Q.37** (1)	**Q.38** (1)	**Q.39** (1)	
Q.40 (3)				

The general equation of a parabola having vertex at the origin and axis along positive Y-axis is

$x^2 = 4ay$ ...(i)

On differentiating Eq. (i), we get

$2x = 4a\frac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{x}{2a} \Rightarrow 2a = \dfrac{x}{dy/dx}$

Putting value of 2a in Eq. (i), we get

$$x^2 = 2\left(\frac{x}{dy/dx}\right)y \Rightarrow x\frac{dy}{dx} = 2y$$

Q.41 (4)

Given equation is $\sqrt{\dfrac{dy}{dx}} - 4\dfrac{dy}{dx} - 7x = 0$

$$\Rightarrow \frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^2 + 49x^2 + 56x\frac{dy}{dx}$$

Obviously, it is first order and second degree differential equaiton.

Q.42 (1)

Given equation cab be rewritten as

$$\left(1+\frac{1}{y}\right)dy = -e^x(\cos^2 x - \sin 2x)\,dx$$

On intergrating both sides, we get

$y + \log y = -e^x \cos^2 x + \int e^x \sin 2x\,dx$

$-\int e^x \sin 2x\,dx + C$

$\Rightarrow y + \log y = -e^x \cos^2 x + C$

At x = 0 and y = 1,

$1 + 0 = -e^0 \cos 0 + C$

$\Rightarrow C = 2$ [given]

$\therefore$ Required solution is

$y + \log y = -e^x \cos^2 x + 2$

$\Rightarrow y + \log y + e^x \cos^2 x = 2$

Q.43 (1)

Given, differential equation

$$\frac{dy}{dx} = \frac{y+1}{x^2 - x} \Rightarrow \frac{dy}{y+1} = \frac{dx}{x^2 - x}$$

On intergrating both sides, we get

$$\int \frac{dy}{y+1} = \int \frac{dx}{x^2 - x}$$

Now, $\dfrac{1}{x^2 - x} = \dfrac{1}{x(x-1)} = \dfrac{A}{x} + \dfrac{B}{x-1}$

$\therefore \dfrac{1}{x(x-1)} = \dfrac{A}{x} + \dfrac{B}{x-1}$ ...(i)

$\Rightarrow 1 = x(x-1) + B(x)$

Putting x = 0, then

$1 = A(0) + B(1)$

$\Rightarrow B = 1$

From Eq. (i), we get

$$\int \frac{dy}{y+1} = \int -\frac{1dx}{x} + \int \frac{1}{x-1}dx$$

$\Rightarrow \log(y+1) = -\log x + \log(x-1) + \log C$

$\Rightarrow \log(y+1) + \log x - \log x - \log(x-1) = \log C$

$\Rightarrow \log\left\{\dfrac{x(y+1)}{y+1}\right\} = \log C$

$\Rightarrow \dfrac{x(y+1)}{x-1} = C$ ...(ii)

On putting x = 2 and y = 1 in Eq. (ii), we get

$\dfrac{2(1+1)}{2-1} = C \Rightarrow \quad C = (2)(2) = 4$

Putting value of C = 4 in Eq. (ii), We get

$\dfrac{x(y+1)}{x-1} = 4$

$\Rightarrow xy + x = 4x - 4 \Rightarrow xy = 3x - 4$

Q.44 (1)

Integral curve satisfying $y' = \dfrac{x^2 + y^2}{x^2 - y^2}$, y(1) = 2, has the slope at the point (1, 2) of the curve, equal to

(1*) $-\dfrac{5}{3}$ (2) – 1 (3) 1 (4) $\dfrac{5}{3}$

Ans. Let $x^2 + y^2 - 2ky = 0$

On differentiating w.r.t x, we get

$$2x + 2y\frac{dy}{dx} - 2k\frac{dy}{dx} = 0$$

$$\Rightarrow \quad k = \frac{x}{\left(\frac{dy}{dx}\right)} + y$$

From Eq. (i),

$$x^2 + y^2 - 2\left(\frac{x}{dy/dx} + y\right)y = 0$$

$$\Rightarrow (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$$

Q.45 (2)

Given, $(x^2 + xy)\,dy = (x^2 + y^2)dx$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{x^2 + x^2v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v^2}{1+v}$$

$$\Rightarrow \frac{xdv}{dx} = \frac{1+v^2}{1+v} - v$$

$$= \frac{1+v^2-v-v^2}{1+v} = \frac{1-v}{1+v}$$

$$\Rightarrow dv\left(\frac{1+v}{1-v}\right) = \frac{dx}{x} \Rightarrow dv\left(-1+\frac{2}{1-v}\right) = \frac{dx}{x}$$

On integrating both sides, we get

$-v-2\log(1-v) = \log x + C$

$$\Rightarrow -\frac{y}{x} - 2\log\left(1-\frac{y}{x}\right) = \log x + C$$

$$\Rightarrow -\frac{y}{x} - 2\log(x-y) + 2\log x = \log x + C$$

$$\Rightarrow \frac{y}{x} + 2\log(x-y) + C = \log x$$

Q.46 (2)

Given, $\frac{dy}{dx} + 2y\tan x = \sin x, 0 < x < \frac{\pi}{2}$

Which is linear differential equation.

Here, P = 2 tan x and Q sin x

IF $e^{\int P\,dx} = e^{2\int \tan x\,dx} = e^{2\log\sec x} = \sec^2 x$

$\therefore$ Required solution of differential equation,

$y.\ IF = \int (Q \times IF)\,dx + C$

$\Rightarrow y\sec^2 x = \int (\sin x \times \sec^2 x)\,dx + C$

$= \int \tan x \sec dx + C$

$\therefore y\sec^2 x = \sec x + C$ ...(i)

As, $y\left(\frac{\pi}{6}\right) = 0$

$$\Rightarrow 0.\sec^2\left(\frac{\pi}{6}\right) = \sec\frac{\pi}{6} + C \Rightarrow C = -\frac{2}{\sqrt{3}}$$

$\therefore \sec^2 x = \sec x - \frac{2}{\sqrt{3}}$ [from Eq.(i)]

$$\Rightarrow y = \cos x - \frac{2}{\sqrt{3}}\cos^2 x$$

$$= -\frac{2}{\sqrt{3}}\left(\cos^2 x - \frac{\sqrt{3}}{2}\cos x\right)$$

$$= -\frac{2}{\sqrt{3}}\left[\cos^2 x - \frac{\sqrt{3}}{2}\cos x + \left(\frac{\sqrt{3}}{4}\right)^2 - \left(\frac{\sqrt{3}}{4}\right)^2 - \left(\frac{\sqrt{3}}{4}\right)^2\right]$$

$$= -\frac{2}{\sqrt{3}}\left[\left(\cos x - \frac{\sqrt{3}}{4}\right)^2 - \frac{3}{4}\right]$$

$$= \frac{2}{2\sqrt{3}} - \frac{2}{\sqrt{3}}\left(\cos x - \frac{\sqrt{3}}{4}\right)^2$$

Minimum value of $\left(\cos x - \frac{\sqrt{3}}{4}\right)$ is 0.

