SEQUENCE AND SERIES

EXERCISE-I (MHT CET LEVEL)

Q.1 (2)

We have $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+......$

$=1\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+........$

$=\sqrt{2}[1+2+3+4+..........\text{upto 24 terms}]$

$=\sqrt{2}\times\frac{24\times 25}{2}=300\sqrt{2}$

Q.2 (3)

Given,

$\frac{2n}{2}\{2.2+(2n-1)3\}$

$=\frac{n}{2}\{2.57+(n-1)2\}$

$=\frac{n}{2}\{2.57+(n-1)2\}$

or $2(6n+1)=112+2n$

or $10n=110,\ \therefore n=11$

Q.3 (4)

$$\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q+1)d]}=\frac{p^2}{q^2}$$

$$\frac{a_1+\left(\frac{p-1}{2}\right)d}{a_1+\left(\frac{q-1}{2}\right)d}=\frac{p}{d}$$ For $\frac{a_6}{a_{21}}$, p = 11, q = 41

$\Rightarrow \frac{a_6}{a_{21}}=\frac{11}{41}$

Q.4 (1)

Lert the progression be, a + d, a +2d

Then $x_4=3x_1 \Rightarrow a+3d=3a$

$\Rightarrow 3d=2a$ .....(i)

Agaian $x_7=2x_3+1$

$\Rightarrow a+6d=2(a+2d)+1$

$\Rightarrow 2d=a+1$ ......(ii)

Solving (i) and (ii), we get

a = 3, d = 2

Q.5 **(3)**

a = 25, d = 22 – 25 = –3.

Let n be the number of terms

Sum = 116; $\text{sum}=\frac{n}{2}[2a+(n-1)d]$

$116=\frac{n}{2}[50+(n-1)(-3)]$

or 232 = n [50 – 3n + 3] = n[53 – 3n]

$=-3n^2+53n$

$\Rightarrow 3n^2-53+232=0 \Rightarrow (n-8)(3n-29)=0$

$\Rightarrow n=8 \text{ or } n=\frac{29}{3}, n'\frac{29}{3} \quad \therefore n=8$

$\therefore$ Now, $T_8=a+(8-1)d=25+7\times(-3)$

= 25 – 21

$\therefore$ Last Term = 4

Q.6 **(3)**

Q.7 (1)

n^{th} term of the series is $20+(n-1)\left(-\frac{2}{3}\right)$.

For sum to be maximum, n^{th} term ≥ 0

$\Rightarrow 20+(n-1)\left(-\frac{2}{3}\right)\geq 0 \Rightarrow n\leq 31$

Thus the sum of 31 terms is maximum and is equal to

$\frac{31}{2}\left[40+30\times\left(-\frac{2}{3}\right)\right]=310$

Q.8 (1)

Series $108+117+........+999$ is an A.P. where a = 108, common difference d = 9,

$n=\frac{999}{9}-\frac{99}{9}=111-11=100$

Hence required sum

$=\frac{100}{2}(108+999)=50\times 1107=55350$

Q.9 (1)

We have $(x+1)+(x+4)+.........+(x+28)=155$

Let n be the number of terms in the A.P. on L.H.S. Then

$x+28=(x+1)+(n-1)3 \Rightarrow n=10$

$\therefore (x+1)+(x+4)+......+(x+28)=155$

$\Rightarrow \frac{10}{2}[(x+1)+(x+28)]=155 \Rightarrow x=1$

Q.10 (3)

$S_{2n}-S_n=\frac{2n}{2}\{2a+(2n-1)d\}-\frac{n}{2}\{2a+(n-1)d\}$

$=\frac{n}{2}\{4a+4nd-2d-2a-nd+d\}=\frac{n}{2}\{2a+(3n-1)d\}$

$=\frac{1}{3}.\frac{3n}{2}\{2a+(3n-1)d\}=\frac{1}{3}S_{3n}$

Q.11 (4)

$\log_{\sqrt{3}} x+\log_{\sqrt[4]{3}} x+\log_{\sqrt[6]{3}} x+......+\log_{\sqrt[16]{3}} x=36$

$\Rightarrow \frac{1}{\log_x \sqrt{3}}+\frac{1}{\log_x \sqrt[4]{3}}+\frac{1}{\log_x \sqrt[6]{3}}+...+\frac{1}{\log_x \sqrt[16]{3}}=36$

$\frac{1}{(1/2)\log_x 3}+\frac{1}{(1/4)\log_x 3}+\frac{1}{(1/6)\log_x 3}+...........$

$...........+\frac{1}{(1/16)\log_x 3}=36$

$\Rightarrow (\log_3 x)(2+4+6+.....+16)=36$

$\Rightarrow (\log_3 x)\frac{8}{2}[2+16]=36$

$\Rightarrow \log_3 x=\frac{1}{2}$

$\Rightarrow x=3^{1/2} \Rightarrow x=\sqrt{3}$

Q.12 (2)

According to the given condition

$\frac{15}{2}[10+14\times d]=390 \Rightarrow d=3$

Hence middle term i.e. 8^{th} term is given by

$5+7\times 3=26$

Q.13 (3)

$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$

$\Rightarrow a^{n+1}-ab^n+b^{n+1}-ba^n=0$

$\Rightarrow (a-b)(a^n-b^n)=0$

If $a^n-b^n=0$. Then $\left(\frac{a}{b}\right)^n=1=\left(\frac{a}{b}\right)^0$. Hence $n=0$.

Q.14 (2)

The resulting progression will have n+2 terms with 2 as the first term and 38 as the last term.

Therefore the sum of the progression

$=\frac{n+2}{2}(2+38)=20(n+2)$.

By hypothesis, $20(n+2)=200 \Rightarrow n=8$

Q.15 (3)

Given $M=\frac{a+b+c+d+e}{5}$

$\Rightarrow a+b+c+d+e=5M$

$\Rightarrow a+b+c+d+e-5M=0$

$\Rightarrow (a-M)+(b-M)+(c-M)+(d-M)+(e-M)=0$

Hence, required value = 0

Q.16 (2)

Let four arithmetic means are A_1, A_2, A_3 and A_4.

So $3, A_1, A_2, A_3, A_4, 23$

$\Rightarrow T_6=23=a+5d \Rightarrow d=4$

Thus $A_1=3+4=7$, $A_2=7+4=11$,

$A_3=11+4=15$, $A_4=15+4=19$

Q.17 (2)

Given that $m=ar^{p+q-1}$ and $n=ar^{p-q-1}$

$r^{p+q-1-p+q+1}=\frac{m}{n} \Rightarrow r=\left(\frac{m}{n}\right)^{1/(2q)}$

and $a=\frac{m}{\left(\frac{m}{n}\right)^{(p+q-1)/2q}}$

Now p^{th} term $=ar^{p-1}=\frac{m}{\left(\frac{m}{n}\right)^{(p+q-1)/2q}}\left(\frac{m}{n}\right)^{(p-1)/2q}$

$=m\left(\frac{m}{n}\right)^{(p-1)/2q-(p+q-1)/(2q)}=m\left(\frac{m}{n}\right)^{-1/2}=m^{1-1/2}n^{1/2}$

$=m^{1/2}n^{1/2}=\sqrt{mn}$

Second Method : As we know each term in a G.P. is geometric mean of the terms equidistant from it. Here $(p+q)^{th}$and $(p-q)^{th}$ terms are equidistant from term at a distance of . Therefore, term will be G.M. of and .

Q.18 (1)

$\therefore a,b,c$ are in G.P

$\therefore \frac{b}{a}=\frac{c}{b}=r \Rightarrow \frac{b^2}{a^2}=\frac{c^2}{b^2}=r^2 \Rightarrow a^2,b^2,c^2$

are in G.P.

Q.19 **(4)**

2a = 1 + P and $g^2 = P$

$\Rightarrow g^2 = 2a - 1 \Rightarrow 1 - 2a + g^2 = 0$

Q.20 **(1)**

$$\text{sum} = \frac{8}{9}\left[9+99+999+\ldots.\text{n terms}\right]$$

$$=\frac{8}{9}\left[(10-1)+(100-1)+(1000-1)+\ldots.\text{n terms}\right]$$

$$=\frac{8}{9}\left[\left(10+10^2+10^3+\ldots.+10^n\right)-n\right]$$

$$=\frac{8}{9}\left[\frac{10\left(10^n-1\right)}{10-1}\right]-n$$

$$=\frac{8}{81}\left[10^{n+1}-9n-10\right]$$

Q.21 **(4)**
Q.22 **(2)**
Q.23 **(3)**
Q.24 **(4)**
Q.25 **(2)**

Q.26 (4)

Given that x, 2x + 2, 3x + 3 are in G.P.

Therefore, $(2x+2)^2 = x(2x+2)^2 = x(3x+3)$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+4)(x+1) = 0 \Rightarrow x = -1, -4$

Now first term a = x

Second term $ar = 2(x+1) \Rightarrow r = \frac{2(x+1)}{x}$

then 4^{th} term $= ar^3 = x\left[\frac{2(x+1)}{x}\right]^3 = \frac{8}{x^2}(x+1)^3$

Putting x = –4

We get $T_4 = \frac{8}{16}(-3)^3 = -\frac{27}{2} = -13.5$

Q.27 (2)

$T_6 = 32$ and $T_8 = 128 \Rightarrow ar^5 = 32$ .....(i)

and $ar^7 = 128$ .....(ii)

Dividing (ii) by (i), $r^2 = 4 \Rightarrow r = 2$

Q.28 (1)

Series is a G.P. with $a = 0.9 = \frac{9}{10}$ and $r = \frac{1}{10} = 0.1$

$$\therefore S_{100} = a\left(\frac{1-r^{100}}{1-r}\right) = \frac{9}{10}\left(\frac{1-\frac{1}{10^{100}}}{1-\frac{1}{10}}\right) = 1-\frac{1}{10^{100}}.$$

Q.29 (2)

Given series $6+66+666+\ldots\ldots.+$ upto n terms

$$=\frac{6}{9}(9+99+999+\ldots..\text{ upto terms})$$

$$=\frac{2}{3}(10+10^2+10^3+\ldots\ldots.+\text{ upto terms})$$

$$=\frac{2}{3}\left(\frac{10(10^n-1)}{10-1}-n\right)=\frac{1}{27}[20(10^n-1)-18n]$$

$$=\frac{2(10^{n+1}-9n-10)}{27}$$

Q.30 (4)

Here $\frac{a}{1-r} = 4$ and $ar = \frac{3}{4}$. Dividing these,

$r(1-r) = \frac{3}{16}$ or $16r^2 - 16r + 3 = 0$

or $(4r-3)(4r-1) = 0$

$r = \frac{1}{4}, \frac{3}{4}$ and $a = 3, 1$ so $(a, r) = \left(3, \frac{1}{4}\right), \left(1, \frac{3}{4}\right)$.

Q.31 (2)

$\frac{a}{1-r} = 20$ .....(i)

$\frac{a^2}{1-r^2} = 100$ .....(ii)

From (i) and (ii),

$\frac{a}{1+r} = 5$, $[\because a = 20(1-r)$ by (i)$]$

$\Rightarrow \frac{20(1-r)}{1+r} = 5 \Rightarrow 5r = 3 \Rightarrow r = 3/5$

Q.32 (3)

If n geometric means $g_1, g_2, \ldots\ldots g_n$ are to be inserted between two positive real numbers a and b, then a, g_1, g_2, ...g_n, b are in G.P. Then $g_1 = ar$, $g_2 = ar^2$ ....$g_n = ar^n$

So $b = ar^{n+1} \Rightarrow r = \left(\frac{b}{a}\right)^{1/(n+1)}$

Now n^{th} geometric mean

$(g_n) = ar^n = a\left(\frac{b}{a}\right)^{n/(n+1)}$

2nd Method : As we have the m^{th} G.M. is given by

$$G_m = a\left(\frac{b}{a}\right)^{\frac{m}{n+1}}$$

Now replace m by we get the required result.

Q.33 **(4)**
The roots of equation are 2 and 3

$\therefore g=\sqrt{xy}=2\Rightarrow xy=4$

$G=\sqrt{(x+1)(y+1)}=3\Rightarrow (x+1)(y+1)=9$

$\therefore x=y=2$

Q.34 (2)

As given $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=(ab)^{1/2}$

$\Rightarrow a^{n+1}-a^{n+1/2}b^{1/2}+b^{n+1}-a^{1/2}b^{n+1/2}=0$

$\Rightarrow (a^{n+1/2}-b^{n+1/2})(a^{1/2}-b^{1/2})=0$

$\Rightarrow a^{n+1/2}-b^{n+1/2}=0\ (\because a\neq b\Rightarrow a^{1/2}\neq b^{1/2})$

$\Rightarrow\left(\frac{a}{b}\right)^{n+1/2}=1=\left(\frac{a}{b}\right)^0\Rightarrow n+\frac{1}{2}=0\Rightarrow n=-\frac{1}{2}$

Q.35 (2)

As given $G=\sqrt{xy}$

$\therefore \frac{1}{G^2-x^2}+\frac{1}{G^2-y^2}=\frac{1}{xy-x^2}+\frac{1}{xy-y^2}$

$=\frac{1}{x-y}\left\{-\frac{1}{x}+\frac{1}{y}\right\}=\frac{1}{xy}=\frac{1}{G^2}$.

Q.36 (3)
2, g_1, g_2, g_3, 32
where

$a=2,\ ar=g_1,\ ar^2=g_2,\ ar^3=g_3$ and $ar^4=32$

Now $2\times r^4=32\Rightarrow r^4=16=(2)^4\Rightarrow r=2$

Then third geometric mean $=ar^3=2\times 2^3=16$

2nd Method :

By formula, $G_3=2.\left(\frac{32}{2}\right)^{3/4}=2.8=16$

Q.37 (2)
Let T_n be the n^{th} term and S the sum upto n terms.

$S=1+3+7+15+31+......+T_n$

Again $S=1+3+7+15+..........+T_{n-1}+T_n$
Subtracting, we get

$0=1+\{2+4+8+...(T_n-T_{n-1})\}-T_n$

$\therefore T_n=1+2+2^2+2^3+.....$up to n terms

$=\frac{1(2^n-1)}{2-1}=2^n-1$

Now $S=\Sigma T_n=\Sigma 2^n-\Sigma 1$

$=(2+2^2+2^3+......+2^n)-n$

$=2\left(\frac{2^n-1}{2-1}\right)-n=2^{n+1}-2-n$

2nd Method : $1+3+7+......+T_n$

$=2-1+2^2-1+2^3-1+..........+2^n-1$

$=(2+2^2+......+2^n)-n=2^{n+1}-2-n$.

Trick : Check the options for n = 1, 2.

Q.38 (4)
Suppose that x to be added then numbers 13, 15, 19 so that new numbers $x+13, 15+x, 19+x$ will be in H.P.

$\Rightarrow (15+x)=\frac{2(x+13)(19+x)}{x+13+x+19}$

$\Rightarrow x^2+31x+240=x^2+32x+247\Rightarrow x=-7$

Trick : Such type of questions should be checked with the options.

Q.39 (1)
Here 5^{th} term of the corresponding
A.P. $=a+4d=45$ …..(i)
and 11^{th} term of the corresponding
A.P. $=a+10d=69$ …..(ii)
From (i) and (ii), we get $a=29, d=4$
Therefore 16th term of the corresponding A.P.
$=a+15d=29+15\times 4=89$.

Hence 16th term of the H.P. is $\frac{1}{89}$.

Q.40 (2)
Here first term of A.P. be 7 and second be 9, then 12th term will be $7+11\times 2=29$.

Hence term of the H.P. be $\frac{1}{29}$.

Q.41 (4)
Considering corresponding A.P.
$a+6d=10$ and $a+11d=25\Rightarrow d=3, a=-8$

Hence term of the corresponding H.P. is $\frac{1}{49}$

Q.42 (1)

$$x_n = \frac{(n+1)ab}{na+b}$$

Sixth H.M. $x_6 = \dfrac{7.3.6/13}{\left(6.3+\dfrac{6}{13}\right)} = \dfrac{126}{240} = \dfrac{63}{120}$

Q.43 (2)

If $(b-c)^2, (c-a)^2, (a-b)^2$ are in A.P.

Then we have

$(c-a)^2 - (b-c)^2 = (a-b)^2 - (c-a)^2$

$\Rightarrow (b-a)(2c-a-b) = (c-b)(2a-b-c)$

…..(i)

Also if $\dfrac{1}{b-c}, \dfrac{1}{c-a}, \dfrac{1}{a-b}$ are in A.P.

Then $\dfrac{1}{c-a} - \dfrac{1}{b-c} = \dfrac{1}{a-b} - \dfrac{1}{c-a}$

$\Rightarrow \dfrac{b+a-2c}{(c-a)(b-c)} = \dfrac{c+b-2a}{(a-b)(c-a)}$

$\Rightarrow (a-b)(b+a-2c) = (b-c)(c+b-2a)$

$\Rightarrow (b-a)(2c-a-b) = (c-b)(2a-b-c)$

which is true by virtue of (i).

Q.44 (2)

Given that a, b, c in A.P. and b, c, d in H.P.

So, $2b = a+c$ and $c = \dfrac{2bd}{b+d}$

$\Rightarrow c(b+d) = 2bd = (a+c)d \Rightarrow bc = ad$

Q.45 (4)

$2\ln(c-a) = \ln(a+c) + \ln(a-2b+c)$

$\Rightarrow (c-a)^2 = (a+c)(a-2b+c)$

$\Rightarrow c^2 + a^2 - 2ac = (a+c)^2 - 2b(a+c)$

$\Rightarrow c^2 + a^2 - 2ac = a^2 + c^2 + 2ac - 2ab - 2bc$

$\Rightarrow b(a+c) = 2ac \Rightarrow b(a+c) = 2ac$

$\Rightarrow b = \dfrac{2ac}{a+c}$

Q.46 (4)

Here $T_n = \dfrac{n(n+1)}{2}$

Therefore $S_n = \dfrac{1}{2}\left\{\Sigma n^2 + \Sigma n\right\} = \dfrac{n(n+1)(n+2)}{6}$

Q.47 (1)

Q.48 (3)

Q.49 **(2)**

Q.50 (2)

$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\dots\dots\dots+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

Q.51 (3)

Sum of cubes of ‘n’ natural number

$$= \frac{n^2(n+1)^2}{4} = \frac{15^2(16)^2}{4} = 14{,}400.$$

Q.52 (4)

$$T_n = \frac{3^n - 1}{3^n} = 1 - \left(\frac{1}{3}\right)^n$$

$$S_n = n - \sum_{n=1}^{n}\left(\frac{1}{3}\right)^n \qquad = n - \frac{\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^n\right]}{\left(1-\frac{1}{3}\right)}$$

$$= n - \frac{1}{2}(1-3^{-n}) = n + \frac{1}{2}(3^{-n}-1)$$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (4)

$S = \dfrac{2p+1}{2}\ [2(p^2+1)+2p]$

$= (2p+1)(p^2+1+p)$

$= 2p^3 + 3p^2 + 3p + 1 = p^3 + (p+1)^3$

Q.2 (1)

$$\frac{1}{a}-\frac{1}{b}=\frac{1}{b}-\frac{1}{c}$$

$$\therefore \left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)$$

$$= \left(\frac{2}{a}-\frac{1}{b}\right)\left(\frac{2}{c}-\frac{1}{b}\right) = \frac{4}{ac} - \frac{1}{b}\left(\frac{2}{a}+\frac{2}{c}\right) + \frac{1}{b^2}$$

$$= \frac{4}{ac} - \frac{2}{b}\left(\frac{2}{b}\right) + \frac{1}{b^2} = \frac{4}{ac} - \frac{3}{b^2}$$

Q.3 (3)

Q.4 **(3)**

As $a_1, a_2, a_3, \ldots\ldots, a_n$, are in A.P. we get,

$a_2 - a_1 = a_3 - a_2 = \ldots\ldots\ldots = a_n - a_{n-1} = d$ (say)

Now, $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} = \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1 - a_2} = \frac{\sqrt{a_1}-\sqrt{a_2}}{-d}$

Similarly,

$$\frac{1}{\sqrt{a_2}+\sqrt{a_3}} = \frac{\sqrt{a_2}-\sqrt{a_3}}{-d}, \ldots\ldots\ldots, \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

$$= \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{-d}$$

$$\therefore \frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots.. + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

$$= \frac{\sqrt{a_1}-\sqrt{a_n}}{-d} = -\frac{1}{d}\left[\frac{a_1 - a_n}{\sqrt{a_1}+\sqrt{a_n}}\right]$$

$$= -\frac{1}{d}\left[\frac{a_1 - \{a_1 + (n-1)d\}}{\sqrt{a_1}+\sqrt{a_n}}\right]$$

[Formula for n^{th} term]

$$= -\frac{1}{d}\left[\frac{-(n-1)d}{\sqrt{a_1}+\sqrt{a_n}}\right] = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

Q.5 (4)

$x \in R$

$5^{1+x} + 5^{1-x}$, $a/2$, $5^{2x} + 5^{-2x}$ are in A.P

$a = (5^{2x} + 5^{-2x}) + (5^{1+x} + 5^{1-x})$

$a = (5^{2x} + 5^{-2x}) + 5(5^{x} + 5^{-x})$

$= (5^{x} - 5^{-x})^2 + 2 + 5\,(5^{x/2} - 5^{-x/2})^2 + 10$

$a = 12 + (5^{x} - 5^{-x})^2 + 5(5^{x/2} - 5^{-x/2})^2$

$\Rightarrow a \geq 12$

Q.6 (4)

$$S = \frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + \ldots\ldots + \frac{1}{\log_{2^n} 4}$$

$$= \frac{1}{2} + \frac{1}{1} + \frac{1}{2/3} + \ldots\ldots\ldots + \frac{1}{2/n}$$

$$= \frac{1}{2} + 1 + \frac{3}{2} + \frac{4}{2} + \ldots\ldots\ldots\ldots\ldots + \frac{n}{2}$$

$$= \frac{n(n+1)}{4}$$ **Ans**

Q.7 (1)

Given that

$S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots.\ 2002^2 + 2003^2$

$= 1 + (3^2 - 2^2) + (5^2 - 4^2) + \ldots. + (2003^2 - 2002^2)$

$= 1 + 2 + 3 + 4 + 5 + \ldots. + 2002 + 2003$

$= \frac{2003}{2}\ [1 + 2003] = 2003\ (1002)$

$= (2000 + 3)\ (1000 + 2) = 2007006$

Q.8 (2)

$$\frac{(54-3)}{n+1} = d$$

$$d = \frac{51}{n+1}$$

$$\frac{A_8}{A_{n-2}} = \frac{3}{5}$$

$$\Rightarrow \frac{3 + 8\frac{51}{n+1}}{3 + (n-2)\frac{51}{n+1}} = \frac{3}{5}$$

$$\Rightarrow \frac{3n + 3 + 408}{3n + 3 + 51n - 102} = \frac{3}{5}$$

$\Rightarrow 15n + 2055 = 162n - 297$

$\Rightarrow 147\,n = 2352$

$n = 16$

Q.9 (1)

Let the meas be $x_1, x_2, \ldots x_m$ so that

$1, x_1, x_2, \ldots x_m, 31 = T_{m+2} = a + (m+1)\,d = 1 + (m+1)d$

$$\therefore d = \frac{30}{m+1} \text{ Given : } \frac{x_7}{x_{m-1}} = \frac{5}{9}$$

$$\therefore \frac{T_8}{T_m}\ \frac{a + 7d}{a + (m-1)d} = \frac{5}{9}$$

$$\Rightarrow 9a + 63d = 5a + (5m - 5)d$$

$$\Rightarrow 4.1 = (5m - 68)\frac{30}{m+1}$$

$$\Rightarrow 2m + 2 = 75m - 1020 \Rightarrow 73m = 1022$$

$$\therefore m = \frac{1022}{73} = 14$$

Q.10 (2)

Let the GP be a, ar^2, ar^3, ...

We know that sum of G.P. is possible $\Rightarrow |\,r\,| < 1$

$$S = \frac{a}{1-r} \Rightarrow r = \left(1 - \frac{a}{S}\right)$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a\left(1 - \left(1 - \frac{a}{S}\right)^n\right)}{\frac{a}{S}} = S\left[1 - \left(1 - \frac{a}{S}\right)^n\right]$$

Q.11 (2)

Given,

$a_1 = 2, \& \ \frac{a_{n+1}}{a_n} = \frac{1}{3} = r,$

$$\sum_{r=1}^{20} a_r = \frac{a_1(1-r^{20})}{1-r} = \frac{2\left(1-\left(\frac{1}{3}\right)^{20}\right)}{\frac{2}{3}} = 3\left(1-\frac{1}{3^{20}}\right)$$

Q.12 (1)

since $|r| > 1, \frac{1}{|r|} < 1$

$$\therefore x = \frac{a}{1-\frac{1}{r}} = \frac{ar}{r-1}$$

Similarly, y $= \frac{b}{1-\left(-\frac{1}{r}\right)} = \frac{br}{r+1}$ and

$$z = \frac{c}{1-\frac{1}{r^2}} = \frac{c^2}{r^2-1}$$

$$\therefore xy = \frac{ar}{r-1} \times \frac{br}{r+1} = \frac{abr^2}{r^2-1}$$

Dividing (2) by (1), we get

$$\frac{xy}{z} = \frac{abr^2}{r^2-1} \times \frac{r^2-1}{cr^2} = \frac{ab}{c}$$

Q.13 (1)

The series is a G.P. with common ratio

$= \left(\frac{1-3x}{1+3x}\right)$ and $|r| = \left|\frac{1-3x}{1+3x}\right|$ is less than 1 since x is

Positive $S_\infty = \frac{a}{t-r} = \frac{\frac{1}{1+3x}}{1-\left\{-\left(\frac{1-3x}{1+3x}\right)\right\}} = \frac{1}{2}$

Q.14 (3)

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...$$

$$= \left(1-\frac{1}{2}\right) + \left(1-\frac{1}{4}\right) + \left(1-\frac{1}{8}\right) + \left(1-\frac{1}{16}\right) + ...$$

$$= n - \frac{\frac{1}{2}\left\{1-\frac{1}{2^n}\right\}}{1-\frac{1}{2}} = n-1+2^{-n}$$

Q.15 (1)

The series is

$$(x^2+x^4+x^6+...) + \left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}+....\right)$$

$+ (2+2+....)$

$$= \frac{x^2(x^{2n}-1)}{x^2-1} + \frac{\frac{1}{x^2}\left(1-\frac{1}{x^{2n}}\right)}{1-\frac{1}{x^2}} + 2n$$

$$= \frac{x^2(x^{2n}-1)}{x^2-1} + \frac{x^{2n}-1}{(x^2-1)\,x^{2n}} + 2n$$

$$= \frac{x^{2n}-1}{x^2-1} \times \frac{x^{2n+2}+1}{x^{2n}} + 2n$$

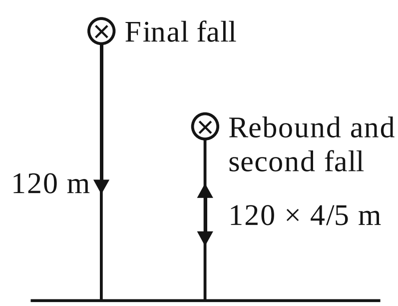
Q.16 (3)

Clearly, the total distance desctibed

$$= 120 + 2\left[\begin{array}{l} 120\times\frac{4}{5} + 120\times\frac{4}{5}\times\frac{4}{5} \\ +120\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5} + ...\text{to } \infty \end{array}\right]$$

Except in the first fall the same ball will travel twice in each step the same distance one upward and seconed downward travel.

$\therefore$ Distnce travelled



$$= 120 + 2\times120\left[\frac{4}{5} + \left(\frac{4}{5}\right)^2 + ...\text{to } \infty\right]$$

$= 120+240\left[\dfrac{\frac{4}{5}}{1-\frac{4}{5}}\right]$

$= 120 + 240 \times 4 = 1080$ m

Q.17 (2)

The given product

$= 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+......} = 2^{s}(\text{say})$

Now $S = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+.....$ .....(i)

$\Rightarrow \frac{1}{2}S = \frac{1}{8}+\frac{2}{16}+\frac{3}{32}+.....$ .....(ii)

Apply ; (i) – (ii)

$\Rightarrow \frac{1}{2}S = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.....$

$= \dfrac{1/4}{1-1/2} = \dfrac{1}{2}$ $\therefore S = 1$

$\Rightarrow$ Product $= 2^1 = 2$

Q.18 (4)

Let GP be $a_1, a_2, ..., a_k, ...$ with first term a & common ratio r,

$a_k = a_{k+1} + a_{k+2}$ $\forall\ a_k > 0$

$\Rightarrow ar^{k-1} = ar^k + ar^{k+1}$ $\Rightarrow$ $r > 0$

$\Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \dfrac{-1 \pm \sqrt{1+4}}{2}$ {r = –ve rejected}

$\Rightarrow r = \dfrac{\sqrt{5}-1}{2} = 2\left(\dfrac{\sqrt{5}-1}{4}\right) = 2 \sin 18^{o}$

Q.19 (3)

$y = 2.\overline{357}$

y = 2.357357357..................(1)

1000 y = 2357.357357...........(2)

so 999y = 2355

$y = \dfrac{2355}{999}$

Q.20 (1)

$3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + ... + \infty = 8$

$a = 3, r = \frac{1}{4}$

Sum of AGP upto ∞

$S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$

$\Rightarrow 8 = \dfrac{3}{(3/4)} + \dfrac{d\left(\frac{1}{4}\right)}{3^2/4^2} \Rightarrow 8 = 4 + \dfrac{4d}{3^2}$

$\Rightarrow 4 = \dfrac{4d}{3^2} \Rightarrow d = 3^2 \Rightarrow d = 9$

Q.21 (3)

$\alpha + \beta = \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} \Rightarrow \alpha + \beta = \dfrac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$\Rightarrow -\dfrac{b}{a} = \dfrac{b^2-2ac}{c^2} \Rightarrow -bc^2 = ab^2 - 2a^2c$

$\Rightarrow ab^2 + bc^2 = 2a^2c \Rightarrow \dfrac{b}{c} + \dfrac{c}{a} = \dfrac{2a}{b}$

So $\dfrac{c}{a}, \dfrac{a}{b}, \dfrac{b}{c}$ are in A.P. $\Rightarrow \dfrac{a}{c}, \dfrac{b}{a}, \dfrac{c}{b}$ are in H.P.

Q.22 (2)

Let H.P, be $\dfrac{1}{a} + \dfrac{1}{a+d} + \dfrac{1}{a+2d} + ...$

$\therefore u = \dfrac{1}{a+(p-1)d}, v = \dfrac{1}{a+(q-2d)},$

$w = \dfrac{1}{a+(p-1)d}$ $\Rightarrow a+(p-1)d = \dfrac{1}{u}$

$a+(q-1)d = \dfrac{1}{v}, a+(r-1)d = \dfrac{1}{w}$

$\Rightarrow (q-r)\{a+(p-1)d\} + (r-p)$

$\{a+(q-1)d\} + ...$

$= \dfrac{1}{u}(q-r) + \dfrac{1}{v}(r-p+...)$

$\Rightarrow (q-r)vw + ... = 0$

Q.23 (2)

It is an arithmetico - geometric series. On multiplying Eq.(i) by 2 and then subtracting it from Eq. (i), we get

$$S = 1 + 2.2 + 3.2^2 + 4.2^3 + \ldots + 100.2^{99}$$

$$\underline{2}S = \underline{1}.2 + 2.2^2 \underline{+} \ldots \underline{+} \ldots \underline{+} 99.2^{99} \underline{+} 100.2^{100}$$

$$-S = 1 + 2 + 2^2 + 2^3 \ldots + 2^{99} - 100.2^{100}$$

$$\Rightarrow \quad -S = \frac{1(2^{100} - 1)}{2 - 1} - 100.2^{100}$$

$$\Rightarrow -S = 2^{100} - 1 - 100.2^{100}$$

$$\Rightarrow -S = -1 - 99.2^{100}$$

$$\Rightarrow S = 99.2^{100} + 1$$

Q.24 (3)

If a is the first term and d is the common difference of the associated A.P.

$$\frac{1}{q} = \frac{1}{a} + (2p-1)d, \frac{1}{p} = \frac{1}{a} + (2q-1)d$$

$$\Rightarrow d = \frac{1}{2pq}$$

I f

$$h \textit{ is the } 2(p+q)^h \text{ term } \frac{1}{h} = \frac{1}{a} + (2p + 2q - 1)d$$

$$= \frac{1}{q} + \frac{1}{p} = \frac{p+q}{pq}$$

Q. 25 (3)

$a^x = b^y = c^z = d^t = k$ and a, b, c, d are in G.P.

a, b, c are in G.P. $\Rightarrow$ So $b^2 = ac$

$$\Rightarrow k^{2/y} = k^{1/x + 1/z} \Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$\Rightarrow$ x, y, z are in H.P.

$\because$ b, c, d are in GP

then $\frac{2}{z} = \frac{1}{y} + \frac{1}{t} \Rightarrow$ y, z, t are in HP

So x, y, z, t are in H.P.

Q.26 (2)

$$AM = A = \frac{a+b+c}{3}$$

$$GM = G = (abc)^{1/3}$$

$$HM = H = \frac{3abc}{ab+bc+ca} = \frac{3G^3}{ab+bc+ca}$$

Equation whose roots are a,b,c

$$\Rightarrow x^3 - (a+b+c)x^2 + (\Sigma ab)x - abc = 0$$

$$\Rightarrow x^3 - 3Ax^2 + \frac{3G^3}{H}.x - G^3 = 0$$ **Ans**

Q.27 (2)

By A.M $\geq$ G.M.

$x^4 + y^4 \geq 2x^2y^2$ and

$$2x^2y^2 + z^2 \geq \sqrt{8}xyz.$$

$$\Rightarrow \frac{x^4 + y^4 + z^2}{xyz} \geq \sqrt{8}$$

Q.28 (4)

Since, product of n positive number is unity.

$$\Rightarrow x_1 x_2 x_3 \ldots\ldots x_n = 1 \qquad \ldots\ldots(i)$$

Using A.M. $\geq$ GM

$$\Rightarrow \frac{x_1 + x_2 + \ldots\ldots + x_n}{n} \geq (x_1 x_2 \ldots\ldots x_n)^{\frac{1}{n}}$$

$$\Rightarrow x_1 + x_n + \ldots\ldots + x_n \geq n(1)^{\frac{1}{n}} [From\, eq^n (i)]$$

Q.29 (2)

$$\text{Let } S = \sum_{r=2}^{\infty} \frac{1}{r^2 - 1}$$

$$= \sum_{r=2}^{\infty} \frac{1}{(r-1)(r+1)} = \frac{1}{2} \sum_{r=2}^{\infty} \left(\frac{1}{r-1} - \frac{1}{r+1} \right)$$

$$= \frac{1}{2} \left[1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \ldots. + \frac{1}{n-1} - \frac{1}{n+1} + \ldots. \right]$$

$$\text{when } n \to \infty \Rightarrow \frac{1}{n+1} \to 0$$

$$\therefore S = \frac{1}{2}\left[1 + \frac{1}{2}\right] = \frac{3}{4}.$$

Q.30 (3)

$$\text{Let } S = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots. \infty = \frac{\pi^2}{6}$$

$$\text{Now } S_{even} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots. \infty$$

$$= \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots.\infty\right] = \frac{1}{2^2} \frac{\pi^2}{6} = \frac{\pi^2}{24}$$

$S_{odd} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... \infty$

$= S - S_{even}$

$= \frac{\pi^2}{6} - \frac{\pi^2}{24} = \frac{3\pi^2}{24} = \frac{\pi^2}{8}$

Q.31 (3)

$1^{st}\ term \to 1, 2^{nd}\ term \to 3,$

$7^{th}\ term \to 4, 11^{th}\ term \to 5, ...$

Series 1,2,4,7,11…

$$a_n = 1 + \frac{n(n-1)}{2} = \frac{n^2 - n + 2}{2}$$

Ifn=14, then a_n =92, Ifn=15m, then a_n =106.

Q.32 (2)

Consider $\frac{3}{4} + \frac{15}{16} + \frac{63}{64} + ...$ upto n terms

$= \frac{2^2-1}{2^2} + \frac{2^4-1}{2^4} + \frac{2^6-1}{2^6}$ upto n terms

$= \left(1 - \frac{1}{2^2}\right) + \left(1 - \frac{1}{2^4}\right) + \left(1 - \frac{1}{2^6}\right) + \cdots$ upto n terms

= (1 + 1 + 1 + ... upto n terms)

$- \left(\frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + ...\text{upto n terms}\right)$

$$= n - \frac{1}{2^2}\left[\frac{1 - \left(\frac{1}{2^2}\right)^n}{1 - \frac{1}{2^2}}\right] = n - \frac{1}{3}\left(1 - 3^{-n}\right)$$

$$= n + \frac{4^{-n}}{3} - \frac{1}{3}$$

Q.33 (2)

$$\sum_{k=1}^{n}(k)(k+1)(k-1) = \sum_{k=1}^{n} k(k^2-1)\sum_{k=1}^{n}(k^3 - k)$$

$$= \left(\frac{n(n-1)}{2}\right)^2 - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left(\frac{n(n-1)}{2} - 1\right)$$

$$= \frac{n^2+n}{2}\left(\frac{n^2+n-}{2}\right)$$

$$= \frac{n^4 + n^3 - 2n^2 - n^3 + n^2 - 2n}{4}$$

$$= \frac{n^4}{4} + \frac{n^2}{4} - \frac{n^2}{2} - \frac{n}{2} \Rightarrow s - \frac{1}{2}$$

Q.34 (3)
Q.35 (3)
Q.36 (2)
Q.37 **(2)**
Q.38 **(4)**
Q.39 (1)

Let S = 1(1!) + 2 (2 !) + 3(3 !) + ... + n (n !)

$$\Rightarrow S = \sum_{r=1}^{n} r(r!) = \sum_{r=1}^{n}(r + 1 - 1)r!$$

$$= \sum_{1}^{n}[(r+1)r! - r!]$$

= (n + 1) ! – 1

Q.40 (3)

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 +$ ......n terms

$= \frac{n(n+1)^2}{2}$, when n is even

$1^2 + 2 . 2^2 + 3^2 + ... 2 . n^2 = n\ \frac{(n+1)^2}{2}$

when n is odd n + 1 is even

$1^2 + 2 . 2^2 + 3^2 + ... n^2 + 2 . (n + 1)^2$

$= (n + 1)\ \frac{(n+2)^2}{2}$

$1^2 + 2 . 2^2 + 3^2 + ... n^2 = (n + 1)\left[\frac{(n+2)^2}{2} - 2(n+1)\right]$

$= \frac{(n+1)\, n^2}{2}$

Q.41 (2)

Given that,

$1^2 + 2^2 + ... n^2 = 1015$

$\frac{n(n+1)(2n+1)}{6} = 1015$

Put n = 15 $\Rightarrow \dfrac{15\times16\times31}{6} = 1240 \Rightarrow n \neq 15$

Put n = 14 $\Rightarrow \dfrac{14\times15\times29}{6} = 1015 \Rightarrow n = 14$

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 [0002]

$2^{1/4}\times4^{1/8}\times8^{1/16}\ldots = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\ldots}$

Now, $S = \dfrac{1}{4}+\dfrac{2}{8}+\dfrac{3}{16}+\ldots$

$\Rightarrow \dfrac{1}{2}S = \dfrac{1}{8}+\dfrac{2}{16}+\ldots$

$\therefore S-\dfrac{1}{2}S = \dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\ldots$

$\Rightarrow \dfrac{1}{2}S = \dfrac{\frac{1}{4}}{1-\frac{1}{2}}$

$\Rightarrow S = 1$

So the given product is 2.

