SOLUTION
THREE DIMENSIONAL GEOMETRY

EXERCISE-I (MHT CET LEVEL)

Q.1 (4)
The planesXOY, YOZ and ZOX, called, respectively the XY-plane, YZ-plane and the ZX-plane, are known as the three coordinate planes.

Q.2 (1)
The point (–3, 1, 2) lies in second octant (i.e., X' OYZ) and the point (–3, 1, –2) lies in octant VI (i.e., X' OYZ') see the table given in solution 16)

Q.3 (4)
Coordinate of any point on the X-axis is (x, 0, 0). Therefore, both y and z-coordinates are zero.

Q.4 **(b)**

Q.5 (2)
Equation of x-axis is $y = 0,\ z = 0$.
Hence y and z remain fixed.

Q.6 (3)
Any point on the XZ-plane will have the coordinate (x, 0, z), the its y-coordinates is 0.

Q.7 (4) It is obvious

Q.8 (3)
Since, L is the foot of perpendicular form P on the x-axis, its y and z-coordinates are zero. The coordinates of L is (3, 0, 0). Similarly, the coordinates of M and N are (0, 4, 0) and (0, 0, 5), respectively.

Q.9 (2)
Here $a = \sqrt{4+4+1} = \sqrt{9},\ b = \sqrt{1+4+4} = \sqrt{9}$
and $c = \sqrt{1+16+1} = \sqrt{18}$.
Obviously it is a right angled and isosceles triangle

Q.10 **(c)**

Q.11 (4) $\frac{-4+1}{5+4} = \frac{2-3}{5-2} = \frac{-2-2}{\lambda+2}$ or $\lambda+2=12$ or $\lambda = 10$.

Q.12 (3)
Distance from origin $= \sqrt{1+4+9} = \sqrt{14}$ and from y-axis $= \sqrt{1+9} = \sqrt{10}$.

Q.13 (2)
From x-axis $= \sqrt{y^2+z^2} = \sqrt{4+9} = \sqrt{13}$
From y-axis $= \sqrt{1+9} = \sqrt{10}$
From z-axis $= \sqrt{1+4} = \sqrt{5}$.

Q.14 (1) Here, $\frac{(3-(-2))}{1-3} = \frac{-6-4}{-2-(-6)} = \frac{-8-7}{-2-(-8)}$
$\Rightarrow -\frac{5}{2} = -\frac{5}{2} = -\frac{5}{2}$.
Obviously, points are collinear.

Q.15 (2)
Let P(x, y, z) Now under given condition, we get
$$\left[\sqrt{(x^2+y^2)}\right]^2 + \left[\sqrt{(y^2+z^2)}\right]^2 + \left[\sqrt{(z^2+x^2)}\right]^2 = 36$$
$\Rightarrow x^2 + y^2 + z^2 = 18$
Then distance from origin to the point (x, y, z) is
$\sqrt{x^2+y^2+z^2} = \sqrt{18} = 3\sqrt{2}$
.

Q.16 (1) Let point be (x, y, z) then $x^2 + y^2 + z^2$
$= (x-a)^2 + y^2 + z^2 = x^2 + (y-b)^2 + z^2 = x^2 + y^2 + (z-c)^2$
Therefore $x = \frac{a}{2},\ y = \frac{b}{2}$ and $z = \frac{c}{2}$.

Q.17 (2)
LetA = (1,1,1) ; B = (–2,4,1); C = (– 1, 5, 5) & D = (2, 2, 5)
AB = $\sqrt{9+9+0}$ = $3\sqrt{2}$, BC = $\sqrt{1+1+16}$ = $3\sqrt{2}$ and CD = $3\sqrt{2}$ and AD = $3\sqrt{2}$. Hence it is a square

Q.18 (1)
Required distance $= \sqrt{3^2+5^2} = \sqrt{34}$.

Q.19 (3)
$AB = \sqrt{(-1-1)^2 + (-1-2)^2 + (-1-3)^2}$
$AB = \sqrt{4+9+16} = \sqrt{29}$

Q.20 (3)
Distance from y-axis is $\sqrt{x^2+z^2}$
$=$ $\sqrt{4^2+5^2}$ $=$ $\sqrt{16+25}$ $=$ $\sqrt{41}$.

Q.21 (2)
It is a fundamental concept.

Q.22 (4)

$$AB = \sqrt{(a-1)^2 + 3^2 + 0} = 5$$

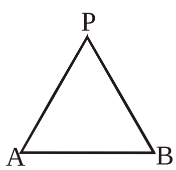
$$\Rightarrow a - 1 = \pm 4 \Rightarrow a = -3, 5$$

$$CD = \sqrt{(a-1)^2 + (a+5)^2 + (a-1)^2} = 6$$

$$\Rightarrow a^2 - 2a - 15 = 0 \Rightarrow a = -3, 5$$

But common solution of (i) and (ii) is – 3.

Q.23 (3)
Let the given poitns are A and B. Let P(x, y, z) the any point equidistant from A and B
$\therefore$ PA = PB
i.e., Distance between P and A = Distance between P and B



$$\Rightarrow \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$$

$$\sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$$

$\Rightarrow$ $(x - 1)^2 + (y - 2)^2 + (z + 1)^2$
$= (x - 3)^2 + (y - 2)^2 + (z + 1)^2$
$\Rightarrow$ $x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z$
$= x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$
$\Rightarrow$ $4x - 8z = 0 \Rightarrow x - 2z = 0$
which is the required equation.

Q.24 (2)
Let the point be P(x, y, z). Then, it is given PA + PB = 10

$$\Rightarrow \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2}$$

$$+ \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

Squaring on both sides, we get
$(x - 4)^2 + y^2 + z^2 = 100 + (x + 4)^2 + y^2 + z^2$

$$-20\sqrt{(x+4)^2 + y^2 + z^2}$$

$\Rightarrow$ $x^2 + 16 - 8x = 100 + x^2 + 16 + 8x$

$$-20\sqrt{(x+4)^2 + y^2 + z^2}$$

$$\Rightarrow -8x - 8x - 100 = -20\sqrt{(x+4)^2 + y^2 + z^2}$$

$$\Rightarrow -16x - 100 = -20\sqrt{(x+4)^2 + y^2 + z^2}$$

$$\Rightarrow 4x + 25 = 5\sqrt{(x+4)^2 + y^2 + z^2}$$

(dividing both sides by – 4)
Again, squarign on both sides, we get
$(4x + 25)^2 = 25\,[(x +)^2 + y^2 + z^2$
$\Rightarrow 16x^2 + 625 + 200x = 25\,[(x + 4)^2 + y^2 + z^2]$
$\Rightarrow 16x^2 + 625 + 200x = 25[x^2 + 16 + 8x + y^2 + z^2]$
$\Rightarrow 16x^2 + 625 + 200x = 25x^2 + 400 + 200x + 25y^2 + 25z^2$
$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$
which is the required equation.

Q.25 (2)
Reterring to he previous solution.
In case R is the mid-point of PQ, then, m : n = 1 : 1, so that $x = \dfrac{x_1 + x_2}{2}, y = \dfrac{y_1 + y_2}{2}$ and $z = \dfrac{z_1 + z_2}{2}$
These are the coordinates of the mid-point of hte segment joining P (x_1, y_1, z_1) and Q (x_2, y_2, z_2).
are $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}\right)$

Q.26 **(c)**

Q.27 (2)
The coordinates of the point R which divides PQ in the ratio k : 1 where coordinates of P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) are obtained by taking $k = \dfrac{m}{n}$ inthe coordinates of hte point R which divides PQ intermally in the ratio m : n, which are as given below.

$$\left(\frac{kx_2 + x_1}{1+k}, \frac{ky_2 + y_1}{1+k}, \frac{kz_2 + z_1}{1+k}\right)$$

Note *Generally, this result is used in solving problems involving a general point on the line passing through two given points.*

Q.28 (4)
Since ZOX plane i.e. y = 0 divides the join of (1,–1, 5) and (2, 3, 4) in the ratio $\lambda : 1$.

$$\therefore \frac{3\lambda - 1}{\lambda + 1} = 0 \Rightarrow \lambda = \frac{1}{3}$$

Q.29 (4)

From $x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$, $y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$,

$$z=\frac{m_1z_2+m_2z_1}{m_1+m_2};\quad x=\frac{20}{7},\ y=\frac{5}{7},\ z=\frac{15}{7}.$$

Q.30 (2)

$$x=\frac{-5+9}{-2}=-2,\ y=\frac{-5(2)+3(-5)}{-2}=\frac{25}{2}$$

$$z=\frac{-5(3)+3(6)}{-2}=-\frac{3}{2}$$

Q.31 (3)

$$0=\frac{a-2+4}{3}\Rightarrow a=-2,\ 0=\frac{1+b+7}{3}\Rightarrow b=-8$$

and $0=\frac{3-5+c}{3}\Rightarrow c=2$.

Q.32 (2)

Required ratio $=-\left(\frac{5}{-2}\right)=\frac{5}{2}$ i.e., 5 : 2.

Q.33 (2)

$$\left[\frac{2+1}{1},\frac{4-0}{1},\frac{-2-1}{1}\right]=(3,\ 4,\ -3)$$

Q.34 (2)

For xy-plane, $z=0\Rightarrow\frac{6\lambda+1}{\lambda+1}=0\Rightarrow\lambda=-\frac{1}{6}$

$$\therefore x=\frac{-5+18}{5}=\frac{13}{5},\ y=\frac{-1+24}{5}=\frac{23}{5}$$

Q.35 (1)

Let point C divides the line AB in the ratio $1:\lambda$.

$$\therefore 5=\frac{9\lambda+3}{\lambda+1}\Rightarrow 4\lambda=2\Rightarrow\lambda=\frac{1}{2}$$

Hence required ratio is 2 : 1.

Q.36 (2)

Suppose P divides QR in the ratio $\lambda=1$.

Then co-ordinates of P are $\left(\frac{5\lambda+2}{\lambda+1},\frac{\lambda+2}{\lambda+1},\frac{-2\lambda+1}{\lambda+1}\right)$.

It is given that the x-coordinate of P is 4. i.e. $\frac{5\lambda+2}{\lambda+1}=4$,

$\Rightarrow\lambda=2$

So, z-coordinate of P is $\frac{-2\lambda+1}{\lambda+1}=\frac{-4+1}{2+1}=-1$.

Q.37 (1)

$$\therefore \text{Ratio}=-\left(\frac{3}{-2}\right)=\frac{3}{2}$$

Required co-ordinates of the points are,

$$\left[\frac{6-6}{5},\frac{10+3}{5},\frac{-14+24}{5}\right]=\left(0,\frac{13}{5},\ 2\right)$$

Q.38 (1)

Let YZ-plane divides the line segment joining A(4, 8, 10) and B(6, 10, –8) at P (x, y, z) in the ratio k : 1 Then, the coordinates of P are

$$\left(\frac{4+6k}{k+1},\frac{8+10k}{k+1},\frac{10-8k}{k+1}\right)$$

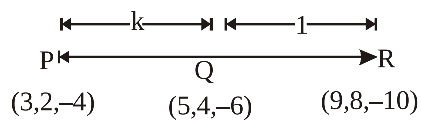
Since, P lies on the YZ-plane, its x-coordinates is zero,

i.e., $\frac{4+6k}{k+1}=0$ or $k=-\frac{2}{3}$

Therefore, YZ–plane divides AB externally in the ratio 2 : 3.

Q.39 (3)

Let Q divides PR in the ratio k : 1.



Here, the point Q divides the line PR internally, so its coordinates are

$$\left[\left(\frac{mx_2+nx_1}{m+n}\right),\left(\frac{my_2+ny_1}{m+n}\right),\left(\frac{mz_2+nz_1}{m+n}\right)\right]$$

$$Q=\left(\frac{k\times9+1\times3}{k+1},\frac{k\times8+1\times2}{k+1},\frac{k(-10)+1\times(-4)}{k+1}\right)$$

$$=\left(\frac{9k+3}{k+1},\frac{8k+2}{k+1},\frac{-10k-4}{k+1}\right)$$

But given, Q = (5, 4, –6)

On comparing the cooresponding coordinates

$$\therefore\ \frac{9k+3}{k+1}=5,\frac{8k+2}{k+1}=4,\frac{-10k-4}{k+1}=-6$$

$\Rightarrow$ $9k+3=5k+5,\ 8k+2=4k+4,$

$-10k-4=-6k-6$

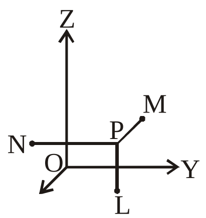
$\Rightarrow$ $4k=2$

$\Rightarrow$ $k=\frac{1}{2}$

Hence, point Q divides PR internally in the ratio 1 : 2

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)
Since, L is the foot of perpendicular segment from P on the XY-plane, z-coordinates is zero in the XY-plane.



Hence, coordinates of L are (3, 4, 0). Similarly, we can find the coordinates of M (0, 4, 5) and N (3, 0, 5).

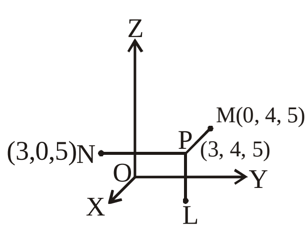
Q.2 (2)
Since, L is the foot of perpendicular from P on the XY-plane, z-coordinate is zero in the XY-plane. Hence, coordinates of L are (6, 7, 0)

Q.3 (1)
Since, L is the foot of perpendicular from P on the X-axis, y and z-coordinates are zero. Hence, the coordinates of L are (6, 0, 0)

Q.4 (1)
Locus of the point y = 0, z= 0 is X-axis both y = 0 and z = 0

Q.5 (3)
Since, L is the foot of perpendicular segment drawn from the point P (3, 4, 5) on the XZ-plane. Since, the y-coordinates of all points in the XZ-plane are zero, coordinates of hte foot of perpendicular are (3, 0, 5)

Q.6 (4)
L is the foot of perpendicular drawn from the point P(3, 4, 5) to the XY-plane. Therefore, the coordinate of the point L is (3, 4, 0). The distance between the point (3, 4, 5) and (3, 4, 0) is 5. Similarly, we can find htelengths of the foot of perpendiculars on YZ and ZX-plane which are 3 and 4 units, respectively.



Q.7 (2)
Three points are collinear if the sum of any two disatnces is equal to the third distance.

$$PQ = \sqrt{(-2-2)^2 + (-2-4)^2 + (-2-6)^2}$$
$$= \sqrt{(16+36+64)} = \sqrt{116} = 2\sqrt{29}$$
$$QR = \sqrt{(6+2)^2 + (10+2)^2 + (14+2)^2}$$
$$= \sqrt{64+144+256} = \sqrt{464} = 4\sqrt{29}$$
$$PR = \sqrt{(6-2)^2 + (10-4)^2 + (14-6)^2}$$
$$= \sqrt{16+36+64} = \sqrt{116} = 2\sqrt{29}$$

Since, QR = PQ + PR. Therefore, given points are collinear.

Q.8 (4)
The point on the X-axis is of form P(x, 0, 0). Since, the $PA^2 = PB^2$, i.e., $(x-3)^2 + (0-2)^2 + (0-2)^2$
$=(x-5)^2 + (0-5)^2 + (0-4)^2$

$\Rightarrow 4x = 25 + 25 + 16 - 17$ i.e., $x = \dfrac{49}{4}$

Thus, the point P on the X–axis is $\left(\dfrac{49}{4}, 0, 0\right)$ which is equidistant from A and B.

Q.9 (1)
Let P be the point on Y-axis. Therefore, it is of hte form P(0, y, 0). The point (1, 2, 3) is at a distance $\sqrt{10}$ from (0, y, 0). Therefore.

$$\sqrt{(1-0)^2 + (2-y)^2 + (3-0)^2} = \sqrt{100}$$

$\Rightarrow y^2 - 4y + 4 = 0 \Rightarrow (y-2)^2 = 0 \Rightarrow y = 2$
Hence, the required point is (0, 2, 0).

Q.10 (1)
Let P (0, 7, 10), Q (–1, 6, 6) and R (–4, 9, 6) be the given three points.

Here, $PQ = \sqrt{1+1+16} = 3\sqrt{2}$

$QR = \sqrt{9+9+0} = 3\sqrt{2}$

$PR = \sqrt{16+4+16} = 6$

Now, $PQ^2 + QR^2 = \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2$

$= 18 + 18 = 36 = (PR)^2$

Therefore, ΔPQR is a right angled triangle at Q. Also, OQ = QR. Hence, ΔPQR is a right angled isosceles triangle.

Q.11 (3)

Let A(5, –1, 1), B(7, –4, 7), C(1, –6, 10) and D(–1, –3, 4) be the four poitns of a quadrialateral. Here,

$AB = \sqrt{4+9+36} = 7.$

$BC = \sqrt{36+4+9} = 7.$

$CD = \sqrt{4+9+36} = 7$

$DA = \sqrt{36+4+9} = 7$

$AC = \sqrt{16+25+181} = \sqrt{122}$

$BD = \sqrt{64+1+9} = \sqrt{74}$

Note the AB = BC = CD = DA and AC ≠ BD. Therefore, ABCD is a rhombus.

Q.12 (2)

Since x-coordinate of every point in YZ-plane is zero. Let P(0, y, z) be a point on the YZ-plane such that PA = PB = PC. Now,

PA = PB

$\Rightarrow (0-2)^2 + (y-0)^2 + (z-3)^2$

$= (0-0)^2 + (y-3)^2 + (z-2)^2$.

i.e. $z - 3y = 0$

and PB = PC

$\Rightarrow y^2 + 9 - 6y + z^2 + 4 - 4z = y^2 + z^2 + 1 - 2z.$

i.e., $3y + z = 6$

Simplifying the two equations, we get y = 1 and z = 3. Here, the coordinate of the point P are (0, 1, 3).

Q.13 (2)

Let L be foot of perpendicular from point P on the Y-axis. Therefore, its x and z-cordinates are zero, i.e., (0, 4, 0), Therefore, distance between the points (0, 4, 0) and (3, 4, 5) is $\sqrt{9+25}$ i.e., $\sqrt{34}$

Q.14 (1)

Let L be the foot of perpendicular drawn from the point P(6, 7, 8) to the XY-plane and the distance of htis foot L from P is z-coordinate of P i.e., B units.

Q.15 (4)

Let M is the foot of perpendicular from P on the Y-axis, therefore, its x and z-coordinates are zero. The coordinates of M is (0, 5, 0). Therefore, the perpendicular distnace of hte point P from Y-axis

$\sqrt{3^2+6^2} = \sqrt{45}$

Q.16 (3)

Let hte joint of P(2, 4, 5) and Q (3, 5, –4) be divided by XZ-plane in the ratio k : 1 at the point R (x, y, z). Therefore,

$$x = \frac{3k+2}{k+1}, y\frac{5k+4}{k+1}, z = \frac{-4k+5}{k+1}$$

Since, the point R(x, y, z) lies on the XZ-plane, the y-coordinate should be zero, i.e.,

$$\frac{5k+4}{k+1} = 0$$

$$\Rightarrow \quad k = -\frac{4}{5}$$

Hence, the required ratio is –4 : 5, i.e., externally in the ratio 4 : 5 .

Q.17 (3)

Let P (x, y, z) be the required point, i.e., divides AB in the ratio 5 : 1. Then,

P(x, y, z) =

$$\left(\frac{5\times10+\times1-2}{5+1}, \frac{5\times-6+1\times0}{5+1}, \frac{5\times-12+1\times6}{5+1}\right)$$

$= (8, -5, -9)$

Q.18 (2)

$$AB = \sqrt{(5-3)^2+(3-2)^2+(2-0)^2} = \sqrt{4+1+4} = 3$$

$$AC = \sqrt{(-9-3)^2+(6-2)^2+(-3-0)^2}$$

$$= \sqrt{144+16+9} = 13$$

Since, AD is the bisector of ∠BAC, we have

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{13}$$

i.e., D divides BC in the ratio 3 : 13

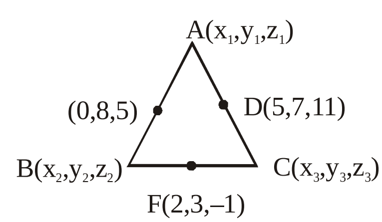
Hence, the coordinates of D are

$$\left(\frac{3(-9)+13(5)}{3+13}, \frac{3(6)+13(3)}{3+13}, \frac{3(-3)+13(2)}{3+13}\right)$$

$$= \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$$

Q.19 (1)

Let the vertices of a triangle are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$



Since, D, E and F are the mid-points of AC, BC and AB,

$$\therefore \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = (0,8,5)$$

$\Rightarrow x_1 + x_2 = 0, y_1 + y_2 = 16, z_1 + z_2 = 10$...(i)

$$\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2},\frac{z_2+z_3}{2}\right)=(2,3,-1)$$

$\Rightarrow$ $x_2 + x_3 = 4, y_2 + y_3 = 6, z_2 + z_3 = -2$ ....(ii)

and $$\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2},\frac{z_1+z_3}{2}\right)=(5,7,11)$$

$\Rightarrow x_1 + x_3 = 10, y_1 + y_3 = 14, z_1 + z_3 = 22$
...(iii)

On adding Eqs. (i) (ii) and (iii) and (iv), we get

$x_3 = 7, x_1 = 3, x_2 = -3$

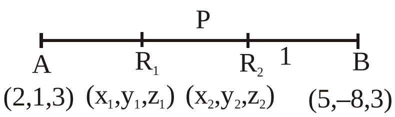
$y_3 = 2, y_1 = 12, y_2 = 4$

and $z_3 = 5, z_1 = 17, z_2 = -7$

Hence, vertices of a triangle are (7, 2, 5), (3, 12, 17) and (–3, 4, –7).

Q.20 (1)

Let R_1 and R_2 are the points with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) which trisect the line segment AB.



Now, $AR_1 : R_1B = 1 : 2$

$$\Rightarrow x_1=\frac{1\times5+2\times2}{1+2}=\frac{9}{3}=3$$

$$y_1=\frac{1\times(-8)+2\times1}{1+2}=\frac{-6}{3}=-2$$

and $$z_1=\frac{1\times3+2\times-3}{1+2}=\frac{-3}{3}=-1$$

$\therefore$ R_1 = = (3, –2, –1)

Similarly, $AR_2 : R_2B = 2 : 1$

$$\Rightarrow x_2=\frac{2\times5+1\times2}{1+2}=\frac{12}{3}=4$$

$$y_2=\frac{2\times(-8)+1\times1}{1+2}=\frac{-15}{3}=-5$$

and $$z_2=\frac{2\times3+1\times(-3)}{1+2}=\frac{3}{3}=1$$

$\therefore$ Coordinates of R_2 are (4, –5, 1).

Q.21 (1)

For centroid of ΔABC

$$x=\frac{a-2+4}{3}=\frac{a+2}{3}$$

$$y=\frac{1+b+7}{3}=\frac{b+8}{3}$$

and $$z=\frac{3-5+c}{3}=\frac{c-2}{3}$$

But, given centroid is (0, 0, 0).

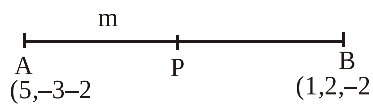
$$\therefore \frac{a+2}{3}=0\Rightarrow a=-2$$

$$\frac{b+8}{3}=0\Rightarrow b=-8$$

$$\frac{c-2}{3}=0\Rightarrow c=2$$

Q.22 (1)

Let point P(x, y, z) divides the line joining the points A and B in the ratio m : 1



Since, point P is on XOZ-plane

$\therefore$ y-coordinate = 0

$$\Rightarrow \frac{2m-3}{m+1}=0\Rightarrow m=\frac{3}{2}$$

Now, $$x=\frac{3+2\times5}{3+2}=\frac{13}{5}$$

and $$z=\frac{3\times(-2)+2\times(-2)}{5}=-2$$

$\therefore$ Required point is $\left(\frac{13}{5},0,-2\right)$

Q.23 (3)

Let the point R divides the line joining the points P(2, 4, 5) and Q (3, 5, –4) in the ratio m : n Then, the coordinates of R are $\left(\frac{3m+2n}{m+n},\frac{5m+4n}{m+n},\frac{-4m+5n}{m+n}\right)$

For yz-plane, x-coordinates will be zero.

$$\therefore \frac{3m+2n}{m+n}=0\Rightarrow\frac{m}{n}=\frac{-2}{3}$$

Alternate Method

The ratio in which YZ-plane divides the line segment $=-x_1 : x_2 = -2 : 3$

EXERCISE-III

Q.1 [6]

$$\frac{x-2}{a}=\frac{y+1}{b}=\frac{z+1}{c} \quad \Rightarrow 4a+b+c=0$$
...(i)

$$2x+y=0=x-y+z \quad \Rightarrow \begin{vmatrix}\hat{i} & \hat{i} & \hat{k}\\ 2 & 1 & 0\\ 1 & -1 & 1\end{vmatrix}=$$

$\hat{i}(1-0)-\hat{j}(2-0)+\hat{k}(-2-1) = \hat{i}-2\hat{j}-3\hat{k}$

$a-2b-3c=0$ ... (ii)

From (i) & (ii)

$4a+b+c=0 \qquad \Rightarrow a-2b-3c=0 \qquad \Rightarrow$

$$\frac{a}{-3+2}=\frac{b}{1+12}=\frac{c}{-8-1} \Rightarrow \frac{a}{-1}=\frac{b}{13}=\frac{c}{-9}$$

$\therefore$ equation of the line

$$\frac{x-2}{-1}=\frac{y+1}{13}=\frac{z+1}{-9} \Rightarrow \frac{3-2}{-1}=\frac{\alpha+1}{13}=\frac{\beta+1}{-9}$$

$\Rightarrow \alpha=-14$ and $\beta=8 \Rightarrow |\alpha+\beta|=6$.

Q.2 [4]

$L_1: \dfrac{x+4}{3}=\dfrac{y+6}{5}=\dfrac{z-1}{-2}=r$; $L_2: 3x-2y+z+5=0$

$=2x+3y+4z-k$

Any point on the first line is $(3r-4, 5r-6, -2r+1)$

As lines are coplanar therefore this point must lie on both the planes representing the second line

$3(3r-4)-2(5r-6)+(-2r+1)+5=0 \qquad \Rightarrow$

$r=2$

and $\qquad 2(3r-4)+3(5r-6)+4(-2r+1)-k=0$

$\Rightarrow k=4$

Q.3 [32]

Since $3(2)+4(-3)+6(1)=0$ and $3(1)+4(2)+6(-3)+7=0$

$\therefore$ the line $\dfrac{x-1}{2}=\dfrac{y-2}{-3}=\dfrac{z+3}{1}$ lies in the plane $3x+4y+6z+7=0$.

In the new position again the line lies in the plane. Let the equation of the new position of the plane be $ax+by+cz=0$, then $2a-3b+c=0$ and $a+2b-3c=0$

$\therefore \dfrac{a}{9-2}=\dfrac{b}{1+6}=\dfrac{c}{4+3}$ i.e. $a=b=c$

$\therefore$ equation of the required plane is $x+y+z=0$

Q.4 [27]

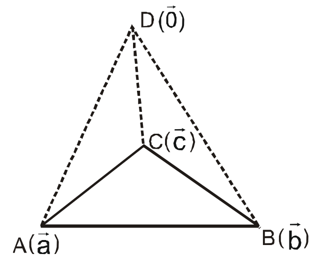
Since tetrahedron is regular AB = BC = AC = DC and angle between two adjcant side = $\pi/3$

consider planes ABD and DBC vector, normal to plane ABD is = $\vec{a}\times\vec{b}$

vector, normal to plane DBC is = $\vec{b}\times\vec{c}$ angle between these planes is angle

between vectors $(\vec{a}\times\vec{b})$ & $(\vec{b}\times\vec{c})$

$$\Rightarrow \cos\theta = \frac{(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c})}{|\vec{a}\times\vec{b}||\vec{b}\times\vec{c}|} = \frac{-\frac{1}{4}|\vec{b}|^2|\vec{a}||\vec{c}|}{\frac{3}{4}|\vec{a}||\vec{b}|^2|\vec{c}|} = -\frac{1}{3}$$



Since acute angle is required $\theta=\cos^{-1}\left(\dfrac{1}{3}\right) \Rightarrow$

$\sec\theta=3 \Rightarrow \sec^3\theta=27$

Q.5 [17]

circum-radius $\equiv$ distance of circum centre from any of the vertex

$\equiv$ distance of $\dfrac{\vec{a}+\vec{b}+\vec{c}}{4}$ from vertex D $(\vec{0})$

[tetrahedron is regular]

Circumradius $=\dfrac{1}{4}|\vec{a}+\vec{b}+\vec{c}|=\dfrac{1}{4}\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})}$

$$=\frac{1}{4}\sqrt{k^2+k^2+k^2+2\left(\frac{k^2}{2}+\frac{k^2}{2}+\frac{k^2}{2}\right)}=\frac{1}{4}\sqrt{6k^2}$$

$$=\sqrt{\frac{3}{8}}\,k$$

$$\frac{r}{R}=\frac{1}{3} \Rightarrow r=\frac{R}{3}=\frac{k}{\sqrt{24}} \Rightarrow R=\sqrt{\frac{3}{8}}\,k \;\&\; r=\frac{k}{\sqrt{24}}$$

$\Rightarrow R^2+r^2=\dfrac{5}{12}k^2$

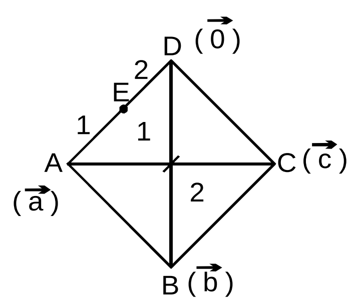
$\Rightarrow$ minimum value of $p+q=17$

Q.6 [13]

$\sqrt{3^2+4^2+12^2}=13$

Q.7 [11]

$|\vec{a}|=|\vec{b}|=|\vec{c}|=|\vec{a}-\vec{b}|=|\vec{c}-\vec{b}|=|\vec{a}-\vec{c}|=a$



On solving we get

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{a}\cdot\vec{c} = \frac{a^2}{2} \Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = a$

$E = \left(\frac{2\vec{a}}{3}\right)$ & $F = \left(\frac{\vec{b}}{3}\right)$ area of $\Delta CEF = \frac{1}{2}|\overrightarrow{CE}\times\overrightarrow{CF}|$

$= \frac{1}{2}\left|\left(\frac{2\vec{a}}{3}-\vec{c}\right)\times\left(\frac{\vec{b}}{3}-\vec{c}\right)\right| =$

$\frac{1}{2}\left|\frac{2}{9}(\vec{a}\times\vec{b})+\frac{2}{3}(\vec{c}\times\vec{a})+\frac{1}{3}(\vec{b}\times\vec{c})\right| =$

$\frac{1}{2}\left|\left(\frac{2+6+3}{9}\right)\vec{a}\times\vec{b}\right|$

$= \frac{1}{2}\cdot\frac{11}{9}|\vec{a}\times\vec{b}| \Rightarrow \frac{11}{18}\cdot|\vec{a}\times\vec{b}| \Rightarrow \frac{11}{18}\cdot|\vec{a}||\vec{b}|\sin 60 \Rightarrow \frac{11}{18}\cdot a^2\cdot\frac{\sqrt{3}}{2} = \frac{11\sqrt{3}}{36}a^2$

Q.8 [9]

Equation of the plane ABC will be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Now d = distance of the plane from origin O = $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} \Rightarrow \frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

and $m = OM = \sqrt{a^2+b^2+c^2}$

So $\left(\frac{m}{d}\right)^2 = (a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right) = 3 + \left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)+\left(\frac{b^2}{c^2}+\frac{c^2}{b^2}\right)+\left(\frac{c^2}{a^2}+\frac{a^2}{c^2}\right)$

$\Rightarrow \left(\frac{m}{d}\right)^2_{Min} = 3+6=9$

By using A.M.–H. M. inequality, we get

$\frac{a^2+b^2+c^2}{3} \geq \frac{3}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}} \Rightarrow (a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\geq 9$

Hence $(a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)_{minimum} = 9$

Q.9 [240]

Volume (V) $= \frac{1}{3}A_1 h_1 \Rightarrow h_1 = \frac{3V}{A_1}$

|||ly $h_2 = \frac{3V}{A_2}$, $h_3 = \frac{3V}{A_3}$ and $h_4 = \frac{3V}{A_4}$

So $(A_1+A_2+A_3+A_4)(h_1+h_2+h_3+h_4) = (A_1+A_2+A_3+A_4)\left(\frac{3V}{A_1}+\frac{3V}{A_2}+\frac{3V}{A_3}+\frac{3V}{A_4}\right)$

$= 3V(A_1+A_2+A_3+A_4)\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)$

Now using A.M.-H.M inequality in A_1, A_2, A_3, A_4, we get

$$\frac{A_1+A_2+A_3+A_4}{4} \geq \frac{4}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)}$$

$\Rightarrow (A_1+A_2+A_3+A_4)\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)\geq 16$

Hence the minimum value of $(A_1+A_2+A_3+A_4)(h_1+h_2+h_3+h_4) = 3V(16) = 48V = 48\times 5 = 240$ **Ans.**

Q.10 [3]

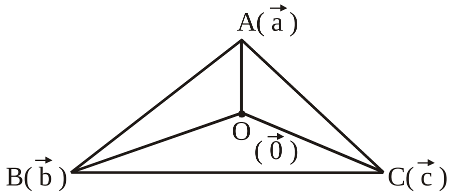
where $t = \frac{17}{5}$ Þ [t] = 3]

Sol. Let position vector of A, B, C be $\vec{a}, \vec{b}, \vec{c}$ respectively.