$\therefore$ Maximum value of $y = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

Q.47 (4)

The given equation cannot be written as a polynomial in all the differentials.

$\therefore$ Degree of the equation is not defined but order = 2

Q.48 (1)

Given, equation of plane passes through (2,5,–3) is

$a(x-2) + b(y-5) + c(z+3) = 0$ ...(i)

Which is perpendicular to the planes,

$x + 2y + 2z = 1$ and $x - 2y + 3z = 4$

Then, $a + 2b + 2c = 0$ ...(ii)

and $a - 2b + 3c = 0$ ...(iii)

Eliminating a,b,c from Eqs. (i), (ii) and (iii), we get

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ 1 & 2 & 2 \\ 1 & -2 & 3 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(6+4) - (y-5)(3-2) + (z+3)(-2-2) = 0$

$\Rightarrow 10x - y - 4z = 27$

Q.49 (3)

Let equation of the circle to

$x^2 + y^2 - 2gx = 0$

Differentiating w.r.t.x,

$$2x + 2y\frac{dy}{dx} - 2g = 0 \Rightarrow 2g = \left(2x + 2y\frac{dy}{dx}\right)$$

Putting 2g in Eq. (i),

$$x^2 + y^2 - \left(2x + 2y\frac{dy}{dx}\right)x = 0 \Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$$

Q.50 (1)

Given, differential equation is

$$(1+y^2) + \left(x - e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$$

$$\Rightarrow (1+y^2)\frac{dx}{dy} = -x + e^{\tan^{-1}}y$$

$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$, which is a linear differential equation.

Here, $P = \frac{1}{1+y^2}, Q = \frac{e^{\tan^{-1}}}{1+y^2}$

IF $= e^{\int P dy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$\therefore$ Solution of differential equation is

X. IF = ∫ Q . If dy + C

$$xe^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2}e^{\tan^{-1}y}dx + \frac{C}{2}$$

$$\therefore\ 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + C$$

Q.51 (1)

We have, $\frac{dy}{dx}=\frac{y}{x+\sqrt{xy}}$

$\Rightarrow \frac{dx}{dy}=\frac{x+\sqrt{xy}}{y}=\frac{x}{y}+\sqrt{\frac{x}{y}}$

Put $x = vy \Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$

$\Rightarrow v+y\frac{dv}{dy}=v+\sqrt{v}\Rightarrow y\frac{dv}{dy}=\sqrt{v}$

$\Rightarrow \frac{dv}{\sqrt{v}}=\frac{dy}{y}\Rightarrow\int\frac{dv}{\sqrt{v}}=\int\frac{dy}{y}$

$\Rightarrow 2\sqrt{v}=\log y+\log C'\Rightarrow 2\sqrt{\frac{x}{y}}=\log(C'y)$

$\Rightarrow C'y=e^{2\left(\sqrt{x/y}\right)}\Rightarrow y=\frac{1}{C'}e^{2\left(\sqrt{x/y}\right)}$

$\Rightarrow y=Ce^{2\left(\sqrt{x/y}\right)}$ [$\because$ 1/C' = C]

Q.52 (1)

We have, $(2x-2y+3)\,dx-(x-y+1)\,dy=0$

$\Rightarrow \frac{dy}{dx}=\frac{2(x-y)+3}{x-y+1}$

Put $x - y = v$

$\Rightarrow 1-\frac{dy}{dx}=\frac{dv}{dx}\Rightarrow\frac{dy}{dx}=1-\frac{dv}{dx}\Rightarrow 1-\frac{dv}{dx}=\frac{2v+3}{v+1}$

$\Rightarrow \frac{dv}{dx}=1-\frac{2v+3}{v+1}=\frac{-(v+2)}{v+1}$

$\Rightarrow \left(\frac{v+1}{v+2}\right)dv=-\,dx$

$\Rightarrow \left(1-\frac{1}{v+2}\right)dv=-\,dx$ ...(i)

On integrating both sides of Eq. (i), We get

$\int\left(1-\frac{1}{v+2}\right)dv=-\int dx$

$\Rightarrow v-\log(v+2)=-x+C$

$\Rightarrow x-y\log(x-y+2)=-x+C$ ...(ii)

On putting x = 0, y = 1 in Eq. (ii), we get C = – 1

$\therefore\ x-y-\log(x-y+2)=-x-1$

$\Rightarrow 2x-y-\log(x-y+2)+1=0$

JEE-MAIN PREVIOUS YEAR'S

Q.1 (4)

$2ye^{x/y^2}dx+(y^2-4xe^{x/y^2})dy=0$

$2e^{x/y^2}(ydx-2xdy)+y^2dy=0$

$2e^{x/y^2}\frac{(y^2dx-2xyd)}{y}+y^2dy=0$

Divide by y^3 both side.

$2e^{x/y^2}\frac{(y^2dx-2xydy)}{y^4}+\frac{1}{y}dy=0$

$2e^{x/y^2}d\left(\frac{x}{y^2}\right)+\frac{dy}{y}=0$

Integrating both side

$\int 2e^{x/y^2}d\left(\frac{x}{y^2}\right)+\int\frac{dy}{y}=0$

$2e^{x/y^2}+\ln y+c=0$

(0,1) lies on it.