Q.2 [2500]

Let 1 + 1/50 = x. Let S be the sum of 50 terms of the given series.

Then,

$S = 1 + 2x + 3x^2 + 4x^3 + \ldots + 49x^{48} + 50x^{49}$ ...(i)

$xS = x + 2x^2 + 3x^3 + \ldots + 49x^{49} + 50x^{50}$ ...(ii)

$(1-x)S = 1 + x + x^2 + x^3 + \ldots + x^{49} - 50x^{50}$

[Subtracting (ii) from (i)]

$\Rightarrow S(1-x) = \dfrac{1-x^{50}}{1-x} - 50x^{50}$

$\Rightarrow S(-1/50) = -50(1-x^{50}) - 50x^{50}$

$\Rightarrow \dfrac{1}{50}S = 50 \quad \Rightarrow \quad S = 2500$

Q.3 [**0003**]

$\alpha + r = \dfrac{4}{A}, \qquad \alpha r = \dfrac{1}{A}$

$\alpha + r = 4\alpha r$

or $\dfrac{1}{\alpha}+\dfrac{1}{r} = 4$ …(i)

Again $\beta+\delta = 6\beta\delta$

or $\dfrac{1}{\beta}+\dfrac{1}{\delta} = 6$ …(ii)

$\therefore \alpha, \beta, \gamma, \delta$ an in H.P.

$\dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\gamma}, \dfrac{1}{\delta}$ A.P.

But the no. eve a – 3d, a – d, a + d, a + 3d

$4a = 6 + 4 = 10$ or $a = \dfrac{5}{2}$

$\dfrac{1}{\alpha}+\dfrac{1}{\gamma} = 4$ (Given) then

$a - 3d + a + d = 4$

$2a - 2d = 4$

$a - d = 2$

$d = \dfrac{1}{2}$

$\therefore \alpha = 1, \beta = 2, \gamma = 3, \delta = 4$

$\lambda = 3, \quad \lambda + 5 = 8$ **Ans.**

Q.4 [**0012**]

If $2\alpha^2, \alpha^4, 2r$ are in A.P. then

$2\alpha^4 = 2\alpha^2 + 24$

$\Rightarrow \alpha^4 = \alpha^2 + 12$

$\Rightarrow \alpha^4 - \alpha^2 - 12 = 0$

Then $\alpha^2 = \dfrac{1\pm\sqrt{49}}{2}$

$\therefore \alpha_1^2 = \alpha_2^2 = 4$

again $1, \beta^2, 6-\beta^2$ are in G.P.

$(\beta^2)^2 = 1\cdot(6-\beta^2)$

$\Rightarrow \beta^4 + \beta^2 - 6 = 0$

$\therefore \beta^2 = \dfrac{-1\pm\sqrt{25}}{2}$

$\Rightarrow 2, -3$

$\beta_1^2 = \beta_2^2 = 2$

$\therefore \alpha_1^2 + \alpha_2^2 + \beta_1^2 + \beta_2^2 = 4\times2 + 2\times2 = 12.$

Q.5 [0003]

Let common ratio is $\dfrac{1}{2^b}$

and $S_\infty = \dfrac{a}{1-r} = \dfrac{\frac{1}{2^a}}{1-\frac{1}{2^b}} = \dfrac{1}{7}$

$\Rightarrow b = 3 \,\&\, a = b$

$\Rightarrow b = 3 \,\&\, a = b$

Hence, a = 3

Q.6 [0001]

$T_2 = 3 + d, T_{10} = 3 + 9d, T_{34} = 3 + 33d$

since $T_2 . T_{10}, T_{34}$ are in G.P

$T_{10}^{\ 2} = T_2 T_{34}$

$\Rightarrow (3+9d)^2 = (3+d)(3+33d)$

$\Rightarrow d = 0,1$

hence d = 1

Q.7 [0012]

According to question,

$\frac{\log_z x}{\log_x y} = \frac{\log_y z}{\log_z x} \Rightarrow (\log x)^3 = (\log z)^3$

$\Rightarrow x = z$

Since $2y^3 = x^3 + z^3 \Rightarrow x^3 = y^3$ or $x = y$

given $xyz = 64$ & $x = y = z$

$\therefore x = y = z = 4$

$\& x + y + z = 12$

Q.8 [900]

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow 3(a_1 + a_{24}) = 225$ (sum of terms equidistant from beginning and end are equal)

$a_1 + a_{24} = 75$

Now $a_1 + a_2 + \ldots\ldots + a_{23} + a_{24} = \frac{24}{2}[a_1 + a_{24}]$

$= 12 \times 75 = 900$

Q.9 **[4]**

We can write the given equation as

$\log_2\left(x^{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots\ldots}\right) = 4$

$\Rightarrow \log_2(x^2) = 4 \Rightarrow x^2 = 2^4 \Rightarrow x = 4$

Q.10 **[0]**

$T_p = AR^{p-1} = x$

$\log x = \log A + (p-1)\log R$

Similary write log y , log z

Multiply by q – r, r – p and p – q and add we get,

$(q-r)\log x + (r-p)\log y + (p-q)\log z = 0$

PREVIOUS YEAR'S

MHT CET

Q.1 (4)
Q.2 (1)
Q.3 (2)
Q.4 (3)
Q.5 (4)
Q.6 (3)
Q.7 (2)
Q.8 (4)
Q.9 (2)
Q.10 (2)
Q.11 (3)

Q.12 (2)

Given, M is the arithmetic mean of I and n.

$\therefore$ I + n = 2M ...(i)

and G_1, G_2, G_3 are geometric means between I and n.I, G_1, G_2, G_3, n are in GP.

$\therefore G_1 = Ir, G_2 = Ir^2, G_3 = Ir^3 n = Ir^4 \Rightarrow r = \left(\frac{n}{I}\right)^{1/4}$

Now, $G_1^4 + 2G_2^4 + G_3^4 = (Ir)^4 + 2(Ir^2)^4 + (Ir^3)^4$

$= I^4 \times r^4 (1 + 2r^4 + r^8) = I^4 \times r^4 (r^4 + 1)^2$

$= I^4 \times \frac{n}{I}\left(\frac{n+I}{I}\right)^2 = In \times 4M^2 = 4IM^2n$

Q.13 (3)

Given, $\log_2 x + \log_2 y \geq 6$

$\Rightarrow \log_2 (xy)$ ³ 6 Þ xy ³ 2^6

$\Rightarrow \sqrt{xy} \geq 2^3$

$\because \frac{x+y}{2} \geq \sqrt{xy}$ or $x + y \geq 2\sqrt{xy} \geq 16$ [$\because$ AM $\geq$ GM]

$\therefore x + y \geq 16$

JEE-MAIN PREVIOUS YEAR'S

Q.1 (3)

$a, A_1, A_2 \ldots\ldots A_n, 100$

We have , 100 = a + (n + 2 – 1) d

$d = \left(\frac{100 - a}{n+1}\right)$

$\frac{A_1}{A_n} = \frac{a + \frac{(100-a)}{(n+1)}}{a + n\frac{(100-a)}{n+1}} = \frac{1}{7}$

$= \frac{an + a + 100 - a}{an + a + 100n - na} = \frac{1}{7}$

$\Rightarrow$ 7an + 700 = a + 100 n

We have a + n = 33 $\Rightarrow$ a = 33 –n

$\therefore$ 7(33 – n) n + 700 = (33 – n) + 100n

$\Rightarrow 231n - 7n^2 + 700 = (33 - n) + 100n$

$\Rightarrow 7n^2 - 132n - 667 = 0$

$\Rightarrow (n - 23)(7n + 29) = 0$

$\Rightarrow$ n = 23 Ans.

Q.2 [41651]

$$S_n = \frac{n^2}{1-\frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{(n+2)}$$

$$S_n = \frac{n(n^2+2n+1)}{(n+2)}$$

$$S_n = \frac{n[n(n+2)+1]}{(n+2)}$$

$$S_n = n\left[n+\frac{1}{n+2}\right]$$

$$S_n = n^2 + \frac{n+2-2}{(n+2)}$$

$$S_n = n^2 + 1 - \frac{2}{(n+2)}$$

$$\text{Now } \frac{1}{26} + \sum_{n=1}^{50}\left[(n^2-n)-2\left(\frac{1}{n+2}-\frac{1}{n+1}\right)\right]$$

$$= \frac{1}{26}+\left[\frac{50\times 51\times 101}{6}-\frac{50\times 51}{2}-2\left(\frac{1}{52}-\frac{1}{2}\right)\right]$$

$= 41651$

Q.3 (2)

$a_{n+2}\ 2a_{n+1} - a_n + 1,$
$a_1 = 1, a_3 = 3, a_4 = 6$ .....

$$\because \frac{a_n+2}{7^n+2} = \frac{2}{7}.\frac{a_n+1}{7^n+1} - \frac{1}{49}.\frac{a_n}{7^n}+\frac{1}{7^{n+2}}$$

$$\text{So } \sum_{n=2}^{\infty}\frac{a_n+2}{7^{n+2}} = \frac{2}{7}\sum_{n=2}^{\infty}\frac{a_n+1}{7^{n+1}} - \frac{1}{49}\sum_{n=2}^{\infty}\frac{a_n}{7^n}$$

$$\text{Let } \sum_{n=2}^{\infty}\frac{a_n}{7^n} = p$$

$$p - \frac{a_2}{7^2} - \frac{a_3}{7^3} = \frac{2}{7}\left(p-\frac{a_2}{7^2}\right) - \frac{1}{49}p + \frac{\frac{1}{7^4}}{\frac{6}{7}}$$

$$p - \frac{1}{49} - \frac{3}{343} = \frac{2}{7}p - \frac{2}{7^3} - \frac{1}{49}p + \frac{1}{6.7^3}$$

$$p = \frac{7}{216}$$

Q.4 (3)

$$s = 1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+...$$

$$\frac{s}{6} = \frac{1}{6}+\frac{5}{6^2}+\frac{12}{6^3}+\frac{22}{6^4}+...$$

$$S-\frac{s}{6} = 1+\frac{4}{6}+\frac{7}{6^2}+\frac{10}{6^3}+\frac{13}{6^4}+...$$

$$\frac{5s}{6} = 1+\frac{4}{6}+\frac{7}{6^2}+\frac{10}{6^3}+\frac{13}{6^4}+...$$

$$\frac{5s}{6^2} = \frac{1}{6}+\frac{4}{6^2}+\frac{7}{6^3}+\frac{10}{6^4}+...$$

$$\frac{5s}{6}-\frac{5s}{6^2} = 1+\frac{3}{6}+\frac{3}{6^2}+\frac{3}{6^3}+\frac{3}{6^4}+.........$$

$$\frac{5s}{6^2}.s = 1+\frac{3}{6}\left[1+\frac{1}{6}+\frac{1}{6^2}+...\right]$$

$$\frac{25}{36}s = 1+\frac{3}{6}\frac{1}{\left(1-\frac{1}{6}\right)} = 1+\frac{3.6}{6.5}$$

$$s = \frac{8}{5}\times\frac{36}{25} = \frac{288}{125}$$

Q.5 [2223]

3, 6, 9, 12, 15, 17, 21 ... upto 78 term
5, 9, 13, 17, ... upto 59 term
Common term of both the series
9, 21, 33, --till 19 terms
$a = 9, d = 12 \Rightarrow a_n = a + (n-1)d$
$a_n = 9 + (19-1)12 = 9 + 18 \times 12$
$a_n = 225$ and $n = 19$

$$S_n = \frac{n}{2}[a+a_n] = \frac{19}{2}(9+225) = \frac{19}{2}\times 234 = 2223$$

Q.6 (2)

$d=1, \sum_{i=1}^{n} a_i = 192$

$a_1 + a_2 + a_3 + ... + a_n = 192$

$\frac{n}{2}(a_1+a_2) = 192$

$n(a_1+a_n) = 384$

also $\sum_{i=1}^{n/2} a_{2i} = 120$

$a_2 + a_4 + a_6 + ... + a_n = 120$

$$\frac{\frac{n}{2}}{2}\left(a_2+a_n\right)=120$$

$$\frac{n}{4}\left(a_2+a_n\right)=120$$

$$n\left(1+a_1+a_n\right)=480 \qquad \{\because a_2 = 1 + a_1\}$$

$$a_1+a_n=\frac{480}{n}-1 \qquad .....(2)$$

from (1) & (2)

$$\frac{384}{n}=\frac{480}{n}-1$$

384 = 480–n

n=96

Q.7 (4)

$x^3y^2 = 2^{15}$

A.M. $\geq$ G. M.

$$\frac{x+x+x+y+y}{5}\geq(x^3y^2)^{\frac{1}{5}}$$

$$3x + 2y \geq 5.\left(2^{15}\right)^{\frac{1}{5}}$$

$3x + 2y \geq 5.2^3$

$3x + 2y \geq 40$

$(3x + 2y)_{min} = 40$ Ans.

Q.8 [1633]

$24 = 2^3 \times 3$

a can't be multiple of 2k or 3k

$\alpha = [1 + 2 + ...+ 100] - [2+ 4 + ... + 100]$

$-[3 + 6 + 9 + ...99]$

$+ [6 + 12 + ...+ 96]$

$$= 5050 - 2\ \frac{(50)(51)}{2}-3\frac{(33)(34)}{2}+6\frac{(16)(17)}{2}$$

= 5050 − 2550 − 1683 + 816

= 2500 −867

= 1633

Q.9 (2)

$S = 1 + 2.3 + 3.3^2 + ...+ 10.3^9$

$3s = \quad 3 + 2.3^2 + ... + 9.3^9 + 10.3^{10}$

$-2s = 1 + 3 + 3^2 + ... + 3^9 - 10(3)^{10}$

$$-2s=\left(\frac{3^{10}-1}{2}\right)-10(3)^{10}$$

$$s=\frac{20(3^{10})-(3^{10}-1)}{2\times 2}$$

$$S=\frac{19\cdot(3^{10})+1}{4}$$

Q.10 (3)

$$A=\sum_{n=1}^{\infty}\frac{1}{3+(-1)^n)^n}$$

$$=\frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}.....$$

$$=\left(\frac{\frac{1}{2}}{1-\frac{1}{4}}\right)+\frac{\frac{1}{16}}{1-\frac{1}{16}}$$

$$=\frac{2}{3}+\frac{1}{15}=\frac{11}{15}$$

$$B=\sum_{n=1}^{\infty}\frac{(-1)^n}{(3+(-1)^n)^n}$$

$$=\frac{-1}{2}+\frac{1}{4^2}-\frac{1}{2^3}-\frac{1}{4^4}.....$$

$$=-\left(\frac{\frac{1}{2}}{1-\frac{1}{4}}\right)+\frac{\frac{1}{6}}{1-\frac{1}{16}}$$

$$=\frac{-2}{3}+\frac{1}{15}=\frac{-9}{15} \qquad =\frac{A}{B}=\frac{-11}{9}$$

Q.11 [40]

$a_2 + a_4 = 2a_3 + 1$

$\Rightarrow a_1r + a_1r^3 = 2a_1r^2 + 1$ ... (1) (r = common ratio)

and $3a_2 + a_3 = 2a_4$

$\Rightarrow 3a_1r + a_1r^2 = 2a_1r^3$

$\Rightarrow 2r^2 - r - 3 = 0$

$(2r - 3)(r + 1)$

$\Rightarrow r = -1, 3/2$

for r = − 1

$-a_1, -a_1 = 2a_1 + 1$

$a_1 = \frac{-1}{4}$ (rejected)

Hence $r = \frac{3}{2}$ put in equation (i)

$$a_1\left(\frac{3}{2}+\frac{27}{8}-\frac{9}{2}\right)=1$$

$$\Rightarrow a_1\left(\frac{12+27-36}{8}\right)=1$$

$$\Rightarrow a_1=\frac{8}{3}$$

$a_2 + a_4 + 2a_5 = a_1r + a_1r^3 + 2a_1r^4$

$$= \frac{3}{2}\left(\frac{8}{3}\right)+\frac{8}{3}\left(\frac{27}{8}\right)+2\left(\frac{8}{3}\right)\left(\frac{81}{16}\right)$$
$= 4 + 9 + 27$
$= 40$

Q.12 (3)

$x = 1 + a + a^2 = ...$

$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$

$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$

$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$

a, b, c are in A.P.

$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z}$ are in A.P.

$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P.

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

Q.13 [276]

$$\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+...$$

$$T_n = \frac{n}{4n^4+1}$$

$$= \frac{n}{(2n^2+1)^2-(2n)^2} = \frac{n}{(2n^2+2n+1)^2-(2n^2-2n+1)}$$

$$= \frac{1}{4}\left[\frac{1}{2n^2-2n+1}-\frac{1}{2n^2+2n+1}\right]$$

$$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{4}\left[\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+...\frac{1}{200+20+1}\right]$$

$$= \frac{1}{4}\left[1-\frac{1}{221}\right] = \frac{1}{4}\times\frac{220}{221} = \frac{55}{221} = \frac{m}{n}$$

$m + n = 55 + 221 = 276$

Q.14 (3)

$$S = 2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+...$$

Considering infinite sequence,

$$S = 2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+... \quad ...(1)$$

$$\frac{S}{7} = \frac{2}{7}+\frac{6}{7^2}+\frac{12}{7^3}+\frac{20}{7^4}+... \quad ...(2)$$

Equation (1) – (2)

$$\Rightarrow \frac{6s}{7} = 2+\frac{4}{7}+\frac{6}{7^2}+\frac{8}{7^3}+\frac{10}{7^4}+..... \quad ....(3)$$

$$\Rightarrow \frac{6s}{7^2} = \frac{2}{7}+\frac{4}{7^2}+\frac{6}{7^3}+\frac{8}{7^4}+...... \quad .....(4)$$

Equation (3) – (4)

$$\Rightarrow \frac{6S}{7}\left(1-\frac{1}{7}\right) = 2+\frac{2}{7}+\frac{2}{7^2}+\frac{2}{7^3}+...$$

$$\Rightarrow \frac{6^2S}{7^2} = \frac{2}{1-\frac{1}{7}} = \frac{2}{6}\times 7$$

$$\Rightarrow S = \frac{2\times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$$

Q.15 (4)

$a_1, a_2, a_3, ...$ A.P.; $a_1 = 2$; $a_{10} = 3$; $d_1 = \frac{1}{9}$

$b_1, b_2, b_3, ...$ A.P.; $b_1 = \frac{1}{2}$; $b_{10} = \frac{1}{3}$; $d_2 = \frac{-1}{54}$

[Using $a_1b_1 = 1 = a_{10}b_{10}$; d_1 & d_2 are common differences respectively]

$$a_4 \cdot b_4 = (2+3d_1)\left(\frac{1}{2}+3d_2\right)$$

$$= \left(2+\frac{1}{3}\right)\left(\frac{1}{2}-\frac{1}{18}\right)$$

$$= \left(\frac{7}{3}\right)\left(\frac{8}{18}\right) = \left(\frac{28}{27}\right)$$

Q.16 (3)

$f(x+y)=2f(x).f(y) \qquad f(1) = 2$

put $x= y=1 \quad f(2)= 2 \cdot 2 \cdot 2 = 2^3$

$x=1, y=2 \quad f(3)= 2 \cdot f(1) \cdot f(2)=2 \cdot 2 \cdot 2^3=2^5$

$T_n=2\{4^{n-1}\}=f(n)$

$f(\alpha+k)= 2.f(\alpha).f(k)$

$$\sum_{k=1}^{10} f(\alpha+k) = 2f(\alpha)\sum_{k=1}^{10} f(k)$$

$$= 2f(\alpha)[2+2^3+2^5.....\text{upto } 10 \text{ terms}] \text{ G.P.}$$

$$= 2f(\alpha)\left[\frac{2[4^{10}-1]}{4-1}\right]$$

$$= \frac{2}{3}f(\alpha)[2(2^{20}-1)] = \frac{512}{3}(2^{20}-1)$$

$\Rightarrow 4f(\alpha) = 512$
$\Rightarrow f(\alpha) = 128$
$\Rightarrow 128 = 2.4^{n-1}$
$\Rightarrow 64 = 4^{n-1} = 4^3$
$\Rightarrow n=4$

Q.17 (4)

$S = \frac{1}{2.3^{10}} + \frac{1}{2^2.3^9} ..... + \frac{1}{2^{10}.3}$ is a G.P.

$\text{First term} = \frac{1}{2.3^{10}}$

$r = \frac{3}{2}, n = 10$

$$S = \frac{1}{2.3^{10}}\left\{\frac{\left(\frac{3}{2}\right)^{10} - 1}{\frac{3}{2} - 1}\right\} = \frac{1}{3^{10}}\left\{\frac{3^{10} - 2^{10}}{2^{10}}\right\}$$

$$= \frac{3^{10} - 2^{10}}{2^{10}.3^{10}}$$

$\therefore k = 3^{10} - 2^{10}$
$3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5) = 211 \times 275$
$= (210 + 1)(270 + 5)$
$= (6\lambda + 1)(6\mu + 5)$
$\therefore$ remainder $= 5$ Ans.

Q.18 [98]

$S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81}$ ..... up to 100 terms

$= \frac{3-2}{3} + \frac{3^2 - 2^2}{3^2} + \frac{3^3 - 2^3}{3^3} + \frac{3^4 - 2^4}{3^4}$ ..... up to 100 terms

$= 100 - \left\{\frac{2}{3} + \frac{2^2}{3^2} + \frac{2^3}{3^3} + \text{....up to 100 terms}\right\}$

$= 100 - 2\left[1 - \left(\frac{2}{3}\right)^{100}\right]$

$S = 98 + 2\left(\frac{2}{3}\right)^{100}$

$\therefore [S] = 98$

Q. 19 [5264]
Sum of elements in $A \cap B$

$$= \underbrace{2 + 4 + 5 + ... + 200}_{\text{Multiple of } 2} - \underbrace{6 + 12 + ... + 198}_{\text{Multiple of } 2\&3 \ i.e. 6}$$

$$- \underbrace{10 + 20 + ... + 200}_{\text{Multiple of } 5\&2 \ i.e. 10} + \underbrace{30 + 60 + ...180}_{\text{Multiple of } 2,5\ \&3 \ i.e. 30}$$

=5264

Q. 20 [1100]

$$A = \sum_{i=1}^{10}\sum_{j=1}^{10} \min\{i, j\}$$

$$B = \sum_{i=1}^{10}\sum_{j=1}^{10} \max\{i, j\}$$

$$A = \sum_{i=1}^{10} \min(i,1) + \min(i,2) + ... \min(i,10)$$

$$\underbrace{1+1+1+1+...+1}_{19 \, times} + \underbrace{2+2+2...+2}_{19 \, times} + \underbrace{3+3+3+...+3}_{15 \, times}$$
....(1)1 times

$$B = \sum_{i=1}^{10} \max(i,1) + \max(i,2) + ... \max(i,10)$$

$$\underbrace{10 + 10 + ... + 10}_{19 \text{ times}} + \underbrace{(9 + 9 + ... + 9)}_{17 \text{ times}} + ... + (1) 1 \text{times}$$

$A+B = 20(1+2+3+...+10)$

$$= 20 \times \frac{10 \times 11}{2} = 10 \times 110 = 1100$$

Q.21 (3)

$A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$

$(A_4)^4 = \frac{1}{1296}$

$A_4 = \frac{1}{6}$ ... (1)

$A_2 + A_4 = \frac{7}{36}$ ... (2)

$A_2 = \frac{1}{36}$

$A_6 = 1 =$ $A_8 = 6 = s$ $A_{10} = 36$
$A_6 + A_8 + A_{10} = 43$

Q.22 [702]
1, $a_1, a_2, a_3, ..., a_{18}$, 77 are in AP
i.e., 1, 5, 9, 13, ..., 77
Hence, $a_1 + a_2 + a_3 + ... + a_{18} = 5 + 9 + 13 + ...$ upto 18 terms
$= 702$

Q.23 [120]

$$\frac{2^3 - 1^3}{1x7} + \frac{4^3 - 3^3 + 2^3 - 1^3}{2x11} +$$

$$\frac{6^3 - 5^3 + 4^3 - 3^3 + 2^3 - 1^3}{3x15} + ... +$$

$= 1 + 2 + 3 + ... + 15$ term

$$\frac{15x16}{2} = 8x15 = 120$$

Q.24 [6993]

3
6 9 12
15 18 21 24 27

In 11^{th} set total no. of elements = 2 × 11 – 1 = 21

Total number of element till 10^{th} group

= 3(1 + 3 + 5 + ...+ 19]

= 300

First element of 11^{th} group = 303

Sum of element of 11^{th} group = $\frac{21}{2}[2 \times 303 + (10) \times 3]$

= 21(303 + 30]

= 6993

Q.25 [57]

$^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P.

$\beta^2 - \frac{5}{2}\beta^3 = -12\beta$

$\beta = \frac{12}{5}$ or $\beta = -2$

$\therefore \beta = \frac{12}{5}$

$d = \frac{-72}{5} - \frac{144}{25} = -\frac{504}{25}$

$\therefore 50 - \frac{2d}{\beta^2} = 57$

Q.26 [12]

$\frac{6}{3^{12}} + 10\left(\frac{1}{3^{11}} + \frac{2}{3^{10}} + \frac{2^2}{3^9} + \frac{2^3}{3^8} + ... + \frac{2^{10}}{3}\right)$

$\frac{6}{3^{12}} + \frac{10}{3^{11}}\left(\frac{6^{11}-1}{6-1}\right)$

$2^{12} \cdot 1$; m · n = 12

Q.27 [38]

$x^2 - 8ax + 2a = 0 \langle^p_r$ & $x^2 + 12bx + 6b = 0 \langle^q_s$

p + r = 8 a q + p = – 12b

pr = 2a qp = 6b

$\frac{1}{p} + \frac{1}{r} = 4$ $\frac{1}{q} + \frac{1}{p} = -2$

Now $\frac{1}{p}, \frac{1}{q'}\frac{1}{r'}\frac{1}{s'}$ ax is AP with common diff. = d & first term = α

$\frac{1}{p} + \frac{1}{r} = 4$ $\Rightarrow \alpha + \alpha + 2d = 4$ $\Rightarrow \alpha + d = 2$

$\alpha + 2d = -1$

――――――

$-d = 3$

s”y $\frac{1}{q} + \frac{1}{s} = -2$ $\Rightarrow \alpha + d + \alpha + 3d = -2$

$\boxed{d = -3}$

& $\boxed{\alpha = 5}$

Now $\frac{1}{p} = 5, \frac{1}{q} = 2, \frac{1}{r} = -1, \frac{1}{s} = -4$

So $2a = pr \Rightarrow 2a = \frac{1}{5}.\frac{1}{-1} \Rightarrow a = \frac{1}{-10}$

$6b = qs \Rightarrow 6b = \frac{1}{2}.\left(\frac{1}{-4}\right) \Rightarrow b = \frac{-1}{48}$

Hence $a^{-1} - b^{-1}$, = – 10 + 48 = 38

Q.28 [27560]

$a_1 = b_1 = 1$, $\underbrace{a_n = a_{n-1} + 2}_{AP}$ & $b_n = a_n + b_{n-1}$

$\forall$ n ≥ 2

$a_1 = 1, a_2 = 3, a_3 = 5, ..., a_n = (2n-1)$

Now $b_2 = a_2 + b_1 = 3 + 1 = 4$

$b_3 = a_3 + b_2 = 5 + 4 = 9$

$b_4 = a_4 + b_3 = 7 + 9 = 16$

$b_5 = a_5 + b_4 = 9 + 16 = 25$

$\Rightarrow \Sigma_{n=1}^{15} a_n . b_n = 1.\ 1^2 + 3.2^2 + 5.3^2 + 7.4^2 + ...+ 29.15^2$

$\Rightarrow S_{15} = \Sigma_{n=1}^{15} [(2n-1).\ n^2]$

$= 2\ \Sigma_{n=1}^{15} n^3 - \Sigma_{n=1}^{15} n^2$

$= 2.\left(\frac{15.16}{2}\right)^2 - \frac{15.16.31}{6}$

= 28800 – 1240

= 27560 Ans.

Q.29 (2)

$a_0 = 0; a_1 = 0$

$a_{n+2} = 3a_{n+1} - 2a_{n+1} : n \geq 0$

$a_{n+2} - a_{n+1} = 2\ (a_{n+1} - a_n) + 1$

n = 0 $a_2 - a_1 = 2(a_1 - a_0) + 1$

n = 1 $a_3 - a_2 = 2(a_2 - a_1) + 1$

n = 2 $a_4 - a_3 = 2(a_3 - a_2) + 1$

n = n $a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1$

$(a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n + 1) = 0$

$a_{n+2} = 2a_{n+1} + (n + 1)$

$n \to n - 2$

$a_n - 2a_{n-1} = n - 1$

Now, $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$

$= (a_{25} - 2a_{24})(a_{23} - 2a_{22}) = (24)(22) = 528$

Q.30 (3)

By splitting

$$\frac{1}{20}\left[\left(\frac{1}{20-a}-\frac{1}{40-a}\right)+\left(\frac{1}{40-a}-\frac{1}{60-a}\right)+...+\left(\frac{1}{180-a}-\frac{1}{200-a}\right)\right]=\frac{1}{256}$$

$(20-a)(200-a) = 256 \times 9$

$a^2 + 220a + 1696 = 0$

$a = 8, 212$

Hence maximum value of a is 212

Q.31 [16]

$$S=\frac{a_1}{2}+\frac{a_2}{2^2}+\frac{a_3}{2^3}+...$$

$$\frac{S}{2}=\frac{a_1}{2^2}+\frac{a_2}{2^3}+...$$

$$\frac{S}{2}=\frac{a_1}{2}+d\left(\frac{1}{2^2}+\frac{1}{2^3}+...\right)$$

$$\frac{S}{2}=\frac{a_1}{2}+d\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)$$

$\therefore$ $S = a_1 + d = a_2 = 4$

or $4a_2 = 16$

Q.32 [286]

$$\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{100.101.102}=\frac{k}{101}$$

$$\frac{4-2}{2.3.4}+\frac{5-3}{3.4.5}+...+\frac{102-100}{100.101.102}=\frac{2k}{101}$$

$$\frac{1}{2.3}-\frac{1}{3.4}+\frac{1}{3.4}-\frac{1}{4.5}+...+\frac{1}{100.101}-\frac{1}{101.102}=\frac{2k}{101}$$

$$\frac{1}{2.3}-\frac{1}{101.102}=\frac{2k}{101}$$

$$\therefore\ 2k=\frac{101}{6}-\frac{1}{102}$$

$\therefore$ $34k = 286$

Q.33 (1)

Consider a case when $\alpha = \beta = 0$ then

$f(x)=\gamma x$ $\qquad$ $g(x)=\frac{x}{\gamma}$

$\frac{1}{n}\sum_{i=1}^{n} f(a_i) \Rightarrow \frac{1}{n}(a_1 + a_2 + ... + a_n) = 0$

$\Rightarrow f(g(0))$ $\qquad$ $\Rightarrow f(0) = 0$

Q.34 (3)

$a_{n+2}\ a_{n+1} - a_{n+1}\ .\ a_{n=2}$

series will satisfy

$a_1a_2,\ a_2a_3,\ a_3a_4,\ a_4a_5$

1.2, 2.2 2.3, 2.4

$$\frac{a_n+\frac{1}{a_{n+1}}}{a_{n+2}}=\frac{a_{n+2}-\frac{1}{a_{n+1}}}{a_{n+2}}$$

$$=1-\frac{1}{a_{n+1}a_{n+2}}=1-\frac{1}{2(r+1)}=\frac{2r+1}{2(r+1)}$$

Now proof is given by

$$=\prod_{r=1}^{30}\frac{(2r+1)}{2(r+1)}$$

$$=\frac{(1.3.5......'61)}{\lfloor 31.2^{30}}\times\frac{2^{30}\times\lfloor 30}{2^{30}\times\lfloor 30}=\frac{\lfloor 61}{2^{60}\lfloor 31\lfloor 30}$$

$\alpha = -60$

Q.35 [50]

$f(x) = 0 \Rightarrow (x-p)^2 - q = 0.$

Roots are $p+\sqrt{q}, p-\sqrt{q}$ absolute difference between roots is $2\sqrt{q}$.

Now $|f(a_i)| = 500$

Let a_1, a_2, a_3, a_4 are a, a +d, a+2d, a+3d

$|f(a_4)| = 500$

$|(a_1 - p)^2 - q| = 500$

$$\Rightarrow \frac{9}{4}d^2 - q = 500 \qquad ...(1)$$

And $|f(a_1)|^2 = |f(a_2)|^2$

$((a_1 - p)^2 - q)^2 = ((a_2 - p)^2 - q)^2$

$\Rightarrow ((a_1 - p)^2 - (a_2 - p)^2)\ ((a_1 - p)^2 - q + (a_2 - p)^2 - q) = 0$

$$\Rightarrow \frac{9}{4}d^2 - q + \frac{d^2}{2} - q = 0$$

$$2q=\frac{10d^2}{4}\Rightarrow q=\frac{5d^2}{4}$$

$$\Rightarrow d^2=\frac{4q}{5}$$

From equation (1) $\frac{9}{4}.\frac{4.q}{5}-q=500$

$$\frac{4q}{5}=500$$

And $2\sqrt{q}=2\times\frac{50}{2}=50$

Q.36 [142]

$$\Sigma x_0^1=\frac{3\left(1-\left(\frac{1}{2}\right)\right)^{20}}{1-\frac{1}{2}}=6\left(1-\frac{1}{2^{20}}\right)$$

$$=\sum_{i=1}^{20}(x_i-i)^2$$

$$=\sum_{i=1}^{20}(x_i)^2+(i)^2-2x_i i$$

$$\text{Now} = \sum_{i=1}^{20}(x_i)^2 = \frac{9\left(1-\left(\frac{1}{4}\right)\right)^{20}}{1-\frac{1}{4}} = 12\left(1-\frac{1}{2^{40}}\right)$$

$$= \sum_{i=1}^{20} i^2 = \frac{1}{6}\times 20\times 21\times 41 = 2870$$

$$= \sum_{i=1}^{20} x_i.i = s = 3 + 2.3\frac{1}{2} + 3.3\frac{1}{2^2} + 4.3\frac{1}{2^3} + ...AGP$$

$$= 6\left(2-\frac{22}{2^{20}}\right)$$

$$\bar{x} = \frac{12-\frac{12}{2^{40}} + 2870 - 12\left(2-\frac{22}{2^{20}}\right)}{20}$$

$$\bar{x} = \frac{2858}{20} + \left(\frac{-12}{2^{40}} + \frac{22}{2^{20}}\right)\times\frac{1}{20}$$

$$[\bar{x}] = 142$$

Q.37 (2)

Give

$$\frac{S_5}{S_9} = \frac{5}{17}$$

$$\frac{\frac{5}{2}[2a_1+(5-1)d]}{\frac{9}{2}[2a_1+(9-1)d]}$$

$$\frac{5}{9}\left[\frac{2a_1+4d}{2a_1+8d}\right] = \frac{5}{17}$$

$$\frac{5}{9}\left[\frac{a_1+2d}{a_1+4d}\right] = \frac{5}{17}$$

$$\frac{1}{9}\left[\frac{a_1+2d}{a_1+4d}\right] = \frac{1}{17}$$

$17a_1 + 34d = 9a_1 + 36d$

$8a_1 = 2d$

$4a_1 = d$

Now

$110 < a_{15} < 120$

$110 < a_1 + (15-1)\,d < 120$

$110 < a_1 + 14d < 120$

$110 < a_1 + 14 \times (4a_1) < 120$

$110 < a_1 + 56a_1 < 120$

$110 < 57a_1 < 120$

$$\frac{110}{57} < a_1 < \frac{120}{57}$$

$1.9 < a_1 < 2.1$

$a_1 \in n$

$a_1 = 2$

Then $4a_1 = d$

$d = 8$

New sum of firnt ten terms

$$S_{10} = \frac{10}{2}[2x(2)+(10-1)x8]$$

$= 5[4 + 9x8]$

$= 5\,[4 + 72]$

$= 380$

Q.38 [166]

$$\sum_{k=1}^{10}\frac{k}{k^4+k^2+1}$$

$$= \sum_{k=1}^{10}\frac{k}{(k^2+k+1)(k^2-k+1)}$$

$$= \sum_{k=1}^{10}\frac{1}{2}\left(\frac{1}{k^2-k+1}-\frac{1}{k^2+k+1}\right)$$

$$= \frac{1}{2}\left[\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{7}....\frac{1}{91}-\frac{1}{111}\right]$$

$$= \frac{1}{2}\left[1-\frac{1}{111}\right]$$

$$= \frac{55}{111} = \frac{m}{n}$$

$m + n = 166$

Q.39 [53]

Let common difference is d and number of terms is n

$199 = 100 + (n-1)\,d$

$$\Rightarrow d = \frac{99}{n-1}$$

n	d
4	33
10	11
12	9

required answer = 33 + 11 + 9 = 53

Q.40 (3)

$$\ln N = ([2.2^2...2^{60}][4.4^2...4^n])^{\frac{1}{60+n}}$$

$$= \left[2^{(1+...+60)}\cdot 4^{(1+2+...+n)}\right]^{\frac{1}{60+n}}$$

$$= \left[2^{(1830)}\cdot 4^{\frac{n(n+1)}{2}}\right]^{\frac{1}{60+n}} \quad = 2^{\frac{1830+n(n+1)}{60+n}} \quad 2^{\left(\frac{225}{8}\right)}$$

$= \frac{1830 + n^2 + n}{60 + n} = \frac{225}{8}$

$\Rightarrow 8n^2 - 217n + 1140 = 0$

$n = 20, \frac{57}{8}$

$\sum_{k=1}^{20} (nk - k^2)$

$(20)\left[\frac{(20)(21)}{2}\right] - \frac{(20)(21)(41)}{6}$

$\frac{(20)(21)}{2}\left[20 - \frac{41}{3}\right]$

$\frac{(20)(21)(19)}{6} (10)(7)(19) = 1330$

Q.41 [1]

$S_{21} = \frac{21}{2}(2A + 20d) = \frac{21}{2}(2.\ 10ar\ + 20{,}10ar^2)$

$(\therefore A = 10\ ar\ \&\ d = 10ar^2)$

$= 21\ (10ar + 10\ .10ar^2)$

$= 21 \text{ x } 10ar\ (1+10r)$

$a_{11} = A + 10\ d = 10ar + 10.\ 10ar^2 = 10\ ar\ (1+10r)$ ......(1)

$S21 = 21xa_{11}$

Q.42 (2)

$\frac{3}{4}\sum_{n=1}^{21} \frac{(4n+3)-(4n-1)}{(4n+3)(4n-1)} = \frac{3}{4}\sum_{n=1}^{21}\left(\frac{1}{4n-1} - \frac{1}{4n+3}\right)$

$= \frac{3}{4}\left(\frac{1}{3} - \frac{1}{87}\right) = \frac{7}{29}$

PERMUTATION & COMBINATION

EXERCISE-I (MHT CET LEVEL)

Q.1 (4)
After fixing 1 at one position out of 4 places 3 places can be filled by $^7P_3$ ways. But some numbers whose fourth digit is zero, so such type of ways $= {}^6P_2$
$\therefore$ Total ways $= {}^7P_3 - {}^6P_2 = 480$

Q.2 (2)

Since $^nC_2 - n = 44 \Rightarrow n = 11$

Q.3 (c)
Rank =(4!×3)+(3!×2)+(2!×2)+1
=72+12+4+1=89

Q.4 (b)
We have : 30 = 2 × 3 × 5. So, 2 can be assigned to either a or b or c i.e. 2 can be assiggned in 3 ways. Similarly, each of 3 and 5 can be assigned in 3 ways. Thus, the number of solution is 3 × 3 × 3 = 27.

Q.5 (d)
No. of word startion with A are 4 ! = 24
No. fo words starting with H are 4 ! = 24
No. of words atarting with L are 4 ! = 24
These account for 72 words Next word is RAHLU and th 74th word RAHUL.