$\therefore\ 2\vec{a}+5\vec{b}+10\vec{c}=0$ ……(i)

Taking cross product with $\vec{a}$ in (i)

$0+5\vec{a}\times\vec{b}+10\vec{a}\times\vec{c}=0$



$\Rightarrow\ \vec{a}\times\vec{b} = 2\vec{c}\times\vec{a}$

Taking cross product with $\vec{c}$ in (i)

$2\vec{a}\times\vec{c} + 5\vec{b}\times\vec{c} + 0 = 0$

$\Rightarrow\ \vec{b}\times\vec{c} = \dfrac{2}{5}\vec{c}\times\vec{a}$

$\because\ t = \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta AOC} =$

$\dfrac{\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\frac{1}{2}\left|\vec{c}\times\vec{a}\right|} =$

$\dfrac{\left|2\vec{c}\times\vec{a}+\frac{2}{5}\vec{c}\times\vec{a}+\vec{c}\times\vec{a}\right|}{\left|\vec{c}\times\vec{a}\right|} = \dfrac{17}{5}$

$\therefore\ |t| = 3$

Q.11 [9]

$\dfrac{x+2}{1} = \dfrac{y-3}{2} = \dfrac{z-k}{3} = \lambda \Rightarrow (\lambda-2, 2\lambda+3, 3\lambda+k)$

for A, $\lambda = 2$

$A(0, 7, 6+k) \Rightarrow$ for B $\lambda = -\dfrac{k}{3} \Rightarrow$

$B\left(-2-\dfrac{k}{3}, 3-\dfrac{2k}{3}, 0\right)$

$\angle AOB = 90° \Rightarrow \overrightarrow{AO}\,.\,\overrightarrow{OB} = 0 \Rightarrow$

$7\left(-3+\dfrac{2k}{3}\right) = 0$ or $k = \dfrac{9}{2} \Rightarrow 2k = 9$

Q.12 [34]

$P = xi + yj$; $\overrightarrow{AP} = (x-1)i + yj$; $\overrightarrow{BP} = (x+1)i + yj$

$\overrightarrow{PA}\cdot\overrightarrow{PB} = x^2 - 1 + y^2$; $\overrightarrow{OA}\cdot\overrightarrow{OB} = -1$

Now $(\overrightarrow{PA}.\overrightarrow{PB}) + 3\,(\overrightarrow{OA}.\overrightarrow{OB}) = 0 \Rightarrow x^2 + y^2 - 4 = 0$

$\Rightarrow x^2 + y^2 = 4$

$|\overrightarrow{PA}|\,|\overrightarrow{PB}| = \sqrt{(x-1)^2+y^2}\ \sqrt{(x+1)^2+y^2} = \sqrt{5-2x}\sqrt{5+2x}$

$|\overrightarrow{PA}|\,|\overrightarrow{PB}| = \sqrt{25-4x^2}$

Now from $x^2 + y^2 = 4$ put $x = 2\cos\theta$ $y = 2\sin\theta$

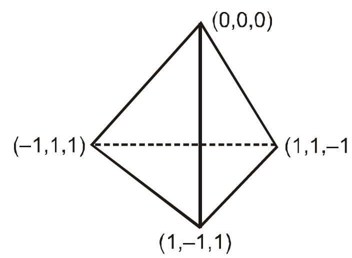
$|\overrightarrow{PA}|\,|\overrightarrow{PB}| = \sqrt{25-16\cos^2\theta}$; $|\overrightarrow{PA}|\,|\overrightarrow{PB}|_{max} = \sqrt{25} = M$

$|\overrightarrow{PA}|\,|\overrightarrow{PB}|_{min} = \sqrt{9} = m$; $M^2 + m^2 = 25 + 9 = 34$

Q.13 [2]

The planes are

$y + z = 0$ ....(1)



$z + x = 0$ ....(2)

$x + y = 0$ ....(3)

$x + y + z = 1$ ....(4)

Solving above equations we get vertices of the tetrahedron as $(0,0,0), (-1,1,1), (1,-1,1)$ and $(1,1,-1)$

$\therefore$ Required volume $= \left|\dfrac{1}{6}\begin{vmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}\right| =$

$\left|\dfrac{1}{6}\begin{vmatrix}0 & 2 & 1\\ 2 & 0 & 1\\ 0 & 0 & -1\end{vmatrix}\right| = \dfrac{4}{6} = \dfrac{2}{3} \Rightarrow t = \dfrac{2}{3} \Rightarrow 3t = 2$

Q.14 [4]

$L_1 : \dfrac{x}{0} = \dfrac{y}{b} = \dfrac{z-c}{-c} = r$; $L_2 : \dfrac{x}{a} = \dfrac{y}{0} = \dfrac{z+c}{c} = \ell$

Dr's of AB are $-a\ell, br, -cr - c\ell + 2c \Rightarrow$ AB is perpendicular to both the lines

$\therefore\ 0(-a\ell) + b.\,br + (-c)(-cr - c\ell + 2c) = 0 \Rightarrow$

$(b^2 + c^2)\,r + c^2\ell = 2c^2$ .....(1)

and $a(-a\ell) + 0(br) + c(-cr - c\ell + 2c) = 0$ $\Rightarrow -$

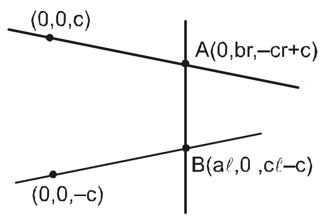
$(a^2 + c^2)\ell - c^2r + 2c^2 = 0$

$(a^2 + c^2)\ell + c^2r = 2c^2$ .....(2)

from (1) & (2)

$$\ell = \frac{2b^2c^2}{a^2b^2 + b^2c^2 + c^2a^2}, \quad r = \frac{2a^2c^2}{a^2b^2 + b^2c^2 + c^2a^2}$$

$$A\left(0, \frac{2a^2bc^2}{a^2b^2 + b^2c^2 + c^2a^2}, c\left(\frac{a^2b^2 + b^2c^2 - c^2a^2}{a^2b^2 + b^2c^2 + c^2a^2}\right)\right)$$



$$B\left(\frac{2ab^2c^2}{a^2b^2 + b^2c^2 + c^2a^2}, 0, c\left(\frac{b^2c^2 - a^2b^2 - c^2a^2}{a^2b^2 + b^2c^2 + c^2a^2}\right)\right)$$

$$d^2 = \frac{4a^2b^4c^4}{(a^2b^2 + b^2c^2 + c^2a^2)^2} +$$

$$\frac{4a^4b^2c^4}{(a^2b^2 + b^2c^2 + c^2a^2)^2} + \frac{4c^2(a^4b^4)}{(a^2b^2 + b^2c^2 + c^2a^2)^2}$$

$$\frac{4}{d^2} = \frac{(a^2b^2 + b^2c^2 + c^2a^2)^2}{a^2b^4c^4 + a^4b^2c^4 + a^4b^4c^2} =$$

$$\frac{a^2b^2 + b^2c^2 + c^2a^2}{a^2b^2c^2} \Rightarrow \frac{4}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

PREVIOUS YEAR'S

MHT CET

Q.1 (c)
Q.2 (c)
Q.3 (a)
Q.4 (a)
Q.5 (d)
Q.6 (a)
Q.7 (a)
Q.8 (b)
Q.9 (a)
Q.10 (a)
Q.11 (a)
Q.12 (d)
Q.13 (b)
Q.14 (d)
Q.15 (b)
Q.16 (c)
Q.17 (a)
Q.18 (b)
Q.19 (d)
Q.20 (b)
Q.21 (c)
Q.22 (d)
Q.23 (d)
Q.24 (a)
Q.25 (c)
Q.26 (d)
Q.27 (d)
Q.28 (2)
Q.29 (d)
Q.30 (c)
Q.31 (d)
Q.32 (b)
Q.33 (a)
Q.34 (a)
Q.35 (d)
Q.36 (b)
Q.37 (c)
Q.38 (b)
Q.39 (d)
Q.40 (c)

JEE-MAIN

Q.1 (D)

Direction ratio of normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

so equation of plane

$18x - y + 7z = d$

It passes through (2,3, – 5)

$36 - 3 - 35 = d \therefore d = -2$

Eq^n of plane

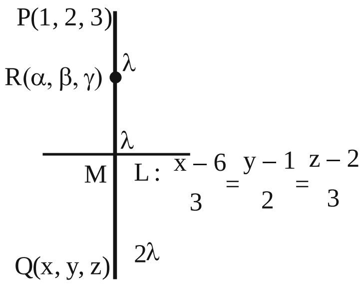
$18x - y + 7z = -2$

$-18x + y - 7z = 2$

$\therefore a = -18, b = 1, c = -7, d = 2$

$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$

Q.2 (125)



Let M be the mid point of PQ

$\therefore$ M = $(3\lambda+6, 2\lambda+1, 3\lambda+2)$

Now, $\overrightarrow{PM} = (3\lambda+5)\hat{i}+(2\lambda-1)\hat{j}+(3\lambda-1)\hat{k}$

$\because \overrightarrow{PM} \perp (3\hat{i}+2\hat{j}+3\hat{k})$

$\therefore 3(3\lambda+5)+2(2\lambda-1)+3(3\lambda-1)=0$

$\lambda = \frac{-5}{11}$

$\therefore M\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$

since R is mid-point of PM

$22(\alpha+\beta+\gamma) = 125$

Q.3 (C)

$$\frac{a-2}{3}=\frac{b-4}{-1}=\frac{c-7}{4}=\frac{-2(6-4+28-2)}{3^2+1^2+4^2}$$

$$\Rightarrow a = \frac{-84}{13}+2, b = \frac{28}{13}+4, \ C = \frac{-112}{13}+7$$

2a + b + 2c = – 6

Q.4 (26)

Points P (1, 2, –1) and Q (2, –1, 3) lie on same side of the plane. Perpendicular distance of point P from plane

is $\frac{-1+2-1-1}{\sqrt{1^2+1^2+1^2}} = \frac{1}{\sqrt{3}}$

Perpendicular distance of point Q from plane is

$$\left|\frac{-2-1+3-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

PQ is parallel to given plane. So, distance between P and Q = distance between their foot of perpendiculars.

$\Rightarrow |\overrightarrow{PQ}| = \sqrt{(1-2)^2+(2+1)^2+(-1-3)^2}$

$= \sqrt{26}$

$d^2 = 26$

Q.5 (B)

$$L : \frac{x-2}{3}=\frac{y+1}{-2}=\frac{2+3}{-1}$$

Plane : px – qy + z = 5

Line is satisfying the plane

So (2, –1, –3) satisfying px – qy + z = 5

P(2) – q(–1) + (–3) = 5 ⇒ 2p + q = 8 ... (i)

The line is parallel to plane

$\therefore$ 3P + 2q – 1 = 0 . . . (ii)

From (i) and (ii)

P = 15, q = –22

Eq. of plane is 15x – 22y + z – 5 = 0

Distance from origin

$$= \frac{5}{\sqrt{15^2+(-22)^2+(1)^2}} = \frac{5}{\sqrt{710}}$$

$$= \sqrt{\frac{25}{710}} = \sqrt{\frac{5}{142}}$$

Q.6 (B)

Image of P(1, 2, 1) in x + 2y + 2z – 16 = 0

Is given by Q(3, 6, 5)

Equation of plane T

$$\begin{vmatrix} x & y & z+1 \\ 3 & 6 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

2x – z = 1

By options, (1, 2, 1) lies on plane T

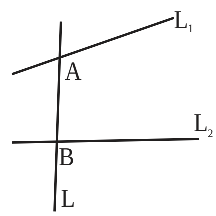
Q.7 **[84]**

$$L_1 \rightarrow \frac{x-7}{3}=\frac{y-1}{-1}=\frac{z+2}{1}$$

so point A(3λ+7, –λ+1, λ–2)

$$L_2 \rightarrow \frac{x}{2}=\frac{y-7}{3}=\frac{z}{1}$$

so point B(2μ, 3μ, 7μ)



$\therefore$ D.R. of AB is

3λ+7–2μ, –λ+1–3μ–7, λ–2–μ

3λ–2μ+7, –λ–3μ–6, λ–μ–2

given DR's of AB are 1, –4, 2

$$\therefore \frac{3\lambda+7-2\mu}{1}=\frac{-\lambda-3\mu-6}{-4}=\frac{\lambda-\mu-2}{2}$$

$$\frac{3\lambda+7-2\mu}{1}=\frac{-\lambda-3\mu-6}{-4}$$

$$\frac{-\lambda-3\mu-6}{-4}=\frac{\lambda-\mu-2}{2}$$

$-12\lambda-28+8\mu=-\lambda-3\mu-6$ $\quad -2\lambda-6\mu-12=-4\lambda+4\mu+8$

$11\lambda-11\mu+22=0$ $\quad 2\lambda-10\mu-20=0$

$\boxed{\lambda-\mu+2=0}$ ...(1) $\quad \boxed{\lambda-5\mu-10=0}$

....(2)

Solving Eq. (1) & (2)

$\lambda=-5$ & $\mu=-3$

$\therefore$ A(–8, 6, –7) & B(–6, –2, –3)

$$AB=\sqrt{(-8+6)^2+(6+2)^2+(-7+3)^2}$$

$$AB=\sqrt{4+64+16}$$

$$(AB)^2=84$$

Q.8 **[2]**

$$L_1:\to \vec{r}=\left(-\hat{i}+3\hat{k}\right)+\lambda\left(\hat{i}-a\hat{j}\right)$$

$$\Rightarrow \frac{x+1}{1}=\frac{y-0}{-a}=\frac{z-3}{0} \quad ....(1)$$

$$L_2:\to \vec{r}=\left(-\hat{j}+2\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$$

$$\frac{x-0}{1}=\frac{y+1}{-1}=\frac{z-2}{1} \quad .....(2)$$

Given shortest distance between L_1 & L_2 is $=\sqrt{\frac{2}{3}}$

$$\left|\frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|=\sqrt{\frac{2}{3}}$$

$$\left|\frac{\begin{vmatrix} -1-0 & 0+1 & 3-2 \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}}{\sqrt{(-a)^2+(1-0)^2+(-1+a)^2}}\right|=\sqrt{\frac{2}{3}}$$

$$\Rightarrow\left|\frac{-1(-a)-1(1)+1(-1+a)}{\sqrt{a^2+1+a^2+1-2a}}\right|=\sqrt{\frac{2}{3}}$$

$$\left|\frac{2a-2}{\sqrt{2a^2-2a+2}}\right|=\sqrt{\frac{2}{3}}$$

$$\frac{\left(2(a-1)\right)^2}{2\left(a^2-a+1\right)}=\frac{2}{3}$$

$\Rightarrow 3(a-1)^2=a^2-a+1$

$\Rightarrow 3a^2-6a+3=a^2-a+1$

$\Rightarrow 2a^2-5a+2=0$

$2a^2-4a-a+2=0$

$2a(a-2)-1(a-2)=0$

$a=1/2$ & $[a=2]$

Ans. 2

Q.9 (a)

$$\text{S.D.}=\frac{[(\bar{a}_2-\bar{a}_1)(\bar{b}_1\bar{b}_2)]}{|\bar{b}_1\times\bar{b}_2|}$$

$$=\frac{\begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix}}=\frac{(15-4\lambda)-2(10-\lambda)+2(5)}{|(15-4\lambda)\hat{i}-\hat{j}(10-\lambda)+\hat{k}(5)|}$$

$$=\frac{1}{\sqrt{3}}=\frac{5-2\lambda}{\sqrt{(15-4\lambda)^2(10-\lambda)^2+5^2}}$$

$=(15-4\lambda)^2+(10-\lambda)^2+25=3(5-2\lambda)^2$

$=225+16\lambda^2-120\lambda+100+\lambda^2-20\lambda+25$

$=75+12\lambda^2-60\lambda$

$5\lambda^2-80\lambda+175=0$

$\lambda^2-16\lambda+35=0$

$\boxed{\lambda_1+\lambda_2=16}$

Q.10 (c)

$(r-(-1, 0, 2))\cdot[6, -4, 2]=0$

$6(x+1)-4(y)+2(z-2)=0$

$\Rightarrow 3x-2y+z+1=0$ ...P_1

$2x+y+3z=1$ ...P_2

$$\cos\theta=\frac{2\times3+1(-2)+3(1)}{\sqrt{14}\cdot\sqrt{14}}$$

$$=\frac{6-2+3}{14}=\frac{1}{2}$$

$\therefore \cos\theta=\frac{1}{2}$ $\quad \Rightarrow\theta=\frac{\pi}{3}$

Q.11 **[51]**

$l_1: 8x+4\sqrt{2}y=1$

$l_2: -8-6\sqrt{3}z=1$

$l_1: a_1=\left(\frac{1}{8}, 0, 0\right)$ and $a_2=\left(0, \frac{1}{4\sqrt{2}}, 0\right)$

$P_1: a_2-a_1=\left\langle\frac{-1}{8}, \frac{1}{4\sqrt{2}}, 0\right\rangle$

$l_2 : b_1 = \left(-\frac{1}{8}, 0, 0\right)$ & $b_2 = \left(0, 0, \frac{-1}{6\sqrt{3}}\right)$

$P_2 : b_2 - b_1 = \left\langle \frac{1}{8}, 0, \frac{-1}{6\sqrt{3}} \right\rangle$

$$\vec{n} = P_1 \times P_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{1}{8} & \frac{1}{4\sqrt{2}} & 0 \\ \frac{1}{8} & 0 & \frac{-1}{6\sqrt{3}} \end{vmatrix}$$

$$\vec{n} = \hat{i}\left(\frac{-1}{24\sqrt{6}}\right) - \hat{j}\left(\frac{1}{48\sqrt{3}}\right) - \hat{k}\left(\frac{1}{32\sqrt{2}}\right)$$

$$\vec{n} = \frac{-1}{96\sqrt{6}}\left\langle 4, 2\sqrt{2}, 3\sqrt{3} \right\rangle$$

$$\vec{c} = (a_1 - b_1) = \left(\frac{1}{4}, 0, 0\right)$$

$$d = \left|\frac{\vec{c}\cdot\vec{n}}{|\vec{n}|}\right|$$

$$= \left|\frac{1}{\sqrt{16+8+27}}\right| = \frac{1}{\sqrt{51}}$$

$$\Rightarrow \frac{1}{d^2} = 51$$

Q.12 **(3)**

Equation of family of planes passing through line of intersection of two planes $2x + y - 5z = 0$ and $3x - y + 4z - 7 = 0$

is $(3x - y + 4z - 7) + \lambda(2x + y - 5z) = 0$

$(3 + 2\lambda)x + (-1 + \lambda)y + (4 - 5\lambda)z - 7 = 0$

....(1) is perpendicular to plane

$2x + y - 5z = 9$

$(3 + 2\lambda).\, 2 + (-1 + \lambda).1 + (4 - 5\lambda)(-5) = 0$

$\Rightarrow 6 + 4\lambda - 1 + \lambda - 20 + 25\lambda = 0$

$\Rightarrow 30\lambda - 15 = 0$

$\Rightarrow \lambda = \frac{15}{30} = \frac{1}{2}$

Put λ in equation (i)

$$\left(3 + 2\times\frac{1}{2}\right)x + \left(-1 + \frac{1}{2}\right)y + \left(4 - \frac{5}{2}\right)z - 7 = 0$$

$\Rightarrow 4x - \frac{y}{2} + \frac{3}{2}z - 7 = 0, \Rightarrow 8x - y + 3z - 14 = 0$

$\Rightarrow$ Point (1, 0, 2) satisfy equation.

Q.13 **[33]**

6 digit numbers formed by digits 1 and 8 only

for a number to be multiple fo 21 it must be divisible by both 3 and 7

To make divisible by '3'

sum must be divisible by 3

possible cases

(i) All digits are 1's $\rightarrow$ 1

(ii) All digits are 8's $\rightarrow$ 1

(iii) 3 1's and 3 8's $\rightarrow \frac{6!}{3!3!} = 20$

To make divisible by 7

|2(last digit)– (remaining number)| = 7k, $k \in Z$

total possibilities = 2^6

total numbers divisible by 21 = 20 + 1 + 1 = 22

$p = \frac{22}{2^6} \Rightarrow 96\,p = \frac{22}{64} \times 96 = 33$

Q.14 (A)

$1 + m - n = 0$

$3l^2 + m^2 + cl(l + m) = 0$

$3l^2 + m^2 + cl^2 + clm) = 0$

$(3 + c)\, l^2 + clm + m^2 = 0$

$(3 + c)\left(\frac{1}{m}\right)^2 + c\left(\frac{1}{m}\right) + 1 = 0$ ...(1)

$\because$ lies are parallel

Roots of (1) must be equal

$\Rightarrow D = 0$

$C^2 - 4(3+c) = 0$

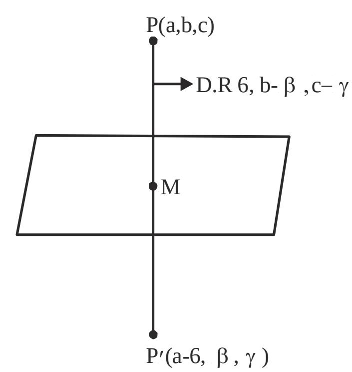
$C^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

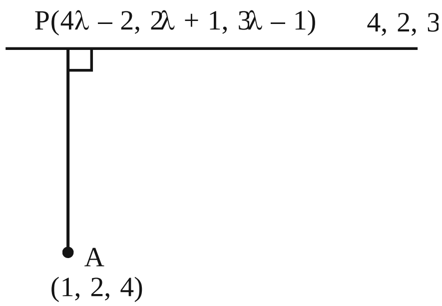
$c = 6$ or $c = -2$

+ve value of c = 6

Q.15 (137)



Q.16 **(A)**



$$\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{4}=\lambda$$

$(x, y, z) = (4\lambda-2, 2\lambda+1, 3\lambda-1)$

$\overrightarrow{AP}=(4\lambda-3)\hat{i}+(2\lambda-1)\hat{j}+(3\lambda-5)\hat{k}$

$\vec{b}=4\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{AP}\cdot\vec{b}=0$

$4(4\lambda-3)+2(2\lambda-1)+3(3\lambda-5)=0$

$29\lambda=12+2+15=29$

$\lambda=1$

$P=(2,3,2)$

$3x+4y+12z+23=0$

$$d=\left|\frac{6+12+24+23}{\sqrt{9+16+144}}\right|$$

$$d=\left|\frac{65}{13}\right|=5$$

Q.17 **(A)**

$$\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$$

$$\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$$

$A=(3,2,1)\ B=(-3,6,5)$

$\vec{n}_1=2\hat{i}+3\hat{j}-\hat{k}$

$\vec{n}_2=2\hat{i}+\hat{j}+3\hat{k}$

$\overrightarrow{BA}=6\hat{i}-4\hat{j}-4\hat{k}$

$$\text{SHORTEST DISTANCE}=\frac{[\overrightarrow{BA}\,\vec{n}_1\,\vec{n}_2]}{[\vec{n}_1\times\vec{n}_2]}$$

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&-1\\2&1&3\end{vmatrix}$$

$=10\hat{i}-8\hat{j}-4\hat{k}$

$[\overrightarrow{BA}\,\vec{n}_1\,\vec{n}_2]=60+32+16=108$

$[\vec{n}_1\times\vec{n}_2]=\sqrt{100+64+16}=\sqrt{180}$

$$\text{S.D}=\frac{108}{\sqrt{180}}=\frac{108}{6\sqrt{5}}=\frac{18}{\sqrt{5}}$$

Q.18 (B)

Equation of L

$$\frac{x-1}{1}=\frac{y+1}{1}=\frac{z+1}{1}=\lambda$$

$x=\lambda+1,\ y=\lambda-1,\ z=\lambda-1$

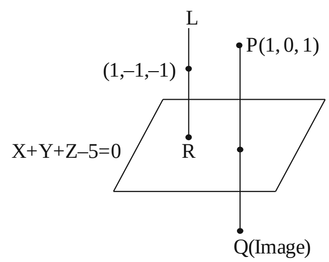
Putting in equation of plane

$\lambda+1+\lambda-1+\lambda-1=5$

$3\lambda=6$

$\lambda=2$

$\therefore R(3,1,1)$



$QR^2=PR^2=4+1+0=5$

Q.19 (5)

L_1: $\vec{r}=(0,0,0)+\lambda(1,2,3)$

L_2: $\vec{r}=(1,3,1)+\mu(1,1,5)$

$$\text{direction of } \vec{n} \text{ of plane}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&3\\1&1&5\end{vmatrix}=7\hat{i}-2\hat{j}-\hat{k}$$

for point of intersection of L_1 and L_2:

$\lambda=1+\mu$ ....(1)

$2\lambda=3+\mu$ ....(2)

$3\lambda=1+5\mu$ ....(3)

Solving (1), (2) and (3),

$\lambda=2,\ \mu=1$

$\therefore S(2,4,6)$

Equation of plane $7(x-2)-2(y-4)-(z-6)=0$

$7x-2y-z-14+8+6=-0$

$7x-2y-z=0$

$a=7, b=-2, d=0$

$a+b+d=5$

Q.20 (2)

$$I_1:\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}$$

$$I_2=\frac{x-1}{1}=\frac{y+3/2}{\alpha/2}=\frac{z+5}{2}$$

$$I_3 = \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z+0}{4}$$

$$I_1 \perp I_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{3}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$$

angle between I_2 & I_3

$$\cos\theta = \frac{|1\times(-3)+(-2)(\alpha/2)+2\times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put $\alpha=3$

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

Q. 21 (1)

Let equation of rotated plane be:

$(2x+3y+z+20)+\lambda(x-3y+5z-8)=0$

$(2+\lambda)x+(3-3\lambda)y+(1+5\lambda)z+20-8\lambda=0$

Above plane is perpendicular to $2x+3y+z=20=0$

So, $(2+\lambda).2+(3-3\lambda).3+(1+5\lambda).1=0 \Rightarrow \lambda=7$

$\Rightarrow$Equation of rotated plane: $x-2y+4z-4=0$

Mirror image of $A\left(2,\frac{-1}{2},2\right)$ in rotated plane is

B (a,b,c)

Equation of AB: $\dfrac{x-2}{1} = \dfrac{y+1/2}{-2} = \dfrac{x-2}{4} = k$

Let coordinate of B be $\left(2+k,\frac{-1}{2}-2k,2+4k\right)$

Mid point of AB is $\left(2+\frac{k}{2},\frac{-1}{2}-k,2+2k\right)$

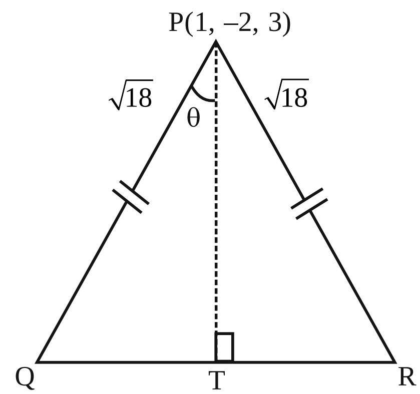
which will lie on the plane $x-2y+4z-4=0$

Hence $k = \dfrac{-2}{3}$

Therfore B is $\left(\frac{4}{3},\frac{5}{6},\frac{-2}{3}\right) \equiv \left(\frac{8}{6},\frac{5}{6},\frac{-4}{6}\right)$

So, $\dfrac{a}{8} = \dfrac{b}{5} = \dfrac{c}{-4}$

Q.22 (B)



$-x+2y-z=0$

$3x-5y+2z=0$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(1) + \hat{k}(-1)$$

$$\vec{n} = -\hat{i} - \hat{j} - \hat{k}$$

Equation of LOI is $\dfrac{x}{1} = \dfrac{y}{1} = \dfrac{z}{1}$

dr's of PT $\to (\alpha-1, \alpha+2, \alpha-3)$

dr's of QR $\to (1, 1, 1)$

$\Rightarrow (\alpha-1)\times 1 + (\alpha+2)\times 1 + (\alpha-3)\times 1 = 0$

$3\alpha = 2$

$$\alpha = \frac{2}{3}$$

$$PT^2 = \frac{1}{9}+\frac{64}{9}+\frac{49}{9}$$

$$PT^2 = \frac{114}{9}$$

$$PT = \frac{\sqrt{114}}{3}$$

$$\cos\theta = \frac{\sqrt{114}}{3}\times\frac{1}{3\sqrt{2}} = \frac{\sqrt{57}}{9} = \frac{\sqrt{19\times3}}{3\times3}$$

$$= \frac{\sqrt{19}}{3\sqrt{3}}$$

$$\cos 2\theta = \frac{2\times19}{27} - 1 = \frac{11}{27}$$

$$\sin 2\theta = \sqrt{1-\left(\frac{11}{27}\right)^2} = \frac{\sqrt{38}\sqrt{16}}{27}$$

$$= \frac{4}{27}\sqrt{38}$$

$$\text{Area} = \frac{1}{2}\times\sqrt{18}\sqrt{18}\times\frac{4}{27}\sqrt{38}$$

$$= \frac{18}{2}\times\frac{4}{27}\sqrt{38} = \frac{36}{27}\sqrt{38} = \frac{4}{3}\sqrt{38}$$

Q.23 (A)

Equation of plane passing through the intersection of planes
$5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ is
$5x + 8y + 13z - 29 + \lambda(8x - 7y + z - 20) = 0$ and if it is passing through (2, 1, 3) then $\lambda = \frac{7}{2}$

P_1 : Equation of plane through intersection of $5x + 8y + 13z - 29 = 0$ and
$8x - 7y + z - 20 = 0$ and the point (2, 1, 3) is

$$5x + 8y + 13z - 29 + \frac{7}{2}(8x - 7y + z - 20) = 0$$

$\Rightarrow 2x - y + z = 6$

Similarly P_2 : Equation of plane through intersection of $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the point (0, 1, 2) is $x + y + 2z = 5$

Angle between planes $= \theta = \cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right) = \frac{\pi}{3}$

Q.24 (B)

Equation of plane passing through line of intersection of planes $P_1 : \vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k}) = 6$ and

$P_2 : \vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k}) = 7$ is

$P_1 + \lambda P_2 = 0$

$$(\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})-6)+\lambda(\vec{r}\cdot(-6\hat{i}+5\hat{j}-\hat{k})-7) = 0$$

and it passes through point $\left(2, 3, \frac{1}{2}\right)$

$$\Rightarrow \left(2+9-\frac{1}{2}-6\right)+\lambda\left(-12+15-\frac{1}{2}-7\right) = 0$$

$\Rightarrow \lambda = 1$

Equation of plane is $\vec{r}\cdot(-5\hat{i}+8\hat{j}-2\hat{k}) = 13$

$|\vec{a}|^2 = 25 + 64 + 4 = 93;\ d = 13$

Value of $\frac{|13\vec{a}|^2}{d^2} = 93$

Q.25 [B]

$$\frac{X+1}{2} = \frac{4-3}{3} = \frac{Z-1}{-1} = \lambda$$

$\theta\ (2\lambda - 1, 3 + 3, -\lambda + 1)$
DR of PQ $2\lambda - 1 - a, 3\lambda - 1, \lambda - 1$
Now PQ and line is 1r
So, $2(2\lambda - 1 - a) + 3(3\lambda - 1)(-\lambda - 1) = 0$

$$\Rightarrow \lambda = \frac{a+2}{7}$$

$$Q\left(\frac{2a-3}{7}, \frac{3a+27}{7}, \frac{5-a}{7}\right)$$

Distance between PQ

$$\sqrt{\left(\frac{2a-3}{7}-a\right)^2+\left(\frac{3a+27}{7}-u\right)^2+\left(\frac{5-Q}{7}-2\right)^2}$$

$= \sqrt{6}$ (given)
$35a^2 + 42a + 91 = 14^2 \times 6$

$3\sqrt{a}^2 + 42a - 1085 = 0$
$5a^2 + 6a - 155 = 0$

$a = 5, a = -\frac{62}{10}\ (\because a > 0)$

so a = 5

$\lambda = \frac{5+2}{7} = 1$

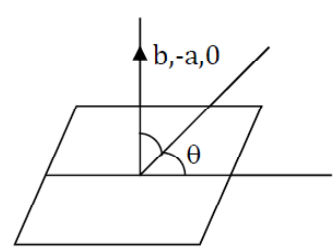
Point Q (1,6,0) is mid point P and R, so

$$1 = \frac{\alpha_1+\alpha}{2} = \frac{\alpha_2+5}{2} \Rightarrow \alpha_1 = -3$$

Similarly $\alpha_2 = 8, \alpha_3 = -2$

$a + \frac{3}{2}\alpha_1 = \alpha + \alpha_1 + \alpha_2 + \alpha_3 = 5 - 3 + 8 - 2 = 8$

Q.26 (1 and 2)



D.R of line is = $\begin{vmatrix} i & j & k \\ a & b & o \\ a & b & c \end{vmatrix}$ = i (bc) + j (ac) + 0k

D.R. of line (b, a o)
$0x + y - z + 2 = 0$
(0,1 , –1)

$\text{Sin } \theta = \dfrac{a}{\sqrt{2}\sqrt{b^2+a^2}} = \dfrac{1}{2}$

$\sqrt{2}a = \sqrt{a^2+b^2}$

$a^2 = b^2 \Rightarrow a = b$ or $a = -b$

$(1,-1,0) = \left(\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}, 0\right)$

Q.27 (4)

Given : $L_1 : \dfrac{x-1}{\lambda} = \dfrac{y-2}{1} = \dfrac{z-3}{2}$

and $L_2 : \dfrac{x+26}{-2} = \dfrac{y+18}{3} = \dfrac{z+28}{\lambda}$

are coplanar

$\Rightarrow \begin{bmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{bmatrix} = 0$

$\Rightarrow \lambda = 3$

Now, normal of plane P, which contains L_1L_2

$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{bmatrix}$

$= -3\hat{i} - 13\hat{j} + 11\hat{k}$

$\Rightarrow$ Equation of required plane P :
$3x + 13y - 11z + 4 = 0$
(0, 4, 5) does not lie on plane P.