$2e^0+\ln 1+c=\Rightarrow c=-2$

Required curve :

$2e^{x/y^2}+\ln y-2=0$

For x (e)

$2e^{x/e^2}+\ln e-2=0\Rightarrow x=-e^2\ \ln 2$

Q.2 (2)

$\frac{dy}{dx}=2\tan x\cos x-2\tan x.y$

$\frac{dy}{dx}+(2\tan x)\,y=2\sin x$

Integrating factor = $e^{\int 2\tan x\,dx}=\frac{1}{\cos^2x}$

$y\left(\frac{1}{\cos^2x}\right)=\int\frac{2\sin x}{\cos^2x}dx$

$y\sec^2x=\frac{2}{\cos x}+C$

$y=2\cos x+C\cos^2x$

Passes through $\left(\frac{\pi}{4},0\right)$

$0=\sqrt{2}+\frac{C}{2}\Rightarrow C=-2\sqrt{2}$

$f(x)=2\cos x-2\sqrt{2}\cos^2x.$

Required curve :

$$\int_0^{\pi/2} ydx = 2\int_0^{\pi/2}\cos xdx - 2\sqrt{2}\int_0^{\pi/2}\cos^2 xdx$$

$$= 2\sin x\Big|_0^{\pi/2} - 2\sqrt{2}\left[\frac{x}{2}+\frac{\sin 2x}{4}\right]_0^{\pi/2}$$

$$= 2-\frac{\pi}{\sqrt{2}}$$

Q.3 (1)

$$\frac{xdy}{dx} - y = \sqrt{y^2+16x^2}$$

$$\frac{dy}{dx} = \frac{y+\sqrt{y^2+16x^2}}{x}$$

Let $y = vx$

$$\frac{dy}{dx} = V + \frac{xdv}{dx}$$

$$V + = V + \frac{xdv}{dx} = \frac{vx+\sqrt{v^2x^2+16x^2}}{x}$$

$$V + x\frac{dv}{dx} = V + \sqrt{v^2+16}$$

$$\int\frac{dv}{\sqrt{v^2+16}} = \int\frac{dx}{x}$$

$$\ell n|v+\sqrt{v^2+16}| = \ln x + \ln C$$

$$\frac{y}{x} + \frac{\sqrt{y^2+16x^2}}{x} = Cx$$

$$y + \sqrt{y^2+16x^2} = Cx^2$$

$y(1) = 3$

$C = 8$

at $x = 2$

$$y + \sqrt{y^2+16x} = 32$$

$y^2 + 64 = (32 - y)^2$

$y^2 + 64 = y^2 - 69y + (32)^2$

$64(1 + y) = 32 \times 32$

$y(2) = 15$

Q.4 (2)

$$\frac{dy}{dx} + \frac{\sqrt{2}}{2\cos^4 x - \cos 2x} y = x\, e^{\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$\text{I.F} = e^{\int\frac{\sqrt{2}dx}{2\cos^4 x - \cos 2x}}$$

$$= e^{\sqrt{2}\int\frac{dx}{\cos^4 x+\sin^4 x}}$$

$$= e^{\sqrt{2}\int\frac{\operatorname{cosec}^4 x}{1+\cot^4 x}dx}$$

$$\text{IF} = e^{-\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$ye^{-\tan^{-1}(\sqrt{2}\cot 2x)} = \int xdx$$

$$ye^{-\tan^{-1}\sqrt{2}\cot 2x} = \frac{x^2}{2} + c$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32} + C \Rightarrow c = 0$$

$$y = \frac{x^2}{2}e^{\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{\tan^{-1}(\sqrt{2}\cot\frac{2\pi}{3})}$$

$$\alpha = \sqrt{\frac{2}{3}}$$

$\Rightarrow 3\alpha^2 = 2$

Q.5 (3)

$$(1+e^{2x})\frac{dy}{dx} + 2(1+y^2)e^x = 0$$

$$\int\frac{dy}{(1+y^2)} = -\int\frac{2e^x}{1+e^{2x}}dx$$

Put $e^x = t \rightarrow e^x \cdot dx = dt$

$$\int\frac{dy}{1+y^2} = -2\int\frac{1}{1+t^2}dt$$

$\tan^{-1}(y) = -2\tan^{-1}t + C$

Given $y(0) = 0 \Rightarrow x = 0, y = 0$

$t = e^x = e^0 = 1$

$\tan^{-1}(0) = -2\tan^{-1}1 + C$

$$0 = -2\frac{\pi}{4} + C \Rightarrow C = \frac{\pi}{2}$$

$$\therefore \tan^{-1} y = -2\tan^{-1} t + \frac{\pi}{2}$$

$$\tan^{-1} y + \tan^{-1}\frac{2t}{1-t^2} = \frac{\pi}{2}$$

$$\tan^{-1} y + \cot^{-1}\frac{1-t^2}{2t} = \frac{\pi}{2}$$

$$\therefore y = \frac{1-t^2}{2t}$$

As $x = \log_e\sqrt{3},\ t = \sqrt{3},\ y = \dfrac{1-(\sqrt{3})^2}{2(\sqrt{3})} = -\dfrac{1}{\sqrt{3}}$

$$\therefore y(\log_e\sqrt{3}) = -\frac{1}{\sqrt{3}}$$

Now, $(1+e^{2x})\cdot\frac{dy}{dx}+2(1+y^2)\cdot e^x=0$

As x = 0, $(1 + e^0)\cdot y'(0) + 2(1 + (y(0))^2) - e^0 = 0$

$(1 + 1)\, y'(0) + 2(1 + 0).\ 1 = 0$

$y'(0) = -1$

$\therefore 6(y'(0)+y(\log_e\sqrt{3})^2$

$=6\left(-1+\frac{1}{3}\right)=6\times-\frac{2}{3}=-4$

Q.6 (14)

$\frac{dy}{dx}+\frac{2x}{x-1}y=\frac{1}{(x-1)^2}$

linear D.E.

$\text{I.F.}=e^{\int\frac{2x}{x-1}dx}$

$=e^{\int\left(\frac{2x-2+2}{x-1}\right)dx}$

$=e^{\int\left(2+\frac{2}{x-1}\right)dx}$

$=e^{2x+2\ln|x-1|}$

$=e^{2x}\cdot e^{\ln(x-1)^2}$

$\Rightarrow \text{I.F}=(x-1)^2\cdot e^{2x}$

$y(x-1)^2e^{2x}=\int\frac{1}{(x-1)^2}(x-1)^2e^{2x}dx$

$\Rightarrow y(x-1)^2e^{2x}=\frac{e^{2x}}{2}+C$

$y(2)=\frac{1+e^4}{2e^4}$

$\Rightarrow\left(\frac{1+e^4}{2e^4}\right)e^4=\frac{e^4}{2}+C$

$\Rightarrow C=\frac{1}{2}$

$y(3)=\frac{e^\alpha+1}{\beta e^\alpha}$

$\Rightarrow\left(\frac{e^\alpha+1}{\beta e^\alpha}\right)\cdot4\cdot e^6=\frac{e^6+1}{2}$

$\Rightarrow\left(\frac{\alpha^\alpha+1}{\beta e^\alpha}\right)=\frac{e^6+1}{8e^6}$