Q.6 (d)
Number form by using 1, 2, 3, 4, 5 = 5! = 120
Number formed by using 0, 1, 2, 4, 5

4	4	3	2	1

$= 4.4.3.2.1 = 96$

Total number formed, divisible by 3 (taking numbers without repetition) = 216
Statement 1 is false and statement 2 is true.

Q.7 (b)
First prize can be given in 5 ways. Then second prize can be given in 4 ways and the third prize in 3 ways (Since a competitior cannot get two prizes) and hence the no. of ways.

Q.8 (4)

Required number of ways $^8C_2 = 28$

Q.9 (3)

Since $^nC_2 - n = 35 \Rightarrow \dfrac{n!}{2!\,(n-2)!} - n = 35$

$\Rightarrow n(n-1) - 2n = 70 \Rightarrow n^2 - 3n = 70$

$\Rightarrow n^2 - 3n - 70 = 0 \Rightarrow (n+7)(n-10) = 0 \Rightarrow$
$n = 10$

Q.10 (3)

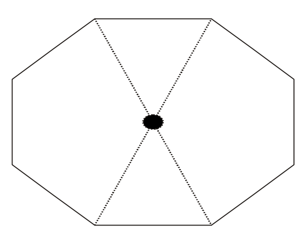
A gets 2, B gets 8; $\dfrac{10!}{2!8!} = 45$

A gets 8, B gets 2; $\dfrac{10!}{8!2!} = 45$

$45 + 45 = 90$

Q.11 (2)
Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the first place with 0 in each of remaining places.
After fixing 1st place, the second place can be filled by any of the 5 numbers. Similarly third place can be filled up in 5 ways and 4th place can be filled up in 5 ways. Thus there will be 5 × 5 × 5 = 125 ways in which 1 will be in first place but this include 1000 also hence there will be 124 numbers having 1 in the first place. Similarly 125 for each 2 or 3. One number will be in which 4 in the first place and i.e. 4000. Hence the required numbers are 124 + 125 + 125 + 1 = 375 ways.

Q.12 (a)
A combination of four vertices is equiva
lent to one interior poin of intersection of diagonals.



$\therefore$ No. of interior points of intersection

$= {}^nC_4 = 70$

$\Rightarrow n(n-1)(n-2)(n-3) = 5.6.7.8$

$\therefore n = 8$

So, number of diagonals $= 8_{C_2} - 8 = 20$

Q.13 (c)

The number of three elements subsets containing a_3 is equal to the number of ways of selecting 2 elements out of n-1 elements. So, the required number of subsets is ${}^{n-1}C_2$

Q.14 (a)

The two letters, the first and the last of the four lettered word can be chosen in $(17)^2$ ways, as repetition is allowed for consonants. The two vowels in the middle are
distinct so that the number of ways of filling up the two places is $= {}^5P_2 = 20.$

Q.15 (a)

A committee of 5 out of 6+4=10 can be made in ${}^{10}C_5 = 252$ ways. If no woman is to be included, thennumber of ways $= {}^5C_5 = 6$

$\therefore$ the required number $= 252 - 6 = 246$

Q.16 **(d)**

Q.17 (3)

Since the 5 boys can sit in 5 ! ways. In this case there are 6 places are vacant in which the girls can sit in ${}^6P_3$ ways. Therefore required number of ways are ${}^6P_3 \times 5\,!$

Q.18 (1)

It is obvious.

Q.19 (3)

Required number of ways = $2^7 - 1 = 127$.

{Since the case that no friend be invited *i.e.*, ${}^7C_0$ is excluded}.

Q.20 (1)

Required number of ways

$= {}^{15}C_1 \times {}^8C_1 = 15 \times 8$

Q.21 (3)

$${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2} = {}^nC_r + {}^nC_{r-1} + {}^nC_{r-1} + {}^nC_{r-2}$$

$$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r$$

Q.22 (2)

${}^nC_2 = 66 \Rightarrow n(n-1) = 132 \Rightarrow n = 12$

Q.23 (2)

${}^nC_2 = 153 \Rightarrow \dfrac{n(n-1)}{2} = 153 \Rightarrow n = 18$

Q.24 (2)

$2\,.\,{}^{20}C_2$ {Since two students can exchange cards each other in two ways}.

Q.25 (2)

Since 5 are always to be excluded and 6 always to be included, therefore 5 players to be chosen from 14. Hence required number of ways are ${}^{14}C_5 = 2002$.

Q.26 (4)

Required number of ways = $2^{10} - 1$

(Since the case that no friend be invited *i.e.*, ${}^{10}C_0$ is excluded).

Q.27 (2)

Required number of ways $= {}^4C_2 \times {}^3C_2 = 18$

Q.28 (4)

The required number of points

$= {}^8C_2 \times 1 + {}^4C_2 \times 2 + \left({}^8C_1 \times {}^4C_1\right) \times 2$

$= 28 + 12 + 32 \times 2 = 104$

Q.29 (1)

${}^{16}C_3 - {}^8C_3 = 504$

Q.30 (2)

Clearly, ${}^nC_3 = T_n$.

So, ${}^{n+1}C_3 - {}^nC_3 = 21 \Rightarrow ({}^nC_3 + {}^nC_2) - {}^nC_3 = 21$

$\therefore\ {}^nC_2 = 21$ or $n(n-1) = 42 = 7.6\ \therefore\ n = 7$

Q.31 (1)

26 cards can be chosen out of 52 cards, in ${}^{52}C_{26}$ ways. There are two ways in which each card can be dealt, because a card can be either from the first pack or from the second. Hence the total number of ways $= {}^{52}C_{26}\,.\,2^{26}$

Q.32 (2)

Required number of ways

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 2^2 - 1 = 63$

Q.33 (1)
It is a fundamental concept

Q.34 (3)
The arrangement can be make as
$.+.+.+.+.+.+.$ *i.e.*, the $(-)$ signs can be put in 7 vacant (pointed) place.
Hence required number of ways $= {}^7C_4 = 35$

Q.35 (1)
The selection can be made in ${}^5C_3 \times {}^{22}C_9$
{Since 3 vacancies filled from 5 candidates in ${}^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9}.

Q.36 (3)
Required number of ways $9! \times 2$
{By fundamental property of circular permutation}.

Q.37 (2)
Since total number of ways in which boys can occupy any place is $(5-1)! = 4!$ and the 5 girls can be sit accordingly in $5!$ ways.
Hence required number of ways are $4! \times 5!$

Q.38 (d)
Leaving one seat vacant between two boys, 5 boys may be seated in 4! ways. Then at
remaining 5 seats, 5 girls any sit in 5! ways. Hence the required number $= 4! \times 5!$

Q.39 (c)
X - X - X - X - X. The four digits 3, 3, 5,5 can be arrabged at (–) places in $\frac{4!}{2!2!} = 6$ ways.
The five digits 2, 2, 8, 8, 8 can be arrabged at (X) places in $\frac{5!}{2!3!}$ ways = 1 ways.
Total no. of arrangements = 6 × 10 = 60 ways

Q.40 (d)
It is obvious by fundamental property of circular permutations.

Q.41 (4)
A garland can be made from 10 flowers in $\frac{1}{2}(9!)$ ways.
{$\because$ *n* flowers' garland can be made in $\frac{1}{2}(n-1)!$ ways}

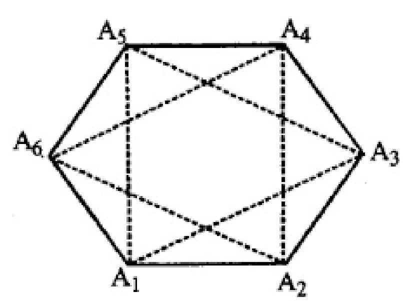
Q.42 (1)
The number of ways in which 5 beads of different colours can be arranged in a circle to form a necklace are $(5-1)! = 4!$.
But the clockwise and anticlockwise arrangement are not different (because when the necklace is turned over one gives rise to another)
Hence the total number of ways of arranging the beads
$= \frac{1}{2}(4!) = 12$

Q.43 (3)
Total number of arrangements are $\frac{6!}{2!} = 360$
The number of ways in which come O's together $= 5! = 120$.
Hence required number of ways $= 360 - 120 = 240$.

Q.44 (2)
It is obvious.

Q.45 (4)
Word 'MATHEMATICS' has $2M, 2T, 2A, H, E, I, C, S$.
Therefore 4 letters can be chosen in the following ways.
Case I : 2 alike of one kind and 2 alike of second kind
i.e., ${}^3C_2 \Rightarrow$ No. of words $= {}^3C_2 \frac{4!}{2!2!} = 18$
Case II : 2 alike of one kind and 2 different
i.e., ${}^3C_1 \times {}^7C_2 \Rightarrow$ No.of words
$= {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$
Case III : All are different
i.e. ${}^8C_4 \Rightarrow$ No. of words ${}^8C_4 \times 4! = 1680$
Hence total number of words are 2454.

Q.46 (c)
Three vertices can be selected in ${}^6C_3$ ways.



The only equilateral triangles possible are

$A_1A_3A_5$ and $A_2A_4A_6$

$$P = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$

Q.47 (a)
Atleast one black ball can be drawn in the following ways
(i) one black and two other colour balls

$= {}^3C_1 \times {}^6C_2 = 3 \times 15 = 45$

(ii) two black and one other colour balls

$= {}^3C_2 \times {}^6C_1 = 3 \times 6 = 18$

(iii) All the three are black $= {}^3C_3 \times {}^6C_0 = 1$

$\therefore$ Req. no. of ways =45+18+1=64

Q.48 (c)

Q.49 (1)
Since, 38808 = 8 × 4851

$= 8 \times 9 \times 539 = 8 \times 9 \times 7 \times 7 \times 11 = 2^3 \times 3^2 \times 7^2 \times 11$

So, number of divisors
$= (3 + 1)(2 + 1)(2 + 1)(1 + 1) = 72.$
This includes two divisors 1 and 38808. Hence, the required number of divisors = 72 – 2 = 70.

Q.50 (1)
Since the total number of selections of r things from n things where each thing can be repeated as many times as one can, is ${}^{n+r-1}C_r$

Therefore the required number $= {}^{3+6-1}C_6 = 28$

Q.51 (a)
First prize may be given to any one of the 4 boys, hence first prize can be distribbuted in 4 ways.
similarly every one of second, third fourth and fiffth prizes can also be given in 4 ways.
$\therefore$ the number of ways of their distribution
$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$

Q.52 (2)
Three letters can be posted in 4 letter boxes in $4^3 = 64$ ways but it consists the 4 ways that all letters may be posted in same box. Hence required ways = 60.

Q.53 (3)
Let E(n) denote the exponent of 3 in n. The greatest integer less than 100 divisible by 3 is 99.

We have $E(100!) = E(1.2.3.4....99.100)$

$= E(3.6.9....99) = E[(3.1)(3.2)(3.3)........(3.33)]$

$= 33 + E(1.2.3......33)$

Now $E(1.2.3......33) = E(3.6.9....33)$

$= E[(3.1)(3.2)(3.3)........(3.11)]$

$= 11 + E(1.2.3.....11)$

and

$E(1.2.3....11) = E(3.6.9) = E[(3.1)(3.2)(3.3)]$

$3 + E(1.2.3) = 3 + 1 = 4$

Thus $E(100!) = 33 + 11 + 4 = 48$.

EXERCISE-II (JEE MAIN LEVEL)

Q.1 **(2)**
As per the given condition, digit 1 should occur at alternate places of the number and at the remaining 5 places either 2, 3, 5 or 7 should appear. Now when the number starts with 1, number of numbers = 4^5 and when the number starts with either 2, 3, 5 or 7, number of numbers = 4^5
So, total number = 2×4^5 = 2048 **Ans.**

Q.2 **(4)**
1 . ② . 3 . ④ . 5 . ⑥ . 7

T					T

${}^3C_2 \cdot 2! \cdot {}^5C_4 \cdot 4! = 6 \times 120 = 720$]

Q.3 **(1)**
$1 + 2 + 3 + ... + 9 = 45 = 0 + 1 + 2 + 3 + ... + 9$
All 9 digit such numbers = 9 !

All 10 digit such numbers when '0' included = 10 ! – 9 !
So, total = 9 ! + (10 ! – 9 !) = (10) ! **Ans.**

Q.4 **(a)**
Total number of 4-digit numbers
$= 5 \times 5 \times 5 \times 5 = 625$
(as each place can be filled by anyone of the numbers 1, 2, 3, 4 and 5)
Number in which no two digits are identical
$= 5 \times 4 \times 3 \times 2 = 120$ (i.e. repetition not allowed)
(as 1^{st} place can be filled in 5 different ways, 2^{nd} place can be filled 4 different ways and so on)
Number of 4-digits numbers in which at least 2 digits are identical
= 625 – 120 = 505 **!**

Q.5 (b)
Total number of arrangements of 10 digits 0, 1, 2, ..., 9 by taking 4 at atime = ${}^{10}C_4 \times 4!$
we observe that in every arrangement of 4 selected digits there is just one arrangement in which the digits are in descending order.

∴ Required number of 4-digit numbers.

$$= \frac{^{10}C_4 \times 4!}{4!} = {}^{10}C_4$$

Q.6 **(b)**

Q.7 **(3)**

Word QUEUE

$E \rightarrow 2, Q, U - 2$

E				

= 18

$\frac{4!}{2!}$

Q	E			

= 3

$\frac{3!}{2!}$

Q	U	E	E	U

= 1

Q	U	E	U	E

= 1 17^{th} rank

Q.8 **(2)**

$${}^{2002}C_{1001} = \frac{(2002)!}{(1001)!(1001)!}$$

no. of zeros in (2002)! are

400 + 80 + 16 + 3 = 499

no. of zeroes in $(1001\,!)^2 = 2(200 + 40 + 8 + 1) = 498$

Hence no. of zeroes is $\frac{(2002)!}{(1001!)^2} = 1$

Q.9 **(3)**

Total number of signals can be made from 3 flags each of different colour by hoisting 1 or 2 or 3 above.

i.e. ${}^3p_1 + {}^3p_2 + {}^3p_3 = 3 + 6 + 6 = 15$

Q.10 **(4)**

Total number of possible arrangements is

${}^4p_2 \times {}^6p_3$.

Q.11 **(4)**

First we have to find all the arrangements of the word 'GENIUS' is

6 ! = 720

number of arrangement which in either started with G ends with S is

(5! + 5! – 4!) = (120 + 120 – 24)= 216

Hence total number of arrangement which is neither started with G nor ends with S is.

(720 – 216) = 504

Q.12 **(1)**

Total no. of arrangement if all the girls do not sit side by side is = [all arrangement – girls seat side by side]

$= 8! – (6! \times 3!) = 6!\,(56 – 6) = 6! \times 50 = 720 \times 50 = 36000$

Q.13 **(1)**

Number of words which have at least one letter repeated = total words – number of words which have no letter repeated = $10^5 – 10 \times 9 \times 8 \times 7 \times 6$ = 69760

Q.14 **(4)**

First we select 3 speaker out of 10 speaker and put in any way and rest are no restriction i.e. total number of

ways = ${}^{10}C_3 . 7!.2\,! = \frac{10!}{3}$

Q.15 **(2)**

upperdeck - 13 seats → 8 in upper deck.

lowerdeck - 7 seats → 5 in lower deck

Remains passengers = 7

Now Remains 5 seats in upper deck and 2 seats in lower deck

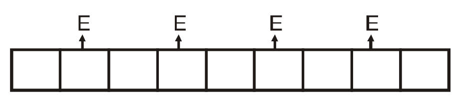
for upper deck number of ways = ${}^7C_5$

for lower deck number of ways = ${}^2C_2$

So total number of ways = ${}^7C_5 \times {}^2C_2 = \frac{7.6}{2} = 21$

Q.16 **(4)**

Even place



There are four even places and four odd digit number

so total number of filling is $\frac{4!}{2!.2!}$ rest are also occupy

is $\frac{5!}{3!.2!}$ ways

Hence total number of ways = $\frac{4!}{2!.2!} \times \frac{5!}{3!.2!} = 60$

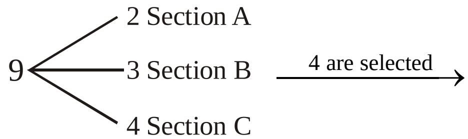
Q.17 **(4)**

Peaches 5 p_1, p_2, p_3, p_4, p_5

Apples 3 a_1, a_2, a_3

Hence number of ways = ${}^3C_1 \times {}^5C_3 = 30$ **Ans.**

Q.18 **(3)**



required number of ways

A	B	C
2	1	1
1	1	2
1	2	1

Hence ${}^2C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^2C_1 \cdot {}^3C_2 \cdot {}^4C_1 + {}^2C_2 \cdot {}^3C_1 \cdot {}^4C_1$
$= 36 + 24 + 12 = 72$ **Ans.**

Alternatively:

$${}^9C_4 - \underbrace{\left[{}^7C_4 + {}^6C_4 + {}^5C_4 + {}^4C_4\right]}_{\text{think!}} = 126 - 56 =$$

72 **Ans.**

Q.19 **(3)**

They can sit in groups of either 5 and 3 or 4 and 4

required number $= \dfrac{8!}{5! \times 3!} \times 1 + \dfrac{8! \times 2!}{4! \times 4! \times 2!} = 126$

Q.20 **(3)**

Total number of ways is

$\dfrac{6! \times 3!}{2!} = 720 \times 3 = 2160$

Q.21 **(1)**

First we select 5 beads from 8 different beads to ${}^8C_5$

Now total number of arrangement is

${}^8C_5 \times \dfrac{4!}{2!} = 672$

Q.22 **(2)**

T										T

Total arrangement is $\dfrac{9!}{2!.2!} = 90720$

Q.23 **(3)**

NINETEEN

$\Rightarrow$ N $\to$ 3 : I, T

E $\to$ 3

First we arrange the word of N, N, N, I and T

then the number of ways $= \dfrac{5!}{3!}$.

Now total 6 number of place which are arrange E is ${}^6C_3$

Hence total number of ways $= \dfrac{5!}{3!} \cdot {}^6C_3$

Q.24 (d)

Q.25 (d)

Q.26 (a)

Q.27 (a)

Q.28 (a)

Q.29 **(b)**

Q.30 **(1)**

Total number of ways of arranging 2 identical white balls.

3 identical red balls and 4 green balls of different shades

$= \dfrac{9!}{2!3!} = 6.7!$

Number of ways when balls of same colour are together $= 3! \times 4! = 6.4!$

$\therefore$ Number of ways of arranging the balls when atleast one ball is separated from the balls of the same colour $= 6.7! - 6.4! = 6(7! - 4!)$

Q.31 **(3)**

only 7, 8 and 9 can be used

Aliter: 9, 9, 9, 9, 9, 9, 7 $\to \dfrac{7!}{6!} = 7$

9, 9, 9, 9, 9, 8, 8 $\to \dfrac{7!}{2!5!} = 21$

Total = 28 **Ans.**

Q.32 **(1)**

Coefficient x^{10} in $(x + x^2 .....x^5)^6$ = coefficient of x^4 in $(x^0 + x^1 ....x^4)^6 = {}^{6+4-1}C_4 = {}^9C_4 = 126$

Alternatively: Give one apple to each child and then for rest 4 apples $= {}^{4+6-1}C_{6-1} = 126$

Q.33 **(3)**

$(x + y + z)^n$ $\to$ use beggar]

Q.34 **(2)**

3 A + 2 O. A. = 3 . 2 = 6 ; 3 A + 2 diff = 3 ;

2 A + 2 O. A. + 1 D = 3 $\Rightarrow$ 12

Q.35 **(3)**

If 1 be unit digit then total no. of number is 3! = 6

Similarly so on if 3, 5, or 7 be unit digit number

then total no. of no. is 3! = 6

Hence sum of all unit digit no. is $= 6\times(1+3+5+7) = 6\times 16 = 96$

Hence total sum is $= 96 \times 10^3 + 96 \times 10^2 + 96 \times 10^1 + 96$

$\times 10^0$
$= 96000 + 9600 + 960 + 96 = 106656 = 16 \times 1111 \times 3!$

Q.36 **(1)**
$(1 + 10 + 10^2 + 10^3) \times 4^3 \times (6 + 7 + 8 + 9) = (1111) \times 64 \times 30$
$= 2133120$

Q.37 **(4)**
Total number of proper divisors is
$(p + 1)(q + 1)(r + 1)(s + 1) - 2$ (Number and 1 are not proper divisor)

Q.38 **(1)**
$N = 2^\alpha .3^\beta 5^\gamma = 2^3 .3^2 .5$
$(\alpha + 1)(\beta + 1)(\gamma + 1) = 4.3.2$
$N = 360 = 2^3 . 3^2 .5$

$$\frac{4.3.2}{2} = 12$$

Q.39 **(1)**
Here $21600 = 2^5 . 3^3 . 5^2 \Rightarrow (2 \times 5) \times 2^4 \times 3^3 \times 5^1$
Now numbers which are divisible by 10 $= (4 + 1)(3 + 1)(1 + 1) = 40$
$(2 \times 3 \times 5) \times (2^4 \times 3^2 \times 5^1)$ now numbers which are divisible by both 10 and 15
$= (4+1)(2+1)(1+1) = 30$
So the numbers which are divisible by only $40 - 30 = 10$

EXERCISE-III

Q.1 [**0485**]

Man – 7 < 4L, 3G ; Wife – 7 < 3L, 4G

3L and 3G are to be invited.

Man's	Wife's	Total Ways
3L	3G	$^4C_3 \cdot {^4C_3} = 16$
3G	3L	$^3C_3 \cdot {^3C_3} = 1$
2L + 1G	1L + 2G	$(^4C_2 \cdot {^3C_1}) \times (^3C_1 \cdot {^4C_2}) = 324$
1L + 2G	2L + 1G	$(^4C_1 \cdot {^3C_2}) \times (^3C_2 \cdot {^4C_1}) = 144$

Total = 485.

Q.2 (0008)

$$\left[\frac{50}{7}\right] + \left[\frac{50}{7^2}\right] = 7 + 1 = 8$$

Q.3 (0008)
We know that a number is divisible by 3. If sum of its digits is divisible by 3.
Hence we must have
$8 + 7 + 6 + 4 + 2 + (x + y) = 3k$
$27 + x + y = 3k$
$\Rightarrow$ x + y is multiple of 3
Hence required (x, y) order pairs
$= (0,3), (0,9), (1,5), (3,0), (3,9), (5,1), (9,0), (9,3)$

Q.4 (0002)
no. of required triangles of 'n' sides polygon is

$$\frac{n(n-4)(n-5)}{6}$$

n = 6

$$\Rightarrow \frac{6(6-4)(6-5)}{6} = 2$$

Q.5 (0005)
No. of different garlands = no.of ways by which we can put 5 identical balls in 3 different boxes = 5
[possibilities are (5,0,0) , (4,1,0), (3,2,0),(2,2,1),(3,1,1)

Q.6 [**0010**]
Here $21600 = 2^5 . 3^3 . 5^2$
$\Rightarrow (2 \times 5) \times 2^4 \times 3^3 \times 5^1$
Now numbers which are divisible by 10
$= (4 + 1)(3 + 1)(1 + 1) = 40$
$(2 \times 3 \times 5) \times (2^4 \times 3^2 \times 5^1)$ now numbers which are divisible by both 10 and 15
$= (4+1)(2+1)(1+1) = 30$
So the numbers which are divisible by only $40 - 30 = 10$

Q.7 [**0126**]
Coefficient x^{10} in $(x + x^2 .....x^5)^6$ = coefficient of x^4 in $(x^0 + x^1 ....x^4)^6 = {^{6+4-1}C_4} = {^9C_4} = 126$
Alternatively: Give one apple to each child and then for rest 4 apples $= {^{4+6-1}C_{6-1}} = 126$

Q.8 [**0672**]
First we select 5 beads from 8 different beads to $^8C_5$
Now total number of arrangement is

$$^8C_5 \times \frac{4!}{2!} = 672$$

Q.9 [0015]
Number divisible by 3 if sum of digits divisible
case-I If $1 + 2 + 3 + 4 + 8 = 18$
Number of ways = 120

case-II If $1 + 2 + 3 + 7 + 8 = 21$
Number of ways = 120
case-III If $2 + 3 + 4 + 7 + 8 = 24$
Number of ways = 120
case-IV If $1 + 2 + 0 + 4 + 8 = 15$
Number of ways = 96
case-V If $1 + 2 + 0 + 7 + 8 = 18$
Number of ways = 96
case-VI If $2 + 0 + 4 + 7 + 8 = 21$
Number of ways = 96
case-VII If $0 + 1 + 3 + 4 + 7 = 15$
Number of ways = 96

Total number 744

Q.10 [10]
Ten digits can be partitioned into four parts as
$1 + 1 + 3 + 5$; $1 + 1 + 1 + 7$; $1 + 3 + 3 + 3$
(each partitioning has odd number of digits)
The number of ways in which these can be placed in the four spaces $= \frac{4!}{2!} + \frac{4!}{3!} + \frac{4!}{3!} = 20$ ways
also numbers of arrangements of vowels = 5 !
Number of arrangements of digits = 10 !
total ways = 20 (10 !) (5 !)

PREVIOUS YEAR'S

MHT CET

Given word is 'HAVANA'(3A, 1H, 1N, 1V)
Total number of ways of arranging the given word
$$= \frac{6!}{3!} = 120$$
Total number of words in which N, V together
$$= \frac{5!}{3!} \times 2! = 40$$
$\therefore$ Required number of ways = 120 – 40 = 80

Q.2 (3)
3 consonants can be selected from 7 consonants $= {}^7C_3$ ways
2 vowels can be selected from 4 vowels $= {}^4C_2$ ways
$\therefore$ Required number of words $= {}^7C_3 \times {}^4C_2 \times 5!$
[selected 5 letters can be arranged in 5! ways, to get a different word]
$= 35 \times 6 \times 120 = 25200$

Q.3 (2)
Since, telephone number start with 67, so two digits is already fixed. Now, we have to do arrangement of three digits from remaining eight digits.
$\therefore$ Possible number of ways $= {}^8P_3$
$$= \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \times 7 \times 6 = 636 \text{ ways}$$

Q.4 (3)
Required number of selections
$= {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$
$= 70 + 56 + 28 + 8 + 1 = 163$

Q.5 (4)
The volwels in the word' COMBINE' are O, I and E which can be arranged at 4 places in ${}^4P_3 \times 4!$
$= 4! \times 4! = 576$

Q.6 (3)
The number of ways in which 4 novels can be selectd $= {}^6C_4 = 15$
4 novles can be arranged in 4! ways.
$\therefore$ The total number of ways $= 15 \times 4! \times 3$
$= 15 \times 24 \times 3 = 1080$

JEE-MAIN

Q.1 [18915]
$b_1 \in \{1, 2, 3......100\}$
Let A = set when $b_1, b_2 b_3$ are consecutive
$$n(A) = \frac{97 + 97 + ... + 97}{98 \text{ times}} = 97 \times 98$$
Simlarly B = set when $b_2 b_3 b_4$ are consecutive
$n(B) = 97 \times 98$
$n(A \cap B) = 97$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
Number of permutation = 18915

Q.2 [1086]
Let abcd is four digit number then first three digit 'abc' should be divisible by last digit 'd'

	No. of such numbers
d = 1,	$9 \times 10 \times 10 = 900$
d = 2,	$4 \times 5 \times 5 = 100$
d = 3,	$3 \times 4 \times 4 = 48$
d = 4,	$2 \times 3 \times 3 = 18$
d = 5,	$1 \times 2 \times 2 = 4$
d = 6, 7, 8, 9	$4 \times 4 = 16$

Total numbers = 1086

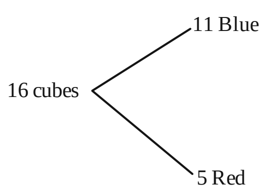
Q.3 **[40]**
$x_1 + x_2 + x_3 + x_4 + x_5 = 5$
only one possibility i.e. 3, 3, 3, –2, –2
$$\therefore \text{ number of ways } = \frac{5!}{3!2!} \times 1 \times 2 = 40$$

Q.4 **[576]**

Sum of even digit – sum of odd digit = 11 n
Case - 1 → Sum of even place = 10
Sum of odd places = 21
Sum of even place = 10
(2,3,5)(1,2,7)(1,4,5)
Sum of odd place = 21
(1,4,7, 9)(3,4,5, 9)(2,3,7,9)
$= 3! \times 4! \times 3 = 144 \times 3 = 432$
Case - 2 → Sum of even place = 21 (5,7,9)
Sum of odd place = 10(1, 2, 3, 4)
$= 3! \times 4!$
$= 144$
Total possible ways as 432 + 144 = 576

Q.5 (243)
Case I : When two zero
a 0 0 $a \in \{1, 2, ..., 9\}$
So much numbers = 9
Case II : When one zero
a 0 a $a \in \{1, 2, ..., 9\}$
a a 0
Such numbers = 9 × 2 = 18
Case III : When no zero
a a b
a b a
b a a
Such numbers = 3 × 9 × 8 = 216
Total = 9 + 18 + 216 = 243

Q.6 (56)



$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$
$x_1\ x_6 \geq 0,\quad x_1, x_3, x_4, x_5 \geq 2$
$x_2 = t_2 + 2$
$x_3 = t_3 + 2$
$x_4 = t_4 + 2$
$x_5 = t_5 + 2$
$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$
No. of solutions $= {}^{6+3-1}C_3 = {}^{8}C_3 = 56$

Q.7 (63)
a+b+c=7,14,21
Case 1 : If a+b+c=7 c =1 a+b=6⇒ 6 Cases
c =3 a+b=4⇒ 4 Cases
c =5 a+b=2⇒ 2 Cases

Case 2 : If a+b+c=14 c =1 a+b=13⇒ 6 Cases
c =3 a+b=11⇒ 8 Cases
c =5 a+b=9⇒ 9 Cases
c =7 a+b=7⇒ 7 Cases
c =4 a+b=5⇒ 5 Cases

Case 3 : If a+b+c=21 c =3 a+b=18⇒ 1 Cases
c =5 a+b=16⇒ 3 Cases
c =7 a+b=14⇒ 5 Cases
c =9 a+b=12⇒ 7 Cases
∴ 63 numbers

Q. 8 **[1120]**
n(B)=10
n(a)=5
The number of ways of forming a group of 3 girls and 3 boys.
$= {}^{10}C_3 \times {}^{5}C_3$

$$= \frac{10 \times 9 \times 8}{3 \times 2} \times \frac{5 \times 4}{2} = 1200$$

The number of ways when two particular boys B_1 of B_2 be the member of group together
$= {}^{8}C_1 \times {}^{5}C_3 = 8 \times 10 = 80$
Number of ways when boys B_1 and B_2 not in the same group together
$= 1200 \times 80 = 1120$

Q.9 (4)
To make a no divisible by 3 we can use the digits 1, 2, 5, 6, 7 or 1, 2, 3, 5, 7
Using 1, 2, 5, 6, 7, number of even numbers is
= 4 × 3 × 2 × 1 × 2 = 48
Using 1, 2, 3, 5, 7 number of even numbers is
4 × 3 × 2 × 1 × 1 = 24
Required answer is 72

Q.10 [17]
${}^{b}C_3 \times {}^{9}C_2 = 168$
$b(b-1)(b-2)(g)(g-1) = 8 \times 7 \times 6 \times 3 \times 2$
b = 8, g = 3
b + 3, g = 17

Q.11 **[1492]**
M A N K I N D
ADIKMNN

A _ _ _ _ _ _ _ _ $= \frac{6!}{2!} = 360$

D _ _ _ _ _ _ _ _ $= \frac{6!}{2!} = 360$

I _ _ _ _ _ _ _ _ $= \frac{6!}{2!} = 360$

K _ _ _ _ _ _ _ _ $= \frac{6!}{2!} = 360$

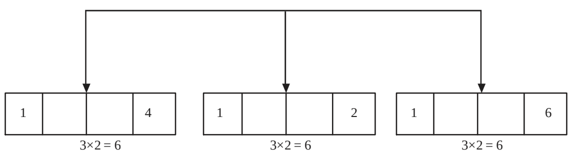
M A D _ _ _ _ _ _ $= \frac{4!}{2!} = 12$

M A I _ _ _ _ _ _ $= \frac{4!}{2!} = 12$

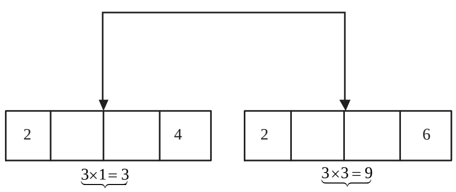
M A K _ _ _ _ _ _ $= \frac{4!}{2!} = 12$

M A N D _ _ _ _ $= 3! = 6$
M A N I _ _ _ _ $= 3! = 6$
M A N K D _ _ $= 2! = 2$
M A N K I D _ _ $= 1! = 1$
M A N K I N D $= 1! = \underline{1}$

Q.12 [30]
case (i) when first digit is 1



case (ii)
When first digit is 2



Total such numbers are 6 + 6 + 6 + 3 + 9 = 30

BINOMIAL THEOREM

EXERCISE-I (MHT CET LEVEL)

Q.1 **(3)**

$\frac{1}{6}=\frac{{}^nC_6(2^{1/3})^{n-6}(3^{-1/3})^6}{{}^nC_{n-6}(2^{1/3})^6(3^{-1/3})^{n-6}}$ or $6^{-1}=6^{-4}.6^{n/3}=6^{n/3-4}$

$\therefore \frac{n}{3}-4=-1 \Rightarrow n=9.$

Q.2 **(3)**

$T_r={}^{15}C_{r-1}(x^4)^{16-r}\left(\frac{1}{x^3}\right)^{r-1}={}^{15}C_{r-1}x^{67-7r}$

$\Rightarrow 67-7r=4 \Rightarrow r=9$.

Q.3 (d)

$T_{r+1}={}^{18}C_r\,(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

$=(-r)^r\ {}^{18}C_r 9^{18\frac{3r}{2}}x^{18\frac{3r}{2}}$

is independent of x provided r = 12 and then a = 1.

Q.4 (c)

Put $\text{fog}_{10}x = y$, the given expression becomes $(x+x^y)^5$.

$T_3={}^5C_2.x^3(x^y)^2=10x^{3+2y}=10^6(\text{given})$

$\Rightarrow (3+2y)\log_{10}x=5\log_{10}10=5$

$\Rightarrow (3+2y)\,y=5 \Rightarrow y=1,-\frac{5}{2}$

$\Rightarrow \log_{10}x=1$ or $\log_{10}x=-\frac{5}{2}$

Q.5 (b)

Given, $\left(x-\frac{1}{x}\right)^7$ and the $(r+1)^{th}$ term in the expansion of $(x+a)^n$ is $T(r+1)={}^nC_r\,(x)^{n-r}\,a^r$

$\therefore 7-2r=3 \Rightarrow r=2$

thus the coefficient of $x^3={}^7C_2(-1)^2=\frac{7\times 6}{2\times 1}=21$

Q.6 **(a)**

General term of the given binomial series is given by:

$T_{r+1}={}^{10}C_r\left\{\frac{x^{1/2}}{3}\right\}^{10-r}\left\{x^{-1/4}\right\}^r$

Put r = 4, we get

$T_5={}^{10}C_4.\frac{1}{3^6}x^3.x^{-1}$

Thus coefficient of $x^2=\frac{70}{243}$.

Q.7 (d)

Q.8 **(c)**

Q.9 **(b)**

Q.10 **(3)**

Let T_{r+1} term containing x^{32}.

Therefore ${}^{15}C_r x^{4r}\left(\frac{-1}{x^3}\right)^{15-r}$

$\Rightarrow x^{4r}x^{-45+3r}=x^{32} \Rightarrow 7r=77 \Rightarrow r=11$.

Hence coefficient of x^{32} is ${}^{15}C_{11}$ or ${}^{15}C_4$

Q.11 **(2)**

x^7, x^8 will occur in T_8 and T_9 .

Coefficients of T_8 and T_9 are equal.

$\therefore {}^nC_7 2^{n-7}\left(\frac{1}{3}\right)^7={}^nC_8 2^{n-8}\left(\frac{1}{3}\right)^8 \Rightarrow n=55$.

Q.12 **(2)**

Here $T_{r+1}={}^9C_r\left(\frac{x^2}{2}\right)^{9-r}\left(\frac{-2}{x}\right)^r$

$={}^9C_r\frac{x^{18-3r}(-2)^r}{2^{9-r}}$, this contains x^{-9} if $18-3r=-9$

i.e. if $r=9$. Coefficient of x^{-9}

$={}^9C_9\frac{(-2)^9}{2^0}=-2^9=-512$.

Q.13 **(3)**

As in Previous question, obviously the term independent of x will be

$$^nC_0.^nC_0 + ^nC_1.^nC_1 + ...^nC_n.^nC_n = C_0^2 + C_1^2 + ... + C_n^2.$$

Q.14 **(2)**

Middle term of $\left(x+\frac{1}{x}\right)^{10}$ is $T_6 = {}^{10}C_5$.

Q.15 (a)

Q.16 **(c)**

Q.17 **(3)**

Middle term = $T_{\frac{2n+2}{2}} = T_{n+1} = {}^{2n}C_n x^n = \frac{2n!}{(n!)^2}.x^n$.

Q.18 **(2)**

Greatest coefficient of $(1+x)^{2n+2}$ is

$$= {}^{(2n+2)}C_{n+1} = \frac{(2n+2)!}{\{(n+1)!\}^2}$$

Q.19 **(4)**

$(1+3x+2x^2)^6 = [1+x(3+2x)]^6$

$= 1 + {}^6C_1x(3+2x) + {}^6C_2x^2(3+2x)^2$

$+{}^6C_3x^3(3+2x)^3 + {}^6C_4x^4(3+2x)^4$

$+{}^6C_5x^5(3+2x)^5 + {}^6C_6x^6(3+2x)^6$

Only x^{11} gets from ${}^6C_6x^6(3+2x)^6$

$\because$ ${}^6C_6x^6(3+2x)^6 = x^6(3+2x)^6$

$\therefore$ Coefficient of = .

Q.20 **(3)**

Trick : Put $n = 1, 2$

At $n=1, {}^1C_0 - \frac{1}{2}{}^1C_1 = 1 - \frac{1}{2} = \frac{1}{2}$

At $n=2, {}^2C_0 - \frac{1}{2}{}^2C_1 + \frac{1}{3}{}^2C_2 = 1 - 1 + \frac{1}{3} = \frac{1}{3}$

which is given by option (c).

Q.21 **(3)**

$$\frac{C_1}{C_0} + 2.\frac{C_2}{C_1} + 3.\frac{C_3}{C_2} + ... + n.\frac{C_n}{C_{n-1}}$$

$$= \frac{n}{1} + 2\frac{n(n-1)/1.2}{n} + 3\frac{n(n-1)(n-2)/3.2.1}{n(n-1)/1.2} + ... + n.\frac{1}{n}$$

$$= n + (n-1) + (n-2).... + 1 = \sum n = \frac{n(n+1)}{2}$$

Trick : Put $n = 1,2,3$ ....., then $S_1 = \frac{{}^1C_1}{{}^1C_0} = 1$,

$$S_2 = \frac{{}^2C_1}{{}^2C_0} + 2\frac{{}^2C_2}{{}^2C_1} = \frac{2}{1} + 2.\frac{1}{2} = 2 + 1 = 3$$

By option, (put n=1,2......) (a) and (b) does not hold condition, but (c) $\frac{n(n+1)}{2}$, put n =1, 2......

$S_1 = 1, S_2 = 3$ which is correct.