Q.28 (2)

Normal of plane P :

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ -1 & 2 & -2 \end{bmatrix} = 4\hat{i} - \hat{j} - 3\hat{k}$

Equation of plane P which passes through (2, 2, –2) is $4x - y - 3z - 12 = 0$
Now, A(3, 0, 0), B(0, –12, 0), C(0, 0, –4)
$\Rightarrow \alpha = 3, \beta = -12, \gamma = -4$
$\Rightarrow p = \alpha + \beta + \gamma = -13$
Now, volume of tetrahedron OABC

$V = \left|\dfrac{1}{6}\overrightarrow{OA} \times \overrightarrow{OC}\right| = 24$

(V, p) = (24, –13)

Q.29 (BONUS)

DR's of line of shortest distance

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j} - 2\hat{k}$

Angle between line and plane is $\cos^{-1}\sqrt{\dfrac{2}{27}} = \alpha$

$\cos\alpha = \sqrt{\dfrac{2}{27}}, \sin\alpha = \dfrac{5}{3\sqrt{3}}$

DR's normal to plane (1,–1,–1)

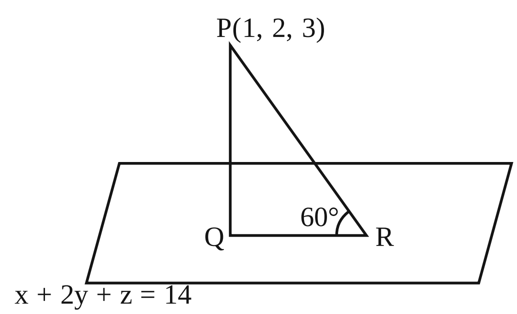
$\sin\alpha = \left|\dfrac{-a-2+2}{\sqrt{4+4+1}\sqrt{a^2+1+1}}\right| = \dfrac{5}{3\sqrt{3}}$

$\sqrt{3}|a| = 5\sqrt{a^2+2}$

$3a^2 = 25a^2 + 50$

Ans. No value of (a) [Bonus]

Q.30 (2)



Length of perpendicular

$PQ = \left|\dfrac{1+4+3-14}{\sqrt{6}}\right| = \sqrt{6}$

$QR = (PQ)\cot 60° = \sqrt{2}$

$\therefore$ Area of $\Delta PQR = \dfrac{1}{2}(PQ)(QR) = \sqrt{3}$

Q.31 (4)

A(2, 3, 9); B(5, 2, 1); C(1, λ, 8); D(λ, 2, 3)

$[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix} = 0$$

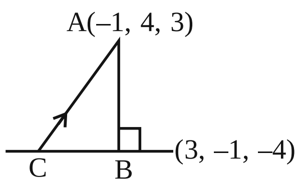
$\Rightarrow [-6(\lambda-3)-1]-8(1-(\lambda-3)(\lambda-2))+(6+(\lambda-2)) = 0$

$3(-6\lambda+17)-8(-\lambda^2+5\lambda-5)+(\lambda+4) = 8$

$8\lambda^2-57\lambda+95 = 0$

$$\lambda_1\lambda_2 = \frac{95}{8}$$

Q.32 (2)



Let B be foot of $\perp$ coordinates of B = $\left(-2, \frac{7}{2}, \frac{3}{2}\right)$

Direction ratio of line AB is < 2, 1, 3 > so

m = 1, n = 3

So, equation of AC is $\frac{x+1}{3} = \frac{y-4}{-1} = \frac{z-3}{-4} = \lambda$

So point C is $(3\lambda-1, -\lambda+4, -4\lambda+3)$. But C lies on the plane, so

$6\lambda-2-\lambda+4-12\lambda+9 = 4$

$\Rightarrow \lambda = 1 \Rightarrow C(2, 3, -1)$

$\Rightarrow AC = \sqrt{26}$

Q.33 (10)

$(a, -4a, -7) \perp$ to $(3, -1, 2b)$

$a = 2b$ ... (1)

$(a, -4a, -7) \perp$ to $(b, a, -2)$

$3a + 4a - 14b = 0$

$ab - 4a^2 + 14 = 0$ ... (2)

From equations (1) and (2)

$2b^2 - 16b^2 + 14 = 0$

$b^2 = 1$

$a^2 = 4b^2 = 4$

$$\frac{x+1}{5} = \frac{y-2}{3} = \frac{z}{1} = k$$

$\alpha = 5k-1, \beta = 3k+2, \gamma = k$

As (α, β, γ) satisfies $x - y + z = 0$

$5k - 1 - (3k+2) + k = 0$

$k = 1$

$\therefore \alpha + \beta + \gamma = 9k + 1 = 10$

Q.34 (450)

DR's of RS $\equiv (\alpha, -1, \beta)$

DR of PQ $\equiv \left(\frac{56}{17}+2, \frac{43}{17}+1, \frac{111}{17}-1\right)$

$\equiv \left(\frac{90}{17}, \frac{60}{17}, \frac{94}{17}\right)$

$\frac{90}{17}\alpha + \frac{60}{17}(-1) + \frac{94}{17}\beta = 0$

$90\alpha + 94\beta = 60$

$$\beta = \frac{60-90\alpha}{94}$$

$$\beta = \frac{30(2-3\alpha)}{94}$$

$$\beta = -30\frac{(3\alpha-2)}{94}$$

$$\beta = \frac{-15}{47}(3\alpha-2)$$

$$\Rightarrow \frac{\beta}{-15} = \frac{3\alpha-2}{47}$$

$\Rightarrow \beta = -15, \alpha = -15$

$\alpha^2 + \beta^2 = 225 + 225$

$= 450$

Q.35 (4)

$2x + ky - 5z = 1$ and $3kx - ky + z = 5, k < 3$ are mutually perpendicular then

$2(3k) + k(-k) + (-5)(1) = 0$

$-k^2 + 6k - 5 = 0$

$\Rightarrow k^2 - 6k + 5 = 0$

$\Rightarrow k^2 - 5k - k + 5 = 0$

$\Rightarrow k(k-5) - (k-5) = 0$

$\Rightarrow k = 1, 5$

$\Rightarrow k = 1$ $\because k < 3$

$\therefore$ given planes are

$2x + y - 5 = 1$ ...(1)

and $3x - y + z = 5$ ...(2)

Now eq^n of plane passing through intersection of (1) and (2) is

$(2x + y - 5z - 1) + \lambda(3x - y + z - 5) = 0$ ...(3)

Now (3) made intercept of unit length on x-axis, i.e., it passes through (1,0,0)

$\Rightarrow (2-1) + \lambda(3-5) = 0$

$\Rightarrow 1 - 2\lambda = 0$

$$\Rightarrow \lambda = \frac{1}{2}$$

At $\lambda = \frac{1}{2}$ in eq^n (3)

$$\left(2+\frac{3}{2}\right)x + \left(1-\frac{1}{2}\right)y + \left(-5+\frac{1}{2}\right)z + \left(-1-\frac{5}{2}\right) = 0$$

$\Rightarrow 7x + y - 9z - 7 = 0$ ...(4)

for finding intercept on y-axis; (y,0,0) satisfies (4),

y = 7
therefore, correct answer is D.

Q.36 [12]

Equation of plane $4ax - y + 5z - 7a + \lambda(2x - 5y - z - 3) = 0$

this satisfies (4,–1,0)

$16a + 1 - 7a + \lambda(8 + 5 - 3) = 0$

$9a + 1 + 10\lambda = 0$

Normal vector of the plane A is $(4a + 2\lambda, -1 - 5\lambda, 5 - \lambda)$

vector along the line which is contained in the plane A is $i - 2j + k$

$\therefore 4a + 2\lambda + 2 + 10\lambda + 5 - \lambda = 0$

$11\lambda + 4a + 7 = 0$......(2)

Solve (1) and (2) to get $a = 1, \lambda = -1$

Now equation of plane

$x + 2y + 3z - 2 = 0$

Let the point in the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4} = t$ is

$(7t + 3, -t + 2, -4t + 3)$ Satisfy the equation of plane A

$\therefore 7t + 3 - 2t + 4 + 9 - 12t - 2 = 0$

$t = 2$

So $\alpha + \beta + \gamma = 2t + 8 = 12$

Q.37 [2]

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$$

$$\vec{r} = (\hat{i} + \hat{j}) + M(-\hat{i} + \hat{j} - a\hat{k})$$

$$\vec{n} - = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix}$$

$$= \hat{i}(-a^2 + 1) - \hat{j}(-a - 1) + \hat{k}(1 + a)$$

$$\langle -a^2 + 1, a + 1, a + 1 \rangle$$

$$(-a^2 + 1)(a - 1) + (a + 1)(y - 1) + (a + 1)z = 0$$

$$\left| \frac{(-a^2 + 1) + 4(a + 1)}{\sqrt{(-a^2 + 1)^2 + (a + 1)^2 + (a + 1)^2}} \right| = \sqrt{3}$$

$\Rightarrow (-a^2 + 4a + 5)^2 = 3((-a^2 + 1)^2 + (a + 1)^2 + (a + 1)^2)$

$\Rightarrow a^4 + 16a^2 + 25 - 8a^3 + 409 - 109^2$

$\Rightarrow 3(a^4 + 1 - 2a^2 + 2a^2 + 2 + 49)$

$\Rightarrow a^4 + 16a^2 + 25 - 8a^3 + 409 - 109^2$

$\Rightarrow 3a^4 + 3 + 6 + 12a$

$\Rightarrow 2a^4 + 8a^3 - 6a^2 - 28a - 16 = 0$

$\Rightarrow a^4 + 4a^3 - 3a^2 - 14a - 8 = 0$

$\Rightarrow (a + 1)^2 (a - 2)(a + 4) = 0$

$\Rightarrow a = -1, 2, -4$

Largest value of a = 2

Q.38 [125]

$$\vec{n}_1 = \ell\hat{i} - \hat{j} + 3(1 - \ell)\hat{k}$$

$$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$$

Direction ratio of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1-\ell) \\ 1 & 2 & -1 \end{vmatrix}$

$$= (6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

3x - 8y + 7z = 4 will contain the line

$$(6\ell - 5)i + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$$

Normal of 3x - 8y + 7z = 4 will be perpendicular to the line

$$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$$

$$\Rightarrow \ell = \frac{2}{3}$$

$\therefore$ direction ratio of line $\left(-1, \frac{5}{3}, \frac{7}{3}\right)$

Angle with axis

$$\cos\theta = \frac{5/3}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}}$$

$$\therefore 415\cos^2\theta = \frac{25}{83} \times 415 = 125$$

Q.39 (1)

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$(\hat{i} - \hat{j}(4) + \hat{k}(-3))$$

$$= \hat{i} - 4\hat{j} - 3\hat{k}$$

Equation of line

$$\frac{x-1}{1} = \frac{y-2}{-4} = \frac{z-4}{-3} = \lambda$$

$\overrightarrow{PM} \cdot (1, -4, -3) = 0$

$\Rightarrow (\lambda, -4\lambda + 4, -3\lambda - 1) \cdot (1, -4, -3) = 0$

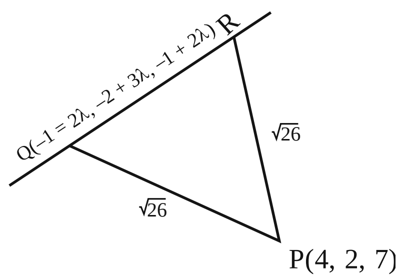
$\Rightarrow \lambda + 16\lambda - 16 + 9\lambda + 3 = 0$

$\Rightarrow 26\lambda = 13$

$$\lambda = \frac{1}{2}$$

$$PM = \sqrt{\frac{1}{4} + 4 + \frac{25}{4}} = \sqrt{\frac{13}{2} + 4} = \sqrt{\frac{21}{2}}$$

Q.40 [153]



$PQ^2 = 26$

$(4+1-2\lambda)^2 + (2+2-3\lambda)^2 + (7-1-2\lambda)^2 = 26$

$(5-2\lambda)^2 + (4-3\lambda)^2 + (6-2\lambda)^2 = 26$

$\Rightarrow 25 + 4\lambda^2 - 20\lambda + 16 + 9\lambda^2 - 24\lambda + 36 + 4\lambda^2 - 24\lambda = 26$

$\Rightarrow 17\lambda^2 - 68\lambda + 77 = 26$

$\Rightarrow 17\lambda^2 - 68\lambda + 51 = 0$

$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$

$\Rightarrow (\lambda - 3)(\lambda - 1) = 0$

$\Rightarrow \lambda = 1, 3$

$\therefore$ Q(1, 1, 3)

R(5, 7, 7)

P(4, 2, 3)

$\overrightarrow{PQ} = 3\hat{i} + \hat{j} + 4\hat{k}$

$\overrightarrow{PR} = \hat{i} + 5\hat{j} + 0\hat{k}$

$$A = \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & 5 & 0 \end{matrix}\right\| = \frac{1}{2}\left|\hat{i}(0-20) - \hat{j}(0-4) + \hat{k}(15-1)\right|$$

$$= \frac{1}{2}\left|-20\hat{i} + 4\hat{j} + 14\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{400+16+196}$$

$$= \frac{1}{2}\sqrt{612}$$

$$\therefore A^2 = \frac{1}{4}(612) = 153$$

Q.41 (3)

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}(-3) - \hat{j}(3) + \hat{k}(0)$$

$$\therefore \vec{n} = -\hat{i} - \hat{j}$$

$\therefore$ Plane is $-1(x-1) - 1(y+1) = 0$

$P : -x - y = 0 \Rightarrow x + y = 0$

Distance from Q(a, a, 2)

$$\Rightarrow \left|\frac{a+a}{\sqrt{2}}\right| = 3\sqrt{2}$$

$\therefore 2|a| = 6$

$a = 3$ or $a = -3$

P(1, –1, 1) & Q(3, 3, 2)

or Q(–3, –3, 2)

$PQ^2 = 4 + 16 + 1 = 21$

Q.42 (1)

$$\frac{x+7}{-6} = \frac{y-6}{7} = \frac{z-0}{1} \Rightarrow \vec{n}_1 = (-6, 7, 1)$$

$$\frac{x-7}{-2} = \frac{y-2}{1} = \frac{z-6}{1} \Rightarrow \vec{n}_2 = (-2, 1, 1)$$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix} = \hat{i}(6) - \hat{j}(-4) + \hat{k}(8) = (3, 2, 4)$$

$\vec{a} = (-7, 6, 0)$, $\vec{b} = (7, 2, 6)$

$$\therefore S_d = \left|\frac{(\vec{b}-\vec{a})\cdot(\vec{n}_1 \times \vec{n}_2)}{|\vec{n}_1 \times \vec{n}_2|}\right|$$

$$= \left|\frac{(14, -4, 6)\cdot(3, 2, 4)}{\sqrt{9+4+16}}\right|$$

$$= \left|\frac{42-8+24}{\sqrt{29}}\right| = \frac{58}{\sqrt{29}} = 2\sqrt{29}$$

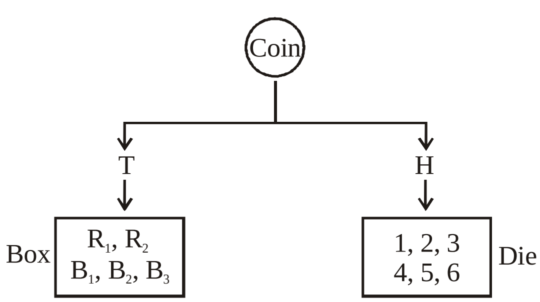
PROBABILITY

EXERCISE-I (MHT CET LEVEL)

Q.1 (3)
In our day-to day life, we perform many activities which have a fixed result no matter any number of times they are repeated. Such as, given any triangle, without knowing the three angles, we can definitely say that the sum of measure of angles is 180º
We also perform many experimental activities. Where the result may not be same, when they are repeated under identical conditions, Such as when a coin is tossed it may turn up a head or a tail, but we are not sure which one of these results will actually be obtained. Such experiments are called random experiments.
An experiment is called random experiment, if it satisfies the following two conditions
(i) It has more than one possible outcome.
(ii) It is not possible to predict the outcome in advance.

Q.2 (1)
Clearly, the coins are distinguishable in the sense that we can speak of the first coin and the second coin. Since, either coin can turn up Head (H) or Tail (T), the possible outcomes may be
Heads on both coins = (H. H) = HH
Head of first coin and tail on the other = (H,T) = HT
Tail on first coin and Head on the other = (T, H) = TH
Tail on both coins = (T,T,) = TT
Thus, the sample space is S = {HH, HT, TH, TT}

Q.3 (1)
Let H and T represent a head and tail.
Let red ball represented by R_1, R_2 and black ball represented by B_1, B_2 and B_3.



The sample space is
S={TR_1, TR_2, TB_1, TB_2, TB_3, H1, H2, H3, H4, H5, H6}
Hence, infinite number of possibilities occur.

Q.4 (3)
The sample space is
S={6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6), (1, 3, 6).....}
Hence, infinite number of possibilities occur.

Q.5 (4)
They are basically independent.

Q.6 (2)
$$P(A_1 \cup A_2) = 1 - [1 - P(A_1)][1 - P(A_2)]$$
$$= P(A_1) + P(A_2) - P(A_1).P(A_2).$$

Q.7 (4) They are mutually independent.

Q.8 (3)
It is obvious.

Q.9 (2)
Since $A \cap \bar{B}$ and $A \cap B$ are mutually exclusive events such that $A = (A \cap \bar{B}) \cup (A \cap B)$
$\therefore\ P(A) = P(A \cap \bar{B}) + P(A \cap B)$
$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A \cap B) = P(A) - P(A)P(B)$
$(\because\ A, B$ are independent)
$\Rightarrow P(A \cap \bar{B}) = P(A)(1 - P(B)) = P(A)P(\bar{B})$
$\therefore$ A and $\bar{B}$ are also independent.

Q.10 (1)
$B \cup C$ is independent to A, so S_1 is true
$B \cap C$ is also independent to A, so S_2 is true.

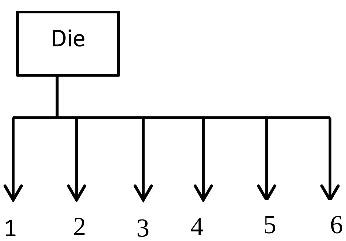
Q.11 (1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$$\frac{5}{6} = \frac{2}{3} + \frac{1}{2} - P(A \cap B) \Rightarrow P(A \cap B) = 0$$
$\therefore$ Events *A* and *B* are mutually exclusive.

Q.12 (3)
Recall that union of two sets A and B denoted by $A \cup B$ contains all those elements which are either in A or in B or in both. When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event

'either A or B or both'. This event 'A$\cup$B' is also called 'A or B'.
Therefore, event 'A or B' = $A \cup B$
$= \{\omega : \omega \in A \text{ or } \omega \notin B\}$

Q.13 (2)
We know that, intersection of two sets $A \cap B$ is the set of those elements which are common to both A and B, i.e., which belong ot both 'A and B'.
If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'.
Thus, $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$
For example, in the experiment of 'throwing a die twice' Let A be the event 'score on the first throw is 6' and B is the event 'sum of two scores is atleast 11'. The,
$A = \{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$
and $B = \{(5, 6), (6, 5), (6, 6)\}$
So, $A \cap B = \{(6, 5), (6, 6)\}$
Note that the set $A \cap B = \{6, 5), (6, 6)\}$ may represent the event 'the score on the first throw is 6 and the sum of the scores is atleast 11'.

Q.14 (4)



Let E = The die shows 4 = {4}.
Let F = The die shows even number
= {2, 4, 6}
$\therefore E \cap F = \{4\} \neq \phi$
Hence, E and F are not mutually exclusive.

Q.15 (1)

Required probability $= \left(\frac{4}{52}\right)^2 = \frac{1}{169}$.

Q.16 (3)
Required probability is
$P(\text{getting } 8) + P(9) + P(10) + P(11) + P(12)$

$$= \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{15}{36} = \frac{5}{12}.$$

Q.17 (1)

Required probability $= \frac{4}{7}$.

Q.18 (4)

The chance of head $= \frac{1}{2}$ and not of head $= \frac{1}{2}$
Since A has first throw, he can win in the first, third, …
$\therefore$ Probability of A's winning

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 . \frac{1}{2} + \left(\frac{1}{2}\right)^4 . \frac{1}{2} + ...$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + ... = \frac{2}{3}.$$

Q.19 (3)

Required probability $= \frac{4}{36} = \frac{1}{9}$.

Q.20 (1) Required probability $= \frac{4}{52} = \frac{1}{13}$.

Q.21 (2)
Here $P(A) = 0.4$ and $P(\bar{A}) = 0.6$

Probability that A does not happen at all $= (0.6)^3$

Thus required Probability $= 1 - (0.6)^3 = 0.784$.

Q.22 (2)

Required probability $= \frac{1}{2}$.

Q.23 (3)
Number of tickets numbered such that it is divisible by 20 are $\frac{10000}{20} = 500$.

Hence required probability $= \frac{500}{10000} = \frac{1}{20}$.

Q.24 (2)
Favourable cases for one are three *i.e.* 2, 4 and 6 and for other are two *i.e.* 3, 6.

Hence required probability $= \left[\left(\frac{3 \times 2}{36}\right) 2 - \frac{1}{36}\right] = \frac{11}{36}$

{As same way happen when dice changes numbers among themselves}

Q.25 (4)
The probability of students not solving the problem are $1 - \frac{1}{3} = \frac{2}{3}$, $1 - \frac{1}{4} = \frac{3}{4}$ and $1 - \frac{1}{5} = \frac{4}{5}$

Therefore the probability that the problem is not solved by any one of them $=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{2}{5}$

Hence the probability that problem is solved $=1-\frac{2}{5}=\frac{3}{5}$.

Q.26 (4)

Required probability $=\frac{4\,.\,4}{52\,.\,51}\times 2=\frac{8}{663}$.

Q.27 (3)

Obviously numbers will be $\begin{pmatrix} \text{I} & \text{II} \\ 5, & 1 \\ 4, & 2 \\ 2, & 4 \\ 1, & 5 \end{pmatrix}$.

Hence required probability $=\frac{4}{6\,.\,5}=\frac{2}{15}$.

Q.28 (1)

Let E_1 be the event that man will be selected and E_2 the event that woman will be selected. Then

$P(E_1)=\frac{1}{4}$ so $P(\bar{E}_1)=1-\frac{1}{4}=\frac{3}{4}$ and $P(E_2)=\frac{1}{3}$

So $P(\bar{E}_2)=\frac{2}{3}$

Clearly E_1 and E_2 are independent events.

So, $P(\bar{E}_1\cap\bar{E}_2)=P(\bar{E}_1)\times P(\bar{E}_2)=\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$.

Q.29 (3)

Since both heads and tails appears, so

$n(S)=\{HHT, HTH, THH, HTT, THT, TTH\}$

$n(E)=\{HTT, THT, TTH\}$

Hence required probability $=\frac{3}{6}=\frac{1}{2}$.

Q.30 (4)

Probability for white ball $P(W)=\frac{4}{15}$

Probability for red ball $P(R)=\frac{6}{15}$

Probability (white or red ball) $=P(W)+P(R)$

$=\frac{4}{15}+\frac{6}{15}=\frac{10}{15}=\frac{2}{3}$.

Q.31 (2)

Required probability $=\frac{52-16}{52}=\frac{36}{52}=\frac{9}{13}$.

Q.32 (1)

Since the total '13' can't be found.

Q.33 (3)

Three dice can be thrown in $6\times6\times6=216$ ways. A total 17 can be obtained as $(5,6,6)$, $(6,5,6)$, $(6,6,5)$. A total 18 can be obtained as $(6,6,6)$.

Hence the required probability $=\frac{4}{216}=\frac{1}{54}$.

Q.34 (4)

Required probability $=\frac{64}{64}$.

Q.35 (2)

It is obvious.

Q.36 (3)

P (tail in 3rd) .P(tail in 4th) $=\frac{1}{2}.\frac{1}{2}=\frac{1}{4}$.

Q.37 (2)

Prime numbers are $\{2,3,5,7,11\}$.

Hence required probability

$=\frac{1+2+4+6+2}{36}=\frac{15}{36}=\frac{5}{12}$.

Q.38 (1)

Required probability

$=\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)=\frac{2}{3}.\frac{3}{4}.\frac{4}{5}=\frac{2}{5}$.

Q.39 (2)

Since favourable ways are 6. Total ways are 36.

Hence probability $=\frac{6}{36}$.

Q.40 (1)

n = total number of ways $= 2^4 = 16$
m = Favourable number of ways = 3
Since the value of determinant is positive when it is

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$. Hence required probability

$= \frac{3}{16}$.

Q.41 (3)

Required probability $= \frac{16}{52} = \frac{4}{13}$

(Since diamond has 13 cards including a king and there are another 3 kings).

Q.42 (4)
Let R stand for drawing red ball B for drawing black ball and w for drawing white ball.
Then required probability

$= P(WWR) + P(BBR) + P(WBR) + P(BWR) + P(WRR) +$
$P(BRR) + P(RWR) + P(RBR)$.

$= \frac{3.2.2}{8.7.6} + \frac{3.2.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.3.2}{8.7.6} + \frac{3.2.1}{8.7.6} + \frac{3.2.1}{8.7.6} + \frac{2.3.1}{8.7.6} + \frac{2.3.1}{8.7.6}$

$= \frac{2}{56} + \frac{2}{56} + \frac{3}{56} + \frac{3}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} + \frac{1}{56} = \frac{1}{4}$.

Q.43 (2)
Favourable ways are $(2, 6), (3, 5), (4, 4), (5, 3)$ and $(6, 2)$. Hence required probability $= \frac{5}{36}$.

Q.44 (2)
It is obvious.

Q.45 (2)
Probability of getting at least one head in n tosses

$= 1 - \left(\frac{1}{2}\right)^n \geq 0.9 \Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1 \Rightarrow 2^n \geq 10 \Rightarrow n \geq 3$

Hence least value of n = 4

Q.46 (2)
Let a sample space of an experiment be
$S = \{\omega_1, \omega_2, ...\omega_n\}$
Let all the outcome are equally likely to occur, i.e., the chance of occurrence of each simple event must be same.
i.e. $P(\omega_i) = p$, for all $\omega_i \in S$, where $0 \leq p \leq 1$

Since, $\sum_{i=1}^{n} P(\omega_i) = 1$, i.e., p + p + ... + p(n times) = 1

OR np = 1 i.e., $p = \frac{1}{n}$

If S be a sample space and E be an event, such that n(S) = n and n(E) = m. If each out come is equally likely, then if follows that

$P(E) = \frac{m}{n} = \frac{\text{Number of outcomes favourable to E}}{\text{Total possible outcomes}}$

Q.47 (4)
To find the probability of event 'A' or 'B', i.e., $P(A \cup B)$. If S is sample space for tossing of three coins, then
S = {HHT, HHH, HTH, HTT, THH, THT, TTH, TTT}
Let A = {HHT, HTH, THH} and B ={HTH, THH, HHH} be two events associated with 'tossing of a coin thrice'
Clearly, $A \cup B$ = {HHT, HTH, THH, HHH}
Now, $P(A \cup B) = P(HHT) + P(PTH) + P(THH) + P(HHH)$
If all the outcome are equally likely, then

$P(A \cup B) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

Also, $P(A) = P(HHT) + P(HTH) + P(THH) = \frac{3}{8}$

and $P(B) = P(HTH) + P(THH) + P(HHH) = \frac{3}{8}$

Therefore, $P(A) + P(B) = \frac{3}{8} + \frac{3}{8} = \frac{6}{8}$

It is clear that $P(A \cup B) \neq P(A) + P(B)$
The points HTH and THH are common to both A and B. In the computation of P(A) + P(B) the probabilities of point HTH and THH, i.e., the elements of $A \cup B$ are included twice. Thus, to get the probability P (A È B) we have to subract the probabilities of the sample point in $A \cup B$ from P(A) + P(B).
i.e., $P(A \cup B) = P(A) + P(B) - \Sigma P(\omega)$, & $\omega_i \in A \cup B$
$= P(A) + P(B) - (A \cap B)$
Thus, we observe that,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Q.48 (2)
We know that, $A \cup B$ denotes the occurence of atleast one of A and B and $A \cap B$ denotes the occurence of both A and B, simultaneously .
Thus, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$
Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow 0.6 = P(A) + P(B) - 0.3$
$\Rightarrow P(A) + P(B) = 0.9$
$[\because P(A) = 1 - P(\bar{A})$ and $P(B) = 1 - P(\bar{B})]$

$\Rightarrow [1 - P(\bar{A})] + [1 - P(\bar{B})] = 0.9$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 2 - 0.9 = 1.1.$

Q.49 (4)
The sample space is S = {HH, HT, TH, TT}
Let E be the event of getting atleast one tail.
E = {HT, TH, TT}
$\therefore$ Required probability P

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{n(E)}{n(S)} = \frac{3}{4}$$

Q.50 (3)

Required probability $= \frac{^{13}C_2}{^{52}C_2} = \frac{13.12}{52.51} = \frac{1}{17}$.

Q.51 (3)

Required probability $= \frac{^{26}C_3 . {}^{26}C_3}{^{52}C_6}$.

Q.52 (3)
Required probability

$$= \frac{^7C_3}{^9C_5} + \frac{^7C_5}{^9C_5} = \frac{56}{126} = \frac{4}{9}.$$

Q.53 (2)

Total ways of arrangements $= \frac{8!}{2!\,.\,4!}$

$\bullet w \bullet x \bullet y \bullet z \bullet$

Now 'S' can have places at dot's and in places of w, x, y, z we have to put $2A$'s, one I and one N.