$(\alpha, \beta) = (6, 8)$

$\alpha+\beta=14$

Q.7 **(1)**

$y\frac{dx}{dy}=2x+y^3(y+1)e^y,\ x(1)=0$

$\frac{dx}{dy}=\frac{2x}{y}+y^2(y+1).e^y$

$\frac{dx}{dy}+\left(-\frac{2}{y}\right)x=y^2(y+1).e^y$ (it is linear differential equation)

$\text{I.F.}=e^{\int-\frac{2}{y}dy}=\frac{1}{y^2}$

$\therefore x\cdot\frac{1}{y^2}=\int\frac{1}{y^2}\cdot y^2(1+y)\cdot e^ydy+C$

$\frac{x}{y^2}=\int(1+y)\cdot e^ydy+C$

$\frac{x}{y^2}=y.e^y+C$ at $x(1)=0\ \therefore C=-e$

$\therefore\ x = y^3.e^y - ey^2$

$\therefore\ x(e) = e^3.e^e - e.e^2$

$= e^3.e^e - e^3$

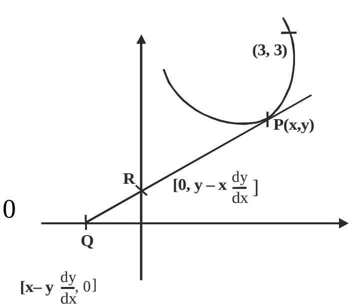
$= e^3(e^e - 1)$

Q.8 (1)

$Y-y=\frac{dy}{dx}(x-x)$

for Q $\quad Y = 0,\ X = x - y\frac{dx}{dy}$

$\frac{x+x-y\frac{dx}{dy}}{2}=0$



$2x-y\frac{dx}{dy}=0$

$2x=y\frac{dx}{dy}\Rightarrow 2\int\frac{dy}{y}=\int\frac{dx}{x}$

$\Rightarrow 2\ \ell ny = \ell nx + \ell nc$

pass (3, 3)

$\Rightarrow c = 3$

$\Rightarrow y^2 = 3x$

length of L.R. = 3

Q.9 (4)

$$-\frac{dx}{dy}=\frac{x^2}{xy-x^2y^2-1}$$

$$\frac{dy}{dx}=\frac{x^2y^2-xy+1}{x^2}$$

$x^2dy = x^2y^2dx – xydx + dx$

$x^2dy + xydx = (x^2y^2+1)dx$

$x[xdy + ydx] = (x^2y^2+1)dx$

$$\int\frac{d(xy)}{1+x^2y^2}=\int\frac{dx}{x}$$

$\tan^{-1}(xy) = \ell nx + c$ = ...(1)

pass (1, 1)

$$\frac{\pi}{4}=c$$

$$\tan^{-1}(xy)=\ell nx+\frac{\pi}{4}$$

put x = e

$$ey(e)=\tan\left[1+\frac{\pi}{4}\right]$$

$$ey(e)=\frac{1+\tan 1}{1-\tan 1}$$

Q.10 **(2)**

$$\frac{dy}{dx}-\frac{y}{x}+\frac{3}{2}\left(\frac{y}{x}\right)^2=0$$

$$y^{-2}\frac{dy}{dx}-\frac{1}{xy}+\frac{3}{2x^2}=0$$

$$\frac{1}{y}=t\Rightarrow-\frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$$

$$-\frac{dt}{dx}-\frac{t}{x}+\frac{3}{2x^2}=0$$

$$\frac{dt}{dx}+\frac{t}{x}=\frac{3}{2x^2}$$

$$\text{I.F.}=e^{\int\frac{dt}{x}}=e^{\ell nx}=x$$

$$tx=\int\frac{3}{2x}dx$$

$$\frac{x}{y}=\frac{3}{2}\ell nx+C$$

$$\Downarrow x=e,\ y=\frac{e}{3}$$

$$\frac{e}{\frac{e}{3}}=\frac{3}{2}+c\Rightarrow c=3-\frac{3}{2}=\frac{3}{2}$$

So, $\frac{x}{y}=\frac{3}{2}\ell nx+\frac{3}{2}$

at $x=1,\ \frac{1}{y}=\frac{3}{2}\Rightarrow y=\frac{2}{3}$

Q.11 **(4)**

$$\frac{dy}{dx}+\frac{2y}{x}=e^x$$

Linear Differential Equation

$$\text{I.F.}=e^{\int\frac{2}{dx}dx}=x^2$$

$yx^2 = \int e^x x^2\,dx$

$yx^2 = x^2e^x – 2xe^x + 2e^x + c$

Put y (1) = 0

$= e – 2e + 2e + c \Rightarrow c = –e$

$z(x) = x^2e^x – 2xe^x + 2e^x – e – e^x$

$= x^2e^x – 2xe^x + e^x – e$

$z'(x) = x^2e^x + 2xe^x – 2e^x – 2xe^x + e^x$

$= x^2e^x – e^x = 0$

$\Rightarrow x = \pm 1$

$z''(x) = 2xe^x + xe^x – e^x$

$z''(1) = 2e + e – e = 2e > 0$

$$z''(-1)=\frac{-2}{e}-\frac{1}{e}-\frac{1}{e}<0.$$

Local maximum at x = –1

$$z(-1)=\frac{1}{e}+\frac{2}{e}+\frac{1}{e}-e=\frac{4}{e}-e$$

Q.12 **(3)**

$$\frac{dy}{dx}+e^x(x^2-2)y-(x^2-2x)(x^2-2)e^{2x}$$

Linear D.E.

$$\text{I.F.}=e^{\int e^x(x^2-2)dx}$$

$$=e^{(x^2-2)e^x-e^x(2x)+2e^x}$$

$$\Rightarrow ye^{e^x(x^2-2x)}=\int e^{e^x(x^2-2x)(x^2-2x)(x^2-2)e^x.e^x}dx$$

Put $e^x(x^2–2x) = t$

$[e^x(2x–2)+e^x(x^2–2x)]dx = dt$

$e^x(x^2–2)dx = dt$

$$ye^{e^x(x^2-2x)}=\int e^t.t\,dt$$

$= te^t – e^t + C$

$$=e^{e^x}(x^2-2x)\left[e^x\left(x^2-2x\right)-1\right]+C$$

Put y = 0

0 = 1[–1]+ C = c = 1

Put = x = 2

y = –1 + 1 = 0

y (2) = 0

Q.13 (4)

$$\frac{dy}{dx}+\frac{2^{x-y}(2^y-1)}{2^x-1}=0$$

x, y > 0, y(1) = 1, y(2) = ?