Q.22 (a)

$$"C_1 + "C_2 = 36 \Rightarrow N = 8$$

$$T_3 = 7T_2 \Rightarrow (2^x)^3 = \frac{1}{2}$$

$$3x = -1 \Rightarrow x = -\frac{1}{2}$$

Q.23 **(a)**

Q.24 **(3)**

Proceeding as above and putting n+1=N.

So given term can be written as

$$\frac{1}{N}\left\{{}^NC_1 + {}^NC_2 + {}^NC_3 + ...\right\}$$

$$= \frac{1}{N}\{2^N - 1\} = \frac{1}{n+1}(2^{n+1} - 1) \qquad (\because N = n+1)$$

Q.25 **(2)**

Multiplying each term by n ! the question reduces to

$$\frac{n!}{1!(n-1)!} + \frac{1}{3!}.\frac{n!}{(n-3)!} + \frac{1}{5!}.\frac{n!}{(n-5)!} + ...$$

$$= {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}.$$

Thus $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ... = \frac{1}{n!}2^{n-1}$.

Q.26 **(3)**

$(1+x+x^2+x^3)^5 = (1+x)^5(1+x^2)^5$

$= (1+5x+10x^2+10x^3+5x^4+x^5)$

$\times(1+5x^2+10x^4+10x^6+5x^8+x^{10})$

Therefore the required sum of coefficients

$= (1+10+5).2^5 = 16 \times 32 = 512$

Note : $2^n = 2^5$ = Sum of all the binomial coefficients in the 2nd bracket in which all the powers of x are even.

Q.27 **(3)**

As we know that

$${}^nC_0 - {}^nC_1^2 + {}^nC_2^2 - {}^nC_3^2 + ... + (-1)^n.{}^nC_n^2 = 0,$$

(if n is odd) and in the question n=15 (odd).

Q.28 **(1)**

$(1.0002)^{3000} = (1+0.0002)^{3000}$

$= 1+(3000)(0.0002)+\frac{(3000)(2999)}{1.2}(0.0002)^2 +$

$\frac{(3000)(2999)(2998)}{1.2.3}(0.0002)^3 +$

We want to get answer correct to only one decimal places and as such we have left further expansion.

$= 1+(3000)(0.0002) = 1.6$

Q.29 (c)

$10^n + 3(4^{n+2})+5$ Taking n=2

$10^2+3\times 4^4+5 = 100+768+5=873$

There fore this is divisible by 9.

Q.30 (c)

The product of r consecutive integers is divisible by r !. Thus $n(n+1)(n+2)(n+3)$ is divisible by

$4! = 24$

Q.31 (a)

Q.32 (b)

Q.33 (a,c,d)

Q.34 **(4)**

We know that $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ and $2<e<3$.

$\therefore\ (1+0.0001)^{10000} < 3$ (By putting $n = 10000$)

Also $(1+0.0001)^{10000} = 1+10000\times 10^{-4}$

$+\frac{10000\times 9999}{2!}\times 10^{-8}+......$ upto10001 terms

$\Rightarrow (1+0.0001)^{10000} > 2$. Hence 3 is the positive integer just greater than $(1+0.0001)^{10000} > 2$.

Hence (d) is the correct option.

Q.35 **(3)**

We have $7^2 = 49 = 50-1$

Now, $7^{300} = (7^2)^{150} = (50-1)^{150}$

$= {}^{150}C_0(50)^{150}(-1)^0 + {}^{150}C_1(50)^{149}(-1)^1 + ...$

$+ {}^{150}C_{150}(50)^0(-1)^{150}$

Thus the last digits of 7^{300} are ${}^{150}C_{150}.1.1$ *i.e.*, 1.

Q.36 **(1)**

111…..1 (91 times)

$= 1+10+10^2+.....+10^{90}$

$= \frac{10^{91}-1}{10-1} = \frac{(10^7)^{13}-1}{10-1} = \frac{t^{13}-1}{9}$, where $t = 10^7$

$= \left(\frac{t-1}{9}\right)(t^{12}+t^{11}+.....+t+1)$

$= \left(\frac{10^7-1}{10-1}\right)(1+t+t^2+....+t^{12})$

$= (1+10+10^2+....+10^6)(1+t+t^2+...+t^{12})$

$\therefore\ 111.....1(91 \text{ times})$ is a composite number.

Q.37 **(2)**

Expansion of $(1-2x)^{3/2}$

$= 1+\frac{3}{2}(-2x)+\frac{3}{2}.\frac{1}{2}.\frac{1}{2}(-2x)^2+\frac{3}{2}.\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{6}(-2x)^3+.....$

Hence 4^{th} term is $\frac{x^3}{2}$

Q.38 **(1)**

$(a+bx)^{-2} = \frac{1}{a^2}\left(1+\frac{b}{a}x\right)^{-2} = \frac{1}{a^2}\left[a+\frac{(-2)}{1!}\left(\frac{b}{a}\right)x+....\right]$

Equating it to $\frac{1}{4}-3x+....$, we get $a=2, b=12$.

Q.39 **(1)**

Given term can be written as $(1+x)^2(1-x)^{-2}$

$= (1+2x+x^2)[1+2x+3x^2+....+(n-1)x^{n-2}$

$+nx^{n-1}+(n+1)x^n+....]$

$= x^n(n+1+2n+n-1)+....$

Therefore coefficient of x^n is $4n$.

Q.40 **(1)**

$$\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$$

$$= \frac{1+\frac{3}{2}x+\frac{\frac{3}{2}.\frac{1}{2}}{2}x^2-\left(1+\frac{3x}{2}+\frac{3.2}{2}\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

$$= \frac{-\frac{3}{8}x^2}{(1-x)^{1/2}} = -\frac{3}{8}x^2(1-x)^{-1/2}$$

$$= -\frac{3}{8}x^2\left(1+\frac{x}{2}+....\right) = -\frac{3}{8}x^2.$$

Q.41 **(2)**

In the expansion of $(y^{1/5}+x^{1/10})^{55}$, the general term is

$T_{r+1} = {}^{55}C_r (y^{1/5})^{55-r}(x^{1/10})^r = {}^{55}C_r . y^{11-r/5} x^{r/10}$.

This T_{r+1} will be independent of radicals if the exponents $r/5$ and $r/10$ are integers, for $0 \le r \le 55$ which is possible only when $r = 0, 10, 20, 30, 40, 50$.

$\therefore$ There are six terms *viz.* $T_1, T_{11}, T_{21}, T_{31}, T_{41}, T_{51}$ which are independent of radicals.

Q.42 **(1)**

We know that $n!$ terminates in 0 for $n \ge 5$ and 3^{4n} terminator in 1, ($\because\ 3^4 = 81$)

$\therefore 3^{180} = (3^4)^{45}$ terminates in 1

Also $3^3 = 27$ terminates in 7

$\therefore\ 3^{183} = 3^{180} 3^3$ terminates in 7.

$\therefore\ 183! + 3^{183}$ terminates in 7

i.e. the digit in the unit place = 7.

Q.43 **(3)**

Let us take

$a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n} = (1 + x + x^2)^n$

Differentiating with respect to x on both sides

$a_1 + 2a_2 x + ... + 2n\, a_{2n} x^{2n-1} = n(1 + x + x^2)^{n-1}(2x + 1)$

Put $x = -1 \Rightarrow a_1 - 2a_2 + 3a_3 - .... + 2n\, a_{2n} = -n$.

EXERCISE-II (JEE MAIN LEVEL)

Q.1 **(3)**

$${}^{2m+1}C_m \left(\frac{x}{y}\right)^{m+1}\left(\frac{y}{x}\right)^m = {}^{2m+1}C_m \left(\frac{x}{y}\right)$$

Dependent upon the ratio $\frac{x}{y}$ and m.

Q.2 **(1)**

$T_2 = {}^nC_1 (a^{1/13})^{n-1}(a^{3/2}) = 14a^{5/2}$

$\Rightarrow n = 14$

$\therefore \frac{{}^nC_3}{{}^nC_2} = 4$

Q.3 (b)

We know by Binormial expansion, that (x + a)^n

$= {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1}.a + {}^nC_2 x^{n-2} a^2$

$+ {}^nC_3 x^{n-3} a^3 + {}^nC_4 x^{n-4}.a^4 + ... + {}^nC_n x^0 a^n$

Given expansion is $\left(x^4 - \frac{1}{x^3}\right)^{15}$

On comparing we get n = 15, x = x^4,

$a = \left(-\frac{1}{x^3}\right)$

$$\therefore \left(x^4 - \frac{1}{x^3}\right)^{15} = {}^{15}C_0 \left(x^4\right)^{15}\left(-\frac{1}{x^3}\right)^0$$

$$+ {}^{15}C_1 \left(x^4\right)^{14}\left(-\frac{1}{x^3}\right) + {}^{15}C_2 \left(x^4\right)^{13}\left(-\frac{1}{x^3}\right)^2$$

$$+ {}^{15}C_3 \left(x^4\right)^{12}\left(-\frac{1}{x^3}\right)^3$$

$$+ {}^{15}C_4 \left(x^4\right)^{11}\left(-\frac{1}{x^3}\right)^4 + ...$$

$$T_{r+1} = {}^{15}C_r \left(x^4\right)^{15-r} . \left(-\frac{1}{x^3}\right)^r$$

$= -{}^{15}C_r x^{60-7r}$

$\Rightarrow x^{60-7r} = x^{32} \Rightarrow 60 - 7r = 32$

$\Rightarrow 7r = 28 \Rightarrow r = 4$

So, 5th term, contains x^{32}

$$= {}^{15}C_4 \left(x^4\right)^{11}\left(-\frac{1}{x^3}\right)^4 = {}^{15}C_4 x^{44} x^{-12}$$

$= {}^{15}C_4 x^{32}$,

Thus, coefficient of $x^{32} = {}^{15}C_4$.

Q.4 (b)

$T_{r+1} = {}^nC_r a^{n-r} . b^r$ where

$a = 2^{\frac{1}{3}}$ and $b = 3^{\frac{-1}{3}}$

T_7 from beginning ${}^nC_6\, a^{n-6} b^6$ and

T_7 from end ${}^nC_6\, b^{n-6} b^6$

$$\Rightarrow \frac{a^{n-12}}{b^{n-12}} = \frac{1}{6}$$

$\Rightarrow 2^{\frac{n-12}{3}} . 3^{\frac{n-12}{3}} = 6^{-1}$

$\Rightarrow n - 12 = -3$

$\Rightarrow n = 9$

Q.5 (b)

Expression = $(1 + x^2)^{40} \cdot \left(x + \frac{1}{x}\right)^{-10}$

$= (1 + x^2)^{30}.x^{10}$

The coefficient of x^{20} in $x^{10}(1 + x^2)^{30}$

= the coefficient of x^{10} in $(1 + x^2)^{30}$

$= {}^{30}C_5 = {}^{30}C_{30-5} = {}^{30}C_{25}$

Q.6 (a)

$$(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2$$

$$+ {}^nC_3x^{n-3}a^3 + {}^nC_4x^{n-4}a^4 + ...$$

$$= ({}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 ....) +$$

$$+ ({}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5) + ...$$

$= A + B$ ......(1)

Similarly, $(x - a)^n = A - B$ ......(2)

Multiplying eqns. (1) and (2), we get

$$(x^2 - a^2)^n = A^2 - B^2$$

Q.7 (d)

Q.8 (b)

Q.9 **(a)**

Q.10 **(d)**

Q.11 **(a)**

Q.12 **(3)**

$$\left(x^{\frac{1}{3}} - x^{-\frac{1}{2}}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r\left(x^{\frac{1}{3}}\right)^{15-r}\left(-x^{-\frac{1}{2}}\right)^r$$

$$T_{r+1} = {}^{15}C_r(-1)^r\,(x)^{\frac{15-r}{3}-\frac{r}{2}} = {}^{15}C_r(-1)^r\,(x)^{\frac{30-5r}{6}}$$

Given if $\frac{30-5r}{6} = 0$ then $T_{r+1} = 5m, m \in N$

$\Rightarrow r = 6 \Rightarrow T_7 = 5m$

$T_7 = {}^{15}C_6\,(-1)^6 = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$

$= 5 \cdot 7 \cdot 13 \cdot 11 \cdot = 5 \cdot (1001) \Rightarrow m = 1001$

Q.13 (2)

G.T. is $T_{r+1} = {}^{100}C_r(2)^{\frac{100-r}{2}}(3)^{\frac{r}{4}}$

The above term will be rational if exponent of 2 & 3 are integers.

i.e. $\frac{100-r}{2}$ and $\frac{r}{4}$ must be integers

the possible set of r is = {0, 4, 8, 16,……, 100}

no. of rational terms is 26

Q.14 (2)

If $n \in N$ & n is even then

$$\frac{1}{1 \cdot (n-1)!} + \frac{1}{3!(n-3)} + \frac{1}{5!(n-5)} + \dots + \frac{1}{(n-1)!\ 1!}$$

$$= \frac{1}{n!}\left[{}^nC_1 + {}^nC_3 + {}^nC_5 + \dots\dots + {}^nC_{n-1}\right]$$

n is even $\Rightarrow$ n – 1 is odd

${}^nC_{n-1}$ second Binomail coeff. from the end

$$= \frac{1}{n!}[C_1 + C_3 + C_5 + \dots\dots + C_{n-1}]$$

$$= \frac{1}{n!} \cdot 2^{n-1} = \frac{2^{n-1}}{n!}$$

Q.15 (2)

middle term $= T_5$

$T_5 = T_{4+1} = {}^8C_4 \cdot k^4 = 1120$

$\Rightarrow k = 2$

Q.16 (4)

$\left(x^k + \frac{1}{x^{2k}}\right)^{3n}$, $n \in N$ Independent of x

$$T_{r+1} = {}^{3n}C_r\left(x^k\right)^{3n-r}\left(\frac{1}{x^{2k}}\right)^r$$

$= {}^{3n}C_r\,x^{3nk - rk - 2kr} = {}^{3n}C_r\,x^{3k(n-r)}$

For Constant term $\Rightarrow 3k(n - r) = 0 \Rightarrow n = r$

$\therefore T_{r+1} = {}^{3n}C_n$ true for any real k or $K \in R$

Q.17 **(1)**

$(3x + 2)^{-1/2}$ has infinite expansion when $\left|\frac{3x}{2}\right| < 1$

$$\Rightarrow x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$$

Q.18 (2)

Coeff of α^t in

$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 \dots\dots + (\alpha + q)^{m-1}$

$\because$ $a \neq -q, p \neq q$

Let $\alpha + P = x$ & $\alpha + q = y$

$= x^{m-1} + x^{m-2}y + x^{m-3}y^2 + \dots\dots y^{m-1}$

$$= x^{m-1}\left[1 - \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 + \dots\dots + \left(\frac{y}{x}\right)^{m-1}\right]$$

$$= x^{m-1}\frac{\left[1-\left(\frac{y}{x}\right)^m\right]}{\left(1-\frac{y}{x}\right)}$$

$$= \frac{x^{m-1}}{x^m}\frac{x^m-y^m}{x-y}\cdot x = \frac{(\alpha+p)^m-(\alpha+q)^m}{\alpha+p-\alpha-q}$$

$$= \frac{1}{(p-q)}\left[(\alpha+p)^m-(\alpha+q)^m\right]$$

$$= \text{coeff of } \alpha^t = \left(\frac{{}^mC_t\, p^{m-t} - {}^mC_t\, q^{m-t}}{p-q}\right)$$

Q.19 (3)

$(2x+5y)^{13}$ greatest form for x = 10, y = 2

$$\frac{n+1}{\left|\frac{x}{y}\right|+1}-1 \leqslant r \leqslant \frac{n+1}{\left|\frac{x}{y}\right|+1}$$

$$\Rightarrow \frac{14}{\left|\frac{2x}{5y}\right|+1}-1 \leqslant r \leqslant \frac{14}{\left|\frac{2x}{5y}\right|+1}$$

$$\Rightarrow \frac{14}{3}-1 \leqslant r \leqslant \frac{14}{3} \Rightarrow \frac{11}{3} \leqslant r \leqslant \frac{14}{3}$$

$\Rightarrow$ 3.66 ...... $\leqslant r \leqslant$ 4.666 $\Rightarrow r = 4$

$\Rightarrow T_5 = {}^{13}C_4 (20)^9 (10)^4$

Q.20 (d)

When exponent is n then total number of terms are n+1. So, total number of terms in

$(2+3x)^4 = 5$

Middle term is 3rd. $\Rightarrow T_3 = {}^4C_2 (2)^2.(3x)^2$

$$= \frac{4\times3\times2\times1}{2\times1\times2}\times4\times9x^2 = 216x^2$$

Q.21 (1)

For numerically greatest term r = $\left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right]$ =

$$\left[\frac{9+1}{1+\left|\frac{4}{9}\right|}\right]$$

$\Rightarrow r = 6$

Numerically greatest term $T_{r+1} = {}^9C_6 (2)^3 \left(\frac{9}{2}\right)^6$

Q.22 (1)

$$\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}} = \sum_{r=0}^{n-1}\frac{r+1}{n+1}$$

$$= \frac{1}{n+1}[1+2+.........+n] = \frac{1}{n+1}\times\frac{n(n+1)}{2} = \frac{n}{2}$$

Q.23 (b)

$${}^{20}C_r = {}^{20}C_{r-10}$$

$$\Rightarrow r+(r-10)=20 \Rightarrow r=15$$

$$\therefore {}^{18}C_r = {}^{18}C_{15} = {}^{18}C_3 = \frac{18.17.16}{1.2.3} = 816$$

Q.24 (a)

$$C_0+(C_0+C_1)+(C_0+C_1+C_2)+...........$$
$$+(C_0+C_1+...........C_{n-1})$$
$$= nC_0+(n-1)C_1+(n-2)C_2+......C_{n-1}$$
$$= C_1+2C_2+3C_3+4C_4......nC_n = n\cdot2^{n-1}$$

Q.25 (d)

let the coefficients of rth, (r + 1)th, and (r +2)th terms be in HP.

Then, $\frac{T}{{}^nC_r} = \frac{1}{{}^nC_{r-1}}+\frac{1}{{}^nC_r+1}$

$$\Rightarrow 2 = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{{}^nC_r}{{}^nC_{r+1}}$$

$$\Rightarrow 2 = \frac{n-r+1}{r}+\frac{r+1}{n-r}$$

$$\Rightarrow n^2-4nr+4r^2+n=0$$

$$\Rightarrow (n-2)^2+n=0$$

which is not possible for any value for n.

Q.26 (b)

$${}^{39}C_{3r-1}-{}^{39}C_{r^2} = {}^{39}C_{r^2-1}-{}^{39}C_{3r}$$

$$\Rightarrow {}^{39}C_{3r-1}+{}^{39}C_{3r} = {}^{39}C_{r^2-1}+{}^{39}C_{r^2}$$

$$\Rightarrow {}^{40}C_{3r} = {}^{40}C_{r^2}$$

$\Rightarrow r^2 = 3r$ or $r^2 = 40 - 3r$

$\Rightarrow r = 0, 3$ or $-8, 5$

3 and 5 are the values as the givehn equation is not defined by r = –8. Hence, the number of valies of r is 2.

Q.27 (d)

$$\sum_{r=0}^{n} \frac{r+2}{r+1}\, nC_r = \frac{2^8-1}{6}$$

$$\sum_{r=0}^{n}\left[1+\frac{1}{r+1}\right]{}^nC_r = \frac{2^8-1}{6}$$

$$\Rightarrow 2^n + \sum_{r=0}^{n}\frac{1}{n+1}.\ {}^{n+1}C_{r+1} = \frac{2^8-1}{6}$$

$$\Rightarrow 2^n + \frac{2^{n+1}}{n+1} = \frac{2^8-1}{6} \Rightarrow \frac{2^n(n+3)-1}{n+1} = \frac{2^8-1}{6}$$

$$\Rightarrow \frac{2^n(n+1+2)-1}{n+1} = \frac{2^5(6+2)-1}{6}$$

Comparing we get n + 1 = 6 $\Rightarrow n = 5$

Q.28 (a)

$$\frac{{}^nC_r}{{}^{r+3}C_r} = 3!\frac{1}{(r+3)(r+2)}\cdot\frac{{}^nC_r}{(r+1)}$$

$$= 3!\frac{1}{(r+3)(r+2)}\cdot\frac{{}^{n+1}C_{r+1}}{(n+1)} \text{ (See formulae)}$$

$$= 3!\frac{1}{(r+3)(r+2)}\cdot\frac{{}^{n+1}C_{r+1}}{r+2}$$

$$= 3!\frac{1}{(r+3)(r+2)}\cdot\frac{{}^{n+2}C_{r+2}}{n+2}$$

$$= 3!\frac{1}{(r+3)(r+2)}\cdot\frac{{}^{n+2}C_{r+2}}{n+3}$$

$$= \frac{3!}{(n+1)(n+2)(n+3)}\ {}^{n+3}C_{r+3}$$

$$\therefore \sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{{}^{r+3}C_r}$$

$$= \frac{6}{(n+1)(n+2)(n+3)}\sum_{r=0}^{n}(-1)^r\ {}^{n+3}C_{r+3}$$

$$= \frac{6}{(n+1)(n+2)(n+3)}$$

$$[{}^{n+3}C_3 - {}^{n+3}C_4 + \ldots\ldots + (-1)^n\ {}^{n+3}C_{n+3}]$$

$$= \frac{6}{(n+1)(n+2)(n+3)}[{}^{n+3}C_0 - {}^{n+3}C_1 + {}^{n+3}C_2]$$

$$[\therefore {}^{n+3}C_0 - {}^{n+3}C_1 + \ldots\ldots + (-1)^{n+3}\times{}^{n+3}C_{n+3} = 0]$$

$$= \frac{6}{(n+1)(n+2)(n+3)}\left(1-n-3+\frac{(n+3)(n+2)}{2}\right)$$

$$= \frac{3}{(n+1)(n+2)(n+3)}(n^2+3n+2) = \frac{3}{n+3}$$

Given, $\frac{3}{n+3} = \frac{3}{a+3}$

$\Rightarrow$ n = a $\Rightarrow$ a – n = 0

Q.29 **(a)**

Q.30 **(2)**

$$\frac{{}^{11}C_0}{1} + \frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \ldots\ldots + \frac{{}^{11}C_{10}}{11}$$

$$= \frac{1}{12}$$

$$\left[\frac{12}{1}.{}^{11}C_0 + \frac{12}{2}.{}^{11}C_1 + \frac{12}{3}.{}^{11}C_2 + \ldots.. + \frac{12}{11}.{}^{11}C_{10}\right]$$

$$= \frac{1}{12}\left[{}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \ldots\ldots. + {}^{12}C_{11}\right]$$

$$= \frac{1}{12}(2^{12}-2) = \frac{2^{11}-1}{6}$$

Q.31 (2)

$$\sum_{k=1}^{n-r}{}^{n-k}C_r = {}^xC_y$$

L.H.S. $= {}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \ldots\ldots + {}^rC_r$

$= {}^rC_r + {}^{r+1}C_r + \ldots\ldots\ {}^{n-2}C_r + {}^{n-1}C_r$

$$\left\{{}^rC_r = \frac{r+1}{r+1}{}^rC_r = {}^{r+1}C_{r+1}\right\}$$

$= {}^{r+1}C_{r+1} + {}^{r+1}C_r + {}^{r+2}C_r + \ldots\ldots + {}^{n-1}C_r$

$= {}^{r+2}C_{r+1} + {}^{r+2}C_r + \ldots\ldots + {}^{n-1}C_r$

$= {}^{r+2}C_{r+1} + \ldots\ldots + {}^{n-1}C_r$

$= {}^{n-1}C_{r+1} + {}^{n-1}C_r$

$= {}^nC_{r+1} = {}^xC_y \Rightarrow x = n, y = r + 1$

Q.32 (3)
$2^{2003} = 8.(16)^{500}$
$= 8\,(17-1)^{500}$
$\therefore$ Remainder = 8

Q.33 (c)
For n=1, we have ;
$$x^{n+1} + (x+1)^{2n-1} = x^2 + (x+1) = x^2 + x + 1,$$
which is divisible by x^2+x+1
For n=2, we have ; $x^{n+1}+(x+1)^{2n-1}$
$$= x^3 + (x+1)^3 = (2x+1)(x^2+x+1),$$
which is divisible by $x^2 + x + 1$

Q.34 (c)
$2^{3n} - 7n - 1$ Taking n=2;
$$2^6 - 7 \times 2 - 1$$
$$= 64 - 15 = 49$$
Therefore this is divisible by 49 .

Q.35 (b)
$$\mathrm{R} = (3+\sqrt{5})^{2n},\ \mathrm{G} = (3-\sqrt{5})^{2n}$$
Let [R] + 1 = 1
($\because$ [.] greatest integer function)
$\Rightarrow \mathrm{R} + \mathrm{G} = 1 (\because 0 < \mathrm{G} < 1)$
$$\Rightarrow (3+\sqrt{5})^{2n} + (3-\sqrt{5})^{2n} = 1$$
seeing the option put n = 1
I = 28 is divisivle by 4 i.e., 2^{n+1}

Q.36 (d)
Q.37 **(*)**
Q.38 **(b)**

Q.39 (4)
$3^{400} = (10-1)^{200}$
${}^{200}C_0 (10)^{200} + \ldots\ldots.. + {}^{200}C_{199} (10)(-1) + {}^{200}C_{200}$
Last two digits = 01

Q.40 (1)
Last two digits in 10! are 00 and third digit = 8

Q.41 (1)
$$\sum_{r=1}^{10} r.\,\frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \sum_{r=1}^{10} n - r + 1$$
$$= (n+1) \times 10 - \frac{10 \times 11}{2}$$
$$= 10n - 45$$

Q.42 (4)
Co-efficient of x^n in $(1-x)^{-2} = {}^{2+n-1}C_1 = n + 1$

Q.43 (4)
coef of x^4 in $(1 - x + 2x^2)^{12}$
$= {}^{12}C_0 (1-x)^{12} (2x^2)^0 + {}^{12}C_1 (1-x)^{11} (2x^2) + {}^{12}C_2 (1-x)^{10} (2x^2)^2$ + above x^4 powers terms of x^4
$= {}^{12}C_0 \cdot {}^{12}C_4 (-x)^4 + {}^{12}C_1\, {}^{11}C_2 (-x)^2\, 2x^2 + {}^{12}C_2\, {}^{10}C_0\, 4x^4$
$= {}^{12}C_4 + 12 \cdot {}^{11}C_2 \cdot 2 + {}^{12}C_2 \cdot 4$
$$= {}^{12}C_4 + 2.3 \cdot \frac{12}{3}\ {}^{11}C_2 + {}^{12}C_2 \cdot 4$$
$$= {}^{12}C_3 + {}^{12}C_2 + 3\left({}^{12}C_2 + {}^{12}C_3\right) + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$$
=
$${}^{12}C_3 + 3({}^{12}C_2 + {}^{12}C_3) + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$$
$$= {}^{12}C_3 + 3\,{}^{13}C_3 + {}^{13}C_3 + {}^{13}C_4$$
$$= {}^{12}C_3 + 3\,{}^{13}C_3 + {}^{14}C_4$$

Q.44 (3)
We have coefficient of x^4 *in* $(1+x+x^2+x^3)^{11}$
= coefficient of x^4 in $(1+x^2)^{11}(1+x)^{11}$
= coefficient of x^4 in $(1+x)^{11}$ + coefficicnt of x^2 in $11.(1+x)^{11}$ + constant term is ${}^{11}C_2 \cdot (1+x)^{11}$
$$= {}^{11}C_4 + 11.{}^{11}C_2 + {}^{11}C_2 = 990$$

Q.45 (4)
$$\text{Let}\ \frac{e^x + e^{5x}}{e^{3x}} = a_0 + a_1 x + a_2 x^2 + a_3 a^3 + \ldots$$
$$= \frac{e^x}{e^{3x}} + \frac{e^{5x}}{e^{3x}} = a_0 + a_1 x + a_2 x^2 + \ldots$$
By using
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots..\ \text{and}$$
$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots..$$
$$e^{-2x} + e^{2x} = 2\left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \ldots..\right]$$
$$= a_0 + a_1 x + a_2 x^2 + a_3 a^3 + \ldots.$$
$$= a_1 = a_3 = a_5 = \ldots. = 0$$
Henece, $2a_1 + 2^3 a_3 + 2^5 a_5 + \ldots\ldots = 0$

Q.46 (4)

$$1+\frac{\log_{e^2} x}{1!}+\frac{\left(\log_{e^2} x\right)^2}{2!}+....$$

$$e^{\log_{e^2} x}=e^{\frac{1}{2}\log_e x}=e^{\log_e \sqrt{x}}=\sqrt{x}$$

Q.47 (2)

The given series is

$$1+\frac{1+a}{2!}+\frac{1+a+a^2}{3!}+\frac{1+a+a^2+a^3}{4!}+...$$

Here, $T_n=\frac{1+a+a^2+a^3+...\text{to n terms}}{n!}$

$$=\frac{1\left(1-a^n\right)}{(1-a)(n!)}=\frac{1}{1-a}\left(\frac{1-a^n}{n!}\right)$$

$$\therefore T_1+T_2+T_3+...\text{to}\,\infty$$

$$=\frac{1}{1-a}\left[\frac{1-a}{1!}+\frac{1-a^2}{2!}+\frac{1-a^3}{3!}+...\text{to}\,\infty\right]$$

$$=\frac{1}{1-a}\left[\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...\text{to}\,\infty\right]-$$

$$\left(\frac{a}{1!}+\frac{a^2}{2!}+\frac{a^3}{3!}+...\text{to}\,\infty\right)\Bigg]$$

$$=\frac{1}{1-a}\left[(e-1)-\left(e^a-1\right)\right]$$

$$=\frac{e-e^a}{1-a}=\frac{e^a-e}{a-1}$$

Q.48 **(3)**

$$\frac{e^{7x}+e^x}{e^{3x}}=e^{4x}+e^{-2x}$$

$$=\left[1+4x+\frac{(4x)^2}{2!}+.......\right]$$

$$+\left[1+(-2x)+\frac{(-2x)^2}{2!}+........\right]$$

$$\therefore \text{coeff . of } x^n=\frac{4^n}{n!}+\frac{(-2)^n}{n!}$$

Q.49 (4)

Q.50 (4)

$(1+x)^2(1-x)^{-2}$

$=(1+x^2+2x)(1-x)^{-2}$

Co-efficient of $x^4={}^5C_4+{}^3C_2+2\,{}^4C_3=16$

Q.51 (1)

$(1+x)^{10}=a_0+a_1+a_2x^2+......+a_{10}x^{10}$

Put $x=i$,

$(1+i)^{10}=a_0-a_2+a_4+....+a_{10}\ +i\,(a_1-a_3+.....+a_9)$

$a_0-a_2+a_4+....+a_{10}=$ real part of $(1+i)^{10}=2^5\cos 10\pi/4$

$a_1-a_3+.......=$ imaginary part of $(1+i)^{10}=2^5\sin 10\pi/4$ ....(2)

$(1)^2+(2)^2=2^{10}$

Q.52 (3)

Sum of the coeff of degree r is

$(1+x)^n(1+y)^n(1+z)^n$

$$=\left(\sum_{f=0}^{n}{}^nC_kx^k\right)\left(\sum_{s=0}^{n}{}^nC_sy^s\right)\left(\sum_{t=0}^{n}{}^nC_tz^t\right)=\sum_{0\leqslant k,s,t\leqslant n}\left({}^nC_k\right)\left({}^nC_s\right)\left({}^nC_t\right)x^ky^sz^t$$

degree $m=k+s+t=r$

sum of coeff $=\sum_{k,s,t\geqslant 0}{}^nC_k\cdot{}^nC_s\cdot{}^nC_t$

= the number of way of choosing a total number r balls out of n white, n block and n red balls.

$={}^{3n}C_r$

EXERCISE-III

Q.1 (0960)

Coefficient of x^7 in $(1-x+2x^3)^{10}$

general term :

${}^{10}C_r(1-x)^{10-r}\cdot(2x^3)^r$; ${}^{10}C_r(1-x)^{10-r}\cdot 2^r\cdot x^{3r}$.

When $r=0$, coefficient of x^7 in ${}^{10}C_0(1-x)^{10}$

$\Rightarrow -{}^{10}C_7$.

When $r=1$, coefficient of x^4 in ${}^{10}C_1(1-x)^9\cdot 2$

$\Rightarrow 20\,({}^9C_4)$

When $r=2$, coefficient of x^1 in ${}^{10}C_2(1-x)^8\cdot 2^2$

$\Rightarrow 180\,(-{}^8C_1)$

coefficent of $x^7=(-{}^{10}C_7+20\cdot{}^9C_4-180\cdot{}^8C_1)=(-120+2520-1440)=960$. **Ans.**

Q.2 (0006)

General term : ${}^{55}C_r\,(y^{1/3})^{55-r}\cdot\left(x^{\frac{1}{10}}\right)^r$

: ${}^{55}C_r\cdot y^{\frac{55-r}{3}}\cdot x^{\frac{r}{10}}$.

Terms free from radical sign, wehn

$r = 0, \frac{55-0}{3} = \frac{55}{3}$ (Not possible)

$r = 10, \frac{55-10}{3} = 15$

$r = 20, \frac{55-20}{3} = \frac{35}{3}$ (Not possible)

$r = 40, \frac{55-40}{3} = 5$

$r = 50, \frac{55-50}{3} = \frac{5}{3}$ (Not possible)

Terms free from radical sign = 2.

Q.3 (0001)

$(27)^{15} = (1+26)^{15} \rightarrow$ Expand

Q.4 (0001)

$2^{60} = (1+7)^{20}$

$= {}^{20}C_0 + {}^{20}C_1.7 + {}^{20}C_2.7^2 + ... + {}^{20}C_{20}.7^{20}$

$\therefore$ The remainder $= {}^{20}C_0 = 1$.

Q.5 (0001)

$3^{400} = (3^4)^{100} = (81)^{100} = (1+80)^{100}$

$= 1 + {}^{100}C_1(80) + {}^{100}C_2(80)^2 + ... + {}^{100}C_{100}(80)^{100}$

= 1 + 8000 + (Last digit in each term is 0)

$\therefore$ Last digit = 1.

Q.6 (0001)

$T_{r+1} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r$ (Ist expansion)

$T_{r+1} = {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2}\right)^r$ (IInd expansion)

x^7 power term in Ist expansion is 6th term and x^{-7} power term in IInd expansion is 7th term, So

${}^{11}C_5 a^6 b^{-5} = {}^{11}C_6 a^5 (-b)^{-6} \Rightarrow ab = 1$

Q.7 (0012)

Since, n is even therefore $\left(\frac{n}{2}+1\right)$ th term is the middle terms

$\therefore T_{\frac{n}{2}+1} = {}^{n}C_{n/2} (x^2)^{n/2} \left(\frac{1}{x}\right)^{n/2} = 924x^6$

$\Rightarrow x^{n/2} = x^6 \Rightarrow n = 12$

Q.8 (0007)

The general term

$= {}^{9}C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(\frac{-1}{3x}\right)^r = (-1)^r\, {}^{9}C_r \frac{3^{9-2r}}{2^{9-r}}.x^{18-3r}$

The term independent of x, (or the constant term) corresponds to x^{18-3r} being

$x^0 \text{ or } 18-3r = 0 \Rightarrow r = 6$

Q.9 (0015)

$S = \sum_{i=0}^{m} {}^{10}C_i\ {}^{20}C_{m-i}$

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + ... + {}^{10}C_{10} x^{10}$ ...(1)

$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + ... {}^{20}C_{20} x^{20}$ ...(2)

$\therefore$ S represents coefficient of x^m in (1) × (2)

Coefficient x^m in $(1+x)^{30} = {}^{30}C_m$

$\therefore$ For this to be maximum

m = 15

Q.10 (0006)

General term : ${}^{55}C_r (y^{1/3})^{55-r} \cdot \left(x^{\frac{1}{10}}\right)^r$

: ${}^{55}C_r \cdot y^{\frac{55-r}{3}} \cdot x^{\frac{r}{10}}$.

Terms free from radical sign, wehn

$r = 0, \frac{55-0}{3} = \frac{55}{3}$ (Not possible)

$r = 10, \frac{55-10}{3} = 15$

$r = 20, \frac{55-20}{3} = \frac{35}{3}$ (Not possible)

$r = 40, \frac{55-40}{3} = 5$

$r = 50, \frac{55-50}{3} = \frac{5}{3}$ (Not possible)

Terms free from radical sign = 2.

PREVIOUS YEAR'S

Q.1 (4)

No restriction on C_1 and C_4

C_2 gets atleast 4 and atmost 7

C_3 gets atleast 2 and atmost 6

Hence Required no. of ways = coefficient of x^{30} in

$= (x^0 + x + x^2 + ... + x^{30}).(x^4 + x^5 + x^6 + x^7).$

$(x^2 + x^3 + x^4 + x^5 + x^6).(x^0 + x + x^2 .... + x^{30})$

$= x^6 (1 + x + x^2 + ... x^{30})^2.(1 + x + x^2 + x^3)(1 + x + x^2 + x^3$

$+ x^4)$
$=$ coefficient of x^{24} in $(1 + x + x^2 + x^2 ..... x^{30})^2 (1 + x + x^2 + x^3) (1 + x + x^2 + x^3 + x^4)$

$$= \text{Coeff. of } x^{24} \text{ in } \frac{(1-x^{31})^2}{(1-x)^2} \cdot \frac{1-x^4}{1-x} \cdot \frac{1-x^5}{1-x}$$

$= \text{Coeff. of } x^{24} \text{ in } (1 + x^{62} - 2x^{31}) (1-x^4 - x^5 + x^9) (1-x)^{-4}$
$= \text{Coeff. of } x^{24} \text{ in } (1 - x^4 - x^5 + x^9) (1 - x)^{-4}$
Coeff. of X^n in $(1-x)^{-r}$ is ${}^{n+r-1}C_r$
$= ({}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3)$
$= 2925 - 1771 - 1540 + 816$
$= 430$ Ans.