Therefore favourable ways $= 5\left(\frac{4!}{2!}\right)$

Hence required probability $= \frac{5.4!\,2!\,4!}{2!\,8!} = \frac{1}{14}$.

Q.54 (4)

Required probability $= \frac{^8C_4}{2^8}$.

Q.55 (1)
In 50 tickets 14 are of prize and 36 are blank. Number of ways both the tickets are blank $= {}^{36}C_2$

Thus the probability of not winning the prize

$$= \frac{^{36}C_2}{^{50}C_2} = \frac{18}{35}.$$

Hence probability of winning the prize $= 1 - \frac{18}{35} = \frac{17}{35}$.

Q.56 (2)

Probability that both balls are white $= \frac{^7C_2}{^{15}C_2} = \frac{1}{5}$

Probability that both balls are black $= \frac{^8C_2}{^{15}C_2} = \frac{4}{15}$

Probability that one ball is white and one is black

$$= \frac{^7C_1 \times {}^8C_1}{^{15}C_2} = \frac{8}{15}.$$

Q.57 (2)
Total number of ways

$= {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1 + {}^6C_5$

$= 60 + 120 + 60 + 6 + 6 = 252$

No. of ways in which at least one woman exist are

$= {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1 = 246$

Hence required probability $= \frac{246}{252} = \frac{41}{42}$.

Q.58 (3)
Total number of ways to form the numbers of three digit with 1, 2, 3 and 4 are ${}^4P_3 = 4! = 24$
If the numbers are divisible by three then their sum of digits must be 3, 6 or 9
But sum 3 is impossible. Then for sum 6, digits are 1, 2, 3
Number of ways $= 3!$
Similarly for sum 9, digits are 2, 3, 4. Number of ways =3!
Thus number of favourable ways $= 3! + 3!$

Hence required probability $= \frac{3! + 3!}{4!} = \frac{12}{24} = \frac{1}{2}$.

Q.59 (2)
m rupee coins and n ten paise coins can be placed in a line in $\frac{(m+n)!}{m!\,n!}$ ways.

If the extreme coins are ten paise coins, then the remaining $n-2$ ten paise coins and m one rupee coins can be arragned in a line in $\frac{(m+n-2)!}{m!(n-2)!}$ ways.

Hence the required probability

$$=\frac{\frac{(m+n-2)!}{m!(n-2)!}}{\frac{(m+n)!}{m!n!}}=\frac{n(n-1)}{(m+n)(m+n-1)}.$$

Q.60 (3)
The total number of functions from A to itself is n^n and the total number of bijections from A to itself is n! {Since A is a finite set, therefore every injective map from A to itself is bijective also}.

$\therefore$ The required probability $=\frac{n!}{n^n}=\frac{(n-1)!}{n^{n-1}}$.

Q.61 (1)

Required probability $=\frac{{}^{12}C_1}{{}^{20}C_1}=\frac{3}{5}$.

Q.62 (3)
Number of ways of selecting two good mangoes = ${}^6C_2=15$. Number of ways that at least one of the two selected mangoes is to be good $={}^6C_1\times{}^9C_1=54$

$\therefore$ Required probability $=\frac{15}{64}=\frac{5}{18}$.

Q.63 (1)

Required probability $=\frac{{}^3C_1}{{}^7C_1}\times\frac{{}^2C_1}{{}^6C_1}=\frac{1}{7}$.

Q.64 (4)

Required probability $=\frac{{}^5C_3}{{}^{16}C_3}=\frac{5\times4\times3}{16\times15\times14}=\frac{1}{56}$.

Q.65 (2)
Required probability

$$=\frac{{}^{20}C_1\times{}^{20}C_1}{{}^{40}C_2}=\frac{20\times20\times2}{40\times39}=\frac{20}{39}.$$

Q.66 (4)
3 cards are drawn out of 26 red cards (favourable)

$$=\frac{{}^{26}C_3}{{}^{52}C_3}=\frac{26!}{3!\,23!}\times\frac{3!\,49!}{52!}=\frac{2}{17}.$$

Q.67 (2)
Four boys can be arranged in $4!$ ways and three girls can be arranged in 3! ways.

$\therefore$ The favourable cases $=4!\times3!$

Hence the required probability

$$=\frac{4!\times3!}{7!}=\frac{6}{7\times6\times5}=\frac{1}{35}.$$

Q.68 (1)

Required probability $=\frac{{}^{88}C_3}{{}^{90}C_5}=\frac{2}{801}$

Q.69 (1)

Total number of ways $={}^{15}C_{11}$

Favourable cases $={}^8C_6\times{}^7C_5$

Required probability $=\frac{{}^8C_6\times{}^7C_5}{{}^{15}C_{11}}$.

Q.70 (4)

P(Black or Red) $=\frac{{}^5C_1+{}^3C_1}{{}^{12}C_1}=\frac{2}{3}$.

Q.71 (3)
The total number of ways in which 2 integers can be chosen from the given 30 integers is ${}^{30}C_2$. The sum of the selected numbers is odd if exactly one of them is even and one is odd.

$\therefore$ Favourable number of outcomes $={}^{15}C_1.{}^{15}C_1$

$\therefore$ Required probability $=\frac{{}^{15}C_1.{}^{15}C_1}{{}^{30}C_2}=\frac{15}{29}$.

Q.72 (3)
The total number of ways in which 3 integers can be chosen from first 20 integers is ${}^{20}C_3$. The product of three integers will be even if at least one of them is even.

$\therefore$ Required probability = 1 – Probability that none is even

$$=1-\frac{{}^{10}C_3}{{}^{20}C_3}=1-\frac{2}{19}=\frac{17}{19}.$$

Q.73 (4)

Total ways = $2!\ {}^6C_2 = 30$

Favourable cases $= 30 - 6 = 24$

$\therefore$ Required probability $= \frac{24}{30} = \frac{4}{5}$.

Q.74 (2)

3 ball can be drawn in ${}^{18}C_3$ ways

Favourable cases $= {}^6C_3$

$\therefore$ Required probability

$= \frac{{}^6C_3}{{}^{18}C_3} = \frac{6\times5\times4}{18\times17\times16} = \frac{5}{204}$.

Q.75 (4)

Required probability $= \frac{4.5.6}{{}^{15}C_3} = \frac{24}{91}$.

Q.76 (1)

Required probability $= \frac{(21)!2!}{(22)!} = \frac{1}{11} = \frac{1}{1+10}$.

$\therefore$ Odds against = 10 : 1.

Q.77 (1)

We have $P(A+B) = P(A) + P(B) - P(AB)$

$\Rightarrow \frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3} \Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

Thus, $P(A).P(B) = \frac{1}{2}\times\frac{2}{3} = \frac{1}{3} = P(AB)$

Hence events A and B are independent.

Q.78 (2)

Since here $P(A+B+C) = P(A) + P(B) + P(C)$

$= \frac{2}{3} + \frac{1}{4} + \frac{1}{6} = \frac{13}{12}$, which is greater than 1.

Hence the statement is wrong.

Q.79 (3)

Since we have $P(A+B) = P(A) + P(B) - P(AB)$

$\Rightarrow 0.7 = 0.4 + P(B) - 0.2 \Rightarrow P(B) = 0.5$.

Q.80 (4)

$P(A+B) = P(A) + P(B) - P(AB) = \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}$.

Q.81 (4)

Required probability is

$P(\text{Red} + \text{Queen}) - P(\text{Red} \cap \text{Queen})$

$= P(\text{Red}) + P(\text{Queen}) - P(\text{Red} \cap \text{Queen})$

$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$.

Q.82 (1)

$P(A \cap B) = \frac{2}{8} + \frac{5}{8} - \frac{6}{8} = \frac{1}{8}$.

Q.83 (4)

$P(\text{neither A nor B}) = P(\bar{A} \cap \bar{B})$

$= P(\bar{A}).P(\bar{B}) = 0.6 \times 0.5 = 0.30$.

Q.84 (1)

It is obvious.

Q.85 (2)

Let A be the event to be multiple of 4 and B be the event to be multiple of 6

So, $P(A) = \frac{25}{100}$, $P(B) = \frac{16}{100}$ and $P(A \cap B) = \frac{8}{100}$

Thus required probability is

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{25}{100} + \frac{16}{100} - \frac{8}{100} = \frac{33}{100}$.

Q.86 (2)

$P(A \cup B) = P(A) + P(B) = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$.

{Since events are mutually exclusive,

So $P(A \cap B) = 0$}

Q.87 (3)

$P(A) = P(A \cap B) + P(A \cup B) - P(B)$

$= \frac{1}{3} + \frac{5}{6} - \frac{2}{3} = \frac{3}{6} = \frac{1}{2}$.

Q.88 (1)

Since we have

$P(A \cup B) + P(A \cap B) = P(A) + P(B) = P(A) + \frac{P(A)}{2}$

$\Rightarrow \frac{7}{8} = \frac{3P(A)}{2} \Rightarrow P(A) = \frac{7}{12}$.

Q.89 (2)

Required probability is
$P[(A$ will die and B alive) or $(B$ will die and A alive)]
$= P[(A \cap B') \cup (B \cap A')]$
Since events are independent, so
Required probability $= P(A).P(B') + P(B).P(A')$
$= p.(1-q) + q(1-p) = p + q - 2pq.$

Q.90 (1)
$$P(A \cap B) = P(A).P(B) = \frac{1}{6}$$
$$P(\bar{A} \cap \bar{B}) = \frac{1}{3} = 1 - P(A \cup B)$$
$$\Rightarrow \frac{1}{3} = 1 - [P(A) + P(B)] + \frac{1}{6} \Rightarrow P(A) + P(B) = \frac{5}{6}.$$
Hence P(A) and P(B)are $\frac{1}{2}$ and $\frac{1}{3}$

Q.91 (1)
Since events are independent.
So, $P(A \cap B') = P(A) \times P(B') = \frac{3}{25}$
$$\Rightarrow P(A) \times \{1 - 2P(B)\} = \frac{3}{25} \qquad .....(i)$$
Similarly, $P(B) \times \{1 - P(A)\} = \frac{8}{25}$ .....(ii)
On solving (i) and (ii), we get
$$P(A) = \frac{1}{5} \text{ and } \frac{3}{5}.$$

Q.92 (4)
$P(A \cup B) = P(A) + P(B) = 0.45 + 0.35 = 0.8.$

Q.93 (3)
It is a fundamental concept.

Q.94 (c)
For mutually exclusive events
$$P(A \cup B) = P(A) + P(B)$$
$$\Rightarrow P(A) = \frac{2}{7}$$

Q.95 **(b)**
From the given problem:
$$P(A \cup B) = \frac{3}{4}, P(A \cup B) = \frac{1}{4}$$
$$P(A^C) = \frac{2}{3} = 1 - P(A) \Rightarrow P(A) = 1 - \frac{2}{3} = \frac{1}{3}$$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow P(B) = P(A \cup B) + P(A \cap B) - P(A)$
$$= \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = 1 - \frac{1}{3} = \frac{2}{3}$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (3)
A coin is tossed once, then the sample space is
S={H, T}.

Q.2 (2)
When three dice tossed together, then the total number of outcomes = 6^3
$= 6 \times 6 \times 6 = 216$

Q.3 (3)
The sample space S is
S = {HH, TH, (HT, 1), (HT, 2), (HT, 3), (HT, 4), (HT, 5), (HT, 6), (TT, 1), (TT, 2), (TT, 3), (TT, 4), (TT, 5), (TT, 6)}
Hence, total number of outcomes =14

Q.4 (3)
The sample space is
S = {T, HT, HHT, HHHT, HHHHT, ...}

Q.5 (2)
The sample space associated with the given random experiment is given by
S = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), 5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}
Now, when outcomes is doublet, then coin is tossed. So, sample space
= {(11, H) (11, 7)
(22, H) (22, T)
(33, H) (33, T)
(44, H) (44, T)
(55, H) (55, T)
(66, H) (66, T)}
Hence, total number of sample points = 42

Q.6 (4)
The sample space for this experiment
S = {RR, RB, BR, BB}

Q.7 (3)
Let W denotes the white ball and R denote the red ball.
We know that, when a die is rolled, then the sample

space = {1, 2, 3, 4, 5, 6}
Now, according to the question, required sample space is
S={(W, W), (W, R), (R, 1), (R, 2), (R, 3), (R, 4), (R, 5), (R, 6)}

Q.8 (3)
I. There are 13 letters in the word 'ASSASSINATION'.
II. There are 10 letters in the word 'NAGATATION'

Q.9 (1)
Let white ball denoted by W and black ball is denoted by B. Now, two balls drawn at random in succession without replacement.
Then, the sample space is
S={WB, BW, BB}

Q.10 (4)
The defective items denoted by D and non-defective items denoted by N, then the sample space is
S={DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

Q.11 (4)
Let the set be S
Then, S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}
Now, let the event E = Getting a prime number when each of the number is equally likely to be selected
E = {2, 3, 5, 7, 11, 13, 17, 19, 23}

Q.12 (2)
A PIN is sequence of four symbols selected from 36(26 letters + 10 digits) symbols
By the fundamental principle of counting there are 36 × 36 × 36 × 36 = 36^4 = 1679616 PINs in all. When repetition is not allowed the multiplication rule can be applied to concluded that there are
$36 \times 35 \times 34 \times 33 = 1413720$ different PINs.
The number of PINS that contain atleast one repeated symbol.
$= 1679616 - 1413720 = 265896$
Thus, the probability that a randomly chosen PIN contains a repeated symbol is

$$= \frac{265896}{1679616} = 0.1583$$

Q.13 (2)
Given that, M and N are two events, then the probability that atleast one of them occurs is
$P(M \cup N) = P(M) + P(N) - P(M \cap N)$

Q.14 (1)
Given that , A and B are two mutually exclusive events.
Then
$P(A \cup B) = P(A) + P(B)$ $[\because (A \cap B) = \phi]$
$P(A) + P(B) \le 1$
$P(A) + 1 - P(\overline{B}) \le 1$
$P(A) \le P(\overline{B})$

Q.15 (1)
Given that, $P(A \cup B) = P(A \cap B)$
$\Rightarrow A = B \Rightarrow P(A) = P(B)$

Q.16 (2)
Three numbers chosen from

$$1 \text{ to } 20 = {}^{20}C_3 = \frac{20 \times 19 \times 18 \times 17!}{17! \times 3 \times 2 \times 1}$$

$= 20 \times 19 \times 3 = 1140$
Now set of three consecutive numbers = 18

$$\text{P(they are consecutive)} = \frac{18}{1140} = \frac{3}{190}$$

$$\text{P(they are not consecutive)} = 1 - \frac{3}{190} = \frac{187}{190}$$

Q.17 (3)
There are 26 red cards and 26 black cards i.e., total number of cards = 52.
P(both cards of different colour)
= P (B) P(R) + P(R) P(B)

$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51} = 2\left(\frac{26}{52} \times \frac{26}{51}\right) = \frac{26}{51}$$

Q.18 (3)
Given, number of persons = 7
Now, treating the two persons as one, we have 6 persons.
Total number of sitting arrangements = 7!
Favourable number of arrangement = 6!
2 persons can be arranged in two ways.
Total number of favourable arrangement = 2(6!)

$$\therefore \text{ Required probability} = \frac{2(6!)}{7!} = \frac{2}{7}$$

Q.19 (3)
Total number of tickets = 10000
Now, number of tickets numbered such that it is divisible by 20.

$$= \frac{10000}{20} = 500$$

$$\text{Required probability} = \frac{500}{10000} = \frac{1}{20}$$

Q.20 (4)

Given numbers = 0, 2, 3, 5.

Th	H	T	U
3	3	2	1

A number is divisible by 5, if the digit at unit place is 0 to 5.
Total numbers are = $3 \times 3 \times 2 \times 1 = 18$
Favourable events are
when 5 is at unit place = $2 \times 2 \times 1 \times 1 = 4$
Total favourable events are $6 + 4 = 10$

$\therefore$ Required probability = $\frac{10}{18} = \frac{5}{9}$

PREVIOUS YEAR'S

MHT CET

Q.1 (4)
Q.2 (2)
Q.3 (2)
Q.4 (3)
Q.5 (3)
Q.6 (4)
Q.7 (3)
Q.8 (2)
Q.9 (4)
Q.10 (2)
Q.11 (3)
Q.12 (2)
Q.13 (4)
Q.14 (4)

LIMIT OF FUNCTIONS

EXERCISE-I (MHT CET LEVEL)

Q.1 (3)

$\lim_{h\to 0^+}[3+h]=3$ and $\lim_{h\to 0^-}[3-h]=2$

$\therefore \lim_{x\to 3}[x]$ does not exist.

Q.2 (1)

$\lim_{x\to n+0}(x-[n]) = \lim_{x\to n+0} x - \lim_{x\to n+0}[n] = n-n=0$.

Q.3 (3)

$\lim_{x\to 3+} f(x)=5-3=2,\ \lim_{x\to 3-} f(x)=\frac{2}{5-3}=1.$

Q.4 (3)

$\lim_{x\to 1}\frac{x-1}{(x-1)(2x-5)}=-\frac{1}{3}.$

Aliter : Apply L-Hospital's rule.

Q.5 (3)

$$\lim_{n\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}=\lim_{n\to\infty}\frac{4n^3+4n^2+n}{n^3+5n^2+5n-2}$$

$$=\lim_{n\to\infty}\frac{n^3\left(4+\frac{4}{n}+\frac{1}{n^2}\right)}{n^3\left(1+\frac{5}{n}+\frac{5}{n^2}-\frac{2}{n^3}\right)}=4$$

Q.6 (2)

$$\lim_{x\to 2}\frac{x^n-2^n}{x-2}=n.2^{n-1}$$

$\Rightarrow n.2^{n-1}=80$

$\Rightarrow n=5$

Q.7 (a)

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x^2}.$$

$$\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$$

$$\Rightarrow \lim_{x\to 0}\frac{1+x^2-1+x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$\Rightarrow \lim_{x\to 0}\frac{2x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$=\frac{2}{\sqrt{1+\sqrt{1}}}=\frac{2}{2}=1$$

Q.8 **(b)**

Q.9 (1)

$$\lim_{x\to 0}\frac{x.2\sin^2 x}{x^2}=2.\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\cdot\lim_{x\to 0} x=0.$$

Q.10 (3)

$$\left[\lim_{x\to\infty}\left(1+\frac{1}{x/2}\right)^{x/2}\right]^2=e^2.$$

Q.11 (1)

$$\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)\times(\sqrt{x}+1)}{(x-1)(2x+3)\times(\sqrt{x}+1)}=\frac{-1}{5.2}=\frac{-1}{10}.$$

Q.12 (4)

$f(x)=\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$, then

$$\lim_{x\to 0+} f(x)=\lim_{h\to 0}\left(\frac{e^{1/h}-1}{e^{1/h}+1}\right)=\lim_{h\to 0}\frac{e^{1/h}\left(1-\frac{1}{e^{1/h}}\right)}{e^{1/h}\left(1+\frac{1}{e^{1/h}}\right)}=1$$

Similarly $\lim_{x\to 0-} f(x)=-1$. Hence limit does not exist.

Q.13 (2)

$$\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)(x+6)}=\frac{3}{7}$$

Q.14 (3)

$$\lim_{x\to 0}\frac{x[{}^5C_1+{}^5C_2x+{}^5C_3x^2+{}^5C_4x^3+{}^5C_5x^4]}{x[{}^3C_1+{}^3C_2x+{}^3C_3x^2]}=\frac{5}{3}.$$

Aliter : Apply L-Hospital's rule.

Q.15 (4)

$$\lim_{x\to 0}\frac{2\sin 4x\cos 2x}{2\sin x\cos 4x}=\lim_{x\to 0}4\left(\frac{\sin 4x}{4x}\right)\left(\frac{x}{\sin x}\right)\frac{\cos 2x}{\cos 4x}=4$$

Aliter : $\lim_{x\to 0}\dfrac{\dfrac{2\sin 2x}{2x}+\dfrac{6\sin 6x}{6x}}{\dfrac{5\sin 5x}{5x}-\dfrac{3\sin 3x}{3x}}=\dfrac{2+6}{5-3}=4.$

Q.16 (2)

$$\lim_{x\to\infty}\left[\frac{1+\frac{b}{x}+\frac{4}{x^2}}{1+\frac{a}{x}+\frac{5}{x^2}}\right]=1.$$

Q.17 (1)

Expand $\sin x$ and then solve.

Aliter : Apply L-Hospital's rule

$$\lim_{x\to 0}\frac{\sin x-x+\frac{x^3}{6}}{x^5}=\lim_{x\to 0}\frac{\cos x-1+\frac{3x^2}{6}}{5x^4}$$

$$=\lim_{x\to 0}\frac{-\sin x+\frac{6x}{6}}{20x^3}=\lim_{x\to 0}\frac{-\cos x+1}{60x^2}=\lim_{x\to 0}\frac{\sin x}{120x}$$

$$=\lim_{x\to 0}\frac{\cos x}{120}=\frac{1}{120}.$$

Q.18 (b)

$$\lim_{x\to 0}\frac{(4^x-1)^3}{\sin\frac{x^2}{4}\log(1+3x)}$$

$$\lim_{x\to 0}\frac{(4^x-1)^3}{x^3}\frac{(x/2)^2}{\sin x^2/4}\frac{3x}{\log(1+3x)}\frac{4}{3}$$

$$=\frac{4}{3}(\log_e 4)^3.1.\log_e(e)=\frac{4}{3}(\log_e 4)^3$$

Q.19 (b)

$$\lim_{n\to\infty}\sqrt{\frac{x-\sin x}{x+\sin^2 x}}=\lim_{n\to\infty}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\sin^2 x}{x}}}$$

$$=\lim_{n\to 0}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\left(\frac{\sin x}{x}\right)\sin x}}=\sqrt{\frac{1-1}{1+1\times 0}}=0$$

Q.20 (b)

Q.21 (1)

$$\lim_{\theta\to\pi/2}\frac{1-\sin\theta}{\cos\theta}=\lim_{\theta\to\pi/2}\frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}=0$$

Q.22 **(3)**

$$\lim_{x\to 0}\frac{\tan 2x-x}{3x-\sin x}=\lim_{x\to 0}\left\{\frac{\frac{2\tan 2x}{2x}-1}{3-\frac{\sin x}{x}}\right\}=\frac{1}{2}.$$

Aliter : Apply L-Hospital's rule

$$\lim_{x\to 0}\frac{\tan 2x-x}{3x-\sin x}=\lim_{x\to 0}\frac{2\sec^2 2x-1}{3-\cos x}=\frac{2-1}{3-1}=\frac{1}{2}.$$

Q.23 (1)

Apply the L-Hospital's rule, $\lim_{x\to a}\dfrac{f(x)}{g(x)}=\lim_{x\to a}\dfrac{f'(x)}{g'(x)}$.

Q.24 (1) $\lim_{x\to\infty}\dfrac{(a^2-b^2)}{(c^2-d^2)}\dfrac{\left[\sqrt{1+\frac{c^2}{x^2}}+\sqrt{1+\frac{d^2}{x^2}}\right]}{\left[\sqrt{1+\frac{a^2}{x^2}}+\sqrt{1+\frac{b^2}{x^2}}\right]}=\dfrac{a^2-b^2}{c^2-d^2}.$

Q.25 (3)

Put $\cos^{-1}x=y$ and $x\to 1\Rightarrow y\to 0$.

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{(\cos^{-1}x)^2}=\lim_{y\to 0}\frac{1-\sqrt{\cos y}}{y^2}$$

Now rationalizing it, we get $\lim_{y\to 0}\dfrac{(1-\cos y)}{y^2(1+\sqrt{\cos y})}$

$$=\lim_{y\to 0}\frac{1-\cos y}{y^2}.\lim_{y\to 0}\frac{1}{1+\sqrt{\cos y}}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}.$$

Q.26 (2) $\lim_{x\to\pi/4}\dfrac{(\sqrt{2}-\sec x)\cos x(1+\cot x)}{\cot x[2-\sec^2 x]}$

$$= \lim_{x\to\pi/4} \frac{\sin x(1+\cot x)}{(\sqrt{2}+\sec x)} = \frac{\frac{1}{\sqrt{2}}(2)}{\sqrt{2}+\sqrt{2}} = \frac{1}{2}.$$

Aliter : Apply L-Hospital's rule.

Q.27 (4)

$\pi - 2x = \theta \Rightarrow x = \frac{\pi}{2} - \frac{\theta}{2}$ and as

$x \to (\pi/2), \theta \to 0$

Now solve yourself.

Q.28 (4)

$$\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x} = \lim_{x\to 0} \frac{|\sin x|}{x}$$

So, $\lim_{x\to 0+} \frac{|\sin x|}{x} = 1$ and $\lim_{x\to 0-} \frac{|\sin x|}{x} = -1$

Hence limit does not exist.

Q.29 (2)

Given limit $= \lim_{n\to\infty} (4^n + 5^n)^{1/n}$

$$= \lim_{n\to\infty} 5\left[\left\{1+\left(\frac{4}{5}\right)^n\right\}^{(5/4)^n}\right]^{(1/n).(4/5)^n} = 5.e^0 = 5.$$

$$\left(\because \left(\frac{4}{5}\right)^n \to 0 \text{ as } n\to\infty\right)$$

Q.30 (3)

Given limit $= \lim_{x\to 0}\left(\frac{1+\tan x}{1-\tan x}\right)^{1/x}$

$$= \lim_{x\to 0} \frac{\{(1+\tan x)^{1/\tan x}\}^{(\tan x)/x}}{\{(1-\tan x)^{1/\tan x}\}^{(\tan x)/x}} = \frac{e}{e^{-1}} = e^2.$$

Q.31 (4)

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{\sin(e^{x-2}-1)}{\log(x-1)}$$

$$= \lim_{t\to 0} \frac{\sin(e^t-1)}{\log(1+t)}, \{\text{Putting } x = 2+t\}$$

$$= \lim_{t\to 0} \frac{\sin(e^t-1)}{e^t-1}.\frac{e^t-1}{t}.\frac{t}{\log(1+t)}$$

$$= \lim_{t\to 0} \frac{\sin(e^t-1)}{e^t-1}.\left(\frac{1}{1!}+\frac{t}{2!}+...\right)\times\left[\frac{1}{\left(1-\frac{1}{2}t+\frac{1}{3}t^2-...\right)}\right]$$

$= 1.1.1 = 1$, ($\because$ As $t\to 0, e^t - 1 \to 0$).

Q.32 (1)

$$\lim_{x\to-\infty} \frac{\sqrt{4x^2+5x+8}}{4x+5}$$

$$= \lim_{h\to 0} \frac{\sqrt{4(-1/h)^2+5(-1/h)+8}}{4(-1/h)+5}$$

$$= \lim_{h\to 0} \frac{(1/h)\sqrt{4-5h+8h^2}}{(1/h)(-4+5h)} = \frac{\sqrt{4}}{-4} = -\frac{1}{2}.$$

Q.33 (4)

Let $y = \lim_{x\to 3} \frac{x^3-x^2-18}{x-3}, \left(\frac{0}{0}\text{form}\right)$

Applying L-Hospital's rule, we get

$y = \lim_{x\to 3} 3x^2 - 2x = (27-6) = 21.$

Q.34 (1)

$$\lim_{x\to\infty} \frac{2x^2+3x+4}{3x^2+3x+4} = \lim_{x\to\infty} \frac{2+\frac{3}{x}+\frac{4}{x^2}}{3+\frac{3}{x}+\frac{4}{x^2}} = \frac{2}{3}$$

Q.35 (1)

$\lim_{x\to 0} g(f(x)) = \lim_{x\to 0} [f(x)]^2 + 1 = \lim_{x\to 0} (\sin^2 x + 1) = 1.$

Q.36 (3)

$$\lim_{x\to 2}\left(\frac{3^{x/2}-3}{3^x-9}\right) = \lim_{x\to 2}\left(\frac{3^{x/2}-3}{(3^{x/2})^2-3^2}\right)$$

$$= \lim_{x\to 2} \frac{1}{3^{x/2}+3} = \frac{1}{6}.$$

Q.37 (3)

Using L-Hospital's rule,

$$\lim_{\theta\to\frac{\pi}{2}} \frac{-1}{-\mathrm{cosec}^2\theta} = 1.$$

Q.38 (1)

$$\lim_{n\to\infty} \frac{1-n^2}{\Sigma n}$$

$$= \lim_{n\to\infty} \frac{(1-n)(1+n)}{\frac{1}{2}n(n+1)} = \lim_{n\to\infty} \frac{2(1-n)}{n}$$

$$= \lim_{n\to\infty} 2\left(\frac{1}{n}-1\right) = 2(0-1) = -2\,.$$

Q.39 (1)

$$y = \lim_{x\to 0} \frac{4^x - 9^x}{x(4^x+9^x)}, \left(\frac{0}{0}\text{form}\right)$$

Using L-Hospital's rule,

$$y = \lim_{x\to 0} \frac{4^x \log 4 - 9^x \log 9}{(4^x+9^x) + x(4^x \log 4 + 9^x \log 9)}$$

$$\Rightarrow y = \frac{\log 4 - \log 9}{2}$$

$$\Rightarrow y = \frac{\log\left(\frac{2}{3}\right)^2}{2} = \log\frac{2}{3}.$$

Q.40 **(a)**

$$\operatorname*{Lim}_{x\to 0}\left\{\frac{\log_e(1+x)}{x^2} + \frac{x-1}{x}\right\}$$

$$\operatorname*{Lim}_{x\to 0}\frac{\log_e(1+x) + x^2 - x}{x^2}$$

$$\operatorname*{Lim}_{x\to 0}\frac{\left(x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - ....\right) + x^2 - x}{x^2} = \frac{1}{2}$$

Q.41 (1)

$$\lim_{x\to 0}\frac{a^x - b^x}{e^x - 1} = \lim_{x\to 0}\frac{a^x - b^x}{x}.\frac{x}{e^x-1}$$

$$= \lim_{x\to 0}\left[\frac{a^x-1}{x} - \frac{b^x-1}{x}\right]\frac{x}{e^x-1}$$

$$= (\log_e a - \log_e b).\frac{1}{\log_e e} = \log_e\left(\frac{a}{b}\right)$$

Trick : Apply L-Hospital's rule.

Q.42 (1) .

$$\lim_{x\to 0}\frac{(1 + nx + {}^nC_2x^2 + \text{...higher powers of } x \text{ to } x^n) - 1}{x} = n$$

Aliter : Apply L-Hospital's rule.

Q.43 (3)

Multiply function by $\dfrac{(1+x)^{1/2} + (1-x)^{1/2}}{(1+x)^{1/2} + (1-x)^{1/2}}$ and solve.

Aliter : Apply L-Hospital's rule,

$$\lim_{x\to 0}\frac{(1+x)^{1/2} - (1-x)^{1/2}}{x} = \lim_{x\to 0}\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{1-x}} = 1$$

Q.44 (1)

$$\lim_{x\to 0}\frac{e^{x^2} - \cos x}{x^2}$$

Now expanding e^{x2} and cos x, we get

$$\lim_{x\to 0}\frac{\frac{3x^2}{2!} + x^4\left(\frac{1}{2!} - \frac{1}{4!}\right) + ...}{x^2} = \frac{3}{2}$$

Aliter : Apply L-Hospital's rule,

$$\lim_{x\to 0}\frac{2xe^{x^2} + \sin x}{2x} = \lim_{x\to 0} e^{x^2} + \lim_{x\to 0}\frac{\sin x}{2x} = 1 + \frac{1}{2} = \frac{3}{2}.$$

Q.45 (2)

$$\lim_{x\to a}\frac{f(a)[g(x) - g(a)] - g(a)[f(x) - f(a)]}{[x-a]}$$

$$= f(a)g'(a) - g(a)f'(a) = 2\times 2 - (-1)(1) = 5.$$

Q.46 (c)
Q.47 (d)
Q.48 **(a)**

Q.49 (3)

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x+1} - 2x - \beta\right) = 0$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^2(1-\alpha) - x(\alpha+\beta) + 1 - b}{x+1} = 0$$

Since the limit of the given expression is zero, therefore degree of the polynomial in numerator must be less than denominator.

$\therefore\ 1-\alpha = 0$ and $\alpha + \beta = 0 \Rightarrow \alpha = 1$ and $\beta = -1$.