$$\frac{dy}{dx}=\frac{-2^x(2^y-1)}{2^y(2^x-1)}$$

$$\int\frac{2^y}{2^y-1}dy=-\int\frac{2^x}{2^x-1}dx$$

$$\frac{1}{\ln 2}\int\frac{2^y\ln 2}{2^y-1}dy=-\frac{1}{\ln 2}\int\frac{2^x\ln 2}{2^x-1}dx$$

$$\frac{1}{\ln 2}\ln|2^y-1|=\frac{-1}{\ln 2}\ln|2x-1|+C$$

At x = 1, y = 1

Putting this values in above relation we get C = 0

$ln|2^y-1| + ln|2^x-1| = 0$

$(2^x-1)(2^y-1) = 1$

$$2^y-1=\frac{1}{2^x-1}$$

At x = 2

$$2^y=\frac{1}{3}+1=\frac{4}{3}$$

$$y=\log_2\frac{4}{3}=\log_2 4-\log_2 3=2-\log_2 3$$

Q.14 **(2)**

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$$

$$\text{IF}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

$$xe^{\tan^{-1}y}=\int\frac{\tan^{-1}y}{1+y^2}e^{\tan^{-1}y}\cdot dy$$

$$x\cdot e^{\tan^{-1}y}=(\tan^{-1}y-1)e^{\tan^{-1}}+c$$

$\because$ (1, 0) lies exit c = 2

For $y=\tan 1\Rightarrow x=\frac{2}{e}$

Q.15 **(320)**

$$(1-x^2)\frac{dy}{dx}=xy+(x^3+2)\sqrt{1-x^2}$$

$$\Rightarrow\frac{dy}{dx}+\left(\frac{-x}{1-x^2}\right)y=\frac{x^3+2}{\sqrt{1-x^2}}$$

$$\text{IF}=e^{\int\frac{-x}{1-x^2}dx}=\sqrt{1-x^2}$$

$$\Rightarrow y(x)\cdot\sqrt{1-x^2}=\int\sqrt{1-x^2}\cdot\frac{x^3+2}{\sqrt{1-x^2}}dx$$

$$y(x)\cdot\sqrt{1-x^2}=\frac{x^4}{4}+2x+c$$

$$y(0)=0\Rightarrow c=0$$

$$\sqrt{1-x^2}\,y(x)=\frac{x^4}{4}+2x$$

$$\Rightarrow y(x)=\frac{\frac{x^4}{4}+2x}{\sqrt{1-x^2}}$$

required value

$$=\int_{-1/2}^{1/2}\left(\frac{x^4}{4}+2x\right)dx=\frac{1}{4}\cdot 2\int_0^{1/2}x^4dx+2\cdot\int_{-\frac{1}{2}}^{\frac{1}{2}}x\,dx$$

$$=\frac{1}{10}(x^5)_0^{1/2}=\frac{1}{320}$$

$k^{-1}=320$

Q.16 (2)

$$(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$$

$$\frac{dy}{dx}-\frac{y}{x+1}=e^{3x}(x+1)$$

$$\text{IF}\,e^{-\int\frac{dx}{x+1}}=\frac{1}{x+1}$$

$$\therefore y\times\frac{1}{x+1}=\int e^{3x}dx$$

$$\frac{y}{x+1}=\frac{e^{3x}}{3}+C$$

Given y(0) = 1/3

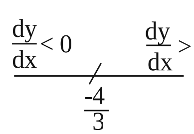
$$\therefore\frac{1}{3}=\frac{1}{3}+C$$

C=0

$$\therefore y=\frac{e^{3x}(x+1)}{3}$$

$$\frac{dy}{dx}=\frac{1}{3}\left\{e^{3x}+3e^{3x}(x+1)\right\}$$

$$=\frac{e^{3x}}{3}\left\{3x+4\right\}$$



So, $x=\frac{-4}{3}$ is a pt. of local minima.

Q.17 (3)

$y^2dx + (x^2 - xy + y^2)\,dy = 0$

$$\frac{dy}{dx}=\frac{-y^2}{x^2-xy+y^2}$$

put y = tx

$$\frac{dy}{dx}=t+x\frac{dt}{dx}$$

$$t+x\frac{dt}{dx}=\frac{-t^2x^2}{x^2-tx^2+t^2x^2}$$

$$t+x\frac{dt}{dx}=\frac{-t^2}{1-t+t^2}$$

$$\left(-\frac{1}{t}+\frac{1}{t^2+1}\right)dt=\frac{dx}{x}$$

$-\log t+\tan^{-1}t=\ln x+C$

$$-\log\left(\frac{y}{x}\right)+\tan^{-1}\left(\frac{y}{x}\right)=\ln x+C$$

Putting x=1,y=1 we get $C=\frac{\pi}{4}$

Put $y=\sqrt{3}x$

$$-\log\left(\sqrt{3}\right)+\tan^{-1}\left(\sqrt{3}\right)=\ln x+\frac{\pi}{4}$$

$$\therefore \ln(\sqrt{3}x)=\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$$

Q.18 (12)

$(4+x^2)dy-2x(x^2+3y+4)dx$

$$(x^2+4)\frac{dy}{dx}=2x^3+6xy+8x$$

$$(x^2+4)\frac{dy}{dx}-6xy=2x^3+8x$$

$$\frac{dy}{dx}-\frac{6x}{x^2+4}y=\frac{2x^3+8x}{x^2+4}$$

$$\text{L.I.}\frac{dy}{dx}+py=\phi$$

$$p=\frac{-6x}{x^2+4}\qquad \phi=\frac{2x^3+8x}{x^2+4}$$

$$\text{I.F.}=e^{-\int\frac{6x}{x^2+4}dx}=e^{-3\log_e(x^2+4)}=\frac{1}{(x^2+4)^3}$$

$$y.\frac{1}{(x^2+4)^3}=\int\frac{2x^3+8x}{(x^2+4)^3(x^2+4)}dx$$

$$\frac{y}{\left(x^2+4\right)^3}=\int\frac{2x(x^2+4)}{(x^2+4)^3(x^2+4)}dx$$

$x^2+4=t$

$2xdx=dt$

$$\frac{y}{(x^2+4)^3}=\int\frac{dt}{t^3}$$

$$\frac{y}{(x^2+4)^3}=\frac{-1}{2(x^2+4)^2}+C$$

Passes through origin (0,0)