Q.2 (2)
General team of

$$\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} = {}^{11}C_r \left(\frac{5}{2}x^3\right)^{11-r} \left(\frac{-1}{5x^2}\right)^r = (-1)^r \, {}^{11}C_r \left(\frac{5}{2}\right)^{11-r} \frac{1}{5^r} x \, 33.5r$$

$$= (-1)^r \, {}^{11}C_r \frac{5^{11-2r}}{2^{11-r}} x^{33-5r}$$

Now, term independent of x in $(1 - x^2 + 3x^3)$

$$\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} \text{ will be,}$$

$=$ Coeff. of x^o in

$$\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} - \text{coeff of } x^{-2} \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right) + 3.\text{Coeff of } x^{-3} \text{in} \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$$

$33 - 5r = 0$ $\quad$ $33 - 5r = -2$ $\quad$ $33 - 5r = -3$
$5r = 33$ $\quad$ $5r = 35$ $\quad$ $5r = 38$

$$r = \frac{33}{5} \text{ (Not possible)} \quad r = 7 \quad r = \frac{38}{5} \text{ (Not possible)}$$

Hence, for r = 7, term is independent of x

$$= -(-1)^7 \, {}^{11}C_7 \frac{5^{-3}}{2^4}$$

$$= \frac{11.10.9.8}{4.3.2.1} \times \frac{1}{2.2.2.2} \times \frac{1}{5.5.5} = \frac{33}{200}$$

Q.3 (4)
General term

$$T_{r+1} = \frac{\lfloor 10}{\lfloor r_1 \lfloor r_2 \lfloor r_3} (3)^{r_1} (-2)^{r_2} (5)^{r_3} (x)^{3r_1 + 2r_2 - 5r_3}$$

$3r_1 + 2r_2 - 5r_3 = 0$ .....(1)
$r_1 + r_2 + r_3 = 10$ .....(2)
From equation (1) and (2)
$r_1 + 2(10 - r_3) - 5r_3 = 0$
$r_1 + 20 = 7r_3$
$(r_1, r_2, r_3) = (1, 6, 3)$

$$\text{constant term} = \frac{10!}{1!\,6!\,3!} (3)^1 (-2)^6 (5)^3$$

$= 2^9 . 3^2 . 5^4 . 7^1$
$\lambda = 9$

Q.4 (3)
Given $9^n - 8n - 1 = 64\alpha$

$$\alpha = \frac{(1+8)^n - 8n - 1}{64} = {}^nC_2 + {}^nC_3 8 + {}^nC_4 8^2 + ...$$

Now, $6^n - 5n - 1 = 25\beta$

$$\beta = \frac{(1+5)^n - 5n - 1}{25}$$

$= {}^nC_2 + {}^nC_3 5 + {}^nC_4 \cdot 5^2 + ...$
$\therefore \alpha - \beta = {}^nC_3(8 - 5) + {}^nC_4(8^2 - 5^2) + ...$

Q.5 (5)

$$\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15} = \left(2x^{\frac{1}{5}} - x^{\frac{-1}{5}}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r \cdot \left(2x^{\frac{1}{5}}\right)^{15-r} \left(x^{\frac{-1}{5}}\right)^r \cdot (-1)^r$$

$$= (-1)^r \, {}^{15}C_r \cdot 2^{15-r} x^{\frac{15-r}{5}} \cdot x^{-\frac{r}{5}}$$

$$= (-1)^r \, {}^{15}C_r \cdot 2^{15-r} x^{\frac{15-2r}{5}}$$

Given coefficient of $x^{-1} = m$

$$\therefore \frac{15-2r}{5} = -1 \Rightarrow 15 - 2r = -5$$

$2r = 20 \Rightarrow r = 10$
$m = (-1)^{10} \cdot {}^{15}C_{10} \; 2^{15-10}$
$m = {}^{15}C_5 \cdot 2^5$
Now for coefficient of x^{-3}

$$\frac{15-2r}{5} = -3 \Rightarrow 15 - 2r = -15$$

$r = 15$
$\therefore (-1)^{15} \cdot {}^{15}C_{15} \cdot 2^{15-15} = -1 = n$
$mn^2 = {}^{15}C_5 \cdot 2^5(-1)^2$
$mn^2 = {}^{15}C_5 \cdot 2^5$
$\therefore r = 5$

Q.6 **(4)**
$3^{2022} = 9^{1011} = (10-1)^{1011}$
$= C_0(10)^{1011} + C_1(10)^{1010}(-1)^1 + ... + C_{1010}(10)^1(-1)^{1010} + C_{1011}(-1)^{1011}$

$$3^{2022} = 10\underbrace{[C_0 10^{1010} + C_1(10)^{1009}(-1)^1 + ... + C_{1010}(-1)^{1010}]}_{K} - 1$$

$3^{2022} = (10 \cdot K - 1) - 4 + 4 = [10K - 5] + 4 = 5(2K - 1) + 4$
So remainder R=4

Q.7 **(4)**
$1 + 3 + 3^2 .... + 3^{2021}$

$$= \frac{3^{2022} - 1}{3 - 1} = \frac{3^{2022} - 1}{2}$$

$= \frac{(3^2)^{1011} - 1}{2}$

$= \frac{[10-1]^{1011} - 1}{2}$

$= \frac{[10^{1011} - (10^{1010}) \cdot 1011_{C_1} + ... + 1011_{C_{1010}}(10) - 1] - 1}{2}$

$= 50(\text{int}) + (1011)(5) - 1$
divide by 50

$\Rightarrow \frac{5055-1}{50} = \frac{5054}{50} \Rightarrow$ Remainder = 4

Q.8 **(1)**

$S = (5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}, (x > 0)$

$$S = (5+x)^{500}\left\{\frac{\left(\frac{x}{x+5}\right)^{501} - 1}{\frac{x}{x+5} - 1}\right\}$$

$$S = (5+x)^{500}\left(\frac{x^{501} - (x+5)^{501}}{-5(x+5)^{500}}\right)$$

$$S = \frac{1}{5}((x+5)^{501} - x^{501})$$

coeff. of $x^{101} = \frac{1}{5}\ {}^{501}C_r x^{501-r}(5)^r$

$501 - r = 101 \Rightarrow r = 400$

$\Rightarrow \frac{1}{5}\ {}^{501}C_{400}(5)^{400}$

$\Rightarrow {}^{501}C_{101}(5)^{400} \times \frac{1}{5} \Rightarrow {}^{501}C_{101}(5)^{399}$

Q.9 **[83]**

$$\left(2x^3 + \frac{3}{x}\right)^{10}$$

$T_{r+1} = 10_{C_r}(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$= 10_{C_r}(2)^{10-r} x^{30-4r} 3^r$

at r = 0, 1, 2, 3, 4, 5, 6, 7 we will get even powers of 'x'

$10_{C_0}(2)^{10} + 10_{C_1}(2)^9 3^1 + 10_{C_2}(2)^8 3^2 + ... + 10_{C_7}(2)^3 3^7$

$\because (2+3)^{10} = 10_{C_0}(2)^{10} + ... + 10_{C_7}(2)^3(3)^7 + 10_{C_8}(2)^2 3^8 + ... + 10_{C_{10}}(3)^{10}$

So, sum of the co-efficients of all the positive even powers of x

$= 5^{10} - \{10_{C_8} 2^2 \times 3^8 + 10_{C_9}(2)^1(3)^9 + 10_{C_{10}}(3)^{10}\}$

$= 5^{10} - 3^9\left\{\frac{10\times 9}{2} \times \frac{4}{3} + 10 \times 2 + 3\right\}$

$= 5^{10} - 3^9(60 + 23) = 5^{10} - 3^9 \times 83$

So, $\beta = 83$

Q.10 **[102]**

${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20} = \frac{m}{n} = ({}^{60}C_{20})$

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$= {}^{41}C_0 + {}^{41}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20}$ $\quad (\because {}^{40}C_0 = {}^{41}C_0)$

$= {}^{42}C_1 + {}^{42}C_2 + ... + {}^{60}C_{20}$

$= {}^{60}C_{19} + {}^{60}C_{20} = {}^{61}C_{20} = \frac{m}{n}\ {}^{60}C_{20}$

$\Rightarrow \frac{61!}{20!41!} = \frac{m}{n}\left(\frac{60!}{20!40!}\right)$

$\Rightarrow \frac{61}{41} = \frac{m}{n}$

$m + n = 102$

Q.11 (5)

$$\left(\frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}}\right)^{60}$$

$T_{r+1} = {}^{60}C_r\left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r}\left(\frac{5^{1/2}}{x^{1/3}}\right)^r$

$= {}^{60}C_r 5^{\frac{3r-60}{4} \times \frac{180-5r}{6}}$

$\frac{180-5r}{6} = 10 \Rightarrow r = 24$

Coeff. of $x^{10} = {}^{60}C_{24}5^3 = \frac{\lfloor 60}{\lfloor 24 \lfloor 36} 5^3$

Powers of 5 in $= {}^{60}C_{24}.\ 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$

Q.12 **(57)**

coefficients and there cumulative sum are :

x^{7n}	$\rightarrow$	${}^7C_0$	1
x^{6n-5}	$\rightarrow$	$2 \cdot {}^7C_1$	1+14
x^{5n-10}	$\rightarrow$	$2^2 \cdot {}^7C_2$	1+14+84
x^{4n-15}	$\rightarrow$	$2^3 \cdot {}^7C_3$	1+14+84+280
x^{3n-20}	$\rightarrow$	$2^4 \cdot {}^7C_4$	1+4+84+280+560=939
x^{2n-25}	$\rightarrow$	$2^5 \cdot {}^7C_5$	

$30 - 20 \geq 0 \cap 2n - 25 < 0 \cap n \in I$

$\therefore 7 \leq n \leq 12$

Sum = 7 + 8 + 9 + 10 + 11 + 12 = 57

Q.13 (286)

$C_1 + 3.2C_2 + 5.3C_3 + ...$ up to 10 terms

$T_r = (2r-1)\ r.C_r = 2r^2 C_r - r.\ C_r$

$S_n = 2\sum r^2C_r - \sum rC_r$
$T_r = 2(r^2 - r + r).C_r - rC_r$
$T_r = (2r(r-1) + 2r)C_r - rC_r$
$T_r = 2r(r-1)C_r + rC_r$
$T_r = 2n(n-1)^{n-2}C_{r-2} + n^{n-1}C_{r-1}$
$S_n = 2n(n-1).2^8 + n.2^9$
$= n.2^9\{(n-1)+1\} = n^2.2^9$

RHS

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} ..... + \frac{C_{10}}{11}$$

$$\int (1+x)^{10}dx = C_0x + \frac{c_1x^2}{2} + \frac{c_2x^3}{3} ..... + \frac{c_{10}x^{11}}{11} + k$$

$$\frac{(1+x)^{11}}{11} = C_0x + \frac{c_1x^2}{2} + ... + \frac{c_{10}x^{11}}{11} + k$$

Putting x=0, we get $k = \frac{1}{11}$

$$x = 1 \Rightarrow \frac{2^{11}}{11} - \frac{1}{11} = C_0 + \frac{c_1}{2} + ... \frac{c_0}{11}$$

$$\therefore 100.2^9 = \frac{2^{11}-1}{11}\frac{(\alpha.2^{11})}{2^\beta - 1}$$

$$2^2.5^2.2^9 = \frac{2^{11}-1}{11}\frac{(\alpha.2^{11})}{2^\beta - 1}$$

$$\therefore \frac{2^\beta - 1}{\alpha} = \frac{2^{11}-1}{25\times 11}$$

$\alpha = 275$
$\beta = 11$
$\therefore \alpha + \beta = 286.$
Infinite solutions are possible.

Q. 14 (3)

$(2021)^{2023} = (7\times 288 + 5)^{2023}$
$= {}^{2023}C_0(7\times 288)^{2023} - ..... {}^{2023}C_{2023}(7\times 288)^0 \times 5$

$$= \frac{5}{7} + \frac{7}{7}k$$

remainder = +5

Q.15 (1)

$$\sum_{k=1}^{31} {}^{31}C_k \cdot {}^{31}C_{k-1}$$

$$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + ... + {}^{31}C_{31} \cdot {}^{31}C_{30}$$

$$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + ... + {}^{31}C_{30} \cdot {}^{31}C_0$$

$$= {}^{62}C_{30}$$

Similarly

$$\sum_{k=1}^{30} {}^{30}C_k \cdot {}^{30}C_{k-1} = {}^{60}C_{29}$$

$$= \frac{60!}{29!\,31!}\left\{\frac{62\cdot 61}{30\cdot 32} - 1\right\}$$

$$= \frac{60!}{30!\,31!}\left(\frac{2822}{32}\right)$$

$$\therefore 16\alpha = 16\times\frac{2822}{32} = 1411$$

Q.16 (2)

$$\left(2x^3 + \frac{3}{x^k}\right)^{12}$$

$$t_{r+1} = {}^{12}C_r(2x^3)^r\left(\frac{3}{x^k}\right)^{12-r}$$

$x^{3r-(12-r)k} \rightarrow$ constant
$\therefore 3r - 12k + rk = 0$

$$\Rightarrow k = \frac{3r}{12-r}$$

$\therefore$ possible values of r are 3, 6, 8, 9, 10 are corresponding values of k are 1, 3, 6, 9, 15
Now ${}^{12}C_r$ = 220, 924, 495, 220, 66
$\therefore$ possible values of k for which we will get 2^8 are 3, 6

Q.17 [23]

$(1+x)^p(1-x)^q$

$$\left[1 + px + \frac{P(p-1)}{2}x^2\right]\left[1 - qx + \frac{q(q-1)}{2}x^2\right]$$

coefficient of x $\Rightarrow (p-q) = -3$ ... (1)

coefficient of $x_2 \Rightarrow \frac{p(p-1)}{2} + \frac{q(q-1)}{2} - pq = -5$

$p^2 + q^2 - p - q - 2pq = -10$
$\Rightarrow (p-q)^2 - (p+q) = -10$
$\Rightarrow p + q = 19$ . . .
(2)
(1) and (2)
p = 8
q = 11
Now, $(1+x)^8(1-x)^{11}$
$\Rightarrow (1-x^2)^8(1-x)^3$
$[1 - 8x^2][1 - 3x + 3x^2 - x^3]$
coefficient of $x^3 = -1 + 24 = 23$

Q.18 (99)

$1 + (1 + 2^{49})(2^{49} - 1) = 2^{98}$
m = 1, n = 98
m + n = 99

Q.19 (6006)

$$y=\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$$

$$T_{r+1}={}^{15}C_r\left(t^2x^{1/5}\right)^{15-r}.\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$$

$$={}^{15}C_r t^{30-3r}.x^{\frac{15-r}{5}}.(1-x)^{r/10}$$

For term ind. of t $\Rightarrow 30-3r=0\Rightarrow r=10$

$T_{11}={}^{15}C_{10}\,x^1(1-x)^1={}^{15}C_{10}\,(x-x^2)$

$$T_{11}={}^{15}C_{10}\left[\frac{1}{4}-\left(x-\frac{1}{2}\right)^2\right]$$

$$(T_{11})_{max}={}^{15}C_{10}\frac{1}{4}\text{at } x=\frac{1}{2}$$

$$K=\frac{15.14.13.12.11}{4\times 5!}$$

$\Rightarrow$ **8K = 6006**

Q.20 (2)

$$\sum_{x=1}^{20}(r^2+1)r!$$

$$=\sum_{x=1}^{20}((r+1)^2-2r)r!$$

$$=\sum_{x=1}^{20}((r+1)(r+1)!-r.r!)-\sum_{r=1}^{20}r.r!$$

$$=\sum_{x=1}^{20}((r+1)(r+1)!-r.r!)-\sum_{r=1}^{20}((r+1)!-r!$$

$$=(21.\underline{|21}-1)-(\underline{|21}-1)$$

$= 20.21! = 22! - 2.21!$

Q.21 (221)

$$\sum_{K=1}^{10}K^2({}^{10}C_K)^2$$

$$\sum_{K=1}^{10}(K.{}^{10}C_K)^2=\sum_{K=1}^{10}(10.{}^9C_{K-1})^2$$

$$=100\sum_{K=1}^{10}C_{K-1}.{}^9C_{10-K}$$

$$100({}^{18}C_9)=100\left(\frac{18!}{9!9!}\right)$$

$\Rightarrow 4862000 = 22000L$

Hence L = 221

Q.22 (84)

$$\frac{T_5}{T_{n-3}}=\frac{{}^nC_4(2^{1/4})^{n-4}(3^{-1/4})^4}{{}^nC_{n-4}(2^{1/4})^4(3^{-1/4})^{n-4}}=\frac{\sqrt[4]{6}}{1}$$

$$\Rightarrow 2^{\frac{n-8}{4}}3^{\frac{n-8}{4}}=6^{1/4}$$

$\Rightarrow 6^{n-8}=6$

$\Rightarrow n-8=1\Rightarrow n=9$

$$T_6={}^9C_5(2^{1/4})^4(3^{-1/4})^5=\frac{84}{\sqrt[4]{3}}$$

$\therefore \alpha = 84$

Q.23 (3)

$7^{2022}+3^{2022}$

$=(49)^{1011}+(9)^{1011}$

$=(50-1)^{1011}+(10-1)^{1011}$

$=5\lambda-1+5K-1$

$=5m-2$

Remainder = 5 – 2 = 3

Q.24 (1)

$(2021)^{2022}+(2022)^{2021}$

$(2023-2)^{2022}+(2023-1)^{2021}$

$(7x-2)^{2022}+(7x-1)^{2021}$

Remiander

$(2^3)^{674}-(1)^{674}$

$(8)^{674}-(1)^{674}$

$\{(7+1)^{674}-1\}$

$=(1)^{674}-1$

$=1-1$

$=0$

Q.25 (1)

$$\sum_{i,j=0}^{n}{}^nC_i{}^nC_j\Rightarrow\sum_{i\neq j}{}^nC_i.{}^nC_j-\sum_{i=j}{}^nC_i.{}^nC_j$$

$$\Rightarrow\sum_{i\neq j}{}^nC_i.{}^nC_j=\sum_{i,j=0}^{n}{}^nC_i{}^nC_j-\sum_{i=j}{}^nC_i{}^nC_j$$

$$=\left(2^n\right)^2-\left(C_0^2+C_1^2+...+C_n^2\right)$$

$=2^{2n}-{}^{2n}C_n$

Q.26 [180]

$(2, 2, 3, 3, 1)\rightarrow\frac{5!}{2!2!}$

$(1, 4, 3, 3, 1)\rightarrow\frac{5!}{2!2!}$

$(2, 2, 1, 9, 1)\rightarrow\frac{5!}{2!2!}$

$(2, 1, 6, 3, 1)\rightarrow\frac{5!}{2!}$

$(4, 9, 1, 1, 1)\rightarrow\frac{5!}{3!}$

$(6, 6, 1, 1, 1)\rightarrow\frac{5!}{2!3!}$

Add all

$\Rightarrow 30+30+30+60+20+10=180$

Q.27 **[24]**

9^{th} term is greatest so

$T_9 > T_8$ & $T_9 > T_{10}$

${}^nC_8 3^{n-8}(6x)^8 > {}^nC_7 3^{n-7}(6x)^7$ & ${}^nC_8 3^{n-8}(6x)^8 > {}^nC_9 3^{n-9}(6x)^9$

$$\frac{{}^nc_8}{{}^nc_7}\ \frac{3^{n-8}(6x)^8}{3^{n-7}(6x)^7} > 1 \,\&\, 1 > \frac{{}^nC_9 3^{n-9}(6x)^9}{{}^nC_8 3^{n-8}(6x)^8}$$

$$\frac{n-8+1}{8}.\frac{1}{3}.6x > 1 \qquad 1 > \frac{n-9+1}{9}.\frac{1}{3}.6x$$

$$\frac{n-7}{8}.\frac{1}{3}.6.\frac{3}{2} > 1$$

$$1 > \frac{n-9+1}{9}.2.\frac{3}{2} \qquad \frac{3(n-7)}{8} > 1 \qquad 9 > 3(n-8)$$

$3(n-7) > 8$ $\quad 9 > 3n-24$

$3n > 29$ $\quad 3n-24 < 9$

$n > \frac{29}{3}$ $\quad 3n < 33$

$\Rightarrow n_0 = 10$

$\frac{29}{3} < n < 1$ $\quad n < 11$

$$k = \frac{{}^{10}C_6 3^{10-6} 6^6}{{}^{10}C_3 3^7 6^3} = \frac{{}^{10}C_6}{{}^{10}C_3}.\frac{6^3}{3^3} = 14$$

$k + n_0 = 10 + 14 = 24$

Q.28 (4)

$(11)^{1011} = (9+2)^{1011} = 9\lambda + 2^{1011}$

$= 9\lambda + (8)^{337}$

$= 9\lambda + (9-1)^{337}$

$= 9\lambda + 9\mu - 1$

$(1011)^{11} = (1011)^2 \times (1011)^9$

So, $(1011)^{11}$ is divisible by 9

$\therefore$ Final number $= 9\lambda + 9\mu - 1 + 9\lambda'$

$= 9k' + 8$

$\therefore$ Remainder is 8

STRAIGHT LINE

EXERCISE-I (MHT CET LEVEL)

Q.1 (1)
The vertices of triangle are the intersection points of these given lines. The vertices of Δ are $A(0,4)$, $B(1,2)$, $C(4,0)$

Now, $AB = \sqrt{(0-1)^2 + (4-1)^2} = \sqrt{10}$

$BC = \sqrt{(1-4)^2 + (0-1)^2} = \sqrt{10}$

$AC = \sqrt{(0-4)^2 + (0-4)} = 4\sqrt{2}$

$\because AB = BC$; $\therefore \Delta$ is isosceles.

Q.2 (c)

Q.3 (2)
The equation of lines are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

$$\Rightarrow y - 4 = \frac{1 \pm \tan 45°}{1 \mp \tan 45°}(x - x_1)$$

$$\Rightarrow y - 4 = \frac{1 \pm 1}{1 \mp 1}(x - 3) \Rightarrow y = 4 \text{ or } x = 3$$

Hence, the lines which make the triangle are $x - y = 2$, $x = 3$ and $y = 4$. The intersection points of these lines are (6, 4), (3, 1) and (3, 4)

$$\therefore \Delta = \frac{1}{2}[6(-3) + 3(0) + 3(3)] = \frac{9}{2}$$

Q.4 (2)

Mid point $\equiv \left(\frac{1+1}{2}, \frac{3-7}{2}\right) = (1,-2)$

Therefore required line is $2x - 3y = k \Rightarrow 2x - 3y = 8$.

Q.5 (b)
Let P(x, y) be the point diveding the join of A and B in the ratio 2 : 3 internally, then

$$x = \frac{20\cos\theta + 15}{5} = 4\cos\theta + 3$$

$$\Rightarrow \cos\theta = \frac{x-3}{4} \text{....(i)}$$

$$y = \frac{20\sin\theta + 0}{5} = 4\sin\theta \Rightarrow \sin\theta = \frac{y}{4} \text{....(ii)}$$

Squaring and adding (i) and (ii), we get the required locis $(x-3)^2 + y^2 = 16$, which is a circle.

Q.6 (b)
Q.7 (c)

Q.8 (1)

Point of intersection $y = -\frac{21}{5}$ and $x = \frac{23}{5}$

$$\therefore 3x + 4y = \frac{3(23) + 4(-21)}{5} = \frac{69 - 84}{5} = -3.$$

Hence, required line is $3x + 4y + 3 = 0$.

Q.9 (2)
Let the co-ordinates of the third vertex be $(2a, t)$.

$$AC = BC \Rightarrow t = \sqrt{4a^2 + (a-t)^2} \Rightarrow t = \frac{5a}{2}$$

So the coordinates of third vertex C are $\left(2a, \frac{5a}{2}\right)$

Therefore area of the triangle

$$= \pm\frac{1}{2}\begin{vmatrix} 2a & \frac{5a}{2} & 1 \\ 2a & 0 & 1 \\ 0 & a & 1 \end{vmatrix} = \begin{vmatrix} a & \frac{5a}{2} & 1 \\ 0 & -\frac{5a}{2} & 0 \\ 0 & a & 1 \end{vmatrix} = \frac{5a^2}{2} \text{sq. units.}$$

Q.10 (d)
Q.11 (a)
Q.12 (a)
Q.13 (d)
Q.14 (b)
Q.15 (d)

Q.16 (2)
It is obvious.

Q.17 (2)

$ax \pm by \pm c = 0 \Rightarrow \frac{x}{\pm c/a} + \frac{y}{\pm c/b} = 1$ which meets on axes at $A\left(\frac{c}{a},0\right)$, $C\left(-\frac{c}{a},0\right)$, $B\left(0,\frac{c}{b}\right)$, $D\left(0,-\frac{c}{b}\right)$.

Therefore, the diagonals AC and BD of quadrilateral ABCD are perpendicular, hence it is a rhombus whose area is given by $= \frac{1}{2}AC \times BD = \frac{1}{2} \times \frac{2c}{a} \times \frac{2c}{b} = \frac{2c^2}{ab}$.

Q.18 (1)

$(h-3)^2 + (k+2)^2 = \left|\frac{5h-12k-13}{\sqrt{25+144}}\right|$.

Replace (h, k) by (x, y), we get

$13x^2 + 13y^2 - 83x + 64y + 182 = 0$, which is the required equation of the locus of the point.

Q.19 (2)

Let point be (x_1, y_1), then according to the condition

$$\frac{3x_1 + 4y_1 - 11}{5} = -\left(\frac{12x_1 + 5y_1 + 2}{13}\right)$$

Since the given lines are on opposite sides with respect to origin, hence the required locus is $99x + 77y - 133 = 0$

Q.20 (1)

Let the point be (x, y). Area of triangle with points $(x, y), (1, 5)$ and $(3, -7)$ is 21 sq. units

$$\therefore \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 5 & 1 \\ 3 & -7 & 1 \end{vmatrix} = 21$$

Solving; locus of point (x, y) is $6x + y - 32 = 0$.

Q.21 (3)

According to question ,

$$x_1 = \frac{2+4+x}{3} \Rightarrow x = 3x_1 - 6$$

$$y_1 = \frac{5-11+y}{3} \Rightarrow y = 3y_1 + 6$$

$$\therefore 9(3x_1 - 6) + 7(3y_1 + 6) + 4 = 0$$

Hence locus is $27x + 21y - 8 = 0$, which is parallel to $9x + 7y + 4 = 0$.

Q.22 (3,4)

Suppose the axes are rotated in the anticlockwise direction through an angle 45^o. To find the equation of L w.r.t the new axis, we replace x by $x\cos\alpha - y\sin\alpha$ and by $x\sin\alpha + y\cos\alpha$, so that equation of line w.r.t. new axes is

$\Rightarrow$

$$1/1(x\cos 45^o - y\sin 45^o) + \frac{1}{2}(x\sin 45^o + y\cos 45^o) = 1$$

Since, p, q are the intercept made by the line on the coordinate axes. we have on putting (p, 0) and then (0, q)

$$\Rightarrow \frac{1}{p} = \frac{1}{a}\cos\alpha + \frac{1}{b}\sin\alpha \Rightarrow \frac{1}{q} = -\frac{1}{a}\sin\alpha + \frac{1}{b}\cos\alpha$$

$$\Rightarrow \frac{1}{p} = \frac{1}{1}\cos 45^o + \frac{1}{2}\sin 45^o$$

$$\Rightarrow \frac{1}{p} = \frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$$

$$\therefore p = \frac{2\sqrt{2}}{3}; \quad \therefore \frac{1}{q} = -\frac{1}{1}\sin 45^o + \frac{1}{2}\cos 45^o$$

$$\frac{1}{q} = \frac{-1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} = -\frac{1}{2\sqrt{2}}, \therefore q = 2\sqrt{2}$$

So intercept made by is assume on the new axis $\left(2\sqrt{2}/3, 2\sqrt{2}\right)$. If the rotation is assume in clockwise direction, so intercept made by the line on the new axes would be $\left(2\sqrt{2}, 2\sqrt{2}/3\right)$.

Q.23 (3)

Here $c = -1$ and $m = \tan\theta = \tan 45^o = 1$

(Since the line is equally inclined to the axes, so $\theta = 45^o$)

Hence equation of straight line is $y = \pm(1.x) - 1$

$\Rightarrow x - y - 1 = 0$ and $x + y + 1 = 0$.

Q.24 (a)

As (-1,1) is a point on 3x-4y+7=0, the rotation is possible. Slope of the given line =3/4. Slope of the line in its new position.

$$= \frac{\frac{3}{4} - 1}{1 + \frac{3}{4}} = -\frac{1}{7}$$

The required equation is

$$y-1=\frac{1}{7}(x+1) or 7y+x-6=0$$

Q.25 (c)

Equations of hte sides of the parallelogram are

$$(x-3)(x-2)=0 \text{ and } (y-5)(y-1)=1$$

i.e. $x=3, x=2; y=5, y=1$

Hence its vertices are : $A(2,1); B(3,1);$

$$C(3,5); D(2,5)$$

Equation of the diagonal AC is

$$y-1=\frac{4}{1}(x-2) \Rightarrow y=4x-7$$

Equation of the diagonal BD is

$$y-1=\frac{4}{1}(x-3) \Rightarrow 4x+y=13$$

Q.26 (b)

Equation of the line making intercepts a and b on the axes is $\frac{x}{a}+\frac{y}{b}=1$ since, it passes through (1,1)

$$\Rightarrow \frac{1}{a}+\frac{1}{b}=1 \quad ....(i)$$

Also the area of the triangle formed by the line and the axes is A.

$$\therefore \frac{1}{2}ab=A \Rightarrow ab=2A \quad ....(ii)$$

From eqs. (i) and (ii), we get, a+b=2A Hence, a and b are the roots of the eq.

$$x^2-(a+b)x+ab=0 \Rightarrow x^2-2Ax+2A=0$$

Q.27 **(d)**

Let A(3, y), B(2, 7), C(–1, 4) and D(0, 6) be the given points.

$$m_1 = \text{slope of } AB = \frac{7-y}{2-3} = (y-7)$$

$$m_2 = \text{slope of } CD = \frac{6-4}{0-(-1)} = 2$$

Since AB and CD are parallel,

$\therefore m1 = m2 \Rightarrow y = 9.$

Q.28 **(d)**

Q.29 **(c)**

Q.30 **(a)**

Q.31 **(a)**

Q.32 (1)

Point of intersection of the lines is (3, –2).

Hence the equation is $2x-7y=2(3)-7(-2)=20$

Q.33 (3)

The required equation which passes through (1, 2) and its gradient is $m=3$, is $(y-2)=3(x-1)$

Q.34 (4)

Here equation of AB is $x+4y-4=0$ .....(i)

and equation of BC is $2x+y-22=0$ .....(ii)

Thus angle between (i) and (ii) is given by

$$\tan^{-1}\frac{-\frac{1}{4}+2}{1+\left(-\frac{1}{4}\right)(-2)}=\tan^{-1}\frac{7}{6}$$

Q.35 (c)

Equation of lines are $\frac{x}{a}-\frac{y}{b}=1$ and $\frac{x}{b}-\frac{y}{a}=1$

$$\Rightarrow m_1=\frac{b}{a} \text{ and } m_2=\frac{a}{b}$$

Therefore

$$\theta=\tan^{-1}\frac{\frac{b}{a}-\frac{a}{b}}{1+\frac{b}{a}\cdot\frac{a}{b}}=\tan^{-1}\frac{b^2-a^2}{2ab}$$

Q.36 (b)

Q.37 **(a)**

Q.38 **(c)**

Q.39 (4)

Here,

Slope of I[st] diagonal= $m_1=\frac{2-0}{2-0}=1 \Rightarrow \theta_1=45^o$

Slope of II[nd] diagonal= $m_2=\frac{2-0}{1-1}=\infty \Rightarrow \theta_2=90^o$

$$\Rightarrow \theta_2-\theta_1=45^o=\frac{\pi}{4}$$

Q.40 (1)

Let the point (h, k) then $h+k=4$ .....(i)

and $1 = \pm \frac{4h + 3k - 10}{\sqrt{4^2 + 3^2}} \Rightarrow 4h + 3k = 15$
…..(ii)
and $4h + 3k = 5$
…..(iii)
On solving (i) and (ii); and (i) and (iii), we get the required points (3, 1) and (–7, 11).
Trick : Check with options. Obviously, points (3, 1) and (–7, 11) lie on x + y =4and perpendicular distance of these points from 4x + 3y = 10 is 1

Q.41 (a)
Q.42 (c)
Q.43 (c)

Q.44 **(d)**
Q.45 **(a)**
Q.46 **(b)**

Q.47 (2)
$L \equiv 3x - 4y - 8 = 0$
$L_{(3,4)} = 9 - 16 - 8 < 0$ and $L_{(2,-6)} = 6 + 24 - 8 > 0$
Hence, the points lie on different side of the line.

Q.48 (4)
Let the distance of both lines are p_1 and $P_2$from origin, then $p_1 = -\frac{8}{5}$ and $p_2 = -\frac{3}{5}$. Hence distance between both the lines $= |p_1 \sim p_2| = \frac{5}{5} = 1$.

Q.49 (a)
The equations of the lines are
$p_1 x + q_1 y - 1 = 0$ ...(i)
$p_2 x + q_2 y - 1 = 0$ ...(ii)
and $p_3 x + q_3 y - 1 = 0$ ...(iii)
As they are concurrent,

$$\begin{vmatrix} p_1 & q_1 & -1 \\ p_2 & q_2 & -1 \\ p_3 & q_3 & -1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

This is also the condition for the points (p_1, q_1), (p_2, q_2) and (q_3, q_3) to be collinear

Q.50 (b)

Q.51 (2)
The set of lines is $4ax + 3by + c = 0$, where $a + b + c = 0$.
Eliminating c, we get $4ax + 3by - (a + b) = 0$
$\Rightarrow a(4x - 1) + b(3y - 1) = 0$
This passes through the intersection of the lines $4x - 1 = 0$ and $3y - 1 = 0$ i.e. $x = \frac{1}{4}, y = \frac{1}{3}$ i.e., $\left(\frac{1}{4}, \frac{1}{3}\right)$.

Q.52 (3)
Required line should be,
$(3x - y + 2) + \lambda(5x - 2y + 7) = 0$ .....(i)
$\Rightarrow (3 + 5\lambda)x - (2\lambda + 1)y + (2 + 7\lambda) = 0$
$\Rightarrow y = \frac{3 + 5\lambda}{2\lambda + 1}x + \frac{2 + 7\lambda}{2\lambda + 1}$
.....(ii)
As the equation (ii), has infinite slope, $2\lambda + 1 = 0$
$\Rightarrow \lambda = -1/2$ putting $\lambda = -1/2$ in equation (i) we have
$(3x - y + 2) + (-1/2)(5x - 2y + 7) = 0 \Rightarrow x = 3$.

Q.53 (a)
Rewritting the equation
$(2x + y + 2)a + (3x - y - 4)b = 0$ and for all a, b the straight lines pass though the inter-section of $2x + y + 2 = 0$ and
$3x - y - 4 = 0$ i.e. the point $\left(\frac{2}{5}, -\frac{14}{5}\right)$

Q.54 (d)
The given system of lines passes through the point of intersection of the straight lines 2x + y – 3 = 0 and 3x + 2y – 5 = 0 [$L_1 + \lambda L_2 = 0$ form], which is (1, 1). The required line will also pass through this point. Further, the line will be farthest from point (4, –3) if it is in direction perpendicular to line joining (1, 1) and (4, –3).
∴ The equation of the required line is

$$y - 1 = \frac{-1}{\frac{-3-1}{4-1}}(x - 1) \Rightarrow 3x - 4y + 1 = 0$$

Q.55 **(a)**

Q.56 (d)

Q.57 (c)

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)

AB = $\sqrt{4+9} = \sqrt{13}$

BC = $\sqrt{36+16} = 2\sqrt{13}$

CD = $\sqrt{4+9} = \sqrt{13}$

AD = $\sqrt{36+16} = 2\sqrt{13}$

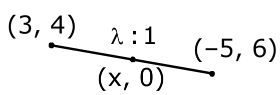
AC = $\sqrt{64+1} = \sqrt{65}$

BD = $\sqrt{16+49} = \sqrt{65}$

its rectangle

Q.2 (1)

$$\frac{-5\lambda+3}{\lambda+3} = x, \quad \frac{6\lambda-4}{\lambda+1} = 0$$



$\Rightarrow \lambda = \frac{2}{3}$

Q.3 (4)

(2a, 3a), (3b, 2b) & (c, c) are collinear

$$\Rightarrow \begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0$$

$\Rightarrow$ (3bc – 2bc) – (2ca – 3ca) + (4ab – 9ab) = 0

$\Rightarrow$ bc + ca + 5ab = 0

$$\Rightarrow \frac{2}{2} \cdot \frac{5}{c} = \frac{1}{a} + \frac{1}{b} \Rightarrow \frac{2}{\left(\frac{2c}{5}\right)} = \frac{1}{a} + \frac{1}{b}$$

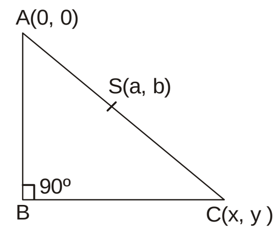
$\Rightarrow$ a, $\frac{2c}{5}$, b are in H.P.

Q.4 (1)

By given information

Since in ΔABC, B is other centre. Hence $\angle$B = 90º

Cercum centre is S (a, b)



$$\frac{x+0}{2} = a \Rightarrow x = 2a$$

$$\frac{y+0}{2} = b \Rightarrow y = 2b$$

Hence, c(x, y) $\equiv$ (2a, 2b)

Q.5 (4)

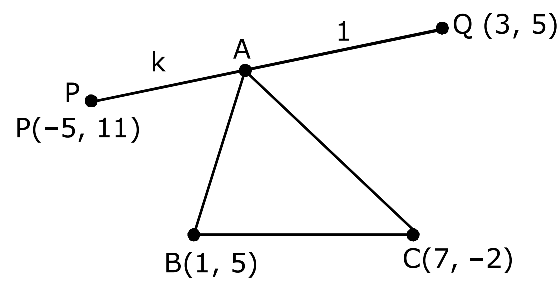
$$\Delta = \left| \frac{1}{2} \begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \\ -a\cos\theta & -b\sin\theta & 1 \end{vmatrix} \right|$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left\| \begin{matrix} 0 & 0 & 2 \\ -a\sin\theta & b\cos\theta & 1 \\ -a\cos\theta & -b\sin\theta & 1 \end{matrix} \right\|$$

$= \frac{1}{2}$. 2 (ab $\sin^2\theta$ + ab $\cos^2\theta$) = ab

Q.6 (3)

$$\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$$



$$\frac{1}{2}\begin{vmatrix} \frac{3k-5}{k+1} & \frac{5k+1}{k+1} & 1 \\ 1 & 5 & 1 \\ 7 & -2 & 1 \end{vmatrix} = |2|$$

$$\Rightarrow 1.(-2-3) - 1.\left(\frac{-6k+10}{k+1} - \frac{35k+7}{k+1}\right) + \left(\frac{15k-25}{k+1} - \frac{5k+1}{k+1}\right) = \pm 4$$

$\Rightarrow$ 6k – 10 + 35 k + 7 + 15k – 25 – 5k – 1 = ± 4 + 37 (k + 1)

$\Rightarrow$ 51 k – 29 = 41 k + 41 or 51 k – 29
= 33k + 33
$\Rightarrow$ 10 k = 70 or 18 k = 62

k = 7 k = $\frac{31}{9}$

Q.7 (1)

AP = $\sqrt{x^2+(y-4)^2}$

BP = $\sqrt{x^2+(y+4)^2}$

$\because$ |AP – BP| = 6
AP – BP = ± 6

$\sqrt{x^2+(y-4)^2} - \sqrt{x^2+(y+4)^2} = \pm 6$

On squaring we get the locus of P
$9x^2 - 7y^2 + 63 = 0$

Q.8 (2)
Let coordinate of mid point is m(h, k)

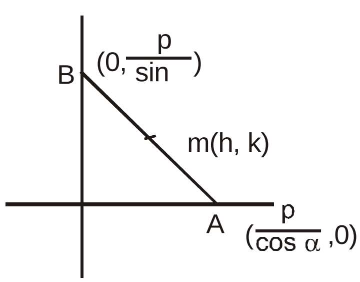
$2h = \frac{p}{\cos d} \Rightarrow \cos\alpha = \frac{p}{2h}$

$2k = \frac{p}{\sin d} \Rightarrow \sin\alpha = \frac{p}{2k}$

Squareing and add.

$\frac{1}{h^2} + \frac{1}{k^2} = \frac{4}{p^2}$

Locus of p(h, k) $\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$



Q.9 (4)

Q.10 (2)

Q.11 **(2)**

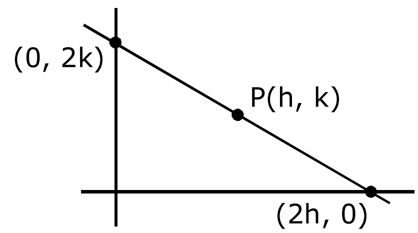
Q.12 **(4)**

Q.13 (3)
Let centroid is (h, k)

then $h = \frac{\cos\alpha + \sin\alpha + 1}{3}$ & $k = \frac{\sin\alpha - \cos\alpha + 2}{3}$

$\cos\alpha + \sin\alpha = 3h - 1$ & $\sin\alpha - \cos\alpha = 3k - 2$
squaring & adding
$2 = (3h-1)^2 + (3k-2)^2$ Locus of (h, k)
$\Rightarrow (3x-1)^2 + (3k-2)^2 = 2$
$\Rightarrow 3(x^2+y^2) - 2x - 4y + 1 = 0$

Q.14 (2)
P is a mid point AB



AB = 10 units
$(2h)^2 + (2k)^2 = 10^2$
$h^2 + k^2 = 25$
Locus of (h, k)
$x^2 + y^2 = 25$

Q.15 (4)
P(1, 0), Q(–1, 0), R(2, 0), Locus of s (h, k) if $SQ^2 + SR^2 = 2SP^2$
$\Rightarrow (h+1)^2 + k^2 + (h-2)^2 + k^2$
$= 2(h-1)^2 + 2k^2$
$\Rightarrow h^2 + 2h + 1 + h^2 - 4h - 4 = 2h^2 - 4h + 2$
$\Rightarrow$ 2h + 3 = 0 Locus of s(h, k)
$\Rightarrow$ 2x + 3 = 0
Parallel to y-axis.