Q.50 (2)

$$\lim_{n\to\infty}\left[\frac{1}{1-n^2}+\frac{2}{1-n^2}+.....+\frac{n}{1-n^2}\right]$$

$$=\lim_{n\to\infty}\frac{\Sigma n}{1-n^2}=\frac{1}{2}\lim_{n\to\infty}\frac{n^2+n}{1-n^2}=-\frac{1}{2}.$$

Q.51 **(1)**

Multipling by log x in dinominater & numanater

$$\lim_{x\to\infty}\frac{\log_x n-[x]}{[x]}=\lim_{x\to\infty}\frac{\log n-[x]\log x}{[x]\log x}$$

$$\lim_{x\to\infty}\frac{\log n}{[x]\log x}-1=-1$$

Q.52 (1)

Calculus is that branch of Mathematics which mainly deals with the study of change in the value of a function with respect to the change in points of its domain.

Q.53 (4)

We have,

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{3(2+h)-3(2)}{h}$$

$$=\lim_{h\to 0}\frac{6+3h-6}{h}=\lim_{h\to 0}\frac{3h}{h}=\lim_{h\to 0}3=3$$

The derivative of the function f(x) =3x at x = 2 is 3.

Q.54 (3)

Let f(x) = sin x. Then,

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{\sin(0+h)-\sin(0)}{h}=\lim_{h\to 0}\frac{\sin h}{h}=1$$

Q.55 (2)

Since, the derivative measures the change in the function, intutively it is clear that the derivative of the constant function must be zero at every point. This is indeed, supported by the following computation.

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{3-3}{h}=\lim_{h\to 0}\frac{0}{h}=0$$

Similarly, $f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{3-3}{h}=0$

Q.56 (2)

It is easy to see that the derivative of the function f(x) = x is the constant function 1. This is because

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{x+h-x}{h}=\lim_{h\to 0}1=1$$

Q.57 (2)

If given function is $6x^{100}-x^{55}+x$. Then, the derivative of function is $6.100.x^{99}-55.\,x^{54}+1$

or $600x^{99}-55x^{54}+1$

Q.58 (2)

Let $y=\frac{a}{x^4}-\frac{b}{x^2}+\cos x$

$\Rightarrow y=a\,x^{-4}-bx^{-2}+\cos x$

Differentiating y w.r.t.x, we get

$$\frac{dy}{dx}=a\frac{d}{dx}\left(x^{-4}\right)-b\frac{d}{dx}\left(x^{-2}\right)+\frac{d}{dx}(\cos x)$$

$=a(-4)\,x^{-4-1}-b\,(-2)\,x^{-2-1}-\sin x$

$$=-\frac{4a}{x^5}+\frac{2b}{x^3}-\sin x\quad\left[\because\frac{d}{dx}\left(x^n\right)=nx^{n-1}\right]$$

Q.59 (3)

Let $y=4\sqrt{x}-2\Rightarrow y=4x^{1/2}-2$

Differentiating w.r.t.x, we get

$$\frac{dy}{dx}=4.\frac{1}{2}x^{\frac{1}{2}-1}-0=2x^{\frac{-1}{2}}=\frac{2}{\sqrt{x}}$$

Q.60 (2)

Let $y=(ax+b)^n$

Differentiating w.r.t.x, we get

$$\Rightarrow\frac{dy}{dx}=n(ax+b)^{n-1}\frac{d}{dx}(ax+b)=n(ax+b)^{n-1}a$$

$$\Rightarrow\frac{dy}{dx}=na(ax+b)^{n-1}$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (4)

$$\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}=\lim_{x\to 0}\frac{|\sin x|}{x}$$

So, $\lim_{x\to 0+}\frac{|\sin x|}{x}=1$ and $\lim_{x\to 0-}\frac{|\sin x|}{x}=-1$

Hence limit does not exist.

Q.2 **(4)**

$\ell\text{im}_{x\to 0^+}\sec x>1$

So ℓimit not exist

Q.3 **(b)**

Q.4 **(3)**

$$\ell im_{x\to 1}(1-x+[x-1]+[1-x])$$

L.H.L.= $\ell im_{x\to 1^-}(1-x+[x-1]+[1-x])$

$= \ell im_{h\to 0}(1-(1-h)+[1-h-1]+[1-1+h])$

$= \ell im_{h\to 0}(h+[-h]+[h])$

$= 0-1+0=-1$

R.H.L. = $\ell im_{x\to 1^+}(1-x+[x-1]+[1-x])$

$= \ell im_{h\to 0}(1-(1+h)+[1+h-1]+[1-(1+h)]$

$= \ell im_{h\to 0}(-h+[h]+[-h])$

$= 0+0-1=-1$

L.H.L. = R.H.L. = -1

so $\ell im_{x\to 1}(1-x+[x-1]+[1-x]) = -1$

Q.5 **(4)**

$$\lim_{x\to\infty}\sec^{-1}\left(\frac{x}{x+1}\right) \qquad \text{Put } x=\frac{1}{y}$$

$$=\lim_{y\to 0}\sec^{-1}\left(\frac{1}{y+1}\right)=\lim_{y\to 0}\cos^{-1}(y+1)\ \&\ y=0^+$$

$$\lim_{y\to 0^+}\cos^{-1}(y+1)=\lim_{h\to 0}\cos^{-1}(1+h)\Rightarrow \text{Not possible}$$

so, Limit does not exists.

Q.6 **(2)**

$(1-x+[x-1]+[1-x])$

L.H.L. = $\lim_{x\to 1^-}(1-x-1)=-1$

R.H.L. = $\lim_{x\to 1^+}(1-x-1)=-1$

L.H.L. = R.H.L. = -1

Q.7 **(2)**

$$\lim_{x\to -1}\frac{x^3-2x-1}{x^5-2x-1}$$

$$\lim_{x\to -1}\frac{(x+1)(x^2-x-1)}{(x+1)(x^4-x^3+x^2-x-1)}=\frac{1}{3}$$

Q.8 **(4)**

$$\lim_{x\to 1}\frac{x\sin(x-[x])}{(x-1)}$$

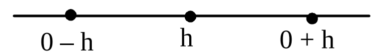
Put $x-1=h\Rightarrow x=h+1$

As $(x-1)\to 0\Rightarrow h\to 0$

$$\lim_{h\to 0}\frac{(1+h)\sin(1+h-[1+h])}{h}$$

$$=\lim_{h\to 0}\frac{(1+h)\sin(1+h-1-[h])}{h}$$

$$=\lim_{h\to 0}\frac{(1+h)\sin(h-[h])}{h}$$



L.H.L = $\lim_{h\to 0^-}f(0-h)$

$$=\lim_{h\to 0^-}\frac{(1+0-h)\sin(0-h-[0-h])}{(0-h)}$$

$$=\lim_{h\to 0^-}\frac{(1-h)\sin(-h+h)}{(-h)}=-1$$

R.H.L. $\lim_{h\to 0^+}f(0+h)=$

$$\lim_{h\to 0^+}\frac{(1+0+h)\sin\left(0+h-[0+h]\right)}{h}$$

$$\lim_{h\to 0^+}\frac{(1+h)\sin(h-h)}{h}=1$$

Since, L.H.L $\neq$ R.H.L

$\therefore$ Limit does not exist

Q.9 **(2)**

$$\lim_{x\to\infty}\frac{x^3+x^2+1}{x^3-x^2+1}$$

$$\lim_{x\to\infty}\frac{1+\frac{1}{x}+\frac{1}{x^3}}{1-\frac{1}{x}+\frac{1}{x^3}}=1$$

Q.10 **(3)**

$$\lim_{x\to\infty}\frac{\sqrt{3}x^5+x^2+13}{x^4+7x^2-\sqrt{17}}$$

$$\lim_{x\to\infty}\frac{\sqrt{3}x+\frac{1}{x^3}+\frac{13}{x^4}}{1+\frac{7}{x^2}-\frac{\sqrt{17}}{x^4}}=\infty$$

Q.11 (d)

Given $f(x)=g(x)h(x)$

$$\Rightarrow h(x)=\frac{f(x)}{g(x)}$$

$$\Rightarrow \lim_{x\to 1}h(x)=\lim_{x\to 1}\frac{f(x)}{g(x)}$$

$$\Rightarrow \lim_{x\to 1}\frac{\left(x^5-1\right)\left(x^3+1\right)}{\left(x^2-1\right)\left(x^2-x+1\right)}$$

$$=\lim_{x\to 1}\frac{x^5-1^5}{x-1}=5\times 1^4=5$$

Q.12 (b)

$$\text{limit} = \lim_{n\to\infty}\frac{1+\left(\frac{b}{a}\right)^n}{1-\left(\frac{b}{a}\right)^n}=1,$$

because $0<\frac{b}{a}<1$ implies

$$\left(\frac{b}{a}\right)^n \to 0 \textit{ as } n\to\infty$$

Q.13 (b)

Consider $\lim_{n\to\infty}\frac{1+2+3+...n}{n^2+100}$

$$=\lim_{n\to\infty}\frac{n(n+1)}{2\left(n^2+100\right)}$$

(By using sum of n natural number

$1+2+3+..+n=\frac{n(n+1)}{2}$)

Take n2 common from N^t and D^r.

$$=\lim_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)}{2n^2\left(1+\frac{100}{n^2}\right)}=\frac{1}{2}$$

Q.14 **(d)**

Q.15 **(3)**

$$\lim_{x\to -\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$$

$$\lim_{x\to -\frac{\pi}{6}}\frac{\left(\sin x-\frac{1}{2}\right)(\sin x+1)}{\left(\sin x-\frac{1}{2}\right)(\sin x-1)}=\frac{3/2}{-1/2}=-3$$

Q.16 **(2)**

$$\lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$$

$$\lim_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}} \qquad =\frac{1}{3}$$

Q.17 **(3)**

$$\lim_{x\to 3}\frac{(x^3+27)\ell n(x-2)}{(x^2-9)}$$

$$=\lim_{x\to 3}\frac{(x+3)(x^2-3x+9)\ell n[1+(x-3)]}{(x-3)(x+3)} = \lim_{x\to 3}$$

$$(x^2-3x+9).\frac{\ell n[1+(x-3)]}{(x-3)}$$

$$=(9-9+9)(1)=9$$

Q.18 **(4)**

$$\operatorname{Lim}_{x\to 0}\frac{\sin(\ell n(1+x))}{\ell n(1+\sin x)}$$

$$=\operatorname{Lim}_{x\to 0}\frac{\sin(\ell n(1+x))}{\ell n(1+x)}\times\frac{\ell n(1+x)}{x}\times\frac{\sin x}{\ell n(1+\sin x)}$$

$$=1\,.\,1\,.\,1=1$$

Q.19 **(3)**

$$\operatorname{Lim}_{x\to\frac{\pi}{2}}\left[\frac{x-\frac{\pi}{2}}{\cos x}\right] \text{ Let } x=\left(\frac{\pi}{2}+h\right)$$

$$=\operatorname{Lim}_{h\to 0}\left[\frac{\frac{\pi}{2}+h-\frac{\pi}{2}}{\cos\left(\frac{\pi}{2}+h\right)}\right]=\operatorname{Lim}_{h\to 0}\left[\frac{h}{-\sinh}\right]=-2$$

Q.20 **(3)**

$$\operatorname{Lim}_{x\to\infty}\frac{x^3\sin\left(\frac{1}{x}\right)+x+1}{x^2+x+1}$$

$$\operatorname{Lim}_{x\to\infty}\frac{x^2\left\{\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}+\frac{1}{x}+\frac{1}{x^2}\right\}}{x^2\left\{1+\frac{1}{x}+\frac{1}{x^2}\right\}}=\frac{1}{1}=1$$

Q.21 (c)

$$\lim_{x\to 0}\frac{\sqrt{1+\sin 3x-1}}{In(1+\tan 2x)}$$

$$=\lim_{x\to 0}\frac{(1+\sin 3x)-1}{\underbrace{\left\{\sqrt{1+\sin 3x+1}\right\}}_{1}\underbrace{In(1+\tan 2x)}_{\sin 3x}}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{1+\sin 3x+1}\right)}\cdot\frac{\sin 3x}{In(1+\tan 2x)^{\frac{1}{\tan 2x}}}$$

$$\times\frac{1}{\tan 2x}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{1+\sin 3x}+1\right)}\cdot\left(\frac{\sin 3x}{3x}\right)\left(\frac{2x}{\tan 2x}\right)$$

$$\times\frac{3}{2}\cdot\frac{1}{In(1+\tan 2x)^{\frac{1}{\tan 2x}}}$$

$$=\left(\frac{1}{1+1}\right)(1)(1)\left(\frac{3}{2}\right)\frac{1}{In\,e}=\frac{3}{4}.$$

Q.22 (c)

Put $x=\frac{\pi}{2}-h$ as $x\to\frac{\pi}{2}$, $h\to 0$

$\therefore$ Given limit

$$=\lim_{h\to 0}\frac{1-\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)}{1+\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)}\cdot\frac{(1-\cosh)}{(2h)^3}$$

$$=\lim_{h\to 0}\tan\frac{h}{2}\,\frac{2\sin^2\frac{h}{2}}{8h^3}$$

$$=\lim_{h\to 0}\frac{1}{4}\cdot\frac{\tan\frac{h}{2}}{\frac{h}{2}\times 2}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\frac{1}{4}$$

$$=\lim_{h\to 0}\frac{1}{32}\cdot\left(\frac{\tan\frac{h}{2}}{\frac{h}{2}\times 2}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2=\frac{1}{32}$$

Q.23 (a)

As given,

$$A=\lim_{x\to\infty}x\sin\left(\frac{1}{x}\right)=\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

Let $t=\frac{1}{x}$ when $x\to\alpha$., $t\to 0$

$$\Rightarrow A=\lim_{x\to\infty}\frac{\sin t}{t}=1$$

$$\left[\because\lim_{t\to 0}\frac{\sin x}{x}=1\right]$$

and $B=\lim_{x\to 0}x\sin\left(\frac{1}{x}\right)$

$$\Rightarrow B=\lim_{x\to 0}x.\ \lim_{x\to 0}\sin\left(\frac{1}{x}\right)$$

$$\Rightarrow B=0$$

$\therefore$ A = 1 and B = 0 is correct

Q.24 (a)

Q.25 (a)

Q.26 **(c)**

Q.27 **(4)**

$$\lim_{n\to\infty}\frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}$$

$$=\lim_{n\to\infty}\frac{5.5^n+3^n-4^n}{5^n+2^n+27.9^n}$$

$$=\lim_{n\to\infty}\frac{5.\left(\frac{5}{9}\right)^n+\left(\frac{3}{9}\right)^n-\left(\frac{4}{9}\right)^n}{\left(\frac{5}{9}\right)^n+\left(\frac{2}{9}\right)^n+27}$$

$$=\frac{0+0-0}{0+0+27}=0$$

Q.28 **(2)**

$$\lim_{x\to\frac{\pi}{4}} (1+[x])^{1/\ell n \tan x}$$

After putting limit [x] becomes zero, so base is dot one hence $1^\infty = 1$

Q.29 **(2)**

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x)}{\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}[1-(0+h)]}{\sqrt{0+h}}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-h)}{\sqrt{h}}$$

Let $1 - h = \cos\theta$

$\sin\theta = \sqrt{1-(1-h)^2}$

$\therefore\ \theta = \sin^{-1}\sqrt{2h-h^2}$

$$= \lim_{h\to 0} \frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{h}}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}\sqrt{2h-h^2}}{\sqrt{2h-h^2}} \cdot \frac{\sqrt{2h-h^2}}{\sqrt{h}}$$

$= 1 \times \sqrt{2} = \sqrt{2}$

Q.30 **(3)**

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$ It is of the form 1^∞, so,

$$\Rightarrow \ell = e^{\lim_{x\to\infty} x\left[\frac{x^2-2x+1}{x^2-4x+2}-1\right]} = e^{\lim_{x\to\infty}\left(\frac{2x^2-x}{x^2-4x+2}\right)}$$

$$= e^{\lim_{x\to\infty}\left(\frac{2-1/x}{1-4/x+2/x^2}\right)} = e^2$$

Q.31 **(3)**

$$\lim_{x\to a}\left(2-\frac{a}{x}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$$ It is of the form 1^∞,

$$\ell = e^{\lim_{x\to a}\tan\frac{\pi x}{2a}\left(2-\frac{a}{x}-1\right)} = e^{\lim_{x\to a}\tan\frac{\pi x}{2a}\left(\frac{x-a}{x}\right)}$$

Put $x = a + h$,

$$\ell = e^{\lim_{h\to 0}\tan\left(\frac{\pi}{2}+\frac{\pi h}{2a}\right).\left(\frac{h}{a+h}\right)} = e^{-\lim_{h\to 0}\cot\left(\frac{\pi h}{2a}\right).\left(\frac{h}{a+h}\right)}$$

$$= e^{-\lim_{h\to 0}\frac{1}{\tan\left(\frac{\pi h}{2a}\right)}\times(\pi h/2a)\times\frac{2a/\pi}{(a+h)}} = e^{-\lim_{h\to 0}\frac{2a}{\pi}\times\frac{1}{a+h}} = e^{-2/\pi}$$

Q.32 **(1)**

$$\lim_{x\to 0}(\cos x)^{\frac{1}{\sin x}}$$ (1^∞ form)

$$= e^{\lim_{x\to 0}\left(\frac{\cos x-1}{\sin x}\right)}$$

$$= e^{\lim_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$= e^{\lim_{x\to 0}-\tan\left(\frac{x}{2}\right)}$$

$= e^0 = 1$

Q.33 **(2)**

$$\lim_{x\to 0}(\cos mx)^{\frac{n}{x^2}}$$ since it is of the form 1^∞, so,

$$\ell = e^{\lim_{x\to 0}\frac{n}{x^2}(\cos mx-1)} = e^{\lim_{x\to 0}\frac{-2n\sin^2\left(\frac{mx}{2}\right)}{x^2\times\frac{m^2}{4}\times\frac{4}{m^2}}}$$

$$= e^{\left(-\frac{m^2 n}{2}\right)}$$

Q.34 **(3)**

$$f(x) = \begin{cases} \dfrac{\tan^2[x]}{(x^2-[x]^2)} & ;\ x>0 \\ 1 & ;\ x=0 \\ \sqrt{\{x\}\cot\{x\}} & ;\ x<0 \end{cases}$$

RHL : $\lim_{x\to 0^-}\sqrt{\{x\}\cot\{x\}} = \sqrt{1\times\cot 1} = \sqrt{\cot 1}$

LHL : $\lim_{x\to 0^+}\dfrac{\tan^2[x]}{(x^2-[x]^2)} = 0 \quad\Rightarrow\quad \cot^{-1}$

$$\left(\lim_{x\to 0^-} f(x)\right)^2 = 1$$

Q.35 **(3)**

$$\ell = \lim_{x\to0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$$

$$\ell = \lim_{x\to0} \frac{(1-\cos x\sqrt{\cos 2x})(1+\cos x\sqrt{\cos 2x})}{x^2(1+\cos x\sqrt{\cos 2x})}$$

$$\ell = \lim_{x\to0} \frac{1-\cos^2 x.\cos 2x}{x^2(1+1\sqrt{1})}$$

$$\ell = \lim_{x\to0} \frac{1-\cos^2 x(1-2\sin^2 x)}{2x^2}$$

$$\ell = \lim_{x\to0} \frac{1-\cos^2 x+2\sin^2 x\cos^2 x}{2x^2}$$

$$\ell = \lim_{x\to0} \frac{\sin^2 x}{2x^2} + \lim_{x\to0} \frac{\sin^2 x}{x^2}.\cos^2 x$$

$$\ell = \frac{1}{2} + 1 = \frac{3}{2}$$

Q.36 **(2)**

$$\lim_{x\to0} \frac{\cos(\sin x)-\cos x}{x^4}$$

$$= \lim_{x\to0} \frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$= \lim_{x\to0} \frac{2\sin\left\{\frac{\sin x+x}{2}\right\}}{\left\{\frac{\sin x+x}{2}\right\}} \times \frac{\left(\frac{\sin x+x}{2}\right)}{x^4} \times \frac{\sin\left(\frac{x-\sin x}{2}\right)}{\frac{\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}}$$

$$= \lim_{x\to0} 2\left(\frac{x+\sin x}{2}\right) \times \left(\frac{x-\sin x}{2}\right) \times \frac{1}{x^4}$$

$$= \lim_{x\to0} \frac{1}{2}\left(\frac{x+\sin x}{x}\right) \left(\frac{x-\sin x}{x^3}\right) =$$

$$\lim_{x\to0} \frac{1}{2}\left(1+\frac{\sin x}{x}\right)\cdot\left(\frac{x-\sin x}{x^3}\right)$$

$$= \frac{1}{2}(1+1)\cdot\left(\frac{1}{6}\right) = \frac{1}{6}$$

Q.37 **(3)**

$$\ell = \lim_{x\to0} \frac{\sin\cdot[\cos x]}{1+[\cos x]}$$

LHL : $\lim_{h\to0} \frac{\sin[\cos(0-h)]}{1+[\cos(0-h)]} \Rightarrow \lim_{h\to0} \frac{\sin 0}{1+0} = 0$

$(\because [\cos(0-h)] = 0)$

RHL : $\lim_{h\to0} \frac{\sin[\cos(0+h)]}{1+[\cos(0+h)]} \Rightarrow \lim_{h\to0} \frac{\sin 0}{1+0} = 0$

so, $\lim_{x\to0} \frac{\sin[\cos x]}{1+[\cos x]} = 0$

Q.38 **(1)**

1^∞ From

$$\ell = \operatorname*{Lim}_{x\to\infty} x\left[\sin\frac{1}{x} + \cos\frac{1}{x} - 1\right]$$

put x = 1/t

$$= \operatorname*{Lim}_{t\to0} \frac{1}{t}[\sin\ t + \cos\ t - 1]$$

$$= \operatorname*{Lim}_{t\to0} \frac{1}{t}\left[2\sin\frac{t}{2}\cos\frac{t}{2} - 2\sin^2\frac{t}{2}\right]$$

$$= \operatorname*{Lim}_{t\to0} \frac{1}{t}\ 2\sin\frac{t}{2}\left[\cos\frac{t}{2} - \sin\frac{1}{2}\right]$$

$= 1 = e^1$

Q.39 **(2)**

$$\lim_{x\to a} \frac{\ell n\{1+\tan(x-a)\}}{\tan(x-a)}\ ;\ \text{as} \lim_{x\to0} \frac{\ell n(1+x)}{x} = 1$$

Q.40 **(3)**

$$\lim_{x\to1} \frac{\cos\left(\frac{\pi x}{2}\right)}{1-x}$$

$$\lim_{x\to1} \frac{\sin\left(\frac{\pi}{2}-\frac{\pi}{2}x\right)}{1-x}$$

$$\lim_{x\to1} \frac{\sin\frac{\pi}{2}(1-x)}{\frac{\pi}{2}(1-x)}.\frac{\pi}{2} = \frac{\pi}{2}$$

Q.41 **(2)**

$$\lim_{x\to\infty} \left(\frac{x}{1+x}\right)^x (1^\infty)$$

$$\Rightarrow e^{\lim_{x\to\infty}(x)\left(\frac{x}{1+x}-1\right)}$$

$$\Rightarrow e^{\lim_{x\to\infty}(x)\left(\frac{x-1-x}{1+x}\right)}$$

$$\Rightarrow e^{\lim_{x\to\infty}\frac{-1}{1+\frac{1}{x}}} = e^{-1}$$

Q.42 (2)

$$\lim_{x\to 0}(1+x)^{1/13x}$$

$$e^{\lim_{x\to 0}\frac{1}{13x}(1+x-1)} = e^{1/13}$$

Q.43 (3)

$$\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x^2}}$$

$$\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{(1+x)(1+\sqrt{x})(1-\sqrt{x})}} = \left(\frac{2}{3}\right)^{1/4}$$

Q.44 (4)

$$\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^3} \quad (1^\infty)$$

$$\text{L.H.L.} = \lim_{x\to 0^-}\left(\frac{\tan x}{x}\right)^{-\infty}$$

$$\text{R.H.L.} = \lim_{x\to 0^+}\left(\frac{\tan x}{x}\right)^{+\infty}$$

L.H.L. $\neq$ R.H.L.

$\Rightarrow$ limit does not exist

Q.45 (3)

$$\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}} \quad (1^\infty)$$

$$\Rightarrow e^{\lim_{x\to 1}\tan\frac{\pi x}{2}(2-x-1)}$$

$$\Rightarrow e^{\lim_{x\to 1}\left(\tan\frac{\pi x}{2}\right)(1-x)}$$

$$\Rightarrow e^{\lim_{x\to 1}\frac{\left(\sin\frac{\pi x}{2}\right)(1-x)}{\sin\left(\frac{\pi}{2}-\frac{\pi}{2}x\right)}}$$

$$\Rightarrow e^{\lim_{x\to 1}\frac{\frac{\pi}{2}(1-x)}{\sin\frac{\pi}{2}(1+x)}\cdot\frac{2}{\pi}} = e^{2/\pi}$$

Q.46 (1)

$$\lim_{x\to 0}\frac{(1-\cos 2x)\sin 5x}{x^2\sin 3x}$$

$$\lim_{x\to 0}\frac{2\sin^2 x.\frac{\sin 5x}{5x}.5}{x^2\frac{\sin 3x}{3x}.3} = \frac{10}{3}$$

Q.47 (1)

$$\lim_{n\to\infty}\left(an-\frac{1+n^2}{1+n}\right) = b$$

$$\lim_{n\to\infty}\left(\frac{an+an^2-1-n^2}{1+n}\right) = b$$

$$\lim_{n\to\infty}\frac{a-1/n+n(a-1)}{1+1/n} = b$$

Ordered pair must be (1,1)

$\therefore$ Option (A) is correct Answer

Q.48 (3)

$$\lim_{x\to\infty}\sqrt{x^2+x+1}-\sqrt{x^2+1}$$

$$= \lim_{x\to\infty}\frac{x^2+x+1-x^2-1}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+1/x^2}} = \frac{1}{2}$$

Q.49 (2)

$$\lim_{x\to 0^+}\left(\frac{e^{x\ell n(2^x-1)}-(2^x-1)^x\sin x}{e^{x\ell nx}}\right)^{1/x}$$

$$\Rightarrow \lim_{x\to 0^+}\left[\frac{(2^x-1)^x-(2^x-1)^x\sin x}{x^x}\right]^{1/x}$$

$$\Rightarrow \lim_{x\to 0^+}\frac{(2^x-1)(1-\sin x)^{1/x}}{x}$$

$$\Rightarrow \ell n2 . \lim_{x\to 0^+}(1-\sin x)^{1/x} \,(1^\infty)$$

$$\Rightarrow \ell n2 . e^{\lim_{x\to 0^+}\frac{1}{x}(1-\sin x-1)}$$

$$\Rightarrow \ell n2 . e^{\lim_{x\to 0^+}\frac{-\sin x}{x}} \Rightarrow \frac{1}{e}\ell n2$$

Q.50 (1)

$$\lim_{x\to\infty}\left(x-x^2\,\ell n\left(1+\frac{1}{x}\right)\right)$$

$$= \lim_{x\to\infty} x - x^2\left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - ......\right)$$

$$= \lim_{x\to\infty} x - x + \frac{1}{2} - \frac{1}{3x} + ... = \frac{1}{2}$$

Q.51 **(2)**

$$\lim_{x\to 0} \frac{e^{\sin x} - 1 - \sin x}{x^2}$$

using expansion of e^x

$$\lim_{x\to 0} \frac{(1 + \sin x + \frac{\sin^2 x}{2!} + \frac{\sin^3 x}{3!} + ...) - 1 - \sin x}{x^2}$$

$$\lim_{x\to 0} \frac{\sin^2 x}{x^2(2!)} + \frac{\sin^3 x}{x^2(3!)} + ... = \frac{1}{2}$$

Q.52 **(4)**

$$\text{Lim}_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}$$

Using expansion of cosx

$$\text{Lim}_{x\to 0} \frac{\left(1 - \frac{\sin^2 x}{2!} + \frac{\sin^4 x}{4!}\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right)}{x^4}$$

using expansion of sinx

$$\text{Lim}_{x\to 0} \frac{-\frac{\left(x - \frac{x^3}{3!}\right)^2}{2!} + \frac{x^2}{2!}}{x^4} + \text{Lim}_{x\to 0} \frac{\sin^4 x}{4!\,x^4}$$

$$- \text{Lim}_{x\to 0} \frac{x^4}{4!\,x^4}$$

$$\text{Lim}_{x\to 0} \frac{-\frac{x^2}{2!} - \frac{x^6}{(3!)^2(2!)} + \frac{2x^4}{2!3!} + \frac{x^2}{2!}}{x^4} + \frac{1}{4!} - \frac{1}{4!}$$

$$\text{Lim}_{x\to 0} \frac{-x^2}{(3!)^2(2!)} + \frac{2}{2!3!} = \frac{1}{6}$$

Q.53 **(2)**

$$\lim_{n\to\infty} n\,\ell n\left(e.\left(1 + \frac{1}{n}\right)^{1-n}\right)$$

$$\lim_{n\to\infty} n\,[1 + (1-n)\,\ell n\,(1 + \frac{1}{n})]$$

$$\lim_{n\to\infty} [n + (n - n^2)]\ell n(1 + \frac{1}{n})]$$

(using expansion of $\ell n\,(1 + x)$)

$$\lim_{n\to\infty} \left[n + (n - n^2)\left(\frac{1}{n} - \frac{1}{2n^2} + ...\right)\right]$$

$$\lim_{n\to\infty} \left[n + \left(1 - n - \frac{1}{2n} + \frac{1}{2} + ...\right)\right]$$

$$1 - 0 + \frac{1}{2} + 0 ....$$

$$1 + \frac{1}{2} = \frac{3}{2}$$

Q.54 **(3)**

$$\lim_{x\to\frac{\pi}{2}} \tan^2 x$$

$$\left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}\right)$$

$$= \lim_{x\to\frac{\pi}{2}} \tan^2 x$$

$$\left(\frac{2\sin^2 x + 3\sin x + 4 - (\sin^2 x + 6\sin x + 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}}\right)$$

$$= \lim_{x\to\frac{\pi}{2}} \tan^2 x \cdot \frac{(\sin^2 x - 3\sin x + 2)}{\sqrt{2+3+4} + \sqrt{1+6+2}}$$ (put

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ & in direction sin x can be taken as 1)

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{6}\left[\frac{\sin^2 x - 3\sin x + 2}{\cos^2 x}\right] \left(\frac{0}{0}\text{ form}\right)$$ (use

L'Hospital rule)

$$= \frac{1}{6} \lim_{x\to\frac{\pi}{2}} \frac{2\sin x\cos x - 3\cos x}{2\cos x(-\sin x)}$$

$$= \frac{1}{6} \lim_{x\to\frac{\pi}{2}} \frac{2\sin x - 3}{-2\sin x}$$

$$= \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$$

Q.55 **(2)**

$$\lim_{x\to 0} \frac{\sin(6x^2)}{\ell n\cos(2x^2 - x)}$$

$$= \lim_{x\to 0} \frac{\sin 6x^2}{6x^2} \cdot \frac{6x^2}{\ell n(\cos(2x^2 - x))}$$

$$= 1. \lim_{x\to 0} \frac{6x^2}{\ell n(\cos(2x^2 - x))} \left(\frac{0}{0} \text{ form}\right)$$ [Using L' Hospital rule]

$$= \lim_{x\to 0} \frac{12x}{\frac{(-\sin(2x^2 - x))(4x-1)}{\cos(2x^2 - x)}}$$

$$= \lim_{x\to 0} \frac{-12\cos(2x^2 - x)}{4x-1} \cdot \frac{x(2x-1)}{\sin(2x^2 - x)} \frac{1}{(2x+1)}$$

$$= = \frac{1-12}{1.1} = -12$$

Q.56 (c)

Let $y = \lim_{x\to 0}(\operatorname{cosec} x)^{1/\log x}$

Taking log on both sides, we get

$$\log y = \lim_{x\to 0} \frac{\log \operatorname{cosec} x}{\log x}\left[\frac{\infty}{\infty} \text{ form }\right]$$

$$= \lim_{x\to 0} \frac{-\cot x}{1/x} \quad \text{(By L'Hospital rule)}$$

$$= -\lim_{x\to 0} \frac{x}{\tan x} \quad \left(\because \cot x = \frac{1}{\tan x}\right)$$

$$\Rightarrow \log y = -1$$

$$\Rightarrow y = e^{-1} = \frac{1}{e}$$

Hence, required limit $= \frac{1}{e}$

Q.57 (d)

Puttig x = $\frac{1}{y}$, we get

$$L = \lim_{y\to 0}\left(\frac{a_1^y + a_2^y + \ldots\ldots + a_n^y}{n}\right)$$

$(\because x \to \infty \; y \to 0)$

$\therefore$

$$\log_e L = \lim_{y\to 0} \frac{n}{y} \cdot \log \frac{1}{nn}\left(a_1^y + a_2^y + \ldots\ldots + a_n^y\right)\left(\frac{0}{0}\right)$$

=

$$n \lim_{y\to 0}\left(\frac{\frac{a_1^y \log a_1 + a_2^y \log a_2 + \ldots\ldots + a_n^y \log a_n}{a_1^y + a_2^y + \ldots\ldots + a_n^y}}{1}\right)$$

[using L'Hospital rule]

$$= n \cdot \frac{\log(a_1 a_2 \ldots\ldots a_n)}{n}$$

$\therefore \log L = \log(a_1 . a_2 \;\; a_n)$

$\Rightarrow L = a_1 . a_2 . a_3 \ldots . a_n$

Q.58 **(c)**

Q.59 **(a)**

Q.60 **(3)**

$$\lim_{x\to 0} \frac{\sin x + \log(1-x)}{x^2}\left(\frac{0}{0}\right)$$

using L Hospital Rule

$$\lim_{x\to 0} \frac{\cos x - \frac{1}{1-x}}{2x}\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0} \frac{-\sin x - \frac{1}{(1-x)^2}}{2} = -\frac{1}{2}$$

Q.61 **(2)**

$$\lim_{x\to -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$$

Let x = cos θ

as $x \to -1^+, \theta \to \pi^-$

$$\lim_{x\to \pi^-} \frac{\sqrt{\pi} - \sqrt{\theta}}{\sqrt{2}\cos\theta/2}\left(\frac{0}{0}\right)$$

using L Hospital rule

$$\lim_{x\to \pi^-} \frac{-\frac{1}{2\sqrt{\theta}}}{-\frac{\sqrt{2}}{2}\sin\frac{\theta}{2}} = \frac{1}{\sqrt{2\pi}}$$

Q.62 **(2)**

$$\lim_{x\to\infty} \frac{\ell nx - [x]}{[x]}$$

since $[x] \le x$

$$\lim_{x\to\infty} \frac{\ell nx - x}{x} \ (\infty/\infty \text{ form})$$

Apply L – H Rule, we get

$$= \lim_{x\to\infty} \frac{\frac{1}{x}.1-1}{1} = \frac{0-1}{1} = -1$$

$\therefore$ Option (2) is correct Answer.