$$0=\frac{-1}{2\times 16}+C$$

$$\frac{y}{(x^2+4)^3}=\frac{-1}{2(x^2+4)^2}+\frac{1}{32}$$

$$y=\frac{-(x^2+4)}{2}+\frac{(x^2+4)^3}{32}$$

$$y(2)=-4+\frac{8\times 8\times 8}{32}$$

$y(2)=-4+16=12$

Q. 19 (42)

$$\frac{dy}{dx}=\frac{1}{1+\sin 2x}$$

$$\int dy=\int\frac{dx}{(\sin x+\cos x)^2}$$

$$y(x)=-\frac{1}{1+\tan x}+C$$

$$y\left(\frac{\pi}{4}\right)=\frac{1}{2}\Rightarrow\frac{1}{2}=-\frac{1}{2}+C\quad C=1$$

$$y(x)=\frac{-1}{1+\tan x}+1$$

$$y(x)=\frac{-1+1+\tan x}{1+\tan x}$$

$$y(x)=\frac{\tan x}{1+\tan x}$$

Solving with $y=\sqrt{2}\sin x$

$$\frac{\tan x}{1+\tan x}=\sqrt{2}\sin x$$

$$\sin x=0,\frac{1}{\sqrt{2}}=\sin x+\cos x$$

$$x = \pi;\ \frac{1}{2} = \sin\left(x + \frac{\pi}{4}\right)$$

$$\sin\frac{\pi}{6} = \sin\left(x + \frac{\pi}{4}\right)$$

$$x + \frac{\pi}{4} = \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} - \frac{\pi}{4}, x = \frac{13\pi}{6} - \frac{\pi}{4}$$

$$x = \frac{7\pi}{12}, x = \frac{23\pi}{12}$$

Sum of solution

$$= \pi + \frac{7\pi}{12} + \frac{23\pi}{12}$$

$$= \frac{12\pi + 7\pi + 23\pi}{12} = \frac{42\pi}{12} = \frac{k\pi}{12}$$

$\Rightarrow k=42$

Q.20 (1)

$$\left(\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right) x\frac{dy}{dx} = x + \left(\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right) y$$

$$\Rightarrow e^{\frac{y}{x}}(xdy - ydx) + \frac{x}{\sqrt{x^2 - y^2}}(xdy - ydx) = xdx$$

Dividing both side by x^2

$$\Rightarrow e^{\frac{y}{x}}\left(\frac{xdy - ydx}{x^2}\right) + \frac{1}{\sqrt{1 - \left(\frac{y}{x}\right)^2}}\left(\frac{xdy - ydx}{x^2}\right) = \frac{dx}{x}$$

$$\Rightarrow e^{\frac{y}{x}} d\left(\frac{y}{x}\right) + \frac{1}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} d\left(\frac{y}{x}\right) = \frac{dx}{x}$$

Integrate both sides

$$\int e^{\frac{y}{x}} d\left(\frac{y}{x}\right) + \int \frac{1}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} d\left(\frac{y}{x}\right) = \int \frac{dx}{x}$$

$$\Rightarrow e^{\frac{y}{x}} + \sin^{-1}\left(\frac{y}{x}\right) = \ln x + c$$

It passes through (1, 0)

$1 + 0 = 0 + c \Rightarrow c = 1$

It passes through $(2\alpha, \alpha)$

$$e^{\frac{1}{2}} + \sin^{-1}\frac{1}{2} = \ln 2\alpha + 1$$

$$\Rightarrow \ln 2\alpha = \sqrt{e} + \frac{\pi}{6} - 1$$

$$\Rightarrow 2\alpha = \exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$$

$$\Rightarrow \alpha = \frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$$

Q.21 (1)

$$x(1 - x^2)\frac{dy}{dx} + (3x^2 y - y - 4x^3) = 0$$

$$x(1 - x^2)\frac{dy}{dx} + (3x^2 - 1)y = 4x^3$$

$$\frac{dy}{dx} + \frac{(3x^2 - 1)}{(x - x^3)} y = \frac{4x^3}{(x - x^3)}$$

$$\frac{dy}{dx} + Py = Q$$

$$IF = e^{\int \frac{3x^2 - 1}{x - x^3} dx} = e^{-1}(|x - x^3|)$$

$$= \frac{1}{x^3 - x} \qquad (\because x > 1)$$

$$y\left(\frac{1}{x^3 - x}\right) = \int \frac{4x^3}{x - x^3} \times \frac{1}{x^3 - x} dx$$

$$= \int \frac{-4x}{(x^2 - 1)^2} dx$$

$$= \frac{2}{x^2 - 1} + c$$

$$y\left(\frac{1}{x^3 - x}\right) = \frac{2}{x^2 - 1} + c$$

$x = 2, y = -2$ gives $c = -1$

$$y\left(\frac{1}{x^3 - x}\right) = \frac{2}{x^2 - 1} - 1$$

Putting $x = 3$; gives $y = -18$

Q.22 [3]

$$\frac{dy}{dx} = \frac{4y^3 + 2yx^2}{3xy^2 + x^3}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \int \frac{3v^2 + 1}{v^3 + v} dv = \int \frac{dx}{x}$$

$\ln|v^3+v| = \ln cx$

$\left(\frac{y}{x}\right)^3 + \frac{y}{x} = cx;\ y(1)=1$

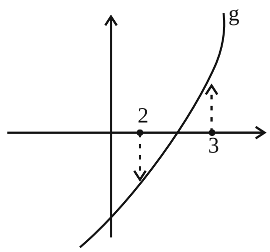
$\therefore c = 2$

$y^3 + yx^2 = 2x^4$

For x = 2, y(2) satisfies $y^3 + 4y = 32$

Let $g(y) = y^3 + 4y - 32$

$g'(y) = 3y^2 + 4 > 0$



$\therefore$ n = 3 Ans.

Q.23 [6]

$\int_3^x f(x)dx = (y/x)^3$

differentiate

$f(x) = 3(y/x)^2 d(y/x)$

$\frac{y}{x}\cdot x = 3\left(\frac{y}{x}\right)^2 d\left(\frac{y}{x}\right)$

$\Rightarrow \int x dx = 3\int\left(\frac{y}{x}\right)d\left(\frac{y}{x}\right)$

$\frac{x^2}{2} = 3\frac{(y/x)^2}{2} + C$

(3, 3)

$\frac{9}{2} = \frac{3}{2}(1) + C \Rightarrow C = 3$

$\frac{x^2}{2} = \frac{3}{2}(y/x)^2 + 3$

$\Rightarrow$ Put $y = 6\sqrt{10}$

x = 6

Q.24 (1)

$\text{I.F.} = e^{\int 2\tan x\,dx}$

$= e^{2(\ln\sec x)}$

$= \sec^2 x$

$y(\sec^2 x) = \int(\sin x)(\sec^2)\,dx$

$y(\sec^2 x) = \sec x + C$ ... (1)