Q.16 (2)

Slope = $\frac{k+1-3}{k^2-5} = \frac{1}{2} \Rightarrow k^2 - 5 - 2k + 4 = 0$

$\Rightarrow k = 1 \pm \sqrt{2}$ $\Rightarrow k^2 - 2k - 1 = 0$ $\Rightarrow$ k

$= \frac{2 \pm \sqrt{4+4}}{2}$

$= \frac{2 \pm 2\sqrt{2}}{2}$

Q.17 (1)
To eliminate the parameter t, square and add the equations, we have

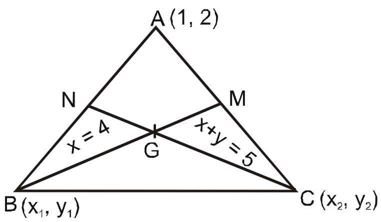
$$x^2 + y^2 = a^2\left(\frac{1-t^2}{1+t^2}\right)^2 + \frac{4a^2t^2}{(1+t^2)^2}$$

$$= \frac{a^2}{(1+t^2)^2}\left[(1-t^2)^2 + 4t^2\right]$$

$$= \frac{a^2(1+t^2)^2}{(1+t^2)^2} = a^2$$

which is the equation of a circle.

Q.18 (2)



$x_1 + y_1 = 5$ ... (i)

$x_2 = 4$ ... (ii)

co - ordinates of G are ≡ (4, 1)

$\Rightarrow \frac{1 + x_1 + x_2}{3} = 4$ ....(iii)

and $\frac{y_1 + y_2 + 2}{3} = 1$ . . .

(iv)

solving above equations, we get B & C.

Q.19 (4)

Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

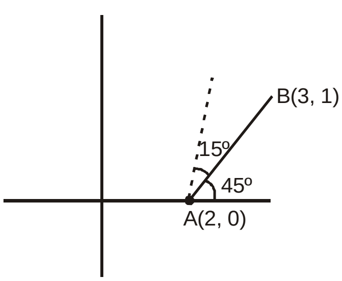
$\frac{a}{2} = 1 \Rightarrow a = 2$

$\frac{b}{2} = 2 \Rightarrow b = 4$

Hence $\frac{x}{2} + \frac{y}{4} = 1 \Rightarrow \quad 2x + y - 4 = 0$

Q.20 (3)

Slope of AB is $\tan\theta = \frac{1-0}{3-2} = 1$



$\theta = 45°$

Hence equation of new line is

$y - 0 = \tan 60°(x - 2)$

$y = \sqrt{3}\,x - 2\sqrt{3}$

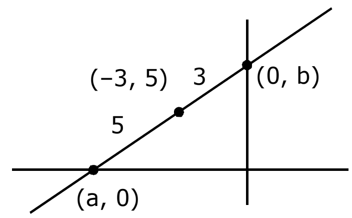
$\Rightarrow \sqrt{3}\,x - y - 2\sqrt{3} = 0$

Q.21 (4)

$-3 = \frac{3a + 0}{5 + 3}$, $5 = \frac{0 + 5b}{5 + 3}$

$\Rightarrow a = -3, b = 8$

$\frac{x}{-8} + \frac{y}{8} = 1$



$-x + y = 8$

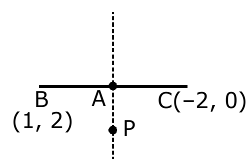
$x - y + 8 = 0$

Q.22 (3)

Perpendicular bisector of slopoe of line BC

$m_{BC} = \frac{2-0}{1+2} = \frac{2}{3}$

$m_{AP} = \frac{-3}{2}$



$A = \left(\frac{1-2}{2}, \frac{2+0}{2}\right) \Rightarrow \left(-\frac{1}{2}, 1\right)$

$y - 1 = \frac{-3}{2}\left(x + \frac{1}{2}\right) \Rightarrow 4y - 4 = -6x - 3$

$\Rightarrow 6x + 4y = 1$

locus of P

Q.23 (3)

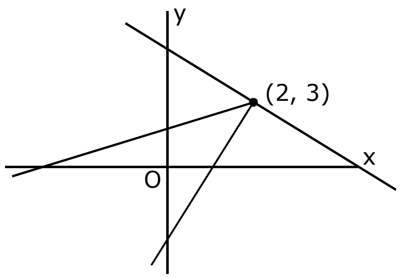
Equation $y - 3 = m(x - 2)$

cut the axis at

$\Rightarrow y = 0 \;\&\; x = \frac{2m - 3}{m}$

$\Rightarrow x = 0 \;\&\; y = -(2m - 3)$

Area $\Delta = 12 = \left|\frac{1}{2} \cdot \frac{(2m-3)}{m}\{-(2m-3)\}\right|$



$(2m-3)^2 = \pm 24m$
$4m^2 - 12m + 9 = 24m$
or $4m^2 - 12m + 9 = -24m$
$4m^2 - 3y\,m + 9 = 0$
$D > 0$
or $4m^2 + 12m + 9 = 0$
$(2m+3)^2 = 0$
two distinct root of m
no. of values of m is 3.

Q.24 (1)

$y - x + 5 = 0$, $\sqrt{3}\,x - y + 7 = 0$

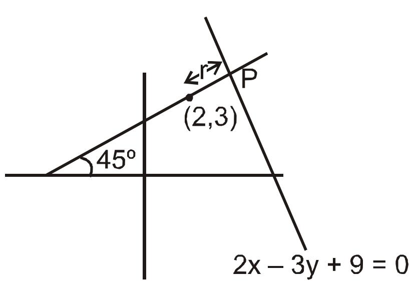
$m_1 = 1, m_2 = \sqrt{3}$
$\theta_1 = 45^\circ, \theta_2 = 60^\circ$
$\theta = 60^\circ - 45^\circ = 15^\circ$

Aliter $\tan\theta = \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$

$\Rightarrow \theta = 15^\circ$

Q.25 **(1)**

Q.26 (2)



Let coordinates of point P by parametric
P$(2 + r\cos 45^\circ, 3 + r\sin 45^\circ)$
It satisfies the line $2x - 3y + 9 = 0$

$$2\left(2+\frac{r}{\sqrt{2}}\right) - 3\left(3+\frac{r}{\sqrt{2}}\right) + 9 = 0 \Rightarrow r = 4\sqrt{2}$$

Q.27 (1)

Let Q(a, b) be the reflection of (4, - 13) in the line 5x + y + 6 =0

Then the mid-point $R\left(\frac{a+4}{2}, \frac{b-13}{2}\right)$ lies on 5a + y + 6 = 0

$$\therefore 5\left(\frac{a+4}{2}\right) + \frac{b-13}{2} + 6 = 0$$

$\Rightarrow 5a + b + 19 = 0$ ...(i)

Also PQ is perpendicular to 5x + y + 6 = 0

Therefore $\frac{b+13}{a-4} \times \left(-\frac{5}{1}\right) = -1$

$\Rightarrow a - 5b - 69 = 0$ ...(ii)

Solving (i) and (ii), we get a = -1, b = -14

Q.28 (4)

The given line is $12(x+6) = 5(y-2)$

$\Rightarrow 12x + 72 = 5y - 10 = 0$

or $12x - 5y + 72 + 10 = 0$

$\Rightarrow 12x - 5y + 82 = 0$

The perpendicular distance from (x_1, y_1) to the line $ax + by + c = 0$

is $\frac{(ax_1 + by_1 + c)}{\sqrt{a^2+b^2}}$

The point (x_1, y_1) is $(-1,1)$ therefore

perpendicular distance from $(-1,1)$ to the line $12x - 5y + 82 = 0$ is

$$= \frac{1-12-5+82|}{\sqrt{12^2+(-5)^2}} = \frac{65}{\sqrt{144+25}}$$

$$= \frac{65}{\sqrt{169}} = 5$$

Q.29 (1)

If D' be the foot of altitude, drawn from origin to the given line, then 'D' is the required point.
Let $\angle OBA = \theta$
$\Rightarrow \tan\theta = 4/3$
$\Rightarrow \angle DOA = \theta$
we have OD = 12/5.
If D is (h, k) then h = OD cos θ, k = OD sinθ
$\Rightarrow$ h = 36/25, k = 48/25.

Q.30 (1)

We have $P_1 =$ length of perpendicular from (0,0) on

$$x\sec\theta + y\,\text{cosec}\,\theta = a$$

i.e. $P_1 = \left|\dfrac{a}{\sqrt{\sec^2\theta + \text{cos}ec^2\theta}}\right| = |a\sin\theta\cos\theta|$

$= \left|\dfrac{a}{2}\sin 2\theta\right| or\ 2P_1 = |a\sin 2\theta|$

$P_2 =$ Length of the perpendicular from (0,0)

on $x\cos\theta - y\sin\theta = a\cos 2\theta$

$$P_2 = \left|\frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = |a\cos 2\theta|$$

Now, $4P_1^2 + P_2^2 = a^2\sin^2 2\theta + a^2\cos^2 2\theta = \alpha^2$

Q.31 (2)

$a^2x + aby + 1 = 0$

origin and (1, 1) lies on same side.

$a^2 + ab + 1 > 0$ $\quad \forall\, a \in R$

$D < 0 \Rightarrow b^2 - 4 < 0$ $\quad \Rightarrow b \in (-2, 2)$

but $b > 0 \Rightarrow b \in (0, 2)$

Q.32 (1)

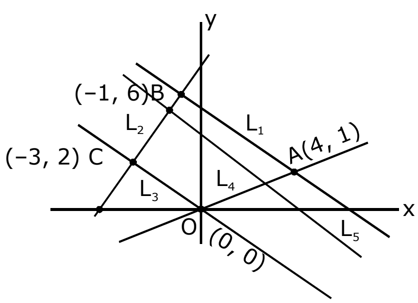
$L_1 : x + y = 5, L_2 : y - 2x = 8$

$L_3 : 3y + 2x = 0, L_4 : 4y - x = 0$

$L_5 : (3x + 2y) = 6$

vertices of quadrilateral

0(0, 0), A (4, 1), B (–1, 6), C(–3, 2)



$L_5(0) = -6 < 0$

$L_5(A) = 12 + 2 - 6 = 8 > 0$

$L_5(B) = -3 + 12 - 6 = 3 > 0$

$L_5(C) = -9 + 4 - 6 = -11 < 0$

O & C points are same side

& A & B points are other same side w.r.t to L_5

So L_5 divides the quadrilateral in two quadrialteral

Aliter :

If abscissa of A is less then abscissa of B

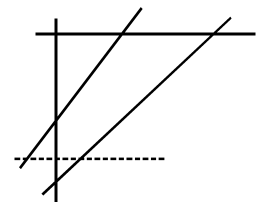
$\Rightarrow$ A lies left of B

otherwise A lies right of B

Q.33 (2)

P(a, 2) lies between

$L_1: x - y - 1 = 0$ &



$L_2: 2(x - y) - 5 = 0$

Method-I

$L_1(P)\,L_2(P) < 0$

$(a - 3)\,(2a - 9) < 0$

$\Rightarrow$ P(a, 2) lies on y = 2

intersection with given lines

$x = 3$ & $x = \dfrac{9}{2}$

$a > 3$ & $a < \dfrac{9}{2}$

(gemetrically)

$a \in \left(3, \dfrac{9}{2}\right)$

Q.34 (4)

$ax + by + c = 0$

$\dfrac{3a}{4} + \dfrac{b}{2} + c = 0$

compare both $(x, y) \equiv \left(\dfrac{3}{4}, \dfrac{1}{2}\right)$

Hence given family passes through $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$

Q.35 (2)

$$\begin{vmatrix} \sin^2 A & \sin A & 1 \\ \sin^2 B & \sin B & 1 \\ \sin^2 C & \sin C & 1 \end{vmatrix} = 0$$

$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin C) = 0$

$\Rightarrow A = B$ or $B = C$ or $C = A$

any two angles are equal

$\Rightarrow \Delta$ is isosceles

Q.36 (4)

$(p + 2q)\,x + (p - 3q)\,y = p - q$

$px + py - p + 2qx - 3qy + q = 0$

$p(x+y-1)+q(2x-3y+1)=0$
passing through intersection of

$x+y-1=0$ & $2x-3y+1=0$ is $\left(\frac{2}{5},\frac{3}{5}\right)$

Q.37 (1)

$$4a^2+b^2+2c^2+4ab-6ac-3bc$$
$$\equiv(2a+b)^2-3(2a+b)c+2c^2=0$$
$$\Rightarrow(2a+b-2c)(2a+b-c)=0\Rightarrow c=2a+b$$
$$\text{or } c=a+\frac{1}{2}b$$

The equation of the family of lines is

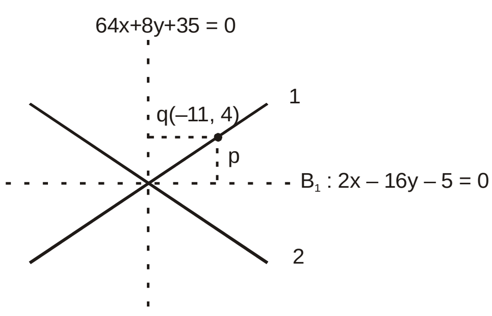
$$a(x+2)+b(y+1)=0 \text{ or } a(x+1)+b\left(y+\frac{1}{2}\right)=0$$

giving the point of consurrence (-2,-1) or

$\left(-1,-\frac{1}{2}\right)$.

Q.38 (1)

$$p=\left|\frac{-22-64-5}{2^2+(-16)^2}\right|=\frac{91}{260}$$



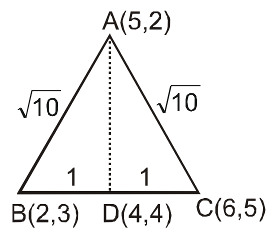
$$q=\left|\frac{-64\times11+8\times4+35}{64^2+8^2}\right|$$

$p<q$ Hence $2x-16y-5=0$ is a cute angle bisector

Q.39 (3)

Equation of AD : $y-4=\frac{2}{-1}(x-4)$

$\Rightarrow y-4=-2x+8$



$\Rightarrow 2x+y=12$

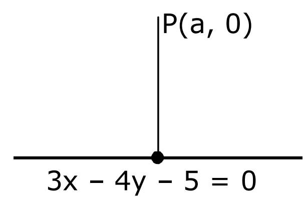
Q.40 (1)

Q.41 (4)

$m=\frac{3}{4}\Rightarrow m_{PQ}=-\frac{4}{3}$

equation of PQ

$y-5=-\frac{4}{3}x$



$4x+3y-15=0$
$\Rightarrow 25x=75$
& $3x-4y-5=0 \Rightarrow x=3$ & $y=1$
Q(3, 1)

Q.42 (4)

$m_1+m_2=-10$

$m_1m_2=\frac{a}{1}$

given $m_1=4m_2\Rightarrow m_2=-2, m_1=-8$,
$a=16$

Q.43 (2)

We have $a=1,\ h=-\sqrt{3},\ b=3,\ g=-\frac{3}{2}$,

$f=\frac{3\sqrt{3}}{2}, c=-4$

Thus $abc+2fgh-af^2-bg^2-ch^2=0$

Hence the equation represents a pair of straight lines.

Again $\frac{a}{h}=\frac{h}{b}=\frac{g}{f}=-\frac{1}{\sqrt{3}}$

$\therefore$ the lines are parallel. The distance between them

$$=2\sqrt{\frac{g^2-ac}{a(a+b)}}=2\sqrt{\frac{\frac{9}{4}+4}{1(1+3)}}=\frac{5}{2}.$$

Q.44 (4)

Q.45 (1)

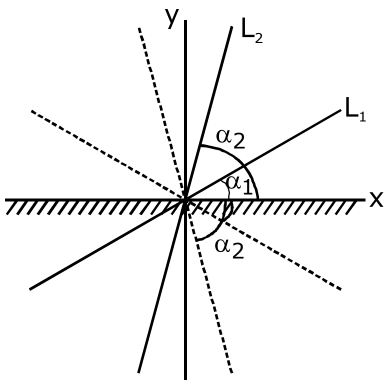
$ax^2 + 2hxy + by^2 = 0$

$m_1 + m_2 = \frac{-2h}{b}, m_1m_2 = \frac{a}{b}$

Relation of slopes of image lines

$(m_1' + m_2') = -(m_1 + m_2)$

$= -\left(\frac{-2h}{b}\right) = \frac{2h}{b}$ $\{m_1' = \tan(\alpha_1)$



$m_1'm_2' = (-m_1)(-m_2)$

$= m_1m_2 = \frac{a}{b}$

$\left(\frac{y}{x}\right)^2 - (m_1' + m_2')\left(\frac{y}{x}\right) + m_1'm_2' = 0$

$\Rightarrow \left(\frac{y}{x}\right)^2 - \frac{2h}{b}\left(\frac{y}{x}\right) + \frac{a}{b} = 0$

$\Rightarrow by^2 - 2hxy + ax^2 = 0$

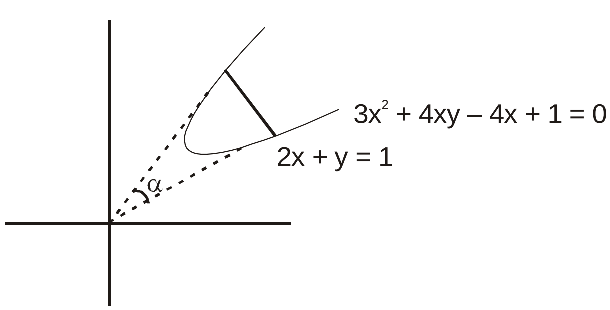
$\Rightarrow ax^2 - 2hxy + by^2 = 0$

Q.46 (1)

Homogenize given curve with given line

$3x^2 + 4xy - 4x(2x + y) + 1(2x + y)^2 = 0$

$3x^2 + 4xy - 8x^2 - 4xy + 4x^2 + y^2 + 4xy = 0$



$-x^2 + 4xy + y^2 =$

coeff. x^2 + coeff. $y^2 = 0$

Hence angle is 90°

EXERCISE-III

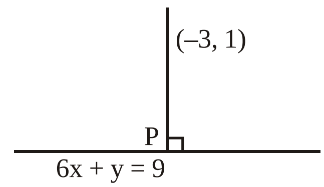
Q.1 **(0100)**

$6x + y = 9$

Equation of perpendicular line from (–3, 1)

$y - 1 = \frac{1}{6}(x + 3)$

$6y - 6 = x + 3$



$\Rightarrow x - 6y - 9 = 0$

$\Rightarrow 6x - 36y + 54 = 0$

$\Rightarrow 6x + y - 9 = 0$

$-37y + 63 = 0$

$y = \frac{63}{37} = \frac{a}{b}$

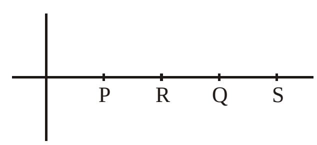
$a + b = 100$

Q.2 **(0002)**

Let P $(x_1, 0)$, Q$(x_2, 0)$, R $(x_3, 0)$ & S $(x_4, 0)$

$x_1 + x_2 = \frac{-2b_1}{a_1}$, $x_1x_2 = \frac{c_1}{a_1}$

$x_3 + x_4 = \frac{-2b_2}{a_2}$, $x_3x_4 = \frac{c_2}{a_2}$



Let R divides PQ internally in ratio k : 1 and S divides externality in k : 1

$\frac{kx_2 + x_1}{k+1} = x_3, \frac{kx_2 - x_1}{k-1} = x_1$

$\Rightarrow kx_2 + x_1 = kx_3 + x_3$ & $kx_2 - x_1 = kx_4 - x_4$

$\Rightarrow k = \frac{(x_3 - x_1)}{x_2 - x_3}$ & $k = \frac{x_1 - x_4}{x_2 - x_4}$

$\Rightarrow \frac{x_3 - x_1}{x_2 - x_3} = \frac{x_1 - x_4}{x_2 - x_4}$

$\Rightarrow x_2x_3 - x_1x_2 - x_3x_4 + x_1x_4 = x_1x_2 - x_2x_4 - x_1x_3 + x_3x_4$

$\Rightarrow -2(x_1x_2 + x_3x_4) = -x_3(x_1 + x_2) - x_4(x_1 + x_2)$

$\Rightarrow 2(x_1x_2 + x_3x_4) = (x_1 + x_2)(x_3 + x_4)$

$\Rightarrow 2\left(\frac{c_1}{a_1} + \frac{c_2}{a_2}\right) = \frac{2b_1}{a_1} \cdot \frac{2b_2}{a_2}$

$\Rightarrow a_1c_2 + a_2c_1 = 2b_1b_2$

Q.3 (0001)

Solving two equations x = 1, y = 1

Put in 2x + ky = 3

$2 + k = 3$

$\Rightarrow k = 3 - 2 = 1$

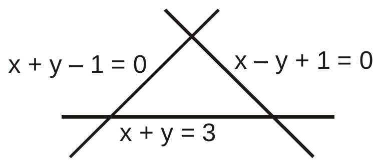
Q.4 (0002)

$x^2 - y^2 + 2y - 1 = 0$

$x^2 - (y^2 - 2y + 1) = 0$

$x^2 - (y - 1)^2 = 0$

$(x + y - 1)(x - y + 1) = 0$



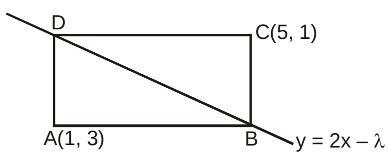
Q.5 (0004)

Mid point of AC lies on BD

so $\dfrac{5+1}{2}, \dfrac{3+1}{2} \equiv (3, 2)$ lies on BD

so $2 = 2(3) - \lambda$

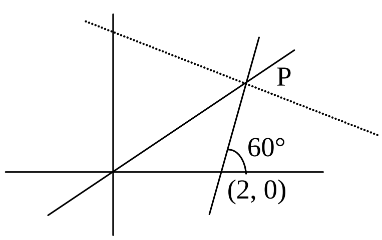
$\lambda = 6 - 2 = 4$



Q.6 **(0006)**

$y - 0 = \sqrt{3}(x - 2)$ it intersect y = x

$\Rightarrow x = \dfrac{2\sqrt{3}}{\sqrt{3}-1}$



So P is $\left(\dfrac{2\sqrt{3}}{\sqrt{3}-1}, \dfrac{2\sqrt{3}}{\sqrt{3}-1}\right)$

So required line is

$$y - \frac{2\sqrt{3}}{\sqrt{3}-1} = -\frac{1}{\sqrt{3}}\left(x - \frac{2\sqrt{3}}{\sqrt{3}-1}\right)$$

it intersect y–axis at x = 0

$$y = \frac{2\sqrt{3}}{\sqrt{3}-1}\left(1 + \frac{1}{\sqrt{3}}\right) = 4 + 2\sqrt{3}$$

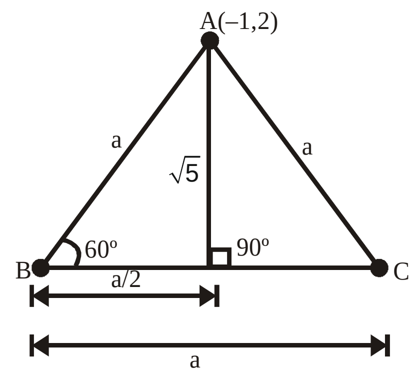
Q.7 **(0008)**

Area of the triangle will be $\dfrac{1}{2}\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

After simplificatiaon it will be

$\dfrac{1}{2}(a - b)(b - c)(c - a) = \dfrac{1}{2}(-2)(-2)\,4 = 8$ sq. units.

Q.8 **(0023)**



Line of BC is 2x – y = 1

$$AD = \left|\frac{2(-1) - 2 - 1}{\sqrt{2^2 + (-1)^2}}\right|$$

$= \sqrt{5}$

$\because \quad \tan 60° = \dfrac{\sqrt{5}}{a/2} = \sqrt{3}$

$\Rightarrow \quad a = \dfrac{2\sqrt{5}}{\sqrt{3}} = \sqrt{\dfrac{20}{3}}$

Q.9 (16)

Here a = a, h = 0, b = – 1, $f = -\dfrac{1}{2}$, g = 2, c = 0

Given equation represent a pair of straight line.

Then, $\begin{vmatrix} a & 0 & 2 \\ 0 & -1 & -1/2 \\ 2 & -1/2 & 0 \end{vmatrix} = 0$

$\Rightarrow a\left[0 - \left(\dfrac{1}{4}\right)\right] - 0 + 2[2] = 0 \Rightarrow a = 16$

Q.10 (25)

The given lines are perpendicular to each other.

$\therefore$ Perpendicular distance = $= \frac{|r_1 - r_2|}{\sqrt{2}} = \sqrt{2}$

$\Rightarrow\ r_1 - r_2 = 2$

The difference between the y-intercepts = 2

This can happen for five combinations{(0, 2), (1, 3), (2, 4), (3, 5), (4, 6)}.

The difference between the x-intercepts = 2

This can happen for five combinations.

Hence, total number of squares = 5 × 5 = 25

MHT CET

Q.1 (3)

Q.2 (3)

Q.3 (3)

Q.4 (1)

Q.5 (1)

Q.6 (1)

Q.7 (1)

Q.8 (4)

Q.9 (2)

Q.10 (1)

Q.11 (1)

Q.12 (3)

Q.13 (2)

Q.14 (3)

Q.15 (1)

Q.16 (4)

Q.17 (1)

Q.18 (3)

Q.19 (3)

Q.20 (2)

Q.21 (2)

Q.22 (1)

Q.23 (1)

Q.24 (1)

Q.25 (2)

Q.26 (1)

Q.27 (4)

Q.28 (4)

Q.29 (1)

Q.30 (3)

Q.31 (1)

Q.32 (3)

Q.33 (4)

Q.34 (1)

Q.35 (2)

Q.36 (3)

Q.37 (3)

Q.38 (1)

Q.39 (2)

Q.40 (3)

Given lines are ax+by c, bx + cy = a and cx + ay = b

On adding the given three equations, we get

ax + by + bx + cy+ cx + ay = a + b + c

$\Rightarrow$ (a + b + c) x + (a + b + c) y = (a + b+ c)

On comparing with 0x + 0y = 0 for collinearity, we get

a + b + c = 0

Q.41 (2)

The equation of line in new position is

$y - 0 = \tan 15^\circ (x - 2)$

$\Rightarrow\ y = \left(2 - \sqrt{3}\right)\left(x - 2\right)$

$\Rightarrow\ \left(2 - \sqrt{3}\right)x - y - 4 + 2\sqrt{3} = 0$

Q.42 (3)

Equation of line is y = mx + 4

$\therefore$ Required distance = $\frac{4}{\sqrt{1 + m^2}}$

Q.43 (4)

Let the equation of perpendicular line to the line 3x – 2y = 6 by 3y + 2x = c ...(i)

Since, it passes through (0,2).

$\therefore\ c = 6$

On putting the value of c in Eq. (i), we get

$3y + 2x = 6$

$\Rightarrow \frac{x}{3} + \frac{y}{2} = 1$

Hence, x intercept is 3.

Q.44 (1)

The point of intersection of the lines $3x + y + 1 = 0$ and $2x - y + 3 = 0$ is $\left(-\frac{4}{5}, \frac{7}{5}\right)$. The equation of line, which makes equal intercepts with axes, is $x + y = a$.

$\therefore -\frac{4}{5} + \frac{7}{5} = a \Rightarrow a = \frac{3}{5}$

Now, equation of line is $x + y - \frac{3}{5} = 0$

$\Rightarrow 5x + 5y - 3 = 0$

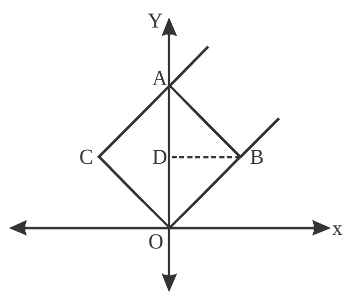
Q.45 (3)

Let lines, OB $\Rightarrow y = mx$

CA $\Rightarrow y = mx + 1$

BA $\Rightarrow y = -nx + 1$

and OC $\Rightarrow y = -nx$



The point of intersection B of OB and aB has x-coordinates $\frac{1}{m+n}$

Now, area of a parallelogram OBAC

$= 2 \times$ area of ΔOBA

$= 2\ \frac{1}{2} \times OA \times DB$

$= 2 \times \frac{1}{2} \times \frac{1}{m+n} = \frac{1}{m+n} = \frac{1}{|m+n|}$

Q.46 (3)

Let coordinates changes from $(x,y) \to (X,Y)$ in new coordinate system whose origin is hn = 3, k = –1

$\therefore x = X + 3, y = Y - 1$

So, $2x - 3y + 5 = 0$

$\Rightarrow 2(X + 3) - 3(Y - 1) + 5 = 0$

$\Rightarrow 2X + 6 - 3Y + 3 + 5 = 0$

$\Rightarrow 2X - 3Y + 14 = 0$

Q.47 (2)

Now, distance of origin from $4x + 2y - 9 = 0$ is

$$\left|\frac{-9}{\sqrt{(4)^2 + (2)^2}}\right| = \frac{9}{\sqrt{20}}$$

and distance of origin from $2x + y + 6 = 0$ is

Hence, the required ratio $= \dfrac{\frac{9}{\sqrt{20}}}{\frac{6}{\sqrt{5}}} = \frac{9}{\sqrt{20}} \times \frac{\sqrt{5}}{6} = \frac{3}{4}$

$= 3 : 4$

Q.48 (4)

Let the points be A (0,0) and B (5,12)

$A(0,0) = (x_1, y_1) \Rightarrow B(5,12) = (x_2, y_2)$

The distance between two points

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow AB = \sqrt{(5-0)^2 + (12-0)^2}$

$= \sqrt{25 + 144} = \sqrt{169}$

$\Rightarrow$ AB = 13 units

Q.49 (3)

Given, equation of line is $7x + 24y - 50 = 0$

Let P be the distance of origin from the line $7x + 24y - 50 = 0$.

Compare with the general form of equation of line $ax + by + c = 0$, we have

$a = 7, b = 24$ and $c = -50$

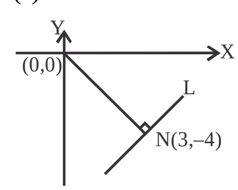
By distance formula, we have

$$P = \left|\frac{C}{a^2 + b^2}\right| = \left|\frac{-50}{\sqrt{49 + 576}}\right| = \left|\frac{-50}{\sqrt{625}}\right|$$

$= \left|\frac{-50}{25}\right| = |-2|$

= 2 units

Q.50 (2)



Slope of line perpendicular to line $L = \frac{-4-0}{3-0} = \frac{-4}{3}$

Now, slope of line $L = \dfrac{-1}{\left(\frac{-4}{3}\right)} = \frac{3}{4}$

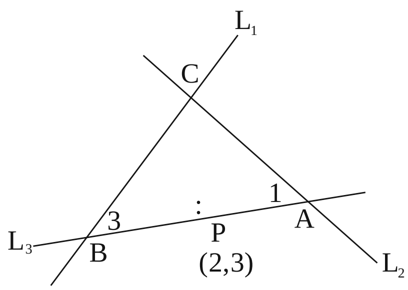
Now, required equation of line L is given by

$(y + 4) = \frac{3}{4}(x - 3)$

$\Rightarrow 4y + 16 = 3x - 9 \Rightarrow 3x - 4y - 25 = 0$

PREVIOUS YEAR'S

Q.1 (2)



Point of Intersection 'C' of L_1 & L_2

$L_1 : 2x + 5y = 10$

$L_2 : -4x + 3y = 12$

Solve to get $c = \left(\frac{-15}{13}, \frac{32}{13}\right)$

Let point A, that lie on $L_2 = \left(\alpha, 4 + \frac{4}{3}\alpha\right)$

and point B, that lie on $L_1 = \left(\beta, 2 - \frac{2}{5}\beta\right)$

P (2,3) divides A and B in 1 : 3 internally

Then, $P(2,3) = P\left(\frac{3\alpha+\beta}{4}, \frac{3\left(4+\frac{4}{3}\alpha\right)+\left(2-\frac{2}{5}\beta\right)}{4}\right)$

$3\alpha + \beta = 8$ and $12 = 12 + 4\alpha + 2 - \frac{2}{5}\beta$

$4\alpha - \frac{2}{5}\beta + 2 = 0$

Solve to get

$\alpha = \frac{3}{13}$ $\quad \beta = \frac{95}{13}$

Hence, $A = \left(\frac{3}{13}, \frac{56}{13}\right)$ and $B = \left(\frac{95}{13} - \frac{12}{13}\right)$

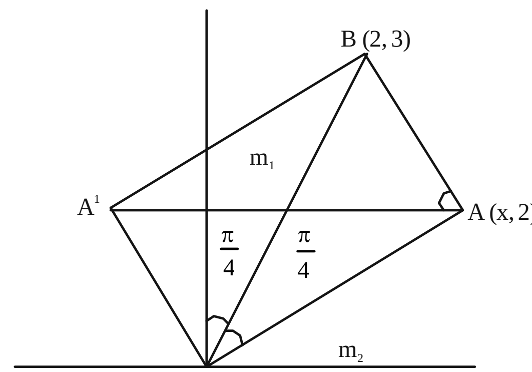
also, $C = \left(\frac{-15}{13}, \frac{32}{13}\right)$

Hence, area $(\Delta ABC) = \frac{1}{2}\left\|\begin{matrix} \frac{3}{13} & \frac{56}{13} & 1 \\ \frac{95}{13} & \frac{-12}{13} & 1 \\ \frac{-15}{13} & \frac{32}{13} & 1 \end{matrix}\right\|$

$= \frac{1}{2(13^3)}\left\|\begin{matrix} 3 & 56 & 1 \\ 95 & -12 & 1 \\ -15 & 32 & 1 \end{matrix}\right\|$

$= \frac{132}{13}$ sq.unit

Q.2 (3)



$m_1 = 3/2, \quad m_2 = 2/x$

$\tan\frac{\pi}{4} = \left|\frac{\frac{3}{2} - \frac{2}{x}}{1 + \frac{6}{2x}}\right| = 1$

$\Rightarrow x_1 = 10, x_2 = -2/5$

$\Rightarrow AA' = 52/5$

Q.3 (3)

Given $H\left(\frac{7}{3}, \frac{7}{3}\right)$

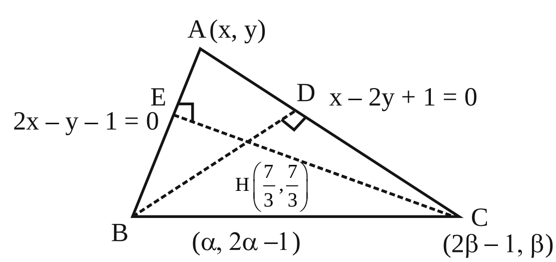
$L_1 : 2x - y - 1 = 0$

$L_2 : x - 2y + 1 = 0$

$x = 1, y = 1$

$\therefore A(1, 1)$

Slope of AC × slope of BD = –1



$$\frac{1}{2}\times\frac{\frac{7}{3}-(2\alpha-1)}{\frac{7}{3}-\alpha}=-1$$

$\alpha = 2$

Now, m(EC) × m(AB) = –1

$$\frac{\frac{7}{3}-\beta}{\frac{7}{3}-2\beta+1}\cdot 2=-1$$

$\beta = 2$

$\therefore$ A(1, 1), B(2, 3), C(3, 2)

$$\text{Centroid} = C_1\left(\frac{1+2+3}{3},\ \frac{1+3+2}{3}\right)=(2,\ 2)$$

$$OC_1=\sqrt{2^2+2^2}=2\sqrt{2}$$

Q.4 (3)

$$\frac{1}{2}\begin{vmatrix}1 & \alpha & 1\\ \alpha & 0 & 1\\ 0 & \alpha & 1\end{vmatrix}=\pm 4$$

$$\frac{1}{2}((-\alpha)-(\alpha)(\alpha)+1(\alpha^2))=\pm 4$$

$$-\frac{\alpha}{2}=\pm 4 = \qquad \Rightarrow \alpha=\pm 8$$

Now

$$\begin{vmatrix}\alpha & -\alpha & 1\\ -\alpha & \alpha & 1\\ \alpha^2 & \beta & 1\end{vmatrix}=0$$

$\alpha(\alpha-\beta)+\alpha(-\alpha-\alpha^2)+(-\alpha\beta-\alpha^3)=0$

$\alpha^2-\alpha\beta-\alpha^2-\alpha^3-\alpha\beta-\alpha^3=0$

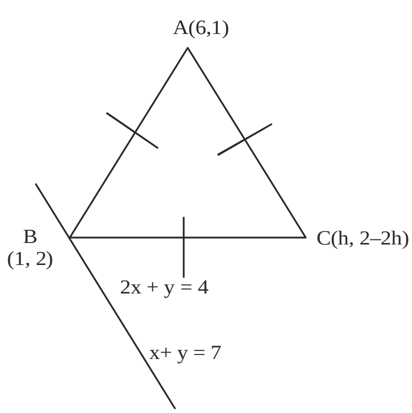
$\alpha^3+\alpha\beta=0$

$\Rightarrow \alpha^2+\beta=0$

$64+\beta=0$

$\beta=-64$

Q.5 (3)



Point B(1, 2)

Now let C be (h, 4 –2h)

(As C lies on 2x + y = 4

$\because \Delta$ is isosceles with base BC

$\therefore$ AB = AC

$$\sqrt{25+1}=\sqrt{(6-h)^2+(2h-3)}$$

$$\sqrt{26}=\sqrt{36+h^2-12h+4h^2+9-12h}$$

$26 = 5h^2 + 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$

$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$

$$h=\frac{19}{5}\ \text{or}\ h=1$$

$$\text{Thus C}\left(\frac{19}{5},\frac{-18}{5}\right)$$

$$\text{Centroid}\left(\frac{6+1+\frac{19}{5}}{3},\frac{1+2-\frac{18}{5}}{3}\right)$$

$$\left(\frac{35+19}{15},\frac{15-18}{15}\right)$$

$$\left(\frac{54}{15},\frac{-3}{15}\right)$$

$$\alpha=\frac{54}{15};\ \beta=\frac{-3}{15}$$

$15(\alpha+\beta)=51$

Q. 6 (4)

$$\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$$

Slope of tangent at (a,b)

$$n.\left(\frac{x}{a}\right)^{n-1}.\frac{1}{a}+n\left(\frac{x}{b}\right)^{n-1}.\frac{1}{b}\frac{dy}{dx}=0$$

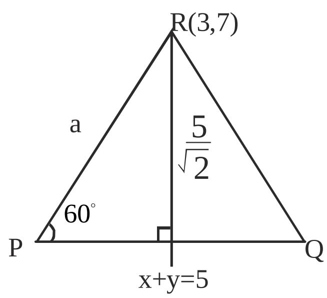
$$\left.\frac{dy}{dx}\right|_{(a,b)}=-\frac{b}{a}$$