Q.63 **(4)**

$$\lim_{n\to\infty} \frac{1}{n^4}\left([1^3x]+[2^3x]+.....+[n^3x]\right)$$

$1^3x-1<[1^3x]\le 1^3x$

$2^3x-1<[2^3x]\le 2^3x$

$3^3x-1<[3^3x]\le 3^3x$

$\vdots \quad \vdots \quad \vdots$

$n^3x-1<[n^3x]\le n^3x$ so,

$(1^3x+...+n^3x)-n<[1^3x]...+[n^3x]\le 1^3x+2^3x+...n^3x$

$$\lim_{n\to\infty}\frac{(1^3x+...+n^3x)-n}{n^4}<\lim_{n\to\infty}\ell\le\lim_{n\to\infty}\frac{1^3x+...+n^3x}{n^4}$$

$$\lim_{n\to\infty}\frac{\left(\frac{n(n+1)}{2}\right)^2x-n}{n^4}<\lim_{n\to\infty}\ \ell\ \le$$

$$\lim_{n\to\infty}\frac{\left(\frac{n(n+1)}{2}\right)^2x}{n^4}$$

$$\lim_{n\to\infty}\left(\frac{n^4\left(1+\frac{1}{n}\right)^2x}{4n^4}-\frac{1}{n^3}\right)<\lim_{n\to\infty}\ell\le\lim_{n\to\infty}\frac{n^4\left(1+\frac{1}{n}\right)^2x}{4n^4}$$

$$\frac{x}{4}-0<\lim_{n\to\infty}\ell<\frac{x}{4} \quad \text{So, } \lim_{n\to\infty}\ell=\frac{x}{4}$$

Q.64 **(4)**

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+......+\frac{1}{\sqrt{n^2+2n}}\right)$$

$$\Rightarrow \lim_{n\to\infty}\frac{(2n+1)}{\sqrt{n^2+2n}}\le\ell\le\frac{(2n+1)}{\sqrt{n^2}}$$

$$\Rightarrow \lim_{n\to\infty}\frac{(2n+1)}{n\sqrt{1+\frac{2n}{n^2}}}\le\ell\le\lim_{n\to\infty}\left(\frac{2n+1}{n}\right)$$

$$\Rightarrow 2\le\ell\le2\Rightarrow\ell=2$$

Q.65 **(1)**

$$\lim_{n\to\infty}\frac{[x]+\frac{1}{2}[2x]+\frac{1}{3}[3x]+....+\frac{1}{n}[nx]}{1^2+2^2+3^3+......+n^2}$$

$x-1<[x]\le x$

$2x-1<[2x]\le 2x\Rightarrow x-\frac{1}{2}<\frac{1}{2}[2x]\le x$

$3x-1<[3x]\le 3x \Rightarrow x-\frac{1}{3}<\frac{1}{3}[3x]\le x$

$4x-1<[4x]\le 4x\Rightarrow x-\frac{1}{4}<\frac{1}{4}[4x]\le x$

$(nx-1)<[nx]\le nx\Rightarrow x-\frac{1}{n}<\frac{1}{n}[nx]\le x$

Adding all terms as :

$(x+x+x+........n\text{ terms})-(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$

$+..................+\frac{1}{n})<$

$[x]+\frac{1}{2}[2x]+\frac{1}{3}[3x]+\frac{1}{4}[4x]+........+\frac{1}{n}[nx]$

$\le(x+x+x+..............+n\text{ terms})$

$$\Rightarrow nx-\sum_{r=1}^{n}\frac{1}{r}<\sum_{r=1}^{n}\frac{1}{r}[rx]\le nx$$

$$\lim_{n\to\infty}\frac{nx}{\frac{n(n+1)(2n+1)}{6}}-\lim_{n\to\infty}\frac{\sum_{r=1}^{n}\frac{1}{r}}{\frac{n(n+1)(2n+1)}{6}}$$

$$<\lim_{n\to\infty}\frac{\sum_{r=1}^{n}\frac{1}{r}[rx]}{\frac{n(n+1)(2n+1)}{6}}\le\lim_{n\to\infty}\frac{nx}{\frac{n(n+1)(2n+1)}{6}}$$

$$\Rightarrow 0-0<\lim_{n\to\infty}\frac{\sum_{r=1}^{n}\frac{1}{r}[rx]}{\frac{n(n+1)(2n+1)}{6}}\le 0$$

$$\lim_{n\to\infty}\frac{\sum_{r=1}^{n}\frac{1}{r}[rx]}{\frac{n(n+1)(2n+1)}{6}}=0$$

Q.66 **(4)**

$$\lim_{x\to\frac{\pi}{4}} [\text{Max}(\sin x, \cos x)]$$

$$\text{L.H.L} \lim_{x\to\frac{\pi}{4}^-} = \cos x = \frac{1}{\sqrt{2}}$$

$$\text{R.H.L.} \lim_{x\to\frac{\pi}{4}} = \sin x = \frac{1}{\sqrt{2}}$$

$$\text{L.H.L.} = \text{R.H.L.} = \frac{1}{\sqrt{2}}$$

Q.67 **(1, 4)**

$$\lim_{x\to\pi^-} [\text{sgn} \sin x] = 1$$

$$\lim_{x\to\pi^+} [\text{sgn} \sin x] = -1$$

$$\lim_{x\to\pi} [\text{sgn} \sin x] = \text{Does not exist}$$

Q.68 **(3)**

$$P_n = \left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)..........\left(1-\frac{1}{n^2}\right)$$

$$= \left(\frac{1}{2}.\frac{3}{2}\right)\left(\frac{2}{3}.\frac{4}{3}\right).\left(\frac{3}{4}.\frac{5}{4}\right)\left(\frac{4}{5}.\frac{6}{8}\right).......\left(\frac{n-1}{n}.\frac{n+1}{n}\right)$$

$$= \frac{n+1}{2n}$$

$$\lim_{n\to\infty} P_n$$

$$\Rightarrow \lim_{n\to\infty} \frac{n+1}{2n}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2} = \frac{1}{2}$$

Q.69 **(3)**

$$f(x) = \lim_{n\to\infty} \{\sin x + 2\sin^2 x + 3\sin^3 x + ... + n\sin^n x\}$$

f(x) = sum of infinite A.G.P.

$$f(x) = \lim_{n\to\infty} \sin x$$

$$\text{Let } S = \{1 + 2\sin x + 3\sin^2 x + ... + n\sin^{n-1} x\}$$

$$S = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$$= \frac{1}{1-\sin x} + \frac{\sin x}{(1-\sin x)^2}$$

$$= \frac{1-\sin x + \sin x}{(1-\sin x)^2}$$

$$= \frac{1}{(1-\sin x)^2}$$

$$\Rightarrow F(x) = \frac{\sin x}{(1-\sin x)^2}$$

$$\Rightarrow (1-\sin x)^2 F(x) = \sin x$$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}} (\sin x)^{\frac{1}{\sin x - 1}} (1^\infty)$$

$$\Rightarrow e^{\lim_{x\to\frac{\pi}{2}} \frac{1}{(\sin x-1)}.(\sin x - 1)} \qquad \Rightarrow e^1$$

Q.70 (2)

For $x > 1$, we have $f(x) = |\log|x|| = \log x$

$$\Rightarrow f'(x) = \frac{1}{x}$$

For $x < -1$, we have

$f(x) = |\log|x|| = |\log(-x)$

$$\Rightarrow f'(x) = \frac{1}{x}$$

For $0 < x < 1$, we have

$$|\log|x|| = -\log x \Rightarrow f'(x) = \frac{-1}{x}$$

For $-1 < x < 0$, we have

$$f'(x) = -\log(-x) \Rightarrow f'(x) = -\frac{1}{x}$$

$$\text{Hence } f'(x) = \begin{cases} \frac{1}{x}, |x|>1 \\ -\frac{1}{x}, |x|<1 \end{cases}$$

Q.71 (3)

$$\text{We have, } f'(x) = \lim_{h\to 0} \frac{f(x-h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan(a(x+h)+b) - \tan(ax+b)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(ax+ah+b)}{\cos(ax+ah+b)} - \frac{\sin(ax+b)}{\cos(ax+b)}}{h}$$

$$= \lim_{h\to 0} \frac{\sin(ax+ah+b)\cos(ax+b) - \sin(ax+b)\cos(ax+ah+b)}{h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0} \frac{a\sin(ah)}{a.h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0} \frac{a}{\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0} \frac{\sin ah}{ah}[ash \to 0, ah \to 0]$$

$$= \frac{a}{\cos^2(ax+b)} = a\sec^2(ax+b)$$

Q.72 (4)

By definition,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{\sin(x+h)}-\sqrt{\sin x}}{h}$$

$$= \lim_{h\to 0}\frac{(\sqrt{\sin(x+h)}-\sqrt{\sin x})(\sqrt{\sin(x+h)}+\sqrt{\sin x}}{h(\sqrt{\sin(x+h)}+\sqrt{\sin x}}$$

$$= \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h(\sqrt{\sin(x+h)}+\sqrt{\sin x}}$$

$$= \lim_{h\to 0}\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}}{2.\frac{h}{2}(\sqrt{\sin(x+h)}+\sqrt{\sin x}}$$

$$= \frac{\cos x}{2\sqrt{\sin x}} = \frac{1}{2}\cot x\sqrt{\sin x}$$

Q.73 (1)

By definition.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(h)}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{\cos(x+h)}-\sqrt{\cos x}}{h}$$

$$= \lim_{h\to 0}\frac{(\sqrt{\cos(x+h)}-\sqrt{\cos x})(\sqrt{\cos(x+h)}+\sqrt{\cos x}}{h(\sqrt{\cos(x+h)}+\sqrt{\cos x}}$$

$$= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h(\sqrt{\cos(x+h)}+\sqrt{\cos x}}$$

$$= \lim_{h\to 0}\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{2.\frac{h}{2}(\sqrt{\cos(x+h)}+\sqrt{\cos x}}$$

$$= \frac{-\sin x}{2\sqrt{\cos x}} = -\frac{\tan x}{2}\sqrt{\cos x}$$

Q.74 (3)

Since, f is an even function

So, f (–x) = f(x) for all x.

Also, f'(0) exists.

So, Rf' (0)= Lf' (0)

$$\Rightarrow \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$\Rightarrow \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$$

$$\Rightarrow \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{f(h)-f(0)}{-h}$$

[using Eq. (i). f(–h) = f(h)]

$$\Rightarrow \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$\Rightarrow 2\lim_{h\to 0}\frac{f(h)-f(0)}{h} = 0 \Rightarrow 2f'(0) = 0 \Rightarrow f'(0) = 0$$

Q.75 (2)

$\therefore$ f(x) = x sin x

$$\Rightarrow f'(x) = \frac{d}{dx}(x\sin x)$$

$$= \sin x\frac{d}{dx}x + x\frac{d}{dx}\sin x = \sin x + x\cos x$$

$$\Rightarrow f'\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} = 1$$

Q.76 (4)

We have, $u = e^x \sin x$

$$\Rightarrow \frac{du}{dx} = e^x\sin x + e^x\cos x = u + v$$

$v = e^x \cos x$

$$\Rightarrow \frac{dv}{dx} = e^x\cos x - e^x\sin x = v - u$$

$$\therefore\ v\frac{du}{dx} - u\frac{dv}{dx} = v(u+v) - u(v-u) = u^2 + v^2$$

$$\frac{d^2u}{dx^2} = \frac{du}{dx} + \frac{dv}{dx} = u + v + v - u = 2v$$

$$\text{and } \frac{d^2v}{dx^2} = \frac{dv}{dx} - \frac{du}{dx} = (v-u)-(v+u) = -2u$$

Q.77 (1)

Let f(x)= $\sqrt{ax+b}$ Then,

$$f(x+h)=\sqrt{a(x+h)+b}=\sqrt{(ax+b)+ah}$$

$$\therefore \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{\sqrt{(ax+b)+ah}-\sqrt{ax+b}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{\sqrt{(ax+b)+ah}-\sqrt{ax+b}}{h}\times\frac{\sqrt{(ax+b)+ah}+\sqrt{ax+b}}{\sqrt{(ax+b)+ah}+\sqrt{ax+b}}$$

$$\Rightarrow \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{(ax+b)+ah-(ax+b)}{h\{\sqrt{ax+b+ah}+\sqrt{ax+b}\}}$$

$$\Rightarrow \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{a}{\sqrt{(ax+b)+ah}+\sqrt{ax+b}}$$

$$\Rightarrow \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{a}{\sqrt{ax+b+ah}+\sqrt{ax+b}}$$

$$\Rightarrow \frac{d}{dx}(f(x))=\frac{a}{\sqrt{ax+b}+\sqrt{ax+b}}$$

Hence, $\frac{d}{dx}(\sqrt{ax+b})=\frac{a}{2\sqrt{ax+b}}$

Q.78 (4)

$$f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$$

Putting x = y = 0, we get f (0) = 0

Putting y = –x, we get f(x) + f (–x) = f(–x) f(0) = (0)

$\Rightarrow$ f(–x) = –f(x)

Also, $\lim_{h\to 0}\frac{f(x)}{x}=2$

Now,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)-f(-x)}{h} \quad \text{[using Eq. (ii) –f(x) = f(–x)]}$$

$$f'(x)=\lim_{h\to 0}\frac{f\left(\frac{x+h-x}{1-(x+h)(-x)}\right)}{h} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\left[\frac{f\left(\frac{h}{1+x(x+h)}\right)}{h}\right]$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)}\times\left(\frac{1}{1+xh+x^2}\right)$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f\left(\frac{h}{1+xh+x^2}\right)}{\left(\frac{h}{1+xh+x^2}\right)}\times\lim_{h\to 0}\frac{1}{1+xh+x^2}$$

$$\left[\text{using } \lim_{h\to 0}\frac{f(x)}{x}=2\right]$$

$$\Rightarrow f'(x)=2\times\frac{1}{1+x^2} \Rightarrow f'(x)=\frac{2}{1+x^2}$$

$$\Rightarrow f'\left(\frac{1}{\sqrt{3}}\right)=\frac{2}{1+\frac{1}{3}}=\frac{2}{4/3}=\frac{6}{4}=\frac{3}{2}$$

EXERCISE-III

Q.1 [2]

$$\ell im_{x\to 0^+} f(x) = \ell im_{x\to 0^+}\frac{\sin^{-1}(1-\{x\}.\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,(1-\{x\})} =$$

$$\ell im_{h\to 0}\frac{\sin^{-1}(1-h).\cos^{-1}(1-h)}{\sqrt{2h}\,(1-h)}$$

$$= \ell im_{h\to 0}\frac{\sin^{-1}(1-h)\sin^{-1}\sqrt{h(2-h)}}{(1-h)\sqrt{2h}} = \ell im_{h\to 0}$$

$$\frac{\frac{\pi}{2}}{1}\;\frac{\sin^{-1}\sqrt{h(2-h)}}{\sqrt{2h-h^2}\,\sqrt{2h}}\;\sqrt{2h-h^2}$$

$$= \ell im_{h\to 0}\frac{\pi}{2}\,.\,1\,.\sqrt{1-\frac{h}{2}}=\frac{\pi}{2}$$

$$\ell im_{x\to 0^-} f(x) = \ell im_{x\to 0^-}\frac{\sin^{-1}(1-\{x\})\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,(1-\{x\})} =$$

$$\lim_{h\to 0} \frac{\sin^{-1} h . \cos^{-1} h}{\sqrt{2(1-h)}\ h} = \frac{\frac{\pi}{2}}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

Q.2 [1]

$$\lim_{x\to\infty} \left(x\sin\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right)\right)$$

$$\lim_{x\to\infty} \left(\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} + \sin\left(\frac{1}{x^2}\right)\right) = 1 + 0 = 1$$

Q.3 [2]

$$\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2} = \lim_{x\to 0}$$

$$\frac{1-\cos^2 x\cos 2x}{(1+\cos x\sqrt{\cos 2x})x^2} = \lim_{x\to 0} \frac{1}{(1+\cos x\sqrt{\cos 2x})} .$$

$$\lim_{x\to 0} \frac{1-\cos^2 x\cos 2x}{x^2}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{1-\left(\frac{1+\cos 2x}{2}\right)\cos 2x}{x^2} = \frac{1}{2}\lim_{x\to 0}$$

$$\frac{2-\cos 2x-(\cos 2x)^2}{2x^2}$$

$$= \frac{1}{4}\lim_{x\to 0} -\frac{1}{x^2}(\cos 2x+2)(\cos 2x-1) = \frac{1}{4}\lim_{x\to 0} (\cos$$

$$2x+2). \lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

$$= \frac{(1+2)}{4} . \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = \frac{3}{4}.2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = \frac{3}{2}$$

$$\Rightarrow \quad \text{limit} = \frac{3}{2} + \frac{1}{2} = 2$$

Q.4 [1]

$\lim_{x\to\infty} f(x)$ exist and is finite & non zero

$$\lim_{x\to\infty} \left[f(x) + \frac{3f(x)-1}{f^2(x)}\right] = 3 \quad \Rightarrow$$

$$\lim_{x\to\infty} f(x) + \frac{3\lim_{x\to\infty} f(x) - 1}{\left[\lim_{x\to\infty} f(x)\right]^2} = 3$$

Let $\lim_{x\to\infty} f(x) = A$

$$A + \frac{3A-1}{A^2} = 3 \quad \Rightarrow \quad A = 1$$

so $\lim_{x\to\infty} f(x) = 1$

Q.5 [0]

$\lim_{x\to 0} f(g(h(x)))$

L.H.L. $x \to 0^-$

$\lim_{x\to 0^-} h(x) = 0^+$

$\lim_{x\to 0^+} f(g(x))$

then $\lim_{x\to 0^+} g(x) = 1^+$

$\lim_{x\to 1^+} f(x) = 1 - 1 = 0$

R.H.L. $x \to 0^+$

$\lim_{x\to 0^+} h(x) = 0^+$

so $\lim_{x\to 0^+} f(g(x)) = 0$

L.H.L. = R.H.L. = 0

Q.6 [1]

$\lim_{x\to 0} g(f(x))$

L.H.L. $= \lim_{x\to 0^-} g[f(x)]$

$\lim_{x\to 0^-} g(\sin x) = \lim_{h\to 0} g(\sinh) = \lim_{h\to 0} (\sin^2 h + 1) = 1$

R.H.L. $= \lim_{x\to 0^+} g[f(x)] = \lim_{x\to 0^+} g(\sin x) = \lim_{h\to 0} g(\sinh)$

$= \lim_{h\to 0} (\sin^2 h + 1) = 1$

L.H.L. = R.H.L. = 1

so $\lim_{x\to 0} g[f(x)] = 1$

Q.7 [2]

$$\lim_{n\to\infty}$$

$$\left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ... + \frac{1}{\sqrt{n^2+2n}}\right)$$

using sandwitch theorem

$$\frac{1}{\sqrt{n^2}} \le \frac{1}{n}$$

$$\frac{1}{\sqrt{n^2+1}} \le \frac{1}{n}$$

$$\vdots \qquad \vdots$$

$$\frac{1}{\sqrt{n^2+2n}} \le \frac{1}{n}$$

adding all these inequilities

$$\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{n^2+2n}} \le \frac{2n}{n}$$

Taking both side $\lim\limits_{n\to\infty}$

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots\ldots\ldots\ldots + \frac{1}{\sqrt{n^2+2n}}\right) = 2$$

Q.8 [1]

$$\lim_{x\to 0^+} x^2\left[\frac{1}{x^2}\right] = \lim_{x\to 0^+} x^2\left(\frac{1}{x^2} - \left\{\frac{1}{x^2}\right\}\right) \Rightarrow$$

$$\lim_{x\to 0^+}\left(1 - x^2\left\{\frac{1}{x^2}\right\}\right) = 1$$

similarly $\lim\limits_{x\to 0^-} f(x) = 1$

Q.9 [11]

$$\lim_{x\to 0}\left(\frac{\sin^{-1}x - \tan^{-1}x}{x^3} + \frac{84x\frac{\pi}{8}}{\sin \pi x}\right) =$$

$$\lim_{x\to 0}\frac{\left[x + \frac{1^2}{3!}x^3 + \ldots\right] - \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\right]}{x^3} + \frac{84}{8}$$

$$= \lim_{x\to 0}\frac{x^3\left(\frac{1}{3!} + \frac{1}{3}\right) + x^5(\ ) + \ldots\ldots}{x^3} + \frac{21}{2} = 11$$

Q.10 [37]

$$\lim_{x\to 0}\frac{x^3}{\sqrt{a+x}(bx - \sin x)} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{\sqrt{a+x}}\lim_{x\to 0}\frac{x^3}{bx - \sin x} = 1$$

$$\Rightarrow \frac{1}{\sqrt{a}}\cdot\lim_{x\to 0}\frac{x^3}{bx - \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\right]} = 1 \Rightarrow$$

$$\frac{1}{\sqrt{a}}\cdot\lim_{x\to 0}\frac{x^3}{(b-1)x + \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots} = 1$$

If limit exists, then $b - 1 = 0 \Rightarrow b = 1$

$$\text{so } \frac{1}{\sqrt{a}}\cdot\lim_{x\to 0}\frac{x^3}{x^3\left[\frac{1}{6} - \frac{x^2}{120} + \ldots\right]} = 1 \Rightarrow$$

$$\frac{1}{\sqrt{a}} \times \frac{1}{\frac{1}{6}} = 1 \Rightarrow a = 36$$

so $a = 36,\ \ b = 1$

Q.11 [20]

$$f(x) = \sum_{\lambda=1}^{n}\left(x - \frac{5}{\lambda}\right)\left(x - \frac{4}{\lambda+1}\right)$$

$$f(0) = \sum_{\lambda=1}^{n}\left(-\frac{5}{\lambda}\right)\left(-\frac{4}{\lambda+1}\right) \Rightarrow f(0)$$

$$= \sum_{\lambda=1}^{n}\left(\frac{20}{(\lambda)(\lambda+1)}\right) \Rightarrow f(0) = 20\sum_{\lambda=1}^{n}\left(\frac{1}{\lambda} - \frac{1}{\lambda+1}\right)$$

$$\Rightarrow f(0) = 20\left(1 - \frac{1}{n+1}\right) \Rightarrow f(0) = \frac{20n}{n+1}$$

Now $\lim\limits_{n\to\infty} f(0) = \lim\limits_{n\to\infty}\dfrac{20n}{n+1} = \lim\limits_{n\to\infty}\dfrac{20}{1+\frac{1}{n}} = \dfrac{20}{1+0} = 20$

Q.12 [12]

$$\text{LHL} = \lim_{h\to 0^+} f(0-h) = \lim_{h\to 0^+} f(-h) = \lim_{h\to 0^+}(-1)^{[h^2]} = 1$$

RHL = $\lim_{h\to0^+} f(0+h) = \lim_{h\to0^+} f(h) = \lim_{h\to0^+}$

$\left(\lim_{n\to\infty}\frac{1}{1+h^n}\right)=1$

Q.13 [21]

$$\lim_{x\to0}\frac{e^{-nx}+e^{nx}-2\cos\frac{nx}{2}-kx^2}{(\sin x-\tan x)}$$

$$\lim_{x\to0} = \frac{2\left[1+\frac{n^2x^2}{2!}+\frac{n^4x^4}{4!}+.....\right]-2\left[1-\frac{n^2x^2}{4.2!}+\frac{n^4x^4}{16.4!}-........\right]-kx^2}{\left(x-\frac{x^3}{3!}+........\right)-\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+.......\right)}$$

$$=\lim_{x\to0}\frac{x^2\left(n^2+\frac{n^2}{4}-k\right)+x^4\left(\frac{2n^4}{4!}-\frac{2n^4}{16.4!}\right)}{x^3\left(-\frac{1}{3!}-\frac{1}{3}\right)+..........}$$

limit exists, if coff. of x^2 is zero.

$\Rightarrow n^2+\frac{n^2}{4}-k=0 \quad \text{Þ} \quad 4k=5n^2$

so the possible value match that is $n=2$

$k=5$

Q.14 [11]

$$\lim_{n\to\infty}\frac{\sum_{r=1}^{n}r^2(n-r+1)}{\sum_{r=1}^{n}r^3}$$

$$=\lim_{n\to\infty}\frac{\sum_{r=1}^{n}(n+1)r^2-\sum_{r=1}^{n}r^3}{\sum_{r=1}^{n}r^3}=$$

$$\lim_{n\to\infty}\left(\frac{(n+1)\frac{(n)(n+1)(2n+1)}{6}}{\frac{n^2(n+1)^2}{4}}-1\right)=\frac{1/3}{1/4}-1=$$

$\frac{4}{3}-1=\frac{1}{3}$

Q.15 [99]

$$\lim_{n\to\infty}\frac{n^{98}}{n^{x-1}\left({}^xC_1-\frac{{}^xC_2}{2n}+\frac{{}^xC_3}{6n^2}-...\right)}=\frac{1}{99}$$

the limit obviously exists if $x-1=98$

Q.16 [1]

$\because \ (1+x)^{\frac{1}{x}}=e\left[1-\frac{x}{2}+\frac{11}{24}x^2-.....\right]$

Now $\ell = \lim_{x\to0}\frac{e-(1+x)^{\frac{1}{x}}}{\tan x} = \lim_{x\to0}$

$$\frac{e-e\left[1-\frac{x}{2}+\frac{11}{24}x^2-.....\right]}{x+\frac{x^3}{3}+\frac{2x^5}{15}+.....}=\frac{e}{2}$$

PREVIOUS YEAR'S

MHT CET

Q.1 (2)
Q.2 (1)
Q.3 (2)
Q.4 (3)
Q.5 (1)
Q.6 (2)
Q.7 (3)
Q.8 (1)
Q.9 (2)
Q.10 (1)
Q.11 (4)
Q.12 (3)

Q.13 (4)

$\lim_{x\to0}\frac{e^{x^2}-\cos x}{x^2}$ $\left[\frac{0}{0}\text{form}\right]$

[using L' Hospital Rule]

$=\lim_{x\to0}\frac{2xe^{x^2}+\sin x}{2x}$ $\left[\frac{0}{0}\text{form}\right]$

$=\lim_{x\to0}\frac{2e^{x^2}+4x^2e^{x^2}+\cos x}{2}$

[using L 'Hospital's rule]

$=\frac{2+0+1}{2}=\frac{3}{2}$

Q.14 (1)

$$\lim_{x\to\sqrt{2}}\frac{x^4-4}{x^2+3\sqrt{2}x-8}$$

$$=\lim_{x\to\sqrt{2}}\frac{\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)\left(x^2+2\right)}{\left(x-\sqrt{2}\right)\left(x+4\sqrt{2}\right)}$$

$$=\lim_{x\to\sqrt{2}}\frac{\left(x-\sqrt{2}\right)\left(x^2+2\right)}{\left(x+4\sqrt{2}\right)}$$

$$=\frac{\left(\sqrt{2}+\sqrt{2}\right)(2+2)}{\left(\sqrt{2}+4\sqrt{2}\right)}=\frac{\left(2\sqrt{2}\right)(4)}{5\sqrt{2}}=\frac{8\sqrt{2}}{5\sqrt{2}}=\frac{8}{5}$$

Q.15 (3)

$$\lim_{x\to0}\frac{1-\cos^3 x}{x\sin x\cos x}$$

$$=\lim_{x\to0}\frac{(1-\cos x)\left(1+\cos^2 x+\cos x\right)}{x^2\cos x.\dfrac{\sin x}{x}}$$

$$=3\lim_{x\to0}\frac{1-\cos x}{x^2}=3x\frac{1}{2}=\frac{3}{2}$$

Q.16 (4)

$$y=\lim_{x\to0}\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x}$$

$$\Rightarrow\ \log y=\lim_{x\to0}\frac{2}{x}\log\left(\frac{a^x+b^x+c^x}{3}\right)$$

$$\Rightarrow\ \log y=\lim_{x\to0}\frac{2\log\left(a^x+b^x+c^x\right)-\log 3}{x}$$

$\Rightarrow$ logy = log $(abc)^{2/3}$

[using L' Hospital's rule]

$\therefore$ y = $(abc)^{2/3}$

Q.17 (2)

$$\because\lim_{n\to\infty}\cos^{2n}x=\begin{cases}1, x=r\pi, r\in I\\ 0, x\neq r\pi, r\in I\end{cases}$$

Here, for $x=10$, $\lim\limits_{x\to\infty}\cos^{2n}(x-10)=1$

and in all other cases it is zero.

$$\therefore\lim_{n\to\infty}\sum_{x=1}^{20}\cos^{2n}(x-10)=1$$

Q.18 (3)

Given $\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2}$

Applying L' Hospital's rule,

$$\Rightarrow\lim_{x\to0}\frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}x^2}=\lim_{x\to0}\frac{\sin x}{2x}$$

Agian applying L' Hospital's rule

$$\Rightarrow\lim_{x\to0}\frac{\frac{d}{dx}\sin x}{\frac{d}{dx}2x}=\lim_{x\to0}\frac{\cos x}{2}=\frac{\cos 0}{2}=\frac{1}{2}$$

Q.19 (2)

Here, $\lim\limits_{x\to0}(\sin x)^{1/x}+\lim\limits_{x\to0}\left(\dfrac{1}{x}\right)^{\sin x}$

$$=0+\lim_{x\to0}e^{\log\left(\frac{1}{x}\right)^{\sin x}}\quad\left[\begin{array}{l}\because\lim\limits_{x\to0}(\sin x)^{1/x}\to0\\ \text{as } 0<\sin x<1\end{array}\right]$$

$$=e^{\lim\limits_{x\to0}\frac{\log\left(\frac{1}{x}\right)}{\operatorname{cosec}x}}=e^{\lim\limits_{x\to0}\frac{x\left(\frac{-1}{x^2}\right)}{-\operatorname{cosec}x\cot x}}$$