Put $x = \frac{\pi}{3},\ y = 0$

c = –2

$y(\sec^2 x) = \sec x - 2$

$y = \cos x - 2\cos^2 x$

$y = -2\left[\left(\cos x - \frac{1}{4}\right)^2 - \frac{1}{16}\right]$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$= y_{max.} = \frac{1}{8}$

Q.25 (4)

$\frac{dy}{dx} + \frac{1}{x^2-1}y = \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}},$

$\frac{dy}{dx} + Py = Q$

$\text{I.F.} = e^{\int P dx} = \left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$

$y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}} = \int\left(\frac{x-1}{x+1}\right)^1 dx$

$= x - 2\log_e|x+1| + C$

Curve passes through $\left(2, \frac{1}{\sqrt{3}}\right)$

$\Rightarrow C = 2\log_e 3 - \frac{5}{3}$

at x = 8,

$\sqrt{7}y(8) = 19 - 6\log_e 3$

Q.26 (1)

Equation of circle passing through (0, –2) and (0, 2) is

$x^2 + (y^2 - 4) + \lambda x = 0, (\lambda \in R)$

Divide by x we get

$\frac{x^2 + (y^2-4)}{x} + \lambda = 0$

Differentiating with respect to x

$\frac{x\left[2x + 2y\cdot\frac{dy}{dx}\right] - [x^2 + y^2 - 4]\cdot 1}{x^2} = 0$

$\Rightarrow 2xy\cdot\frac{dy}{dx} + (x^2 - y^2 + 4) = 0$

Q.27 (1)

$(x - y^2)\,dx + y(5x + y^2)\,dy = 0$

$y\frac{dy}{dx} = \frac{y^2 - x}{y^2 + 5x}$ Let $y^2 = t$

$$\frac{1}{2}\frac{dt}{dx} = \frac{t-x}{t+5x} \mid \text{HDE}$$

$t = vx$

$$v + x\frac{dv}{dx} = 2\frac{(V-1)}{(V+5)}$$

$$x\frac{dv}{dx} = \frac{2v-2-v^2-5v}{v+5}$$

$$\int\frac{(v+5)dv}{v^2+3v+2} = -\int\frac{1}{x}dx$$

$$\int\left(\frac{4}{v+1}-\frac{3}{v+2}\right)dv = -\int\frac{dx}{x}$$

$$4\ell_n(v+1)-3\ell_n(v+2) = -I_n x + C$$

$$\ell n\left(\frac{(v+1)^4}{(v+2)^3}.x\right) = C$$

$$(y^2+x)^4 = C(y^2+2x)^3$$

Q.28 (1)

$$\frac{dy}{dx} = \frac{x+y-2}{x-y} = \frac{(x-1)+(y-1)}{(x-1)-(y-1)}$$

$x-1 = X,\ y-1 = Y$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$Y = VX$

$$\frac{dY}{dX} = V + X\frac{dV}{dX}$$

$$V + X\frac{dV}{dX} = \frac{1+V}{1-V} \qquad X\frac{dV}{dX} = \frac{V^2+1}{1-V}$$

$$\int\frac{1-V}{1+V^2}dV = \int\frac{dX}{X}$$

$$\int\frac{dV}{1+V^2} - \frac{1}{2}\int\frac{2VdV}{1+V^2} = \int\frac{dX}{X}$$

$$\tan^{-1}v \frac{-1}{2}\ln(1+v^2) = \ln x + c$$

$$\tan^{-1}\left(\frac{Y}{X}\right) - \frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right) = \ln(X) + c$$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\frac{(y-1)^2}{(x-1)^2}\right) = \ln(x-1) + c$$

Passes through (2, 1)

$$0 - \frac{1}{2}\ln 1 = \ln 1 + c$$

$\therefore c = 0$

Passes through (k + 1, 2)

$$\therefore \tan^{-1}\left(\frac{1}{k}\right) - \left(\frac{1}{2}\right)\ln\left(1+\frac{1}{k^2}\right) = \ln k$$

$$2\tan^{-1}\left(\frac{1}{k}\right) = \ln\left(\frac{1+k^2}{k^2}\right) + 2\ln k$$

$$2\tan^{-1}\left(\frac{1}{k}\right) = \ln(1+k^2)$$

Q.29 (2)

$$\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{x+3}{x+1}$$

$\int p(x)\,dx$ $\qquad$ $\text{I.F.} = e^{\int p(x)dx}$

$$\int p(x)dx = \int\frac{(2x^2+11x+13)\,dx}{(x+1)(x+2)(x+3)}$$

Using partial fraction

$$\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$$

$$A = \frac{4}{2} = 2$$

$B = 1$

$C = -1$

$$\because \int p(x)dx = A\ln(x+1) + B\ln(x+2) + C\ln(x+3)$$

$$= \ln\left(\frac{(x+1)^2(x+2)}{x+3}\right)$$

$$\text{I.F.} = e^{\int p(x)dx} = \frac{(x+1)^2(x+2)}{(x+3)}$$

Solution $y(\text{IF}) = \int Q.(\text{IF})dx$

$$y\left(\frac{(x+1)^2(x+2)}{x+3}\right) = \int\left(\frac{x+3}{x+1}\right)\frac{(x+1)^2(x+2)}{(x+3)}dx$$

$$y\left(\frac{(x+1)^2(x+2)}{x+3}\right) = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$$

Passes through (0, 1) $C = \frac{2}{3}$

Now, put $x = 1$

$$\Rightarrow y(1) = \frac{2}{3}$$

Q.30 (2)

$$\frac{dy}{dx}+y=\frac{1}{1+e^{2x}}$$

So, integrating factor is $e^{\int 1.dx}=e^x$

So, solution is y. $e^x = \tan^{-1}(e^x) + c$

Now as curve is passing through $\left(0, \frac{\pi}{2}\right)$ so

$$\Rightarrow c=\frac{\pi}{4}$$

$$\Rightarrow \lim_{x\to\infty}(y\cdot e^x)=\lim_{x\to\infty}\left(\tan^{-1}(e^x)+\frac{\pi}{4}\right)=\frac{3\pi}{4}$$