$\therefore$ Equation of tangent

$$y-b=-\frac{b}{a}(x-a)$$

$$\frac{x}{a}+\frac{y}{b}=2\ \forall n\in N$$

Q. 7 (4)

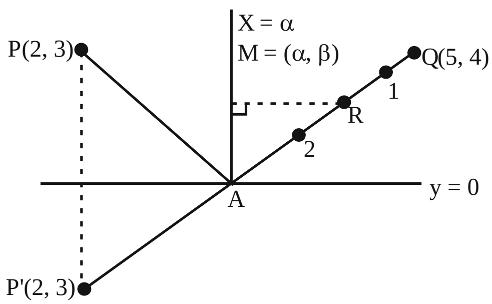


$$\sin 60^o = \frac{5/\sqrt{2}}{a}$$

$$a = \frac{5\sqrt{2}}{3}$$

$$Area = \Delta PQR = \frac{\sqrt{3}}{4}a^2 = \frac{25}{2\sqrt{3}}$$

Q.8 (31)



By observation we see that A(α, 0)
and β = y-co-ordinate of R

$$= \frac{2\times 4 + 1\times 0}{2+1} = \frac{8}{3} \quad \text{... (1)}$$

Now P' is image of P in y = 0 which will be P'(2, –3)

$\therefore$ Equation of P' Q is $(y + 3) = \frac{4+3}{5-2}(x-2)$

i.e., $3y + 9 = 7x - 14$

$A \equiv \left(\frac{23}{7}, 0\right)$ by solving with y = 0

$$\therefore \ \alpha = \frac{23}{7} \quad \text{... (2)}$$

By (1), (2)
$7\alpha + 3\beta = 23 + 8 = 31$

Q.9 (3)
$m > 1$ & A: (4,3)

L: $y - 3 = m(x-4)$ & $L_1: x - y = 2$
Let – $m_L = m = \tan\theta$ & B on L_1
$\Rightarrow$ B: $(\lambda, \lambda - 2)$

Given $AB = \frac{\sqrt{29}}{3} \Rightarrow \sqrt{(\lambda-4)^2 + (\lambda-2-3)^2} = \frac{\sqrt{29}}{3}$

$$\Rightarrow (\lambda-4)^2 + (\lambda-5)^2 = \frac{29}{3}$$

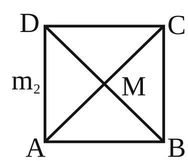
$$\Rightarrow \lambda = \frac{51}{9} \quad \lambda = \frac{10}{3}$$

$$\Rightarrow B: \left(\frac{51}{9}, \frac{33}{9}\right) \quad \text{or} \quad \left(\frac{10}{3}, \frac{4}{3}\right)$$

Now check options **Ans. 3**

Q.10 (2)
$m_1 m_2 = -1$

$$a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$$



Eq. of AC
AC = (cosα – sinα)x + (sinα + cosα)y = 10
BD = (sinα – cosα)x + (sinα – cosα)y = 0
(10(cosα – sinα), 10 (sinα – cosα)

Slope of AC = $\left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)$ = tanθ = M

Eq. of line making an angle $\frac{\pi}{4}$ with AC

$$m_1, m_2 = \frac{m \pm \tan\frac{\pi}{4}}{1 \pm m\tan\frac{\pi}{4}}$$

$$= \frac{m+1}{1-m} \text{ or } \frac{m-1}{1+m}$$

$$\frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} + 1}{1 - \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)}, \frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} - 1}{1 + \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)}$$

$m_1, m_2 = \tan\alpha, \cot\alpha$
mid point of AC & BD
= M(5(cosα – sinα), 5(cosα + sinα))
B(10(cosα – sinα), 10(cosα + sinα))

$a = AB = \sqrt{2}\ BM = \sqrt{2}(5\sqrt{2}) = 10$

a = 10

$$\because a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$$

$100 + 110 + 3(\tan^2\alpha + \cot^2\alpha) = 220$

Hence, $\tan^2\alpha = 3$, $\tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3}$ or $\frac{\pi}{6}$

Now, $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$

$$= 72\left(\frac{9}{16} + \frac{1}{16}\right) + 100 - 30 + 13$$

$$= 72\left(\frac{5}{8}\right) + 83 = 45 + 83 = 128$$

Q.11 (2)

$$A(\alpha, -2) : B(\alpha, 6) : C\left(\frac{\alpha}{4}, -2\right)$$

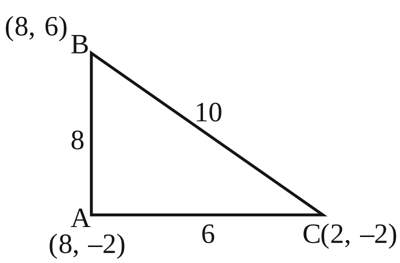
since AC is perpendicular to AB

So, ΔABC is right angled at A

Circumcentre = mid point of BC = $\left(\frac{5\alpha}{8}, 2\right)$

$$\therefore \frac{5\alpha}{8} = 5 \;\&\; \frac{\alpha}{4} = 2$$

a = 8



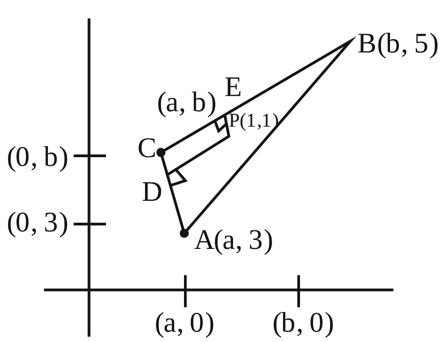
Area $= \frac{1}{2}(6)(8) = 24$

Perimeter = 24

Circumradius = 5

Inradius $= \frac{\Delta}{s} = \frac{24}{12} = 2$

Q.12 (2)



$m_{AC} \to \infty$

$m_{PD} = 0$

$$D\left(\frac{a+a}{2}, \frac{b+3}{2}\right)$$

$$D\left(a, \frac{b+3}{2}\right)$$

$m_{PD} = 0$

$$\frac{b+3}{2} - 1 = 0$$

$b + 3 - 2 = 0$

$b = -1$

$$E\left(\frac{b+a}{2}, \frac{5+b}{2}\right) = \left(\frac{a-1}{2}, 2\right)$$

$m_{CB} \cdot m_{EP} = -1$

$$\left(\frac{5-b}{b-a}\right) \cdot \left(\frac{2-1}{\frac{a-1}{2} - 1}\right) = -1$$

$$\left(\frac{6}{-1-a}\right) \cdot \left(\frac{2}{a-3}\right) = -1$$

$12 = (1 + a)(a - 3)$

$12 = a^2 - 3a + a - 3$

$\Rightarrow a^2 - 2a - 15 = 0$

$(a - 5)(a + 3) = 0$

$a = 5$ or $a = -3$

Given $ab > 0$

$a(-1) > 0$

$-a > 0$

$a < 0$

$a = -3$ Accept

AP line A(–3, 3), P(1, 1)

$$y - 1 = \left(\frac{3-1}{-3-1}\right)(x - 1)$$

$-2y + 2 = x - 1$

$\Rightarrow x + 2y = 3$ Applying ... (1)

Line BC, B(–1, 5)

C(–3, –1)

$$(y - 5) = \frac{6}{2}(x + 1)$$

$y - 5 = 3x + 3$

$y = 3x + 8$ ... (2)

Solving (1) and (2)

$x + 2(3x + 8) = 3$

$\Rightarrow 7x + 16 = 3$

$7x = -13$

$$x = -\frac{13}{7}$$

$$y = 3\left(-\frac{13}{7}\right) + 8$$

$$= \frac{-39+56}{7}$$

$$y = \frac{17}{7}$$

$$x + y = \frac{-13+17}{7} = \frac{4}{7}$$

Q.13 (2)

s≡ sint, c ≡ cost

Let orthocenter be (h,k)

Since it is an equilateral triangle hence orthocenter coincides with centroid.

$\therefore$ a + s + c = 3h, b + s – c = 3k

$\therefore (3h - a)^2 + (3k - b)^2 = (s + c)^2 + (s-c)^2 = 2\,(s^2 + c^2) = 2$

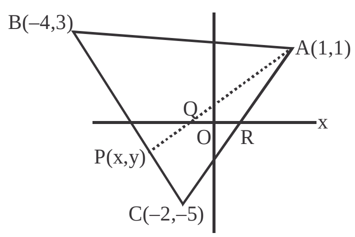
$$\therefore \left(h - \frac{a}{3}\right)^2 + \left(k - \frac{b}{3}\right)^2 = \frac{2}{9}$$

Circle center at $\left(\frac{a}{3}, \frac{b}{3}\right)$

Gives, $\frac{a}{3} = 1, \frac{b}{3} = \frac{1}{3}$ $\Rightarrow$ a = 3, b = 1

$\therefore a^2 - b^2 = 8$

Q.14 (3)



Given $\Delta_1 = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix}$

$\&\Delta_2 = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -4 & 3 & 1 \\ -2 & -5 & 1 \end{vmatrix}$

Given

$$\frac{\Delta_1}{\Delta_2} = \frac{4}{7} \Rightarrow \frac{-2x-5y+7}{36} = \frac{4}{7} \Rightarrow 14x + 35y = -95 \quad ...(1)$$

Equatin of BC is 4x + y = –13 ...(2)

Solve equation (1) & (2)

Point P $\left(\frac{-20}{7}, \frac{-11}{7}\right)$

Here point Q $\left(\frac{-1}{2}, 0\right)$ &R $\left(\frac{1}{2}, 0\right)$

So Area of triangle AQR = $\frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Q.15 (2)

Let P(h, k)

$[(h-1)^2 + (k-2)^2] + [(h+2)^2 + (k-1)^2 = 14$

$h^2 + k^2 + h - 3k = 2$

$x^2 + y^2 + x - 3y - 2 = 0$

If y = 0

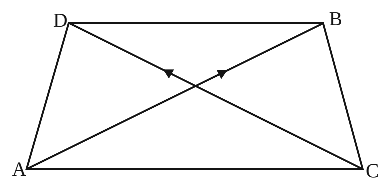
$x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0$

x = –2, 1

A(–2, 0), B(1, 0)

$y^2 - 3y - 2 = 0$

$$y = \frac{3 \pm \sqrt{17}}{2}$$

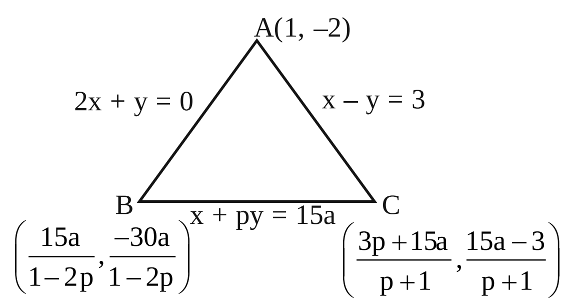


$$\Rightarrow C = \left(0, \frac{3-\sqrt{17}}{2}\right)$$

$$D = \left(0, \frac{3+\sqrt{17}}{2}\right)$$

Area of quadrilateral $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{CD}| = \frac{3\sqrt{17}}{2}$

Q.16 [3]



Orthocenter n = (2, a)

$$m_{AH} = \frac{9+2}{1} = p$$

... (1)

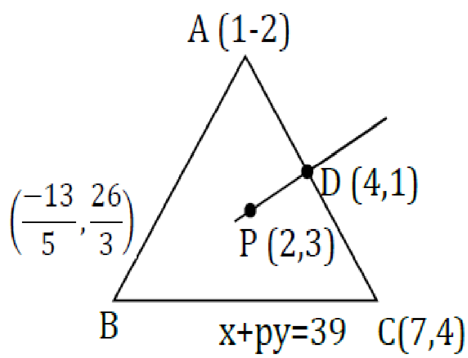
$m_{BH} = -1 \Rightarrow 31a - 3ab = 15a + 4p - 2$ . . . (2)

from (1) & (2)

a = 1

p = 3

Q.17 (4)



AB : 2x + y = 0... (i)
BC : x + py = 39...(ii)
CA : x-y =3.... (iii)
Equation of perpendicular bisector AC = y-3 = - (x-2)
$\Rightarrow$ x+y= 5 ... (iv)
Solving equations (iii) and equations (iv) we get ponit (4) $\equiv$ [4,1]
Now point c = (7,4)
Point C satisfy the equation x + py = 39 then p = 8
So, equation of BC $\equiv$ x + 8y = 39 .....(v)

Now solving the equation (i) and (v) get

Ponit B $\equiv \left(-\frac{13}{5}, \frac{26}{5}\right)$

$(AC)^2 = 72 = 9 \text{ x } p = 9 \text{ x } 8 = 72$
$(AC)^2 + P^2 = 72 + 8^2 = 72 + 64 = 136$

Now , area of Δ ABC = $\frac{1}{2}$ base x height

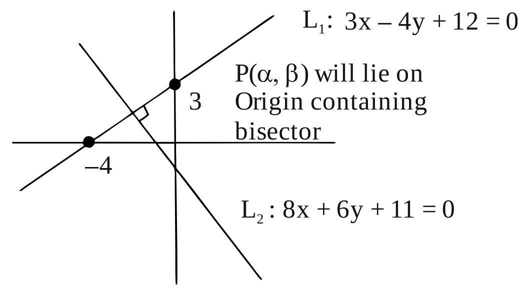
$= \frac{1}{2}(AC) \times$ (Perpendicular distance from B to AC)

$= \frac{1}{2} \times \sqrt{72} \times \frac{54}{5\sqrt{2}} = \frac{27.6\sqrt{2}}{5\sqrt{2}} = \frac{162}{5} = 32.4$
$\Delta = 32.4$

Q.18 (3)

Distance between points $(3\sqrt{2}, 0)$ and $(0, p\sqrt{2}) = 5\sqrt{2}$
(For distance to be minimum points must be collinear)
$18 + 2p^2 = 25 \times 2$
$2p^2 = 50 - 18$
$2p^2 = 32$
$p^2 = 16$
$p = 4$

Q.19 (4)



$3\alpha - 4\beta + 12 = 5 \ \& \ 8\alpha + 6\beta + 11 = 10$
$3\alpha - 4\beta = -7 \Rightarrow 18\alpha - 24\beta = -42$
$8\alpha + 6\beta = -1 \Rightarrow 32\alpha + 24\beta = -4$
$\Rightarrow 50\alpha = -46$

$\therefore \alpha = \frac{-23}{25}$

$\therefore \frac{-69}{25} + 7 = 4\beta$

$\therefore \beta = \frac{106}{100}$

$\therefore \alpha + \beta = \frac{106}{100} - \frac{23}{25} = \frac{106-92}{100}$

$\therefore 100(\alpha + \beta) = 14$

CONIC SECTIONS

EXERCISE-I (MHT CET LEVEL)

CIRCLE

Q.1 (4)

Obviously the centre of the given circle is $(1,-2)$. Since the sides of square are parallel to the axes, therefore, first three alternates cannot be vertices of square because in first two (*a* and *b*) y = –2 and in (3) x = 1, which passes through centre $(1,-2)$ but it is not possible. Hence answer (4) is correct.

Q.2 (3)

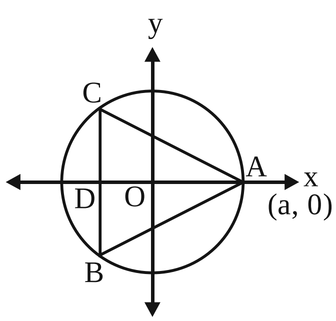
Since the equilateral triangle is inscribed in the circle with centre at the origin, centroid lies on the origin.

So, $\frac{AO}{OD}=\frac{2}{1}\Rightarrow OD=\frac{1}{2}AO=\frac{A}{2}$

So, other vertices of triangle have coordinates,

$\left(-\frac{a}{2},\frac{\sqrt{3}a}{2}\right)$ and $\left[-\frac{a}{2},-\frac{\sqrt{3}}{2}a\right]$

$\left(-\frac{a}{2},\frac{\sqrt{3}}{2}a\right)$



$\left(-\frac{a}{2},\frac{\sqrt{3}}{2}a\right)$

$\therefore$ Equation of line BC is :

$x=-\frac{a}{2}\Rightarrow 2x+a=0$

Q.3 (2)

As the circle is passing through the point (4, 5) and its centre is (2, 2) so its radius is

$\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$.

$\therefore$ The required equation is:

$(x-2)^2+(y-2)^2=13$

Q.4 **(2)**

The diagonal = R

Thus the area of recrtangle

$=\frac{1}{2}\times R\times R=\frac{R^2}{2}$

Q.5 (1)

Q.6 (2)

Q.7 (3)

Q.8 (2)

Q.9 (4)

Q.10 **(1)**

Q.11 **(1)**

Q.12 **(2)**

Q.13 (1)

Centre (3, –1). Line through it and origin is $x+3y=0$.

We get *h* and *k* from (i) and (ii) solving simultaneously as (4, 3). Equation is $x^2+y^2-8x-6y+16=0$.

Trick : Since the circle satisfies the given conditions.

Q.14 (2)

Let the centre of the required circle be (x_1, y_1) and the centre of given circle is (1, 2). Since radii of both circles are same, therefore, point of contact (5, 5) is the mid point of the line joining the centres of both circles. Hence $x_1 = 9$ and $y_1 = 8$. Hence the required equation is $(x-9)^2+(y-8)^2=25$

$\Rightarrow x^2+y^2-18x-16y+120=0$.

Trick : The point (5, 5) must satisfy the required circle. Hence the required equation is given by (2).

Q.15 (1)

Circle is $x^2+y^2-2x-2y+1=0$ as centre is (1, 1) and radius = 1.

Q.16 (4)

Q.17 (4)

Here the centre of circle (3, –1) must lie on the line $x+2by+7=0$.

Therefore, $3-2b+7=0 \Rightarrow b=5$.

Q.18 (2)

Q.19 (4)

Trick : Since both the circles given in option (1) and (2) satisfy the given conditions.

Q.20 (2)

The diameter of the circle is perpendicular distance between the parallel lines (tangents)

$3x-4y+4-0$ and $3x-4y-\frac{7}{2}=0$ and so it is equal to

$$\frac{4}{\sqrt{9+16}}+\frac{\frac{7}{2}}{\sqrt{9+16}}=\frac{3}{2}$$. Hence redius is $\frac{3}{4}$

Q.21 (3)

Q.22 (1)

Q.23 (3)

Equation of pair of tangents is given by $SS_1 = T^2$. Here

$S = x^2+y^2+20(x+y)+20,\ S_1 = 20$

$T = 10(x+y)+20$

$\therefore SS_1 = T^2$

$\Rightarrow 20\{x^2+y^2+20(x+y)+20\} = 10^2(x+y+2)^2$

$\Rightarrow 4x^2+4y^2+10xy=0 \Rightarrow 2x^2+2y^2+5xy=0$.

Q.24 (2)

Since normal passes through the centre of the circle.

$\therefore$ The required circle is the circle with ends of diameter as (3, 4) and (– 1, – 2).

It's equation is $(x-3)(x+1)+(y-4)(y+2)=0$

$\Rightarrow x^2+y^2-2x-2y-11=0.$

Q.25 (2)

Length of tangents is same *i.e.*, $\sqrt{S_1}=\sqrt{S_2}=\sqrt{S_3}$.

We get the point from where tangent is drawn, by solving the 3 equations for *x* and *y*.

i.e., $x^2+y^2=1$,

$x^2+y^2+8x+15=0$ and $x^2+y^2+10y+24=0$

or $8x+16=0$ and $10y+25=0$

$\Rightarrow x=-2$ and $y=-\frac{5}{2}$

Hence the point is $\left(-2,-\frac{5}{2}\right)$.

Q.26 (1)

Q.27 (2)

Suppose (x_1, y_1) be any point on first circle from which tangent is to be drawn, then

$x_1^2+y_1^2+2gx_1+2fy_1+c_1=0$ ….(i)

and also length of tangent

$=\sqrt{S_2}=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$ ….(ii)

From (i), we get (ii) as $\sqrt{c-c_1}$.

Q.28 (3)

Q.29 **(4)**

Q.30 (1)

$T=S_1 \Rightarrow x(4)+y(3)-4(x+4)$
$=16+9-32$

$\Rightarrow 3y-9=0 \Rightarrow y=3$

Q.31 **(2)**

$T=S_1 \Rightarrow x(4)+y(3)-4(x+4)=16+9-32$

$\Rightarrow 3y-9=0 \Rightarrow y=3$

Q.32 (4)

Q.33 (1)

$S_1 = x^2+y^2+4x+1=0$

$S_2 = x^2+y^2+6x+2y+3=0$

Common chord $\equiv S_1-S_2=0 \Rightarrow 2x+2y+2=0$

$\Rightarrow x+y+1=0$

Q.34 (1)

We know that the equation of common chord is $S_1-S_2=0$, where S_1 and S_2 are the equations of given circles, therefore

$(x-a)^2+(y-b)^2+c^2-(x-b)^2-(y-a)^2-c^2=0$

$\Rightarrow 2bx-2ax+2ay-2by=0$

$\Rightarrow 2(b-a)x-2(b-a)y=0 \Rightarrow x-y=0$

Q.35 (2)
Since locus of middle point of all chords is the diameter, perpendicular to the chord.

Q.36 (1)
$SS_1 = T^2$

$\Rightarrow (x^2+y^2-2x+4y+3)(36+25-12x-20y+3)$

$= (6x-5y-x-6+2(y-5)+3)^2$

$\Rightarrow 7x^2+23y^2+30xy+66x+50y-73=0$.

Q.37 **(4)**
Equation of pair of tangents is given by $SS_1 = T^2$,
or $S = x^2+y^2+20(x+y)+20$, $S_1 = 20$,
$T = 10(x+y)\ 20 = 0$
$\therefore$ SS1 = T^2
$\Rightarrow 20(x^2+y^2+20(x+y)+20) = 10^2(x+y+2)^2$
$\Rightarrow 4x^2+4y^2+10xy = 0$
$\Rightarrow 2x^2+2y^2+5xy = 0$

Q.38 (1)
$C_1(1, 2), C_2(0, 4), R_1 = \sqrt{5}, R_2 = 2\sqrt{5}$

$C_1C_2 = \sqrt{5}$ and $C_1C_2 = |R_2 - R_1|$
Hence circles touch internally.

Q.39 (2)
Q.40 (1)
Q.41 (2)
Q.42 (3)
Q.43 (1)
Q.44 (3)
Q.45 (4)
$C_1 = (3, 1), C_2(-1, 4), R_1 = 3, R_2 = 2$

$C_1C_2 = \sqrt{16+9} = 5, R_1+R_2 = C_1C_2$
Hence circles touch externally.

Q.46 (3)
Equation of radical axis, $S_1 - S_2 = 0$
i.e.,
$(2x^2+2y^2-7x)-(2x^2+2y^2-8y-14)=0$

$\Rightarrow -7x+8y+14=0$, $\therefore 7x-8y-14=0$

Q.47 (1)
Common chord $= S_1 - S_2$
$10x-3y-18=0$

Q.48 (2)
Given circle is $\left(2, \frac{3}{2}\right), \frac{5}{2} = r_1$ (say)

Required normals of circlres are $x+3=0, x+2y=0$

which intersect at the centre $\left(-3, \frac{3}{2}\right)$, $r_2 =$ radius (say).
2nd circle just contains the 1st

i.e., $C_2C_1 = r_2 - r_1 \Rightarrow r_2 = \frac{15}{2}$.

Q.49 (4)
Co-axial system $x^2+y^2+2gx+c=0$,
(g variable)
L.H.S. $= \Sigma(g_2-g_3)(h^2+k^2-c+2g_1h) = 0$
Since $\Sigma(g_2-g_3) = 0$ and $\Sigma g_1(g_2-g_3) = 0$.

Q.50 (4)
The equation of polar to circle (i) is $x-5y+13=0$
and equation of polar to circle (ii) is $x+y-1=0$
Clearly, polars intersect at a point.

Q.51 (4)
Q.52 **(1)**

Q.53 (2)
Let pole be (x_1, y_1) then polar
will be $xx_1+yy_1=1$ comparing with $lx+my+n=0$

$\Rightarrow x_1 = -\frac{l}{n}, y_1 = -\frac{m}{n}$.

Q.54 (3)
Polar is $\lambda x+\mu y+c=0$. The condition of tangency $p = r$ gives the result (3).

Q.55 (2)
The required polar is $x(1)+y(2)=7$ or $x+2y=7$.

PARABOLA

Q.56 (3)
Vertex $=(2,0) \Rightarrow$ focus is $(2+2,0)=(4,0)$.

Q.57 (3)
The point $(-3,2)$ will satisfy the equation $y^2 = 4ax$

$\Rightarrow 4 = -12a$, $\Rightarrow 4a = -\frac{4}{3} = \frac{4}{3}$

(Taking positive sign).

Q.58 (3)

$x^2 = -8y \Rightarrow a = -2$ So, focus $= (0, -2)$

Ends of latus rectum $= (4, -2), (-4, -2)$.

Trick : Since the ends of latus rectum lie on parabola, so only points $(-4, -2)$ and $(4, -2)$ satisfy the parabola.

Q.59 (3)

Q.60 (4)

Q.61 (2)

Q.62 **(4)**

Q.63 **(4)**

Q.64 (4)

It is a fundamental concept.

Q.65 (1)

Check the equation of parabola for the given points.

Q.66 (1)

$(x+1)^2 = 4a(y+2)$

Passes through (3, 6) $\Rightarrow 16 = 4a.8 \Rightarrow a = \frac{1}{2}$

$\Rightarrow (x+1)^2 = 2(y+2) \Rightarrow x^2 + 2x - 2y - 3 = 0$

Q.67 (4)

The parabola is $(x-2)^2 = (3y-6)$. Hence axis is $x - 2 = 0$.

Q.68 (2)

Always eccentricity of parabola is .

Q.69 (2)

Parametric equations of $y^2 = 4ax$ are $x = at^2, y = 2at$

Hence if equation is $y^2 = 8x$, then parametric equations are $x = 2t^2, y = 4t$.

Q.70 (3)

Q.71 (4)

It is obvious.

Q.72 (3)

Semi latus rectum is harmonic mean between segments of focal chords of a parabola.

$\therefore\ b = \frac{2ac}{a+c} \Rightarrow a, b, c$ are in H.P.

Q.73 (4)

Let point of contact be (h, k), then tangent at this point is $ky = x + h$. $x - ky + h = 0 \equiv 18x - 6y + 1 = 0$ or $\frac{1}{18} = \frac{k}{6} = \frac{h}{1}$ or $k = \frac{1}{3}$, $h = \frac{1}{18}$.

Q.74 (1)

Equation of parabola is $y^2 = -4ax$. Its focus is at $(-a, 0)$.

Q.75 (1)

Any point on $y^2 = 4ax$ is $(at^2, 2at)$, then tangent is

$2aty = 2a(x + t^2) \Rightarrow yt = x + at^2$

Q.76 (2)

Let point be (h,k). Normal is $y - k = \frac{-k}{4}(x - h)$ or

$-kx - 4y + kh + 4k = 0$

Gradient $= -\frac{k}{4} = \frac{1}{2} \Rightarrow k = -2$

Substituting (h,k) and , we get

Hence point is .

Trick : Here only point satisfies the parabola .

Q.77 **(3)**

Equation of parabola is

$y^2 = 4ax$

$\Rightarrow 2y\frac{dy}{dx} = 4a$ (On differentiating w.r.t 'x')

$\therefore \frac{dy}{dx} = \frac{2a}{y}$, [slope of tangent]

So, slope of normal $= -\left(\frac{dx}{xy}\right)_{(at^2, 2at)}$

$= -\left(\frac{y}{2a}\right) = -\frac{2at}{2a} = -t$

Q.78 (4)

It is obvious.

Q.79 (3)

Normal is $y - 2at_1 = \frac{-2at}{2a}(x - at^2)$

Therefore, slope = –t.

Q.80 (3)

$$y - \frac{2a}{m} = -\frac{2a/m}{2a}\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow y - \frac{2a}{m} = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow m^3y + m^2x - 2am^2 - a = 0.$$

Q.81 (3)

Since the semi-latus rectum of a parabola is the harmonic mean between the segments of any focal chord of a parabola, therefore SP, 4, SQ are in H.P.

$$\Rightarrow 4 = 2.\frac{SP.SQ}{SP + SQ} \Rightarrow 4 = \frac{2(6)(SQ)}{6 + SQ} \Rightarrow SQ = 3.$$

Q.82 (4)

We have $t_2 = -t_1 - \frac{2}{t_1}$

Since $a = 2, t_1 = 1 \therefore t_2 = -3$

$\therefore$ The other end will be $(at_2^2, 2at_2)$ *i.e.*, (18, –12).

Q.83 (4)

The given point (–1, –60) lies on the directrix $x = -1$ of the parabola $y^2 = 4x$. Thus the tangents are at right angle.

Q.84 (1)

Q.85 (3)

Equation of tangent at (1, 7) to $y = x^2 + 6$

$\frac{1}{2}(y + 7) = x.1 + 6 \Rightarrow y = 2x + 5$ .....(i)

This tangent also touches the circle

$x^2 + y^2 + 16x + 12y + c = 0$ .....(ii)

Now solving (i) and (ii), we get

$\Rightarrow x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0$

$\Rightarrow 5x^2 + 60x + 85 + c = 0$

Since, roots are equal so

$b^2 - 4ac = 0 \Rightarrow (60)^2 - 4 \times S \times (85 + c) = 0$

$\Rightarrow 85 + c = 180 \Rightarrow 5x^2 + 60x + 180 = 0$

$\Rightarrow x = -\frac{60}{10} = -6 \Rightarrow y = -7$

Hence, point of contact is (–6, 7)

Q.86 (3)

Equation of tangent to parabola

$ty = x + at^2$ .....(i)

Clearly, $lx + my + n = 0$ is also a chord of contact of tangents.

Therefore $ty = x + at^2$ and $lx + my + n = 0$ represents the same line.

Hence, $\frac{1}{l} = -\frac{t}{m} = \frac{at^2}{n}$ Þ $t = \frac{-m}{l}$, $t^2 = \frac{n}{la}$

Eliminating *t*, we get, $m^2 = \frac{nl}{a}$ *i.e.*, an equation of parabola.

Q.87 (3)

Equation of chord of contact of tangent drawn from a point (x_1, y_1) to parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$ so that $5y = 2 \times 2(x + 2)$ $\Rightarrow$ $5y = 4x + 8$. Point of intersection of chord of contact with parabola $y^2 = 8x$ are $\left(\frac{1}{2}, 2\right), (8, 8)$, so that length $= \frac{3}{2}\sqrt{41}$.

Q.88 (2)

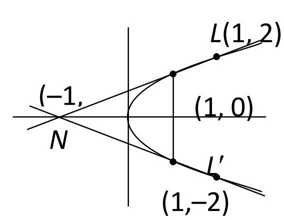
Any line through origin (0,0) is $y = mx$. It intersects $y^2 = 4ax$ in $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

Mid point of the chord is $\left(\frac{2a}{m^2}, \frac{2a}{m}\right)$

$x = \frac{2a}{m^2}, y = \frac{2a}{m} \Rightarrow \frac{2a}{x} = \frac{4a^2}{y^2}$ or $y^2 = 2ax$,

which is a parabola.

Q.89 (2)



Equation of the tangent at (x_1, y_1) on the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$

$\therefore$ In this case, $a = 1$

The co-ordinates at the ends of the latus rectum of the parabola $y^2 = 4x$ are $L(1, 2)$ and $L_1(1, -2)$

Equation of tangent at L and L_1 are $2y = 2(x+1)$ and $-2y = 2(x+1)$, which gives $x = -1$, $y = 0$. Thus, the required point of intersection is (–1, 0).

Q.90 (1)

$$\frac{(y-2at_2)}{(2at_2-2at_1)} = \frac{x-at_2^2}{(at_2^2-at_1^2)};$$

As focus i.e., (a, 0) lies on it,

$$\Rightarrow \frac{-2at_2}{2a(t_2-t_1)} = \frac{a(1-t_2^2)}{a(t_2-t_1)(t_2+t_1)} \Rightarrow -t_2 = \frac{(1-t_2^2)}{(t_2+t_1)}$$

$$\Rightarrow -t_2^2 - t_1t_2 = 1 - t_2^2 \Rightarrow t_1t_2 = -1$$

ELLIPSE

Q.91 (2)

$$\frac{x^2}{(48/3)} + \frac{y^2}{(48/4)} = 1$$

$$a^2 = 16, b^2 = 12 \Rightarrow e = \sqrt{1-\frac{b^2}{a^2}} = \frac{1}{2}$$

Distance is $2ae = 2 \cdot 4 \cdot \frac{1}{2} = 4$.

Q.92 (2)

Vertex (0,7), directrix $y = 12$, $\therefore$ $b = 7$

Also $\frac{b}{e} = 12 \Rightarrow e = \frac{7}{12}, a = 7\sqrt{\frac{95}{144}}$

Hence equation of ellipse is $144x^2 + 95y^2 = 4655$.

Q.93 (2)

$4(x-2)^2 + 9(y-3)^2 = 36$

Hence the centre is (2, 3).

Q.94 (1)

The ellipse is $4(x-1)^2 + 9(y-2)^2 = 36$

Therefore, latus rectum $= \frac{2b^2}{a} = \frac{2.4}{3} = \frac{8}{3}$

Q.95 (b)

We have ae=5 [Since focus is $(\pm ae, 0)$]

and $\frac{a}{e} = \frac{36}{5}\left[\text{since directrix is } x = \pm\frac{a}{e}\right]$

On solving we get a = 6

And $e = \frac{5}{6}$

$$\Rightarrow b^2 = a^2(1-e^2) = 36\left(1-\frac{25}{36}\right) = 11$$

Thus, the required equation of the ellipse

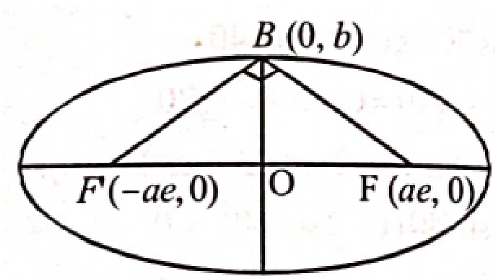
is $\frac{x^2}{36} + \frac{y^2}{11} = 1$

Q.96 (1)

$$\because \angle FBF' = 90^\circ \Rightarrow FB^2 + F'B^2 = FF'^2$$

$$\left(\sqrt{a^2e^2+b^2}\right)^2 + \left(\sqrt{a^2e^2+b^2}\right)^2 = (2ae)^2$$

$$\Rightarrow 2\left(a^2e^2+b^2\right) = 4a^2e^2 \Rightarrow e^2 = \frac{b^2}{a^2} \ldots\text{(i)}$$



Also, $e^2 = 1 - b^2/a^2 = 1 - e^2$

(By using equation (i)) $\Rightarrow 2e^2 = 1 \Rightarrow e = \frac{1}{\sqrt{2}}$

Q.97 (b)

$$e = \frac{1}{2}.\text{Directrix}, x = \frac{a}{e} = 4$$

$$\therefore a = 4 \times \frac{1}{2} = 2. \quad \therefore b = 2\sqrt{1-\frac{1}{4}} = \sqrt{3}$$

Equation of ellipse is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$$

Q.98 (3)

Let eq. ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, length of semi-latus rectum

$$=\frac{b^2}{a}=\frac{a^2(a-e^2)}{a}=a(a-e^2)$$

Given $a(1-e^2)=\frac{1}{3}(2a)$

$$\Rightarrow 1-e^2=\frac{2}{3} \Rightarrow 1-\frac{2}{3}=\frac{1}{3} \Rightarrow e=\frac{1}{\sqrt{3}}$$

Q.99 (4)

We have $\frac{81}{a^2}+\frac{25}{b^2}=1....(1)$

$$\frac{144}{a^2}+\frac{16}{b^2}=1....(2)$$

From eq. (2) –eq. (1) :

$$\frac{63}{a^2}-\frac{9}{b^2}=0 \Rightarrow \frac{b^2}{a^2}=\frac{1}{7}$$

$$e=\sqrt{1-\frac{1}{7}}=\sqrt{\frac{6}{7}}$$

Q.100 (4)
Q.101 (3)
Q.102 (1)

Q.103 **(3)**

Q.104 **(b)**

Q.105 (3)

$$3x^2-12x+4y^2-8y=-4 \Rightarrow 3(x-2)^2+4(y-1)^2=12$$

$$\Rightarrow \frac{(x-2)^2}{4}+\frac{(y-1)^2}{3}=1 \Rightarrow \frac{X^2}{4}+\frac{Y^2}{3}=1$$

$$\therefore\ e=\sqrt{1-\frac{3}{4}}=\frac{1}{2}.\ \therefore \text{ Foci are } \left(X=\pm 2\times\frac{1}{2}, Y=0\right)$$

i.e., $(x-2=\pm 1,\ y-1=0)=(3,1)$ and $(1,1)$.

Q.106 (2)

$$\because ae=\pm\sqrt{5} \Rightarrow a=\pm\sqrt{5}\left(\frac{3}{\sqrt{5}}\right)=\pm 3 \Rightarrow a^2=9$$

$$\therefore b^2=a^2(1-e^2)=9\left(1-\frac{5}{9}\right)=4$$

Hence, equation of ellipse

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \Rightarrow 4x^2+9y^2=36$$

Q.107 (1)

Centre is (3, 0), a = 8, b = $\sqrt{64\left(1-\frac{1}{4}\right)}=4\sqrt{3}$

Now $x = 3 + 8\cos\theta$

$y = 4\sqrt{3}\ \sin\theta$

$(3 + 8\cos\theta,\ 4\sqrt{3}\ \sin\theta)$

Q.108 (1)

Since $S_1 > 0$. Hence the point is outside the ellipse.

Q.109 (2)

$$y=3x\pm\sqrt{\frac{3.5}{3.4},9+\frac{5}{3}\times\frac{4}{4}}$$

$$\Rightarrow y=3x\ \pm\sqrt{\frac{155}{12}}$$

Q.110 (4)

Q.111 (4)

For $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, equation of normal at point (x_1,y_1),

$$\Rightarrow \frac{(x-x_1)a^2}{x_1}=\frac{(y-y_1)b^2}{y_1}$$

$$\therefore (x_1,y_1)\equiv(0,3),\ a^2=5,\ b^2=9$$

$\Rightarrow \frac{(x-0)}{0}\ 5=\frac{(y-3).9}{3}$ or $x = 0$ i.e., *y*-axis.

HYPERBOLA

Q.112 (1)

$$e=\sqrt{1+\frac{b^2}{a^2}} \Rightarrow e^2=\frac{a^2+b^2}{a^2}$$

$$e_1=\sqrt{1+\frac{a^2}{b^2}} \Rightarrow e_1^2=\frac{b^2+a^2}{b^2} \Rightarrow \frac{1}{e_1^2}+\frac{1}{e^2}=1.$$

Q.113 (4)

Q.114 (1)

The hyperbola is $\frac{x^2}{16}-\frac{y^2}{9}=1$. We have difference of focal distance = 2a =8

Q.115 **(2)**
The given equation of hyperbola is

$$16x^2-9y^2=144 \Rightarrow \frac{x^2}{9}-\frac{y^2}{16}=1$$

$$\therefore L.R. = \frac{2b^2}{a}=\frac{2.16}{3}=\frac{32}{3}.$$

Q.116 (1)

Directrix of hyperbola $x=\frac{a}{e}$,

where $e=\sqrt{\frac{b^2+a^2}{a^2}}=\frac{\sqrt{b^2+a^2}}{a}$

Directrix is, $x=\frac{a^2}{\sqrt{a^2+b^2}}=\frac{9}{\sqrt{9+4}} \Rightarrow x=\frac{9}{13}$

Q.117 (1)

$$(x-2)^2+(y-1)^2=4\left[\frac{(x+2y-1)^2}{5}\right]$$

$$\Rightarrow 5[x^2+y^2-4x-2y+5]$$

$$=4[x^2+4y^2+1+4xy-2x-4y]$$

$$\Rightarrow x^2-11y^2-16xy-12x+6y+21=0$$

Q.118 (3)

Hyperbola is $\frac{x^2}{9}-\frac{y^2}{5}=1$

Hence point of contact is $\left[\frac{-9(1)}{\sqrt{9-5}},\frac{-5}{\sqrt{9-5}}\right]\equiv\left[\frac{-9}{2},\frac{-5}{2}\right]$

Trick : Since the point $\left(-\frac{9}{2},-\frac{5}{2}\right)$ satisfies both the equations.