[by L' Hospital's rule]

$$=e^{\lim\limits_{x\to0}\frac{\sin x}{x}\tan x}=e^0=1$$

Q.20 (3)

Given, $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\left[1-\tan\left(\frac{x}{2}\right)\right](1-\sin x)}{\left[1+\tan\left(\frac{x}{2}\right)\right](\pi-2x)^3}$

Let $x=\dfrac{\pi}{2}-h$ as $x\to\dfrac{\pi}{2}$, $h\to0$

$$=\lim_{h\to0}\frac{1-\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)}{1+\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)}\frac{1-\cos h}{(2h)^3}$$

$$=\lim_{h\to0}\tan\frac{h}{2}.\frac{2\sin^2\frac{h}{2}}{8h^3}\left[\because\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}\right]$$

$$=\lim_{h\to0}\frac{1}{4}.\frac{\tan\frac{h}{2}}{\frac{h}{2}\times2}\times\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)\times\frac{1}{4}=\frac{1}{32}$$

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Q.1 (11)

$$\lim_{x\to 1}\frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b}=-2$$

For finite limit

$a+b-5=0$ ______ (1)

Apply L' H rule

$$\lim_{x\to 1}\frac{\cos(3x^2-4x+1)(6x-4)-2x}{(6x^2-14x+a)}=-2$$

For finite limit

$6-14+a=0$

$a=8$

From (1) $b=-3$

Now $(a-b)=11$

Q.2 (D)

$$\lim_{x\to 1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}=\frac{0}{0}\text{form}$$

$$\lim_{x\to 1}\frac{(x^2-1)\sin^2(\pi x)}{(x^4-1)-(2x^3-2x)}$$

$$\lim_{x\to 1}\frac{(x^2-1)(\sin^2\pi x)}{(x^2+1)(x^2-1)-2x(x^2-1)}$$

$$\lim_{x\to 1}\frac{(x^2-1)(\sin^2\pi x)}{(x^2-1)(x^2+1-2x)}=\lim_{x\to 1}\frac{(\sin^2\pi x)}{(x-1)^2}$$

$$=\lim_{x\to 1}\left(\frac{\sin\pi x}{x-1}\right)^2 \qquad \text{Put } x=1+h$$

$$=\lim_{h\to 0}\frac{\sin^2\pi(1+h)}{h^2}=\lim_{h\to 0}\frac{(-\sin\pi h)^2}{h^2}$$

$$=\lim_{h\to 0}\left(\frac{\sin\pi h}{\pi h}\right)^2\cdot\pi^2=(1)^2\cdot\pi^2=\pi^2$$

Q.3 **(1)**

$$\lim_{x\to\frac{\pi}{2}}\tan^2 x\left\{\frac{2\sin^2+3\sin x+4-\sin^2 x-6\sin x-2}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}\right\}$$

$$=\lim_{x\to\frac{\pi}{2}}\tan^2 x\left\{\frac{\sin^2 x-3\sin x+2}{3+3}\right\}$$

$$=\frac{1}{6}\lim_{h\to 0}\tan^2\left(\frac{\pi}{2}-h\right)\left\{\sin^2\left(\frac{\pi}{2}-h\right)-3\sin\left(\frac{\pi}{2}-h\right)+2\right\}$$

$$=\frac{1}{6}\lim_{h\to 0}\frac{1}{\tan^2 h}\{\cos^2 h-3\cos h+2\}$$

$$=\frac{1}{6}\lim_{h\to 0}\frac{(\cos h-1)(\cos h-2)}{h^2}=\frac{1}{12}$$

Q.4 **(3)**

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$=\lim_{x\to 0}\frac{\left(\frac{x+\sin x}{2}\right)\left(\frac{x-\sin x}{2}\right).2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4\times\left(\frac{x+\sin x}{2}\right)\left(\frac{x-\sin x}{2}\right)}$$

$$=\lim_{x\to 0}\frac{x^2-\sin^2 x}{2x^4}$$

$$=\lim_{x\to 0}\frac{x^2-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}+.....\right)^2}{2x^4}$$

$$=\lim_{x\to 0}\frac{x^2-(x^2-\frac{2x^4}{6}+...)}{2x^4}$$

$$=\lim_{x\to 0}\frac{\left(\frac{x^4}{3}\right)}{2x^4}=\frac{1}{6}$$

Q.5 (A)

$$\lim_{x\to 7}\frac{18-[1-x]}{[x]-3a}$$

$$\text{L.H.L.}\lim_{x\to 7^-}\frac{18-[1-x]}{[x]-3a}$$

$$=\frac{18-(-6)}{6-3a}$$

$$=\frac{24}{6-3a}$$

$$\text{R.H.L.}\lim_{x\to 7^+}\frac{18-[1-x]}{[x]-3a}$$

$$=\frac{18-(-7)}{7-3a}=\frac{25}{7-3a}$$

Now, L.H.L. = R. H. L.

$$\frac{24}{6-3a}=\frac{25}{7-3a}$$

$\Rightarrow 168-72a=150-75a$

$\Rightarrow 18=-3a$

$\Rightarrow a=-6$

Q.6 **(3)**

$$f(x)=|1+x|+\frac{a^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$$

$$\lim_{x\to 0^-} f(x) = \alpha - \frac{4}{3} \Rightarrow 0 + \frac{\alpha^{-1}-2}{-1} = \alpha - \frac{4}{3}$$

$$\Rightarrow 2 - \frac{1}{\alpha} = \alpha - \frac{4}{3}$$

$$\Rightarrow \alpha + \frac{1}{\alpha} = \frac{10}{3}$$

$$\Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$$

$$\Rightarrow \alpha = 3;\ \alpha \in I$$

Q. 7 (4)

$$\lim_{x\to\frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1-\tan(\cos^{-1} x)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}} \frac{\sin(\sin^{-1}\sqrt{1-x^2}) - x}{1-\tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}} \frac{\sqrt{1-x^2} - x}{1-\left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}} (-x) = -\frac{1}{\sqrt{2}}$$

Q.8 (3)

$$\lim_{x\to 0} \frac{\alpha\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\right)+\beta\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\right)+\gamma\left(x-\frac{x^3}{3!}+\ldots\right)}{x^3}$$

constant terms should be zero

$\Rightarrow \alpha + \beta = 0$

coeff of x should be zero

$\Rightarrow \alpha + \beta + \gamma = 0$

coeff of x^2 should be zero

$$\Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} = 0$$

$$\lim_{x\to 0} \frac{x^3\left(\frac{\alpha}{3!} - \frac{\beta}{3!} - \frac{\gamma}{3!}\right) + x^4\left(\frac{\alpha}{3!} - \frac{\beta}{3!} - \frac{\gamma}{3!}\right)}{x^3} = \frac{2}{3}$$

$$\frac{\alpha}{6} - \frac{\beta}{6} - \frac{\gamma}{6} = 2/3$$

$\Rightarrow \alpha = 1, \beta = -1, \gamma = -2$

Q.9 (1)

$$\lim_{x\to 0} \left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{\frac{100}{x}}$$

From 1^∞

$$= \lim_{e^{x\to 0}}\left[\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right) - 1\right]\times\frac{100}{x}$$

$$= \lim_{e^{x\to 0}} \left[\frac{100}{x}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x) - ((x+2)^3 + 2(X+2)^3 + 3\sin(x+2)))}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)\right]$$

$$= e^{\frac{100}{16+3\sin 2}\lim\limits_{x\to 0}\frac{3(x+2\cos x)^2 x(1+2\sin x) - 3(x+2)^2 - 4(x+2\cos x)}{x(1-2\sin x) - 4(x+2) + 3\cos(x+2\cos x)\times(1-2\sin x) - 3\cos(x+2)}}$$

$$= e^{\frac{100}{16+3\sin 2}\left(\frac{12-3(4)+8\times 1-8+3\cos 2-3\cos 2}{1}\right)}$$

Using L'H rule;

$= e^0 = 1$

Q.10 (1)

$$\lim_{x\to\frac{\pi}{4}} \frac{8\sqrt{2} - (\cos x + \sin x)^7}{\sqrt{2} - \sqrt{2}\sin 2x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{8\sqrt{2} - (\sqrt{2})^7\left(\sin\left(x+\frac{\pi}{4}\right)\right)^7}{\sqrt{2}\,(1-\sin 2x)}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{8\sqrt{2}\left[1-\sin^7\left(x+\frac{\pi}{4}\right)\right]}{\sqrt{2}(1-\sin 2x)}$$

$$= 8\lim_{h\to 0}\left(\frac{1-\cos^7 h}{1-\cos 2h}\right) \qquad \left(\because \text{ put } x = \frac{\pi}{4} + h\right)$$

$$= 8\lim_{h\to 0} \frac{7\cdot\cos^6 h\cdot \sin h}{+2\sin 2h} \qquad (\because \text{ by L'Hospital})$$

$$= \frac{8}{4}\times 7\lim_{h\to 0}\frac{\cos^6 h\cdot \sin h}{\sin h\cdot \cos h} = 14$$

MATHEMATICAL REASONING

EXERCISE-I (MHT CET LEVEL)

Q.1 (a)

$\sim p \vee q$: Raju is not tall or he is intelligent.

Q.2 (b)

Let p: Paris is in France, q: London is in England

$\therefore$ We have $P \wedge q$

Its negation is $\sim(P \wedge q) = \sim P \vee \sim q$

i.e., Paris is not in France or London is not in England.

Q.3 (d)

$\sim(p \vee q) \vee (\sim p \wedge q)$

$(\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow \sim p \wedge (-q \vee q)$

$\Rightarrow \sim p \wedge t \equiv \sim p$

Q.4 **(b)**

Q.5 **(b)**

Q.6 **(a)**

Q.7 **(a)**

Q.8 **(3)**

Here option A, B, & D is mathematical acceptable sentance so these are statement but option C is interogative sentance so it is nto statement.

Q.9 **(3)**

A, B $\rightarrow$ imperative sentence

D $\rightarrow$ exclametry sentence

C $\rightarrow$ Mathematically acceptable statement it is univossal fact

so the sun is a star is a statement.

Q.10 **(3)**

$\sim(p \wedge q) = \sim p \vee \sim q$

$\sim(2 + 3 = 5 \text{ and } 8 < 10) = 2 + 3 \neq 5 \text{ or } 8 \nless 10$

Q.11 **(3)**

$\sim(p \vee q) = \sim p \wedge \sim q$

so monu is not in class X or Anu is not in class XII

Q.12 **(2)**

If p then q is false

p	q	$p \rightarrow q$
T	T	T
T	P	F
F	T	T
F	F	T

$p \rightarrow q$: F

p : T , q : F

Q.13 **(c)**

Q.14 **(3)**

$(\sim p \vee q) \wedge (\sim p \wedge \sim q)$ is

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim p \wedge \sim q$	$(\sim p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

$\therefore$ neither tautology nor contradiction

Q.15 (4)

Fundamental concept of distribution law

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.

Q.16 **(2)**

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \rightarrow \sim p$	$p \rightarrow q \quad \Rightarrow \sim q \rightarrow \sim p$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

hence

$p \rightarrow q \quad \Rightarrow \sim q \rightarrow \sim p$ is tautology

Q.17 (1)

Ram is smart and Ram is intelligent $\Rightarrow (p \wedge q)$

Q.18 (d)

TTTT

Q.19 (b)

Q.20 (d)

Q.21 (d)
Q.22 (d)

Q.23 (2)
It is a fundamental concept.

Q.24 (3)
Contrapositive of $p \Rightarrow \sim q$ is $q \Rightarrow \sim p$

Q.25 (4)
ΔABC is equilateral triangle if each angle is 60° $p \Leftrightarrow q$.

Q.26 (3)
$\sim (p \vee q) \Rightarrow \sim p \wedge \sim q$

Q.27 (3)
$s = p \Rightarrow q \wedge \sim q$ is contradiction

p	s	$p \to s$
T	F	F
F	F	T

neither tautology nor contradiction.

Q.28 (3)
$\sim (p \wedge q)$ v $\sim (q \Leftrightarrow p)$

p	q	$\sim (p \wedge q)$	$\sim (q \Leftrightarrow p)$	s
T	T	F	F	F
T	F	T	T	T
F	T	T	T	T
F	F	T	F	T

Q.29 (2)
Equations are not a statement but 5 is natural no. is a statement.

Q.30 (1)

Q.31 (b)
Q.32 **(b)**
Q.33 **(d)**

Q.34 (1)

p	q	~ p	pvq	~ (pvq)	~ p ^ q	~ (pvq)v(~ p ^ q)
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	F	T
F	F	T	F	T	T	T

So, $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to $\sim p$

Q.35 (2)
$p \to q$ is false only when p is true and q is false.
$p \to (\sim p \vee q)$ is false only when p is true and $(\sim p \vee q)$ is false.
$\sim p \vee q$ is false if q is false, because $\sim p$ is false.

Q.36 (1)

p	q	$\sim p$	$p \Leftrightarrow q$	$\sim p \wedge (p \Leftrightarrow q) = s$	$\sim s = p \vee q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	T	T	F

Q.37 (a)

PREVIOUS YEAR'S

MHT CET

Q.1 (b)

Q.2 (d)

Q.3 (b)

Q.4 (b)

Q.5 (a)

Q.6 (a)

Q.7 (b)

Q.8 (a)

Q.9 (c)

Q.10 (b)

Q.11 (b)

Q.12 (d)

Q.13 (a)

Q.14 (d)

Q.15 (b)

Q.16 (c)

Q.17 (a)

Q.18 (c)

Q.19 (b)
Q.20 (c)

Q.21 (a)
Q.22 (d)
Q.23 (d)
Q.24 (b)

Q.25 (d)

Q.26 (b)

Q.27 (c)

Q.28 (a)

Q.29 (a)

Q.30 (4)
I. p only if q. This says that number is a multiple of 9 only, if it is a multiple of 3.
II. q is a necessary condition for p.
This says that when a number is a multiple of 9, it is necessarily a multiple of 3.
III. ~ q impilies ~ p.
This says that if a number is not a multiple of 3, then it is not a multiple of 9.

Q.31 (1)
Let P : 7 is greater than 4
and q: 6 is less than 7.
Then, the given statement is disjunction $p \vee q$.
Here, ~ p : 7 is not greater than 4.
and ~ q : 6 is not less than 7.
$\therefore$ ~ $(p \vee q)$: 7 is not greater than 4 and 6 is not less than 7.

Q.32 (2)
Let p and q be two propositions given by
p: $2^2 = 5$,q: I get first class.
Then, given statements if $p \to q$.
The contrapositive of this statement is
$\sim q \to \sim p$, i.e. if I do not get first class, then $2^2 \neq 5$.

Q.33 (1)
Let p: Patna is in Bhiar and q : 5 + 6 = 111
Then, the given statement is disjunction $p \vee q$.
Since, p is true and q is false.
\ The disjunction $p \vee q$ is true.
Hence, truth value of given statement is true.

Q.34 (2)
$\because$ p: A man is happy.
q : A man is rich.
Symbolic form of A man is not happy is ~ p.
Symbolic form of A man is not rich is ~ q.
$\therefore$ symbolic form of 'A man is neither happy nor rich' is
$\sim p \wedge \sim q \equiv \sim (p \vee q)$

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Q.1 (9)
If we take

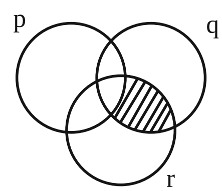
p	q	r	s
F	F	T	F

Truth values of all compound propositions will come true.
Hence, all compound propositions can be made true.

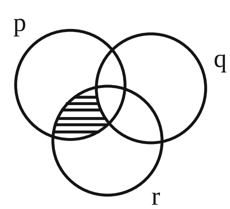
Q.2 (C)
$p \vee q \Rightarrow q$
$\Rightarrow \sim (p \vee q) \vee q$
$\Rightarrow (\sim p \wedge \sim q) \vee q$
$\Rightarrow (\sim p \vee q) \vee (\sim q \vee q)$
$\Rightarrow (\sim p \vee q) \vee t = \sim q \vee q$
Now by taking option C
$(p \wedge q) \Rightarrow \sim p \vee q$
$\Rightarrow \sim p \vee \sim q \vee \sim p \vee q$
$= t$

Q.3 (C)
$(p \vee q) \Rightarrow (-r \vee p)$
We know $p \Rightarrow q = \sim p \vee q$
$\sim(p \vee q) \vee (\sim r \vee p)$
Take negation
$\sim[\sim(p \vee q) \vee (\sim r \vee p)]$
$(p \vee q) \wedge \sim(\sim r \vee p)$
$(p \vee q) \wedge (r \wedge \sim p)$

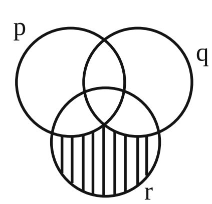
Apply distributive low



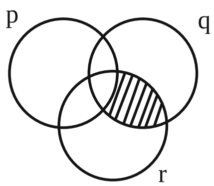
$((p \vee q) \wedge r)((p \vee q) \wedge \sim p)$
$((p \vee q) \wedge r) \wedge \sim p$
Option (A) $p \wedge (\sim q) \wedge r$



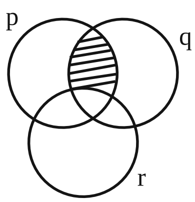
Option (B) $(\sim p) \wedge (\sim q) \wedge r$
$\sim(p \vee q) \wedge r$



Option (C) $(\sim p) \wedge q \wedge r$



Option (D) $p \wedge q \wedge (\sim r)$



Hence correct option is (C)

Q.4 **(2)**

$$(p \wedge q) \rightarrow \underset{(p \wedge \sim q)}{s} \; V \; \overbrace{\{(\sim p) \wedge q\}}^{t}$$

p	q	~ q	~ p	p∧ ~ q	~ p ∧ q	s ∨ t	p ∧ q	p ∧ q → (s ∨ t)	
T	T	F	F	F	F	F	T	F	
T	F	T	F	T	F	T	F	T	(no tautology)
F	T	F	T	F	T	T	F	T	
F	F	T	T	F	F	F	F	T	

$$(p \vee q) \rightarrow \; (p \vee \sim q) \; (\sim p \vee q)$$

p ∨ q	$\overbrace{p\vee \sim q}^{s}$	$\overbrace{\sim p \vee q}^{t}$	s ∨ t	p ∨ q →	s ∨ t
T	T	T	T	T	
T	T	F	T	T	(it is tautology)
T	F	T	T	T	
F	T	T	T	T	

$$(p \rightarrow q) \; \rightarrow \; (p \rightarrow \sim q) \vee \; (\sim p \rightarrow q)$$

p → q	p →~ q	~ p → q	(p →~ q) ∨ (~ p → q)	(p → q) → (p →~ q) ∨ (~ p → q)	
T	F	T	T	T	
F	T	T	T	T	(it is tautology)
T	T	T	T	T	
T	T	F	T	T	

$$(p \leftrightarrow q) \rightarrow \; (p \leftrightarrow \sim q) \vee \; (\sim p \leftrightarrow q)$$

p ↔ q	p ↔~ q	~ p ↔ q	(p ↔~ q) ∨ (~ p ↔ q)	(p ↔ q) → (p ↔~ q) ∨ (~ p ↔ q)	
T	F	F	F	F	
F	T	T	T	T	(no tautology)
F	T	T	T	T	
T	F	F	F	F	

Q.5 (b)

$\sim[(A \cap C) \rightarrow B]$

$\sim[\sim (A \cap C) \vee B]$

$(A \cap C) \wedge \sim B$

$\sim B \wedge (A \cap C)$

Q.6 **(3)**

$$\sim [(\overbrace{(\sim q) \wedge p}^{P}) \Rightarrow (\overbrace{(\sim p) \vee q}^{Q})]$$

$P \wedge \sim Q \Rightarrow (\sim q \wedge p) \wedge \sim \{(\sim p) \vee q\}$

$= (\sim q \wedge p) \wedge (p \wedge \sim q)$

$= (p \wedge \sim q) \wedge (p \wedge \sim q)$

$= (p \wedge \sim q) = \sim (\sim p \vee q)$

$= \sim (p \Rightarrow q)$

Q.7 **(3)**

(A)

p =r	q	~p	r∨□~ p)	(p∧q)	(p∧q)∨r
T	F	F	T	F	T
F	T	T	T	F	F
T	T	F	T	T	T
F	F	T	T	F	F

(B)

p	~q	r∨□~p)	q=r	(p∧q)	(p∧q) ∨r	r∨□~p) ⇒ (p∧q) ∨r
T	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	F

(C)

p	q	r=~p	r∨? ~p)	(p∧q)	(p∧q) ∨r	r∨(~p)⇒ (p∧q) ∨r
T	T	F	F	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	F	T	T	F	T	T

(D)

~p	p	q	r∨(~p)	r = ~q	(p∧q)	(p∧q)∨r	r∨(~p)⇒ (p∧q) ∨r
F	T	T	F	F	T	T	T
F	T	F	T	T	F	T	T
T	F	T	T	F	F	F	F
T	F	F	T	T	F	T	T

Q.8 (D)

$(\sim(p \wedge q) \vee q$

$= (\sim p \vee q) \vee q$

$= \sim p \vee \sim q) \vee q$

$= \sim p \vee t$

= this statement is a tautology option d

$p \Rightarrow (p \vee q)$ is also a tautology.

OR

p	q	P^q	~(P^q)	~(P^q)∨q	p ∨ q	p→(p ∨ q)
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

Q.9 **(C)**
(A) $(\sim q \wedge p) \wedge q) = (\sim q \wedge q) \wedge p = f$
(B) $(\sim q \wedge p) \wedge (p \wedge \sim p) = \sim q \wedge (p \wedge \sim p) = f$
(C) $(\sim q \wedge p) \vee (p \vee \sim p) = (\sim q \wedge p) \vee (t) = t$
(D) $(p \wedge q) \wedge (\sim (p \wedge q)) = f$

Q.10 (C)
$P_1 : \sim (p \rightarrow \sim q)$
$P_2 : (p\wedge \sim q) \wedge ((\sim p) \vee q)$
$p \rightarrow ((\sim p) \vee q)$ is false
$\therefore$ p is true and (~p)v q false
$\therefore$ q is false
p_1 is false
p_2 is false

Q. 11 (1)
Case-I If $\Delta \equiv \nabla \equiv \wedge$
$(p\wedge q) \rightarrow ((p\wedge q)\wedge r)$
it can be flase if r is false,
so not a tautology
Case-II If $\Delta \equiv \nabla \equiv \wedge$
$(p\vee q) \rightarrow ((p\vee q)\vee r) \equiv$ tautology
then $(p\vee q)\vee r \equiv (p\Delta r)\vee q$
Case-III If $\Delta = \vee$, $\nabla = \wedge$
then $(p\wedge q) \rightarrow \{(p\vee q)\wedge r\}$
Not a tautology
(Check $p \rightarrow T$, $q \rightarrow T$, $r \rightarrow F$)
Case-IV If $\Delta = \wedge$, $\nabla = \vee$
then $(p\wedge q) \rightarrow \{(p\wedge q)\vee r\}$
Not a tautology

Q.12 (C)
$s_1 : (\sim p \vee q) \vee (\sim p \vee r)$
$= \sim p \vee (q \vee r)$
$s_2 : p \rightarrow (q \vee r)$
$\equiv \sim p \vee (q \vee r) \rightarrow$ By conditional law
$s_1 \equiv s_2$

Q.13 (4)
$\sim (A \Leftrightarrow B) \equiv (A\wedge \sim B) \vee (\sim A \wedge B)$
$\therefore \ \sim [(P \wedge \bar{R}) \Leftrightarrow \bar{Q}]$
$[(P \wedge \bar{R}) \wedge Q] \vee [(\sim P \vee R) \wedge \bar{Q}]$

Q.14 (4)
$p \equiv$ Ramesh listens to music
$\sim q \equiv$ He is in village
$r \vee s \equiv$ Saturday or Sunday
$p \Rightarrow ((\sim q) \wedge (r \vee s))$

Q.15 (4)

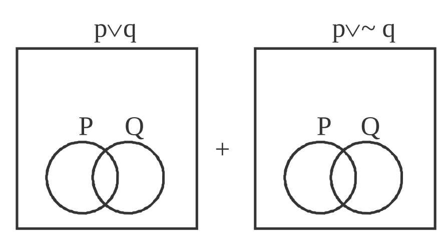
p	q	~p	~q	~p v q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Check opiton Ans.4

Q.16 (2)
$(p \rightarrow q) \vee (p \rightarrow r)$
$(\sim p \vee q) \vee (\sim p \vee r)$
$= \sim p \vee (q \vee r)$
$= p \rightarrow (q \vee r) \equiv (3)$ is true
Now, (1) $(p \wedge \sim r) \rightarrow q$
$1 \sim (p \wedge \sim r) \vee q = (\sim p \vee r) \vee q$
$= \sim p \vee (r \vee q) = p \rightarrow (q \vee r)$
(4) $(p \wedge \sim q) \rightarrow r = p \rightarrow (q \vee r)$

Q.17 (4)
$(p \wedge q) \Rightarrow (p \wedge r)$
$\sim (p \wedge q) \vee (p \wedge r)$
$(\sim p \vee \sim q) \vee (p \wedge r)$
$(\sim p \vee (p \wedge r)) \vee \sim q$
$(\sim p \vee p) \wedge (\sim p \vee r) \vee \sim q$
$(\sim p \vee r) \vee \sim q$
$(\sim p \vee \sim q) \vee r$
$\sim (p \wedge q) \vee r$
$(p \wedge q) \Rightarrow r$

Q.18 (B)
Well check each option
For (A) $* = \vee$ of $\odot = \Lambda$
$(p \vee q) \vee (p\vee \sim q)$
$\equiv p \vee (q\wedge \sim q)$
$\equiv p\vee$ (contradiction) $\equiv p$
For B: $* = \vee, \odot = \vee$
$(p\vee q) \vee (p\vee \sim q) \equiv t$
using Venn diagrams



Q.19 (3)
$(p\Lambda r) \leftrightarrow (p\Lambda \sim q) \equiv \sim P$
When r = ?

P	~p	q	~q	pΛ~q	pΛq	p↔ (pΛ~q)	f↔ (pΛ~q)	(pΛq)↔ (pΛ~q)
T	F	T	F	F	T	F	T	F
T	F	F	T	T	F	T	F	F
F	T	T	F	F	F	T	T	T
F	T	F	T	F	F	T	T	T

(A) r = p

$(p\Lambda r) \leftrightarrow (p\Lambda \sim q) \equiv P \leftrightarrow (p\Lambda - q)$

(B) r = ~ P

$p\Lambda \sim p = F$

$F \leftrightarrow (p\Lambda \sim q)$

(C) r = q

$(p\Lambda q) \leftrightarrow (p\Lambda \sim q)$

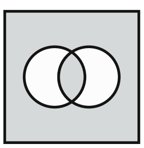
Option (c) is correct

Q.20 (3)

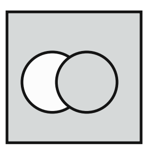
Q.21 (4)

$(\sim(p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



$p \Leftrightarrow q$



$p \rightarrow q$

Q.22 **[4]**

$X \Rightarrow Y$ is false

When X is true and Y is false

So $P \rightarrow T, Q \rightarrow F, R \rightarrow F$

(A) $P \vee Q \rightarrow \sim R$ is T

(B) $R \vee Q \rightarrow \sim P$ is T

(C) $\sim (P \vee Q) \rightarrow \sim R$ is T

(D) $\sim (R \vee Q) \rightarrow \sim P$ is F

STATISTICS

EXERCISE-I (MHT CET LEVEL)

Q.1 (b)

Mean $\overline{x} = \frac{\sum x}{n}$ or $\sum x = n\,\overline{x}$

$\sum x = 25 \times 78.4 = 1960$

But this $\sum x$ is incorrect as 96 was misread as 69.

$\therefore$ correct $\sum x = 1960 + (96 - 69) = 1987$

$\therefore$ correct mean $= \frac{1987}{25} = 79.48$

Q.2 (a)
First ten odd numbers are
1,3,5,7,11,13,15,17,19 respectively. So
A.M.

$(\overline{x}) = \frac{1+3+5+7+9+11+13+15+17+19}{10}$

$= \frac{100}{10} = 10$

Q.3 (1)

Data x	Mean $\overline{x}$
$x = ap + bQ$	$\overline{x} = a\overline{p} \times b\overline{Q}$

Q.4 (1)

$\sum (x_i - \overline{x}) = \sum x_i - n\overline{x}$

$= n\overline{x} - \overline{x} \cdot n$

$= 0$

Q.5 (2)
$P = P_1 . P_2 \dots P_n$

Q.6 (1)

$n\overline{x} = n_1\overline{x}_1 + n_2\overline{x}_2$

$12 \times 6 = 6 \times 8 + 6 \times \overline{x}_2$

$\overline{x}_2 = \frac{72-48}{6} = \frac{24}{6} = 4$

Q.7 (4)
According to question x_2 is replaced by t then

$\overline{x} = \frac{n\overline{x} - x_2 + t}{n}$

Q.8 (1)

Q.9 (4)

x_i	$(x_{(i+1)})x_i$
1	$(1+1)_1$
2	$(2+1)_2$
3	$(3+1)_3$
n	$(n+1)_n$

$\frac{\sum (x_i+1)x_i}{n(n+1)} = \frac{2+6+12+....(n+1)^n}{n(n+1)}$

Q.10 (4)
Mode = 3 median – 2 Mean
121 = 3 median – 2 × 91

$\frac{121+182}{3} = \frac{303}{3} = 101$

Q.11 (1)

x_i	S.D.(s)
$x_i \pm \lambda$	s
λx_i	$\lvert\lambda\rvert s$
$\frac{x_i}{\lambda}$	$\frac{s}{\lvert\lambda\rvert}$

S.D of px + q is |p| s

Q.12 (2)

Q.13 (1)

x_i	s
$x_i \pm \lambda$	s
$\lvert\lambda\rvert x_i$	$\lvert\lambda\rvert s$
$\frac{x_i}{\lvert\lambda\rvert}$	$\frac{s}{\lvert\lambda\rvert}$

S.D. of $\frac{a_x + b}{c}$ is $\left|\frac{a}{c}\right|s$

Q.14 (b)

$-\frac{1}{2}$, We have $S_K = = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$

$\frac{41-45}{8} = -\frac{1}{2}$

Q.15 (b)

Mode = 3 Median -2 Mean

$$\therefore \text{ Median } = \frac{1}{3}(\text{mode} + 2\,\text{mean}) = \frac{1}{3}(60 + 2\times 66) = 64$$

Q.16 (b)

We know that,

Mode = 3 Medium –2 Mean

= 3(22) – 2(21)

= 66 – 42 = 24

Q.17 **(c)**

Q.18 **(b)**

Q.19 **(a)**

Q.20 (3)

$$\sigma = \frac{\sum f_i x_i^2}{\Sigma f_i} - \left(\frac{\sum f_i x_i}{\Sigma f_i}\right)^2$$

Q.21 (2)

$$n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$$

$$= n_1\frac{k}{n_1} + n_2$$

$$\boxed{n_2 = n\bar{x} - K}$$

Q.22 (4)

x_i	$\bar{x}$
$\frac{x_i}{\lambda}$	$\frac{\bar{x}}{\lambda}$

then new mean after each number is divided by 3 is $\frac{\bar{x}}{3}$

Q.23 (3)

x_i	w_i	$x_i wi$
0	0	0
1	1	1^2
2	2	2^2
3	3	3^2
4	4	4^2
⋮	⋮	⋮
n	n	n^2

$$\frac{\sum x_i w_i}{\sum w_i} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \boxed{\frac{2n+1}{3}}$$

Q.24 (2)

A.M. = of 1 + 2 + 4 + 8 + 16 +...... 2^n

$$= \frac{2^{n+1} - 1}{n+1}$$

Q.25 (1)

In central tendency we measure mean, mode, median.

Q.26 (1)

Most stable measure of central tendency is mean.

Q.27 (3)

$$n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$$

$$10\bar{x} = 7\times 10 + 3\times 5$$

$$\bar{x} = \frac{70+15}{10} = \frac{85}{10} = 8.5$$

Q.28 (3)

A statistical measure which can not be determined graphically is harmonic mean it is a fandomental concept.