Q.31 (2)

$$x\,dy=\left(\sqrt{x^2+y^2}+y\right)dx$$

$$x\,dy-ydx=\sqrt{x^2+y^2}dx$$

$$\frac{xdy-ydx}{x^2}=\sqrt{1+\frac{y^2}{x^2}}.\frac{dx}{x}$$

$$\frac{d(y/x)}{\sqrt{1+\left(\frac{y}{x}\right)^2}}=\frac{dx}{x}$$

$$\ln\left(\frac{y}{x}+\sqrt{\left(\frac{y}{x}\right)^2+1}\right)=\ln x+R$$

$$\frac{y+\sqrt{y^2+x^2}}{x}=cx$$

$$y+\sqrt{y^2+x^2}=cx^2$$

$x=1, y=0 \Rightarrow 0+1=C \Rightarrow C=1$

Curve is $y+\sqrt{x^2+y^2}=x^2$

$x=2, y=\alpha$

$$\alpha+\sqrt{4+\alpha^2}=4$$

$$4+\alpha^2=16+\alpha^2-8\alpha^2$$

$$\alpha=\frac{3}{2}$$

Q.32 (1)

Given differential equation can be re-written as $\frac{dy}{dx}$ +

$$(8+4\cot 2x)\,y=\frac{2e^{-4x}}{\sin^2 2x}(2\sin x+\cos 2x)$$

Which is a linear diff. equation

$$\text{I.F.}=e^{\int(8+4\cot 2x)dx}=e^{8x+2\ln(\sin 2x)}$$

$= e^{8x} . \sin 2x + C$

$\therefore$ Solution is

$$y(e^{8x}.\sin^2 2x)=\int 2e^{4x}(2\sin 2x+\cos 2x)dx+C$$

$= e^{4x}. \sin 2x + C$

Given $y\left(\frac{\pi}{4}\right)=e^{-\pi}\Rightarrow C=0$

$$\therefore y=\frac{e^{-4x}}{\sin 2x}$$

$$\therefore y\left(\frac{\pi}{6}\right)=\frac{e^{-4\frac{\pi}{6}}}{\sin\left(2.\frac{\pi}{6}\right)}=\frac{2}{\sqrt{3}}e^{-\frac{2\pi}{3}}$$

Q.33 (1)

$$\frac{dy}{dx}=x+y \quad \begin{matrix}\nearrow y_1(x)\\ \searrow y_2(x)\end{matrix}$$

$y_1(0)=0,\ y_2(0)=1$

$$\frac{dy}{dx}-y=x$$

$$\text{I.F.}=e^{\int -1.dx}=e^{-x}$$

$$\therefore ye^{-x}=\int xe^{-x}.dx$$

$ye^{-x} = -e^{-x}(x+1) + C$

$y = -x - 1 + Ce^x$

$y_1(0) = 0 \Rightarrow C = 1$

$y_2(0) = 1 \Rightarrow C = 2$

So, $y_1(x) = -x - 1 + e^x$

$y_2(x) = -x - 1 + 2e^x$

$\therefore$ For intersection of $y_1(x)$ & $y_2(x)$

$-x - 1 + e^x = -x - 1 + 2e^x$

$\Rightarrow e^x = 0$

which is not possible

So, number of points of intersection of $y_1(x)$ & $y_2(x)$ is 0.

Q.34 [1]

$$\sin(2x^2)\ \text{In}(\tan x^2)\,dy+\left(4xy-4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)\right)dx=0$$

$$\text{In}(\tan x^2)\,dy+\frac{4xydx}{\sin(2x^2)}-\frac{4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)}{\sin(2x^2)}dx=0$$

$$d(y.\ \text{In}(\tan x^2))-4\sqrt{2}x\frac{(\sin x^2-\cos x^2)}{\sqrt{2}\times 2\sin x^2\cos x^2}dx=0$$

$$d(y.\ln(\tan x^2)) - \frac{4x(\sin x^2 - \cos x^2)}{(\sin x^2 + \cos^2 x) - 1}dx = 0$$

$$\Rightarrow \int d(y.\ln(\tan x^2)) + 2\int \frac{dt}{t^2-1} = C$$

$$\Rightarrow y.\ln(\tan x^2) + 2.\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| = C$$

$$y.\ln(\tan x^2) + \ln\left(\frac{\sin x^2 + \cos x^2 - 1}{\sin x^2 + \cos x^2 + 1}\right) = C$$

Put y = 1 and $x = \sqrt{\frac{\pi}{6}}$

$$1.\ln\left(\frac{1}{\sqrt{3}}\right) + \ln\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-1\right)}{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)} = C$$

Now $x = \sqrt{\frac{\pi}{3}} \Rightarrow y.(\ln\sqrt{3}) + \ln\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-1\right)}{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)}$

$$= \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{\sqrt{3}-1}{\sqrt{3}+3}\right)$$

$$y.(\ln\sqrt{3}) = \ln\left(\frac{1}{\sqrt{3}}\right)$$

$\Rightarrow y = -1$

$|y| = 1$

Q.35 (2)

I.F. $= e^{\int\frac{xdx}{x^2-1}}$

$$\Rightarrow e^{-\frac{1}{2}\int\frac{2x}{1-x^2}} = e^{\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$$

Solution is,

$$y\sqrt{1-x^2} = \int \frac{x^4+2x}{\sqrt{1-x^2}}\sqrt{1-x^2}dx$$

$$y\sqrt{1-x^2} = \frac{x^5}{5} + x^2 + C$$

$\Downarrow (0,0)$

$C = 0$

$$\Rightarrow y = \frac{x^5+5x^2}{5\sqrt{1-x^2}}$$

$$\Rightarrow y = \frac{x^5}{5\sqrt{1-x^2}} + \frac{x^2}{\sqrt{1-x^2}}$$

$$\int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx \qquad x = \sin\theta$$

$dx = \cos\theta d\theta$

$$= 2\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{3}} (1-\cos 2\theta)d\theta$$

$$\Rightarrow \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\frac{\pi}{3}}$$

$$\Rightarrow \frac{\pi}{3} - \frac{1}{2}\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

Q.36 [3]

$$\frac{dy}{dx} - y = 2 - e^{-x}$$

is linear differential equation

I.F. $= e^{\int(-1)dx}$

$= e^{-x}$

$$ye^{-x} = \int e^{-x}(2-e^{-x})dx$$

$$= \int(2 - e^{-x} - e^{-2x})dx$$

$$= -2e^{-x} + \frac{e^{-2x}}{2} + C$$

$$\Rightarrow y = -2 + \frac{e^{-x}}{2} + ce^x$$

$\lim_{x\to\infty} y(x) = \text{finite}$

$$\left(y + \frac{3}{2}\right) = \frac{-1}{2}(x-0)$$

$x + 2y + 3 = 0$

$a = -3$

$b = \frac{-3}{2}$

$a - 4b = -3 + 6 = 3$