Q.29 (1)

The measure which takes into account all the data item is mean it is a fandamental concept of account

Q.30 (3)

$$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \Sigma x = n\bar{x}$$

= 15 × 154

= 2310

Σx = 2310 – 145 + 175

= 2340

correct mean = $\frac{2340}{15} = 156$ c.m.

Q.31 (2)

For median arrange

scored in order

0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

Median is $\left(\frac{n+1}{2}\right)^{th}$ term

$\frac{11+1}{2}$ = 6^{th} term = 27.

Q.32 (1)

Total $\Rightarrow \Sigma x = n\bar{x}$ = 10 × 12.5 = 125

First six $\Rightarrow \Sigma x = n\bar{x}$ = 6 × 15 = 90

Last five $\Rightarrow \Sigma x = n\bar{x}$ = 5 × 10 = 50

Last four 125 – 90 = 35

6^{th} no is 50 – 35 = 15

Q.33 (1)

$$AM = \frac{a+b}{2} = 10$$

$$G.M. = \sqrt{ab} = 8$$

$$H.M = \frac{2ab}{a+b} = ?$$

$$H.M. = \frac{(G.M.)^2}{A.M.}$$

$$= \frac{64}{10} = 6.4$$

And number are 16, 4

Q.34 (3)

n = 10

$\overline{x} = 12$

$\Sigma x^2 = 1530$

$$\sigma^2 = \frac{1}{n}\Sigma\left(x_1^2 - \overline{x}^2\right)$$

$$\sigma^2 = \frac{1}{10}\,[1530 - 10(144)] = \frac{90}{10} = 9$$

σ = 3

$\overline{x} = 12$

$$C.O.V. = \frac{\sigma}{x} \times 100 = \frac{3}{12} \times 100 = 25\%$$

Q.35 (1)

$$AM = \frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}{n+1}$$

$$= \frac{2^n}{n+1}$$

Q.36 (3)

$\overline{x}_1 = 50$	$\sigma_1^2 = 15$
$\overline{x}_2 = 48$	$\sigma_2^2 = 12$
$\overline{x}_3 = 12$	$\sigma_3^2 = 2$

Most consistant is kapil

Q.37 (2)

$$\overline{x} = \frac{\Sigma x_i}{n} = \frac{\Sigma(x_i + 2i)}{n}$$

$$= \frac{\Sigma x_i}{n} + \frac{2\Sigma i}{n}$$

$$= \overline{x} + \frac{2n(n+1)}{2n}$$

$$= \overline{x} + (n+1)$$

Q.38 (2)

$$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$= \frac{1^2 + 2^2 + 3^2 + ... + n^2}{n} - \left(\frac{1+2+3+....+n}{n}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)}{2n}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4n^2}$$

$$= \frac{n^2 - 1}{12}$$

Q.39 (1)

$$\overline{x} = \frac{\Sigma x}{n} \Rightarrow M = \frac{\Sigma x}{n} \Rightarrow \Sigma x = nM$$

sum of n – 4 observations is a

mean of remaing 4 observation is $\frac{nM - a}{4}$

Q.40 (3)

Mean of series is

$$\overline{x} = \frac{a + (a+d) + (a+2d) + ... + (a+2nd)}{(2n+1)}$$

$$\overline{x} = a + nd$$

$$\therefore \sum_{i=0}^{2n} |x_i - \overline{x}| \Rightarrow \frac{2d(n)(n+1)}{2} \Rightarrow n(n+1)d$$

$$\therefore \text{ Mean deviation} = \frac{n(n+1)d}{(2n+1)}$$

Q.41 (3)

$$\overline{x} = \frac{\Sigma x_i}{n}$$

$$= \frac{x_1 + 1 + x_2 + 2 + ...}{n}$$

$$= \frac{x_1 + x_1 + ... + x_n}{n} + \frac{1 + 2 + ...n}{n}$$

$$= \overline{x} + \frac{n(n+1)}{2n}$$

$$= \overline{x} + \left(\frac{n+1}{2}\right)$$

Q.42 (4)

Quartile deviation = $\frac{\theta_3-\theta_1}{2}=\frac{40-20}{2}=10$

Q.43 (3)

If x_1, x_2 .... x_n are n observations with frequencies f_1, f_2 .... f_n, then mean deviation from mean (m) is given by

Mean deviation = $\frac{1}{N}\Sigma f_i\,|x_i - M|$

Q.44 (4)

x_i	σ
x	4
$\frac{x}{4}$	$\frac{4}{\|4\|}=1$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (a)

Marks	No.of students	c.f.
10 – 20	12	12
20 – 30	30	42
30 – 40	?	42 + x
40 – 50	65	107 + x
50 – 60	?	107 + x + y
60 – 70	25	132 + x + y
70 – 80	18	150 + x + y
Total	229	

$\frac{N}{2}=\frac{229}{2}=114.5,$ Median=46

$\therefore$ Median class=40-50

$\therefore$ l =40, c.f 42+x, f = 65, h = 10

Median= $l+\left(\frac{\frac{n}{2}-c.f.}{f}\right)\times h$

$$46=40+\frac{114.5-(42+X)}{65}\times 10$$

Or 46-40= $\frac{(114.5-42-x)}{13}\times 2$

$$6=\frac{(72.5-x)}{13}\times 2\ or\ 78=145-2x$$

2x=145-78=67 or $x=\frac{67}{2}=33.5$

$\therefore$ x=34

($\therefore$ Number of students cannot be in fraction) Now

$\sum f_1 = 29 \therefore x+y+150=229$

x+y=229-150=79 …(i)

Putting the value of x in (i), we get

34x + y = 79 $\Rightarrow$ y=79-34=45

$\therefore$ x=34, y=45

Q.2 (3)

x_i	f_i	$x_i f_i$
1	2	2
2	2	4
3	2	6
$\vdots$	$\vdots$	$\vdots$
n	2	2n

$$\frac{\sum x_i f_i}{\sum f_i}=\frac{2+4+6+.......2n}{2+2+......2}$$

$$=\frac{2(1+2+3+....n)}{2n}$$

$$=\frac{2\frac{(n(n+1))}{2}}{2n}$$

$$=\frac{n+1}{2}$$

Q.3 (d)

Since, Mean $=\frac{\sum f_i x_i}{\sum f_i}$ where xi are observations with frequencies f_i, i = 1, 2, ...n

The required mean is given by

$$\bar{X}=\frac{0.1+1.{}^nC_1+2.{}^nC_2+....+n.{}^nC_n}{1+{}^nC_1+{}^nC_2+....+{}^nC_n}$$

$$=\frac{\sum_{r=0}^{n} r.{}^nC_r}{\sum_{r=0}^{n}{}^nC_r}=\frac{\sum_{r=0}^{n} r.\frac{n}{r}\,{}^{n-1}C_{r-1}}{\sum_{r=0}^{n}{}^nC_r}$$

$$= \frac{n\sum_{r=0}^{n} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^{n}C_{r}} = \frac{n.2^{n-1}}{2^n} = \frac{n}{2}$$

Q.4 (a)

We construct the following table taking assumed mean a = 55 (step deviation method).

Class	x_i	f_i	c.f.	$u_i = \frac{x_i - a}{10}$	f_iu_i
10-20	15	2	2	–4	–8
20-30	25	3	5	–3	–9
30-40	35	4	9	–2	–8
40-50	45	5	14	–1	–5
50-60	55	6	20	0	0
60-70	65	12	32	1	12
70-80	75	14	46	2	28
80-90	85	10	56	3	30
90-100	95	4	60	4	16
Total		60			56

$\therefore$ The mean = $a + \frac{\sum f_iu_i}{\sum f_i} \times h$

$$= 55 + \frac{56}{60} \times 10 = 55 + \frac{56}{6} = 64.333$$

Here n = 60 $\Rightarrow \frac{n}{2} = 30$, therefore, 60 – 70 is themedian class

Using the formula :

$$M = \ell + \frac{\frac{n}{2} - C}{f} \times c = 60 + \frac{30 - 20}{12} \times 10$$

$$= 60 + \frac{100}{12} = 60 + 8.333 = 68.333$$

Q.5 (2)

x_i	w_i	x_iw_i
1	1^2	1^3
2	2^2	2^3
3	3^2	3^3
⋮	⋮	⋮
n	n^2	n^3

$$\bar{x} = \frac{\sum xiwi}{\sum wi} = \frac{1^3 + 2^3 + 3^3 + ... + n^3}{1^2 + 2^2 + 3^2 + ... + n^2}$$

$$= \frac{\left[\frac{n(n+1)}{2}\right]^2}{\frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{n^2(n+1)^2}{4} \times \frac{6}{n(n+1)}\ (2n+1)$$

$$= \frac{3n(n+1)}{2(2n+1)}$$

Q.6 (3)

x_i	f_i
1	1
2	1
3	1
:	:
n	1

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1 + 2 + 3 + ...n}{n}$$

$$= \frac{n(n+1)}{2n}$$

$$= \left(\frac{n+1}{2}\right)$$

Q.7 (a)

$$\because \quad \delta_x^2 = \frac{\sum d^2 i}{n}$$

But both A and B have 100 observations, then both the sample A and B have same standard deviation and the same veriance.

Hence, $\frac{V_A}{V_B} = 1$

Q.8 (c)

Occurance (x_i)	Frequency (f_i)	Freq* (x_i)	(x_i-mean)	(x_i-mean)^2	f_i (x_i-mean)^2
3.5	3	10.5	-3.59	12.887	38.661
4.5	7	31.5	-2.59	6.707	46.952
5.5	22	121	121	2.528	55.609
6.5	60	390	-0.59	0.348	20.876
7.5	85	637.5	0.41	0.168	14.298
8.5	32	272	1.41	1.988	63.632
9.5	8	76	2.41	5.809	46.47
Total	217	1538.5	-	-	286.498

$$\text{Variance } \sigma^2 = \frac{\Sigma f_i\left(x_i - \overline{x}\right)^2}{\Sigma f_i} = \frac{286.49}{217}$$

$$= 1.32$$

Q.9 (a)

We know that $Q \cdot D = \frac{5}{6} \times M \cdot D \cdot = \frac{5}{6} \times 12 = 10$

$$\therefore \text{S} \cdot \text{D} = \frac{3}{2} \times Q \cdot D \cdot = \frac{3}{2} \times 10 \Rightarrow S \cdot D \cdot = 15$$

Q.10 (1)

$\Sigma x = n\overline{x} = 100 \times 50 = 5000$

S.D. = $\sqrt{\sigma^2}$

$4 = \sqrt{\sigma^2}$

$= \sqrt{\frac{1}{n}\Sigma x_i^2 - \overline{x}^2}$

$= \sqrt{\frac{\Sigma x^2}{100} - (50)^2}$

$16 = \frac{\Sigma x_i^2}{100} - 2500$

$(16 + 2500) \cdot 100 = \Sigma x_i^2$

$\boxed{251600 = \Sigma x_i^2}$

Q.11 (1)

S.D. = $\sqrt{\frac{1}{N}\Sigma x_i^2 - \overline{x}^2}$

x_i	f_i
1	${}^nC_0$
a	${}^nC_1$
a^2	${}^nC_2$
⋮	⋮
a^n	${}^nC_n$

$$\overline{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{{}^nC_0 + {}^{an}C_1 + {}^{a^2 n}C_2 + {}^{a^n n}C_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}$$

$$\frac{\Sigma f_i x_i^2}{N} = \frac{{}^nC_0 + {}^{a^2 n}C_1 + {}^{a^4 n}C_2 + {}^{a^6 n}C_3 + ... a^{2n\,n}C_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n}$$

Q.12 (a)

We have $\frac{1}{n}\sum_{i=1}^{n}\left(x_i + 2\right)^2 = 18$ and

$$\frac{1}{n}\sum_{i=1}^{n}\left(x_i - 2\right)^2 = 10$$

$$\Rightarrow \sum_{i=1}^{n}\left(x_i - 2\right)^2 = 18n \text{ and}$$

$$\sum_{i=1}^{n}\left(x_i - 2\right)^2 = 10n$$

$$\Rightarrow \sum_{i=1}^{n}\left(x_i + 2\right)^2 = 18n + \sum_{i=1}^{n}\left(x_i - 2\right)^2 = 28n$$

$$\text{and } \sum_{i=1}^{n}\left(x_i + 2\right)^2 - \sum_{i=1}^{n}\left(x_i - 2\right)^2 = 8n$$

$$\Rightarrow 2\sum_{i=1}^{n}\left(x_i + 4\right)^2 = 28n \text{ and } 2\sum_{i=1}^{n} 4x_i^{\,2} = 8n$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 + 4n = 14n \text{ and } \sum_{i=1}^{n} x_i = n$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 + 10n \text{ and } \sum_{i=1}^{n} x_i = n$$

$$\therefore \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1'}^{n} x_i\right)^2} = \sqrt{\frac{10n}{n} - \left(\frac{n}{n}\right)^2} = 3$$

Q.13 (1)

Arrange is accending order

$$\Rightarrow \; t - \frac{7}{2}, t - 3, t - \frac{5}{2}, t - 2, t - \frac{1}{2}, t + \frac{1}{2}, t + 4, t + 5$$

$$\Rightarrow \; \frac{1}{2}\left[4^{th} + 5^{th} \text{ value}\right]$$

$$\Rightarrow \; \frac{1}{2}\left[2t - \frac{5}{2}\right]$$

$\Rightarrow \quad t - \frac{5}{4}$

Q.14 **(c)**

Q.15 (3)

$n_1 = 100$ | $n_2 = 150$

$\bar{x}_1 = 50$ $\quad \bar{x}_2 = 110$

$\sigma_1^2 = 5$ $\quad \sigma_2^2 = 6$

$n\bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$

$= 100 \times 50 + 150 \times 40$

$= 5000 + 6000$

$\bar{x} = \frac{11000}{250} = 44$

$\sigma^2 = n_1 \frac{(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$

$d_1 = 50 - 44 = 6$

$d_2 = 40 - 44 = -4$

$\sigma^2 = 100 \frac{(25+36)+150(36+16)}{250}$

$= \frac{6100+7800}{250} = 55.6$

$\sigma = \sqrt{55.6} = 7.46$

Q.16 (1)

$CV_1 = 58\%$ $\quad CV_2 = 69\%$

$\sigma_1 = 21.2$ $\quad \sigma_1 = 15.6$

$CV = \frac{\sigma}{x} \times 100$

$CV_1 = \frac{\sigma_1}{x_1} \times 100 \Rightarrow \bar{x}_1 = \frac{\sigma_1 \times 100}{CV_1} = \frac{21.2 \times 100}{58}$

$= \frac{2120}{58} = 36.55$

$CV_2 = \frac{\sigma_2}{x_2} \times 100 \Rightarrow \bar{x}_2 = \frac{\sigma_2 \times 100}{CV_2} = \frac{15.6 \times 100}{69}$

$= 22.60$

Q.17 **(c)**

Q.18 **(a)**

Q.19 (1)

x_i	S.D.
$x_i \pm \lambda$	s
λx_i	$\lvert\lambda\rvert s$
$\frac{x_i}{\lambda}$	$\frac{s}{\lvert\lambda\rvert}$

then S.D. of ax + b is |a| s

where s is stindered deviation.

Q.20 (1)

r = range

$\text{S.D.} = S^2 = \frac{1}{n-1} \sum_{i=0}^{n} (x_i - \bar{x})^2$

then $S \le r\sqrt{\frac{n}{n-1}}$

EXERCISE-III

Q.1 [11]

Mean of $1^2, 2^2, 3^2, \ldots\ldots, n^2$ is

$\frac{1^2 + 2^2 + 3^2 + \ldots.n^2}{n} = \frac{\Sigma n^2}{n}$

$\frac{46n}{11} = \frac{n(n+1)(2n+1)}{6n}$

$\Rightarrow 22n^2 + 33n + 11 - 276n = 0$

$\Rightarrow (n - 11)(22n - 1) = 0$

$\Rightarrow n = 11$ and $n \ne \frac{1}{22}$

Q.2 [10.1]

$\bar{x} = \frac{\text{sum of quantities}}{n} = \frac{\frac{n}{2}(a+1)}{n}$

$= \frac{1}{2}[1+1+100d] = 1+50d$

$\therefore \quad MD = \frac{1}{n}\Sigma\lvert x_i - \bar{x}\rvert$

$\Rightarrow 255 = \frac{1}{101}[50d + 49d + \ldots.. + d + 0 + d + 49d + 50d]$

$= \frac{2d}{101}\left[\frac{50 \times 51}{2}\right]$

$\Rightarrow d = \frac{255 \times 101}{50 \times 51} = 10.1$

Q.3 [16]

$\bar{x} = 5$

$\text{Variance} = \frac{1}{n}\Sigma x_i^2 - (\bar{x}^2)$

$$0=\frac{1}{n}\cdot 400-25$$

$$\Rightarrow\ n=\frac{400}{25}=16$$

Q.4 [10]

Given that, $n_1=10, \bar{x}_1=12, n_2=20, \bar{x}_2=9$

$$\therefore\ \bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}=\frac{10\times 12+20\times 9}{10+20}$$

$$=\frac{120+180}{30}=\frac{300}{30}=10$$

Q.5 [30.1]

Given that, $\Sigma_{i=1}^{20}\left(x_i-30\right)=2$

$$\Rightarrow\ \sum_{i=1}^{20}x_i-\sum_{i=1}^{20}(30)=2$$

$$\Rightarrow\ \bar{x}=\frac{20.30}{20}+\frac{2}{20}$$

$= 30 + 0.1 = 30.1$

Q.6 [24]

Given series is 148, 146, 144, 142, ... whose first term and common difference is

$a = 148$, $d = (146 – 148) = –2$

$$S_n=\frac{n}{2}\left[2a+(n+1)d\right]=125\text{ (given)}$$

$$\Rightarrow 125n=\frac{n}{2}\left[2\times 148+(n-1)\times(-2)\right]$$

$\Rightarrow n^2 – 24n = 0 \Rightarrow n(n – 24) = 0$

$\Rightarrow\ n = 24\ (n \neq 0)$

Q.7 [5]

Given that, $n_1=4,\ \bar{x}=7.5,\ n_1+n_2=10,\ \bar{x}=6$

$$\therefore\ 6=\frac{4\times 7.5+6\times\bar{x}_2}{10}$$

$$\Rightarrow\ 60=30+6\bar{x}_2$$

$$\Rightarrow\ x_2=\frac{30}{6}=5$$

Q.8 [44.46]

Total of corrected observations

$= 4500 – (91 + 13) + (19 + 31)$

$= 4446$

$$\therefore \text{Mean}=\frac{4446}{100}=44.46$$

Q.9 [63]

Total marks of 10 failed students

$= 28 \times 10 = 280$

and Total marks of 50 students = 2800

$\therefore$ Total marks of 40 passed students

$= 2800 – 280 = 2520$

$\therefore$ Average marks of 40 passed students

$$=\frac{2520}{40}=63$$

PREVIOUS YEAR'S

MHT CET

Q.1 (4)

$$\bar{x}=5,\ \text{variance}=\frac{1}{2}\Sigma x_i^2-\left(\bar{x}\right)^2$$

$$\Rightarrow\ 0=\frac{1}{n}.400-25$$

$$\Rightarrow\ n=\frac{400}{25}=16$$

Q.2 (4)

Variance of first n natural numbers

$$=\frac{\Sigma n^2}{n}-\left(\frac{\Sigma n}{n}\right)^2$$

$$=\frac{n(n+1)(2n+1)}{6n}-\left(\frac{n(n+1)}{2n}\right)^2$$

$$=(n+1)\left[\frac{2n+1}{6}-\frac{(n+1)}{4}\right]$$

$$=\frac{(n+1)}{12}\left[4n+2-3n-3\right]$$

$$=\frac{(n+1)}{12}\times(n-1)=\frac{n^2-1}{12}$$

Q.3 (4)

Standard deviation

$$\sqrt{\frac{\sum_{j=1}^{18}\left(x_j-8\right)^2}{n}-\left(\frac{\sum_{j=1}^{18}\left(x_j-8\right)}{n}\right)^2}$$

$$= \sqrt{\frac{45}{18} - \left(\frac{9}{18}\right)^2} = \sqrt{\frac{45}{18} - \frac{1}{4}}$$

$$= \sqrt{\frac{81}{36}} = \frac{9}{6} = \frac{3}{2}$$

Q.4 (2)

Let $x_1, x_2, ..., x_n$ be n variates. Then, their arithmetic mean will be

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad ...(i)$$

Now, the sum of deviation of the variates from the AM, i.e. SD $(\bar{x})$

$$= \sum_{i=1}^{n}(x_i - n\bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$$

$$= \sum_{i=1}^{n} x_i - n\bar{x} = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i \text{ [from Eq. (i)]}$$

$= 0$

Q.5 (3)

Given that, $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$

$\because \quad \sigma^2 \geq 0$

$$\therefore \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \geq 0 \Rightarrow \frac{400}{n} - \frac{6400}{n^2} \geq 0$$

$\therefore \quad n \geq 16$

Q.6 (1)

According to the question,

$$6.80 = \frac{(6-a)^2 + (6-b)^2 + (6-5)^2 + (6-8)^2 + (6-10)^2}{5}$$

$\Rightarrow 34 = (6 - a)^2 + (6 - b)^2 + 1 + 4 + 16$

$\Rightarrow (6 - a)^2 + (6 - b)^2 = 13 = 9 + 4$

$\Rightarrow (6 - a)^2 + (6 - b)^2 = 3^2 + 2^2$

$\therefore \quad a = 3, b = 4$

Q.7 (3)

We known that,

$$\bar{x} = \frac{\text{Sum of quantities}}{n} = \frac{\frac{n}{2}(a + I)}{n}$$

$$= \frac{1}{2}(1 + 1 + 100d) = 1 + 50\,d$$

$$\therefore \text{ Mean deviation } = \frac{1}{n}\Sigma|x_i - \bar{x}|$$

$$\Rightarrow 255 = \frac{1}{101}[50d + 49d + 48d + ... + d + 0\,d + ... + 50d$$

$$= \frac{2d}{100}\left(\frac{50 \times 51}{2}\right)$$

$$\therefore \quad d = \frac{255 \times 101}{50 \times 51} = 10.1$$

Q.8 (2)

$$\text{Variance, } \sigma^2 = \frac{1}{n}\Sigma x_i^2 - (\bar{x})^2$$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{2 + 4 + 6 + 8 + ... + 100}{50}$$

$$= \frac{50 \times 51}{50} \text{ [} \because \Sigma 2n = n(n+1)\text{, here n = 50]}$$

$= 51$

$$\therefore \quad \sigma^2 = \frac{1}{50}(2^2 + 4^2 + ... + 100^2) - (51)^2 = 833$$

JEE-MAIN

Q.1 (0)

$$\bar{x} = \frac{\sum x_i}{7} = 62 \qquad \Rightarrow \sum x_i = 434$$

$$\sigma^2 = \frac{\sum_{i=1}^{7}(x_i - \bar{x})^2}{7} = 20$$

$$\Rightarrow \sum_{i=1}^{7}(x_i - 62)^2 = 140$$

$(x_1 - 62)^2 + (x_2 - 62)^2 + (x_3 - 62)^2.....(x_7 - 62)^2 = 140$

For a student to fail, sets be less than 50 marks. So, $(x_i - 62)^2 > 144$ but summation is < 144. Hence, no student can fail.

Q.2 (B)

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$$

$x_1 + x_2 + x_3 + x_4 + x_5 = 24$ .....(1)

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} = \left(\frac{24}{25}\right)^2 = \frac{194}{25}$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 154$$

...(2)

$$\text{Now, } \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$$

$x_1 + x_2 + x_3 + x_4 = 14$
...(3)

$$\frac{x_1^2+x_2^2+x_3^2+x_4^2}{4}=\left(\frac{7}{2}\right)^2 = a$$

$$\frac{154-x_5^2}{4}-\frac{49}{4} = a$$

$$\frac{154}{4}-\frac{49}{4}-\frac{x_5^2}{4} = a$$

$\Rightarrow 4a + x_5^2 = 105$

From equation (1) and (3)

$x_5 = 10; 4a = 5$

$4a + x_5 = 15$

Q.3 (A)

$$\overline{x}=\frac{3+7+12+\alpha+43-\alpha}{5}=13$$

$$\text{Variance} = \frac{\sum xi^2}{N}-\left(\frac{\sum xi}{N}\right)^2$$

$$=\frac{9+49+144+\alpha^2+(43-a)^2}{5}-13^2 \in N$$

$$\Rightarrow \frac{2\alpha^2-\alpha+1}{4}\in N$$

$2\alpha^2 - \alpha + 1 - 5n = 0$

This equation must have solution as natural numbers

$D = (-1)^2 - 4(2)(1 - 5n)$

$D = 40n - 7$

D always have 3 at unit place

$\therefore$ D can't be perfect square

So α can't be integer

Q.4 **[21]**

$$\text{Mean} = \frac{1+2+...+n}{n}=\frac{n+1}{2}$$

$n = 2k - 1 \Rightarrow$ Mean = k

$$\text{M.D.(Mean)} = \frac{|1-k|+|2-k|+...+|n-k|}{n}$$

$$=\frac{2((k-1)+(k-2)+...+1)+0}{n}$$

$$=\frac{2(k-1)k}{2n}=\frac{k(k-1)}{n}$$

$$\Rightarrow \left(\frac{n+1}{2}\right)\left(\frac{n-1}{2n}\right)=\frac{5(n+1)}{n}$$

$\Rightarrow (n - 1) = 20$

$\Rightarrow n = 21$

Q.5 **(B)**

$$\text{mean } \overline{x}=\frac{4+5+6+6+7+8+x+y}{8}=6$$

$\Rightarrow x + y = 48 - 36 = 12$

Variance

$$=\frac{1}{8}(16+25+36+36+49+64+x^2+y^2)-36=\frac{9}{4}$$

$\Rightarrow x^2 + y^2 = 80$

$\therefore$ x = 4; y = 8

$x^4 + y^2 = 256 + 64 = 320$

Q. 6 (1)

$$\sigma^2=\frac{\sum_{i=1}^{5}(x_i-\overline{x})^2}{n}$$

Mean=6

$$\frac{a+b+8+5+10}{5}=6$$

a+b=7

b=7–a

$$6.8=\frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$$

$34 = (a - 6)^2 + (7 - a - 6)^2 + 4 + 1 + 16$

$a^2 - 7a + 12 = 0 \Rightarrow a=4$ or $a=3$

a=4	a=3
b=3	b=4

$$M=\frac{\sum_{i=1}^{5}|x_i-x|}{n}$$

$$M=\frac{|a-6|+|b-6|+|8-6|+|5-6|+|10-6|}{5}$$

When a = 3, b = 4

$$M=\frac{3+2+2+1+4}{5}$$

$$M=\frac{12}{5}$$

When a=4, b=3

$$M=\frac{2+3+2+1+7}{5}$$

$$M=\frac{12}{5}$$

$$25M = 25\times\frac{12}{5}=60$$

Q.7 (17)

We have Variance $= \dfrac{\sum_{r=1}^{15} x_r^2}{15} - \left(\dfrac{\sum_{r=1}^{15} x_r}{15}\right)^2$

Now, as per information given in equation

$\dfrac{\sum x_r^2}{15} - 8^2 = 3^2 \Rightarrow \sum x_r^2 = 15 \times 73 = 1095$

Now, the new $\sum x_r^2 = 1095 - 5^2 + 20^2 = 1470$

And, new $\sum x_r = (15 \times 8) - 5 + (20) = 135$

Variance $= \dfrac{1470}{15} - \left(\dfrac{135}{15}\right)^2 = 98 - 81 = 17$

Q.8 **(3)**

$\bar{x} = \dfrac{\sum x_i}{50} = 15$

$\Rightarrow \sum x_i = 750$

$\dfrac{\sum x_i^2}{50} - (\bar{x})^2 = \sigma^2$

$\Rightarrow \dfrac{\sum x_i^2}{50} - (15)^2 = 4$

$\Rightarrow \sum x_i^2 = 11450$

New $\bar{x} = 16$

$\sum x_{i_{new}} = 16 \times 50 = 800$

let a be the incorrect observation

then correct observation = a + 50

a + (a + 50) = 70

$\Rightarrow$ a = 10

correct observation a + 50 = 10 + 50 = 60

New variance = $\dfrac{11450 - 10^2 + 60^2}{50} - (16)^2$

= 299 – 256 = 43

Q.9 [4]

$\Sigma x_1 = 15 \times 20 = 300$ ... (i)

$\dfrac{\Sigma x_1^2}{20} - (15)^2 = 9$ ... (ii)

$\Sigma x_1^2 = 234 \times 20 = 4680$

$\dfrac{\Sigma(x_1 + \alpha)^2}{20} = 178 \Rightarrow \Sigma(x_1 + \alpha)^2 = 3560$

$\Rightarrow \Sigma x_1^2 + 2\alpha \Sigma x_1 + \Sigma \alpha^2 = 3560$

$4680 + 600\alpha + 20\alpha^2 = 3560$

$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$

$\Rightarrow (\alpha + 28)(\alpha + 2) = 0$

a = –2, –28

Square of maximum value of α is 4

Q.10 [2]

n = 10

$\mu = 15 \Rightarrow \dfrac{\Sigma xi}{10} = 15 \Rightarrow$ sum of 10 observ = 150

$\sigma^2 = 15$

$\Rightarrow \dfrac{\Sigma x_i^2}{n} - \mu^2 = 15$

$\Rightarrow \dfrac{\Sigma x_i^2}{10} = 15 + 225 = 240$

$\Rightarrow \Sigma x_i^2 = 2400$

New mean

$\Rightarrow$ (Sum of 10 obs) – 25 + 15 = 150 –10 = 140

$\mu_n = \dfrac{140}{10} = 14$

Also

$\left(\Sigma x_i^2\right)_{new} = \Sigma x_i^2 - (25)^2 + (15)^2 + = 2400 - 625 + 225 =$

2400 – 400 = 2000

$\sigma^2_{New} = \dfrac{\left(\Sigma x_i^2\right)_{New}}{10} - \mu^2_{new} = \dfrac{2000}{10} - (14)^2$

= 200 –196

= 4

Correct S.D = $\sqrt{4} = 2$

Q.11 [238]

$\bar{X} = 30$

$\sum x_i = 30 \times 40 = 1200$

$\bar{X}_{new} = \dfrac{\Sigma x_i - \text{sum of incorrect observations}}{38}$

$= \dfrac{1200 - 22}{38} = 31$

$\sigma^2 = \dfrac{\Sigma x_i^2}{n} - (\bar{X})^2$

$\Rightarrow 25 = \dfrac{\Sigma x_i^2}{40} - (30)^2$

$\sum x_i^2 = (925)40$

$= 37000$

$\sum x_i^2 \text{new} =$

$\sum x_i^2 - \dfrac{\text{Sum of squares of incorrect observation}}{38}$

$= 3700 - (10^2 + 12^2)$

$= 36756$

$$\sigma^2_{new} = \frac{\Sigma x^2_{i\,new}}{38} - \left(\overline{X}_{new}\right)^2$$

$$= \frac{36756}{38} - (31)^2$$

$$= \frac{238}{38}$$

$$38\sigma^2_{new} = 238$$

Q.12 (3)

$np = \alpha$ ... (1)

$npq = \dfrac{\alpha}{3}$ ... (2)

(2) $\div$ (1)

q = 1/3

p = 2/3

n = $3\alpha/2$

$$= {}^nC_1\left(\frac{2}{3}\right)'\left(\frac{1}{3}\right)^{n-1} = \frac{4}{243}$$

$$n\left(\frac{1}{3}\right)^{n-1} = \frac{2}{81}$$

n = 6

$$P(x = 4 \text{ or } x = 5) = {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{2}{3}\right)^2$$

$$= \frac{15\times16}{36} + \frac{(6)(32)}{36}$$

$$= \frac{240+192}{36}$$

$$= \frac{432}{36} = \frac{144}{3^5} = \frac{48}{3^4} = \frac{16}{27}$$

Q.13 (4)

$$\text{Median } = \frac{2K+12}{2} = K+6$$

$$\text{M.D}(M) = \left(\frac{K+3+K+1+K-1+6-K+6-K+10-K+15-K+18-K}{8}\right)$$

$$6 = \frac{-2K+58}{8}$$

48 = – 2K + 58

2K = 10 $\Rightarrow$ K = 5

$\therefore$ Median = 11