Q.119 (1)

The equation is $(x-0)^2+(y-0)^2=a^2$.

Q.120 (4)

The given ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$. The value of the expression $\frac{x^2}{9}+\frac{y^2}{4}-1$ is positive for $x=1, y=2$ and negative for $x=2, y=1$. Therefore *P* lies outside *E* and *Q* lies inside *E*. The value of the expression x^2+y^2-9 is negative for both the points *P* and *Q*. Therefore *P* and *Q* both lie inside *C*. Hence *P* lies inside *C* but outside *E*.

Q.121 (2)
It is obvious.

Q.122 (3)
If $y=2x+\lambda$ is tangent to given hyperabola, then

$$\lambda=\pm\sqrt{a^2m^2-b^2}=\pm\sqrt{(100)(4)-144}=\pm\sqrt{256}=\pm 16$$

Q.123 (4)

Q.124 (1)

The equation of the tangent to $4y^2=x^2-1$ at (1,0) is $4(y\times 0)=x\times 1-1$ or x–1 =0 or x =1

Q.125 (2)
The equation of chord of contact at point (h,k) is $xh-yk=9$
Comparing with $x=9$, we have $h=1, k=0$
Hence equation of pair of tangent at point (1,0) is $SS_1=T^2$

$$\Rightarrow (x^2-y^2-9)(1^2-0^2-9)=(x-9)^2$$

$$\Rightarrow -8x^2+8y^2+72=x^2-18x+81$$

$$\Rightarrow 9x^2-8y^2-18x+9=0$$

Q.126 (1)

Tangent to $y^2=8x \Rightarrow y=mx+\frac{2}{m}$

Tangent to $\frac{x^2}{1}-\frac{y^2}{3}=1 \Rightarrow y=mx\pm\sqrt{m^2-3}$
On comparing, we get
m = ± 2 or tangent as $2x\pm y+1=0$.

Q.127 (2)

According to question, $S \equiv 25x^2 - 16y^2 - 400 = 0$

Equation of required chord is $S_1 = T$ .....(i)

Here, $S_1 = 25(5)^2 - 16(3)^2 - 400$

$= 625 - 144 - 400 = 81$

and $T \equiv 25xx_1 - 16yy_1 - 400$, where $x_1 = 5, y_1 = 3$

$= 25(x)(5) - 16(y)(3) - 400 = 125x - 48y - 400$

So from (i), required chord is

$125x - 48y - 400 = 81$ or $125x - 48y = 481$.

Q.128 (4)

Given, equation of hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ and equation of asymptotes $2x^2 + 5xy + 2y^2 + 4x + 5y + \lambda = 0$ .....(i), which is the equation of a pair of straight lines. We know that the standard equation of a pair of straight lines is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

Comparing equation (i) with standard equation, we get

$a = 2, b = 2,\ h = \frac{5}{2}, g = 2, f = \frac{5}{2}$ and $c = \lambda$.

We also know that the condition for a pair of straight lines is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.

Therefore $4\lambda + 25 - \frac{25}{2} - 8 - \frac{25}{4}\lambda = 0$

or $-\frac{9\lambda}{4} + \frac{9}{2} = 0$ or $\lambda = 2$. Substituting value of λ in equation (i), we get $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$.

Q.129 (4)

Q.130 (2)

$xy = c^2$ as $c^2 = \frac{a^2}{2}$. Here, co-ordinates of focus are

$(ae\cos 45°, ae\sin 45°) \equiv (c\sqrt{2}, c\sqrt{2})\ \{\because e = \sqrt{2}, a = c\sqrt{2}\}$

Similarly other focus is $(-c\sqrt{2}, -c\sqrt{2})$

Note : Students should remember this question as a fact.

Q.131 (4)

Since it is a rectangular hyperbola, therefore eccentricity $e = \sqrt{2}$.

Q.132 (3)

Multiplying both, we get $x^2 - y^2 = a^2$. This is equation of rectangular hyperbola as $a = b$.

Q.133 (2)

Tangent at $(a\sec\theta, b\tan\theta)$ is,

$\frac{x}{(a/\sec\theta)} - \frac{y}{(b/\tan\theta)} = 1$ or

$\frac{a}{\sec\theta} = 1, \frac{b}{\tan\theta} = 1$

$\Rightarrow a = \sec\theta\ \ b = \tan\theta$ or (a, b)lies on $x^2 - y^2 = 1$

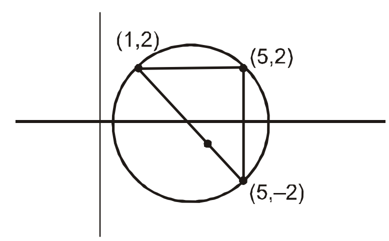
Q.134 (3)

Since eccentricity of rectangular hyperbola is $\sqrt{2}$.

EXERCISE-II (JEE MAIN LEVEL)

CIRCLE

Q.1 (4)



diameter = $4\sqrt{2}$

r = $2\sqrt{2}$

Q.2 (2)

Equation of circle $(x - 0)(x - a) + (y - 1)(y - b) = 0$

it cuts x-axis put y = 0 $\Rightarrow x^2 - ax + b = 0$

Q.3 (4)

Redius ≤ 5

$\frac{\lambda^2}{4} + \frac{(1-\lambda)^2}{4} - 5 \le 5$

$\Rightarrow \lambda^2 + (1-\lambda)^2 - 20 \le 100$

$\Rightarrow 2\lambda^2 - 2\lambda - 119 \le 0$

$\therefore \frac{-\sqrt{239}}{2} \le \lambda \le \frac{1+\sqrt{239}}{2} \Rightarrow -7.2 \le \lambda \le 8.2$

(approx.)

$\therefore \lambda = -7, -6, -5, ..., 7, 8,$ in all 16 values

Q.4 (1)

Q.5 (4)

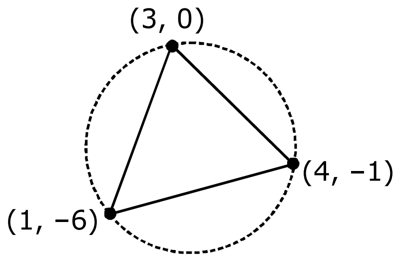
Q.6 (4)

Q.7 (4)

Let the centre (a, b)

$(a-3)^2+(2)^2=(a-1)^2+(b+6)^2$

$=(a-4)^2+(b+1)$



(i) & (ii)

$-6a+9=-2a+1+12b+36$

$\Rightarrow 4a+12b+28=0 \qquad \Rightarrow a+3b+7=0$

(i) & (iii)

$-6a+9=-8a+16+2b+1$

$\Rightarrow 2a-2b=8 \qquad \Rightarrow a-b=4$

$$a=\frac{5}{4},\ b=-\frac{11}{4} \quad r=\sqrt{\frac{49}{16}+\frac{121}{16}}=\frac{\sqrt{170}}{4}$$

$$g=-\frac{5}{4},\ f=\frac{11}{4},\ c=\frac{25}{16}+\frac{121}{16}-\frac{170}{16}$$

$$=\frac{-24}{16}=\frac{-3}{2}$$

$$x^2+y^2-2\cdot\frac{5}{4}x+2\cdot\frac{11}{4}y-\frac{3}{2}=0$$

$2x^2+2y^2-5x+11y-3=0$

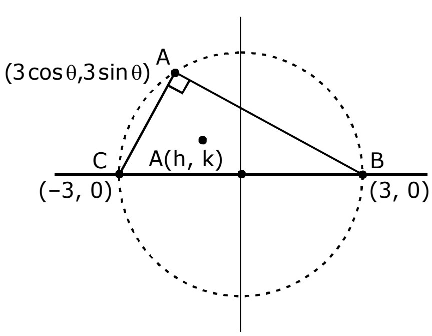
Q.8 (1)

Circle is

$x^2+y^2=9$

$\therefore$ co-ordinate of point

A $(3\cos\theta, 3\sin\theta)$



centroid of ΔABC is P(h, k) whose coordinate is

$$\left(\frac{3+3\cos\theta-3}{3},\frac{0+0+3\sin\theta}{3}\right)\equiv(\cos\theta, \sin\theta)$$

$h=\cos\theta,\ k=\sin\theta$

$h^2+k^2=1 \Rightarrow x^2+y^2=1$

Q.9 (1)

Point on the line $x+y+13=0$ nearest to the circle $x^2+y^2+4x+6y-5=0$ is foot of $\perp$ from centre

$$\frac{x+2}{1}=\frac{y+3}{1}=-\left(\frac{-2-3+13}{1^2+1^2}\right)=-4$$

$x=-6, \quad y=-7$

Q.10 (2)

From centre (2, –3), length of perpendicular on line $3x+5y+9=0$ is

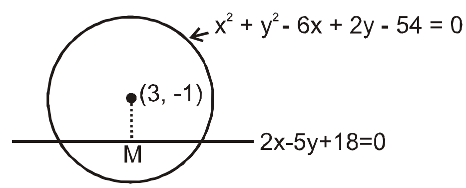
$$p=\frac{6-15+9}{\sqrt{25+9}}=0 \text{ ; line is diameter.}$$

Q.11 (1)

Required point is foot of $\perp$

$$\frac{x-3}{2}=\frac{y+1}{-5}=-\left(\frac{6+5+8}{4+25}\right)=-1 \Rightarrow x=-2+3=1$$

& $y=5-1=4$



$x=1,\ y=4$

Q.12 (3)

$\ell x+my+n=0,\ x^2+y^2=r^2$

$$r=\left|\frac{n}{\sqrt{\ell^2+m^2}}\right| \Rightarrow r^2(\ell^2+m^2)=n^2$$

Q.13 **(2)**

Q.14 (4)

Equation of tangent $x-2y=5$

Let required point be (α,β)

$\alpha x+\beta y-4(x+\alpha)+3(y+\beta)+20=0$

$x(\alpha-4)+y(\beta+3)-4\alpha+3\beta+20=0$

Comparing

$$\frac{\alpha-4}{1}=\frac{\beta+3}{-2}=\frac{4\alpha-3\beta-20}{5}$$

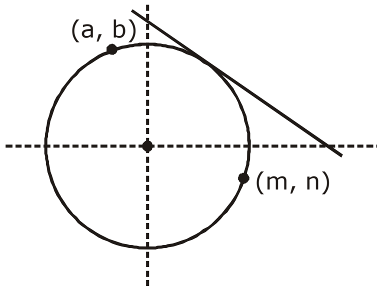
Similarly $(\alpha,\beta)\equiv(3,-1)$

Q.15 (1)

Given $a^2+b^2=1,\ m^2+n^2=1$

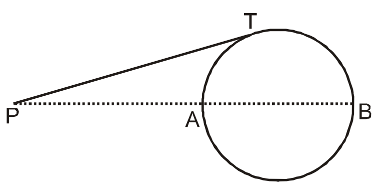
i.e. points (a, b) & (m, n) on the circle

$x^2 + y^2 = 1$ tangent at (a, b)



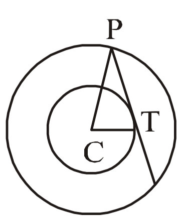
$ax + by - 1 = 0$ point (0, 0) & (m, n)
so lie some side of the tangent
$(0, 0) \Rightarrow -1 < 0$
$\therefore (m, n) \Rightarrow am + bn - 1 < 0 \Rightarrow am + bn < 1$
(m, n) & (a, b) can be equal
$\therefore \quad am + bn \le 1$
(m, n) & (a, b) can be negative
$\therefore \ |am + bn| \le 1$

Q.16 (3)
As we know
$PA.PB = PT^2 = (\text{Length of tangent})^2$



Length of tangent = $\sqrt{16 \times 9}$ = 12

Q.17 (1)
Let the radius of the first circle be $CT = r_1$.
Also let the radius of the second circle be $CP = r_2$.
In the triangle PCT, T is a right angle

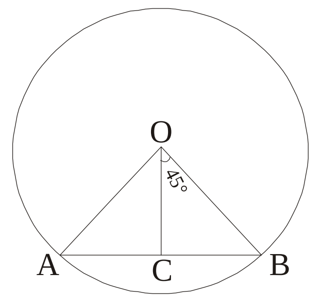


So, $PT = \sqrt{PC^2 - CT^2} = \sqrt{r_1^2 - r_2^2}$

$$= \sqrt{(f^2 - \lambda) - (f^2 - \mu)} = \sqrt{\mu - \lambda}$$

Q.18 (2)
Let point on line be
(h, 4 – 2h) (chord of contact)
$hx + y(4 - 2h) = 1$
$h(x - 2y) + 4y - 1 = 0$ Point $\left(\frac{1}{2}, \frac{1}{4}\right)$

Q.19 (3)



Let AB be the chord of length $\sqrt{2}$. Let O be the centre of the circle and let OC be the perpendicular from O on AB.

Then, $AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

In DOBC, we have
OB = BC cosec 45°

$$= \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$$

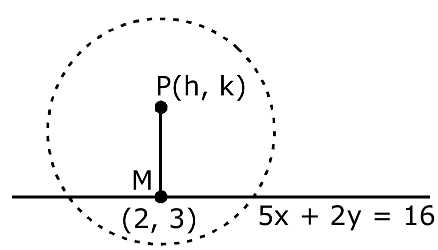
$\therefore$ Area of the circle $= \pi(OB)^2 = \pi$ sq units

Q.20 (1)

Q.21 (2)

Q.22 (1)
Let the centre P(h, k)

$$m_{PH} = \frac{-1}{m_2} = \frac{-1}{-\frac{5}{2}} = \frac{2}{5}$$

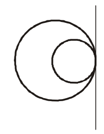


$$\frac{k-3}{h-2} = \frac{2}{5}$$

$2h - 5k + 11 = 0$
$2x - 5y + 11 = 0 \rightarrow$ Line PM.

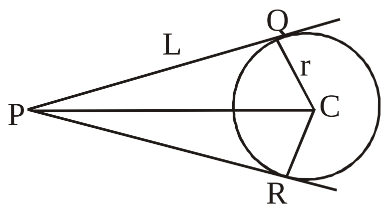
Q.23 (2)

$C_1C_2 = 5$, $r_1 = 7_1$ $r_2 = 2$



$C_1C_2 = |r_1 - r_2| \Rightarrow$ one common tangent

Q.24 (1)

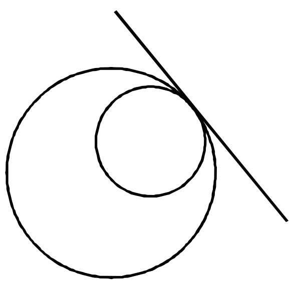


Q.25 (1)

$S_1 \Rightarrow C_1(1, 0),\ r_1 = \sqrt{2}$

$S_2 \Rightarrow C_2(0, 1), r_2 = 2\sqrt{2}$

$C_1C_2 = \sqrt{1^2 + 1^2} = \sqrt{2}$



$C_1C_2 = | r_2 - r_1 |$

$\sqrt{2} = \sqrt{2}$

Internally touch $\therefore$ common tangent is one.

Q.26 (4)

Here circles are $x^2 + y^2 - 2x - 2y = 0$

$x^2 + y^2 = 4$

Now, $C_1(1,1), r_1 = \sqrt{1^2 + 1^2} = \sqrt{2}$

$C_2(0,0), r_2 = 2$

If θ is the angle of intersection then

$$\cos\theta = \frac{n^2 + r_2^2 - (c_1c_2)^2}{2nr_2}$$

$$= \frac{2 + 4 - (\sqrt{2})^2}{2 \cdot \sqrt{2.2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

Q.27 (3)

$S_1 - S_3 = 0 \Rightarrow 16y + 120 = 0$

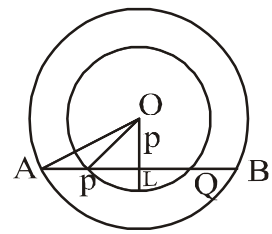
$\Rightarrow y = \frac{-120}{16} \qquad \Rightarrow y = -\frac{15}{2} \Rightarrow x = 8$

Intersection point of radical axis is

$\left(8, \frac{-15}{2}\right)$

Q.28 (3)

The given circles are concentric with centre at (0, 0) and the length of the perpendicular from (0, 0) on the given line is p. Let OL = p



then, $AL = \sqrt{OA^2 - OL^2} = \sqrt{a^2 - p^2}$

and $PL = \sqrt{OP^2 - OL^2} = \sqrt{b^2 - p^2}$

$\Rightarrow AP = \sqrt{a^2 - p^2} - \sqrt{b^2 - p^2}$

Q.29 (2)

Let the two circles be

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

where $g_1 = \frac{5}{2}, f_1 = \frac{3}{2}, c_1 = 7,$

$g_2 = -4, f_2 = 3$ and $c_2 = k$

If the two circles intersects orthogonally, Then

$2(g_1g_2 + f_1f_2) = c_1 + c_2$

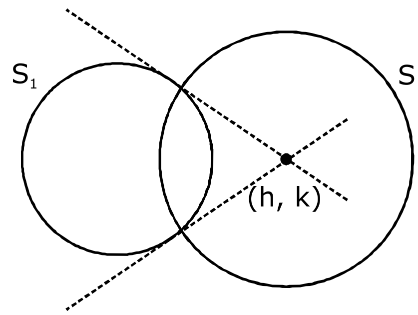
$\Rightarrow 2\left(-10 + \frac{9}{2}\right) = 7 + k$

$\Rightarrow 11 = 7 + k$

$\Rightarrow k = -18$

Q.30 (1)

Let point of intersection of tangents is (h, k) family of circle.



$x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$

Common chord is $S - S_1 = 0$

$\Rightarrow -(\lambda + 6)x + (8 - 2\lambda)y - 2 = 0$

$\Rightarrow (\lambda + 6)x + (2\lambda - 8)y + 2 = 0$ ....(i)

C.O.C. from (h, k) to S_1: $x^2 + y^2 = 1$ is

$hx + ky = 1$ ....(ii)

(i) & (ii) are same equation

$\frac{\lambda + 6}{h} + \frac{2(\lambda - 4)}{k} = \frac{2}{-1}$

$\Rightarrow \lambda = -2h - 6, \quad \lambda = -k + 4$

$\therefore -2h - 6 = -k + 4$

$\Rightarrow 2h - k + 10 \Rightarrow$ Locus : $2x - y + 10 = 0$

Q.31 (1)

$S_1 - S_2 = 0 \Rightarrow 7x - 8y + 16 = 0$

$S_2 - S_3 = 0 \Rightarrow 2x - 4y + 20 = 0$

$S_3 - S_1 = 0 \Rightarrow 9x - 12y + 36 = 0$

On solving centre (8, 9)

Length of tangent

$= \sqrt{S_1} = \sqrt{64 + 81 - 16 + 27 - 7} = \sqrt{149}$

$= (x - 8)^2 + (y - 9)^2 = 149$

$= x^2 + y^2 - 16x - 18y - 4 = 0$

Q.32 **(3)**

Two cicles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 \ 2g_2x + 2f_2y + c_2 = 0$ cuts orthogonally if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Given equations of two circles are

$x^2 + y^2 + 2\lambda x + 6y + 1 = 0$ …(i)

$x^2 + y^2 + 4x + 2y = 0$ …(ii)

On comparing (i) and (ii) with original equation, we get $g_1 = \lambda, f_1 = 3, c_1 = 1$ and $g_2 = 2, f_2 = 1, c_2 = 0$

So, from orhogonality condition, we have

$4\lambda + 6 = 1 \Rightarrow 4\lambda = -5$

$\therefore \lambda = \frac{-5}{4}$

Q.33 (1)

Q.34 (3)

Q.35 (4)

PARABOLA

Q.36 (4)

Eq. of the parabola is

$\sqrt{(x + 3)^2 + y^2} = |x + 5|$

$x^2 + 6x + 9 + y^2 = x^2 + 25 + 10x$

$y^2 = 4(x + 4)$

Q.37 (4)

$(x - 2)^2 + (y - 3)^2 = \left|\frac{3x - 4y + 7}{5}\right|^2$

$\therefore$ focus is (2, 3) & directrix is $3x - 4y + 7 = 0$

latus rectum = $2 \times \perp_r$ distance from focus to directrix

$= 2 \times \frac{1}{5} = 2/5$

Q.38 (a)

Given eq^n of parabola is $y^2 - kx + 6 = 0$

$\Rightarrow y^2 = kx - 6 \Rightarrow y^2 = k\left(x - \frac{6}{k}\right)$

Now, directrix, $x - \frac{6}{k} = -\frac{k}{4}$

$\Rightarrow x = \frac{6}{k} - \frac{k}{4}$...(i)

But directrix is given $\Rightarrow x = \frac{1}{2}$...(ii)

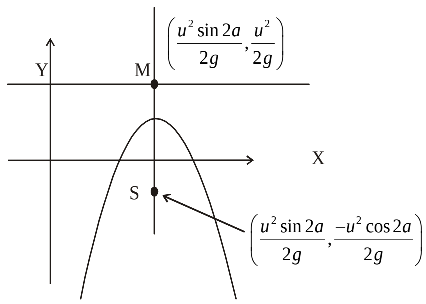
$\Rightarrow k^2 + 2k - 24 = 0$

$\Rightarrow (k + 6)(k - 4) = 0$

$\Rightarrow k = -6, k = 4$

Q.39 (d)

According to the figure, the length of latus rection is



Q.40 (c)

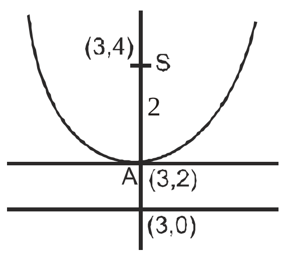
Q.41 **(b)**

Q.42 (1)

$y^2 - 12x - 4y + 4 = 0$

$y^2 - 4y = 12x - 4$

$(y - 2)^2 = 12x$



$Y^2 = 12X$

$x^2 = 4ay$

$(X - 3)^2 + 4x^2 (Y - 2)$

$x^2 - 6x + 9 = 8y - 16$

$x^2 - 6x - 8y + 25 = 0$

Q.43 (3)

Directrix : $x + y - 2 = 0$

Focus to directrix distance = 2a

$$2a = \left|\frac{0 + 0 - 2}{\sqrt{2}}\right|$$

$2a = \sqrt{2}$

$LR = 4a = 2\sqrt{2}$

Q.44 (2)

$x^2 - 2 = -2 \cos t$, $y = 4 \cos^2 \frac{t}{2}$

$\cos t = \frac{x^2 - 2}{-2}$, $y = 4 \cos^2 \frac{t}{2}$

$$y = 2\left(2\cos^2 \frac{t}{2}\right)$$

$y = 2(1 + \cos t)$

$$y = 2\left(1 + \frac{x^2 - 2}{-2}\right)$$

$y = 2 + 2 - x^2$

$y = 4 - x^2$

Q.45 (1)

Length of chord = $\frac{4}{m^2}\sqrt{a(a - mc)(1 + m^2)}$

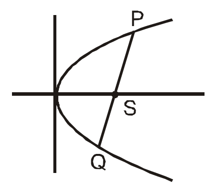
$m = \tan 60° = \sqrt{3}$

Length of chord = $\frac{4}{3}\sqrt{3(3 - \sqrt{3} \times 0)(1 + 3)}$

$$= \frac{4}{3}\sqrt{36} = 8$$

Q.46 (a)

Q.47 (1)



From the property $\frac{1}{PS} + \frac{1}{QS} = \frac{1}{a}$

$$\frac{1}{3} + \frac{1}{2} = \frac{1}{a}$$

$$a = \frac{6}{5}$$

$\therefore$ Latus rectum = $4a = \frac{24}{5}$

Q.48 (4)

Slope of tangent = $\frac{1 - 0}{4 - 3} = 1$

also $\frac{dy}{dx} = 2(x - 3)$

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2(x_1 - 3) = 1 \Rightarrow x_1 - 3 = \frac{1}{2}$$

$$x_1 = \frac{7}{2}$$

$$\therefore \; y_1 = \left(\frac{7}{2} - 3\right)^2 = \frac{1}{4}$$

Equation of tangent is

$$y - \frac{1}{4} = 1\left(x - \frac{7}{2}\right)$$

$4y - 1 = 2(2x - 7)$
$4x - 4y = 13$

Q.49 (b)

$$\text{Given } x = \frac{3y+k}{2} \quad .....(1)$$

$$\text{and } y^2 = 6x \quad .....(2)$$

$$\Rightarrow y^2 = 6\left(\frac{3y+k}{2}\right)$$

$$\Rightarrow y^2 = 3(3y+k) \Rightarrow y^2 - 9y - 3k = 0 \quad .....(3)$$

If line (1) touches parabola (2) then rootsof quadratic equation (3) is equal

$$\therefore (-9)^2 = 4 \times 1 \times (-3k) \Rightarrow k = -27/4$$

Q.50 (c)
Any tangent to parabola $y^2 = 8x$ is

$$y = mx + \frac{2}{m} \quad ......(i)$$

It touches the circle $x^2 + y^2 - 12x + 4 = 0$
if the length of perpendicular from the centre (6, 0) is equal to radius $\sqrt{32}$.

$$\therefore \frac{6m + \frac{2}{m}}{\sqrt{m^2+1}} = \pm\sqrt{32} \Rightarrow \left(3m \frac{1}{m}\right)^2 = 8(m^2+1)$$

$(3m^2 + 1)^2 = 8(m^4 + m^2)$
Hence, the required tangents are $y = x + 2$ and $y = -x - 2$.

Q.51 (b)
Q.52 (c)
Q.53 **(d)**

Q.54 (3)
Let the equation of tangent to the parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m} \quad ...(1)$$

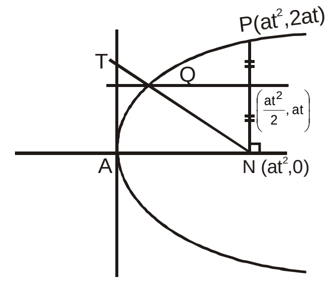
solving equation (1) with parabola $x^2 = 4y$

$$\Rightarrow x^2 = 4\left(mx + \frac{1}{m}\right)$$

Now put D = 0 & find the value of m

Q.55 (2)
$N(at^2, 0)$
solve $y = at$ with
curve $y^2 = 4ax$

$$x = \frac{at^2}{4}$$



$$Q\left(\frac{at^2}{4}, at\right)$$

$$\text{Equation of QN } y = \frac{dt}{\left(\frac{at^2}{4} - at^2\right)}(x - at^2)$$

$$\text{put } x = 0 \;\; y = \frac{4}{3}at$$

$$T\left(0, \frac{4}{3}at\right) \quad AT = \frac{4}{3}at$$

$PN = 2at$

$$\frac{AT}{PN} = \frac{4/3\,at}{2at} = \frac{2}{3} \text{ so } k = \frac{2}{3}$$

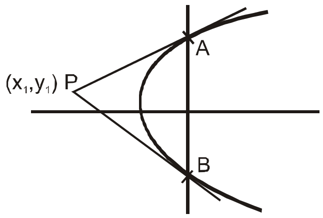
Q.56 (1)
Equation of normal to the parabola $y^2 = 4ax$ at points $(am^2, 2am)$ is
$y = -mx + 2am + am^3$

Q.57 (3)
Line : $y = -2x - \lambda$
Parabola : $y^2 = -8x$
$c = -2am - am^3$ (condition for line to be normal to parabola)
$-\lambda = -2 \times -2 \times -2 - (-2)(-8)$
$-\lambda = -8 - 16$
$\lambda = 24$

Q.58 (a)

Q.59 (3)
Use $T^2 = SS_1$
$\Rightarrow [y.0 - 4(x+2)]^2 = (y^2 - 8x)(0 - 8(-2))$
$\Rightarrow \cancel{16}(x-2)^2 = \cancel{16}(y^2 - 8x)$
$\Rightarrow y = \pm(x+2)$

Q.60 (3)



Eq. of AB is :
T = 0
$yy_1 = 2(x + x_1)$
$2x - yy_1 + 2x_1 = 0$ ...(1)
$4x - 7y + 10 = 0$ . . . .
(2)
equ. (1) & (2) are identical

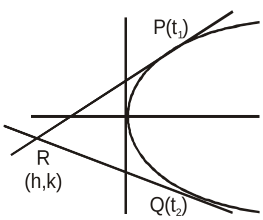
$\therefore \ \frac{2}{4} = \frac{y_1}{7} = \frac{2x_1}{10}$

$y_1 = \frac{7}{2}$ & $x_1 = \frac{5}{2}$

Q.61 (1)
$y^2 = 4ax$

Slope = $\frac{1}{t}$

$\frac{1}{t_1} = \frac{2}{t_2}$



$\Rightarrow t_2 = 2t_1$ ......(1)
$R[at_1t_2, a(t_1 + t_2)]$
$h = at_1 t_2$, $k = a(t_1 + t_2)$

$k = 3at_1 \Rightarrow t_1 = \frac{k}{3a}$

$h = 2at_1^2$

$h = 2a \frac{k^2}{9a^2} \Rightarrow k^2 = \frac{9}{2}ah$ $\Rightarrow y^2 = \frac{9}{2}ax$

Q.62 (4)
$y^2 + 4y - 6x - 2 = 0$

$y^2 + 4y + 4 - 6x - 6 = 0$; $a = \frac{3}{2}$

$(y + 2)^2 = 6(x + 1)$
$Y^2 = 6X$ vertex (–1, –2)
POI of tangents $t_1 t_2 = -1$

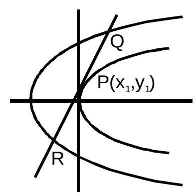
$[at_1t_2, a(t_1 + t_2)]$
$h + 1 = at_1t_2$

$h + 1 = -\frac{3}{2}$

$2h + 2 = -3$
$2h + 5 = 0 \Rightarrow 2x + 5 = 0$

Q.63 (3)
Tangent at P of $y^2 = 4ax$
$yy_1 = 2a(x + x_1)$ .....(1)
Let Mid point (h, k)
$T = S_1$
$yk - 2a(x + h) - 4ab = k^2 - 4a(h + b)$
$yk - 2ax - 2ah + 4ah - k^2 = 0$
$yk - 2ax + 2ah - k^2 = 0$ ....(2)
(1) & (2) are same

$\frac{k}{y_1} = \frac{-2a}{-2a} = \frac{2ah - k^2}{-2ax_1}$



$k = y_1$; $-2ax_1 = 2ah - k^2$
$-2ax_1 = 2ah - y_1^2$; $y_1^2 = 4ax_1$
Mid point $-2ax_1 = 2ah - 4ax_1$
(x_1, y_1) $2ah = 2ax_1$
$h = x_1$

Q.64 (a)
The parametric equations of the parabola $y^2 = 8x$ are $x = 2t^2$ and $y = 4t$. and the given equation of circle of is $x^2 + y^2 - 2x - 4y = 0$
On putting $x = 2t^2$ and $y = 4t$ in circle we get

$$4t^4 + 16t^2 - 4t^2 - 16t = 0$$

$$\Rightarrow 4t^2 + 12t^2 - 16t = 0$$

$$\Rightarrow 4t(t^3 + 3t - 4) = 0$$

$$\Rightarrow t(t - 1)(t^2 + t + 4) = 0$$

$$\Rightarrow t = 0, t = 1$$

$$\left[\because t^2 + t + 4 \neq 0\right]$$

Thus the coordinates of points of intrsection of the circle and the parabola are Q(0, 0) and P(2, 4). clearly on the circle. The coordinates of the focus S of the parabola are (2, 0) whic lies on the circle.

$$\because \text{Area of } \Delta PQS = \frac{1}{2} \times QS \times SP = \frac{1}{2} \times 2 \times 4$$

= 4 sq. units.

Q.65 (c)

Given parabola is $y^2 = 4x$ .....(1)

Let $P \equiv (t_1^2, 2t_1)$ and $Q \equiv (t_2^2, 2t_2)$

Slope of $OP = \frac{2t_1}{t_1^2} = \frac{2}{t_1}$ and $slope\,of\; OQ = \frac{2}{t_2}$

since $OP \perp OQ, \therefore \frac{4}{t_1 t_2} = -1$

or $t_1 t_2 = -4$ ......(2)

Let $R(h,k)$ be the middle point of PQ, then

$h = \frac{t_1^2 + t_2^2}{2}$ ...(3) and $k = t_1 + t_2$ ...(4)

From $(4), k^2 = t_1^2 + t_2^2 + 2t_1 t_2 = 2h - 8$ [From (2) and (3)] Hence locus of $R(h,k)\; is\; y^2 = 2x - 8$

Q.66 (1)

From the property : the feet of the ⊥r will lie on the tangent at vertex of the parabola.

$y = (x-1)^2 - 3 - 1$

$(x-1)^2 = (y+4)$

Tangent at vertex of above parabola is y + 4 = 0.

Q.67 (b)

Q.68 (4)

$(x-1)^2 = 8y$; a = 2 $\quad x - 1 = 0, y = 2$

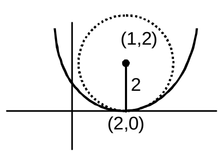
$x^2 = 8y$; x = 1, y = 2

vertex (1, 0)

Focus (1, 2)

Radius of circle = 2

$(x-1)^2 + (y-2)^2 = 4$



$x^2 + y^2 - 2x - 4y + 1 = 0$

Q.69 (3)

$y^2 = 4a(x = \ell_1)$ $\qquad x^2 = 4a(y - \ell_2)$

let the POC (h, k)

2yy' = 4a $\qquad$ 2x = 4ay'

$y' = \left.\frac{2a}{y}\right|_{(h,k)} = \frac{2a}{k}$ ...(1) $\qquad y' = \left.\frac{x}{2a}\right|_{(h,k)}$

(1) and (2) are equal = $\frac{h}{2a}$ ....(2)

$\frac{2a}{k} = \frac{h}{2a}$

$hk = 4a^2$

$xy = 4a^2$

ELLIPSE

Q.70 (1)

PS = ePM

$\sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{2}\left|\frac{x-y-3}{\sqrt{1^2+1^2}}\right|$

Squaring, we have

$7x^2 + 7y^2 + 7 - 10x + 10y + 2xy = 0$

Q.71 (4)

$4x^2 + 9y^2 + 8x + 36y + 4 = 0$

$4(x^2 + 2x + 1) + 9[y^2 + 4y + 4] = 36$

$4(x+1)^2 + 9(y+2)^2 = 36$

$\frac{(x+1)^2}{9} + \frac{(y+2)^2}{4} = 1$

$\Rightarrow e = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

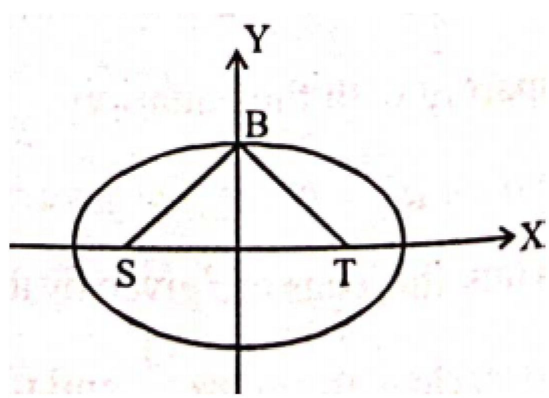
Q.72 (3)

$9x^2 + 4y^2 = 1$

$\frac{x}{1/9} + \frac{y^2}{1/4} = 1 \Rightarrow$ Length of latusrectun $= \frac{2a^2}{b} = \frac{4}{9}$

Q.73 (c)

S is $(-ae, 0)$, T is $(ae, 0)$ and B is $(0, b)$.



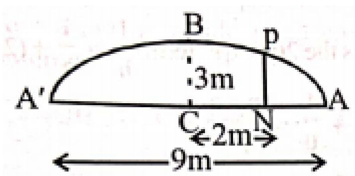
$\Rightarrow SB = \sqrt{(0+ae)^2 + b^2}$

Also $SB^2 = ST^2 \Rightarrow 4a^2e^2 = a^2e^2 + b^2$

$$\Rightarrow 3a^2e^2 = a^2\left(1-e^2\right) = a^2 - a^2e^2$$

$$\Rightarrow 4a^2e^2 = a^2 \Rightarrow e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$$

Q.74 (b)



The equation of the ellipse is

$$\frac{x^2}{\left(\frac{9}{2}\right)^2} + \frac{y^2}{9} = 1$$

Where centre is assumed as origin and base as x-axis. Put x=2, we get

$$\frac{16}{81} + \frac{y^2}{9} = 1 \Rightarrow y = \frac{\sqrt{65}}{3} \approx \frac{8}{3}m \text{ (approximately)}$$

Q.75 (b)

Q.76 (1)

$e = \frac{5}{8}$; $2ae = 10 \Rightarrow 2a = \frac{10}{e} \Rightarrow 2a = 16$

Latus rectum $= \frac{2b^2}{a} = \frac{2a^2(1-e^2)}{a}$

$= 2a(1-e^2) = 16\left(1 - \frac{26}{64}\right) = \frac{39}{4}$

Q.77 (1)

$x = 3(\cos t + \sin t)$ $y = 4(\cos t - \sin t)$

$\Rightarrow \frac{x}{3} = \cos t + \sin t$; $\frac{y}{4} = \cos t - \sin t$

square & add $\frac{x^2}{9} + \frac{y^2}{16} = 2$

Ellipse Equation $\frac{x^2}{18} + \frac{y^2}{32} = 1$

Q.78 (2)

Max. area $= \frac{1}{2} \times 2ae \times b = \frac{1}{2} \times 2 \times 3 \times 4 = 12$

Q.79 (3)

$4(x^2 - 4x + 4) + 9(y^2 - 64 + 9) = 36$

$4(x-2)^2 + 9(y-3)^2 = 36$

$$\frac{(x-2)^2}{9} + \frac{(y-3)^2}{4} = 1.$$

Equation of major axis y = 3.

Equation of minor axis x = 2

Q.80 (b)

Q.81 (c)

Q.82 (c)

Q.83 (d)

Q.84 (2)

Let eccentric angle be θ, then equation of tangent is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \qquad ...(1)$$

given equation is

$$\frac{x}{a} + \frac{y}{b} = \sqrt{2} \qquad ...(2)$$

comparing (1) and (2)

$$\cos\theta = \sin\theta = \frac{1}{\sqrt{2}}$$

$\Rightarrow \theta = 45°$

Q.85 (4)

$3x^2 + 4y^2 = 1$

$3xx_1 + 4yy_1 = 1$

given $3x + 4y = -\sqrt{7}$

comparing

$$\because \frac{3x_1}{3} = \frac{4y_1}{4} = \frac{1}{-\sqrt{7}}$$

$$x_1 = -\frac{1}{\sqrt{7}}$$

$$y_1 = -\frac{1}{\sqrt{7}}$$

Q.86 (c)

Clearly ax + by = 1

i.e $y = -\frac{a}{b}x + \frac{1}{b}$ is tangent to

$$cx^2 + dy^2 = 1 \Rightarrow \frac{x^2}{\frac{1}{c}} + \frac{y^2}{\frac{1}{d}} = 1$$

$$\therefore \left(\frac{1}{b}\right)^2 = \left(\frac{1}{c}\right)\left(-\frac{a}{b}\right)^2 + \left(\frac{1}{d}\right)$$

$$\Rightarrow 1 = \frac{a^2}{c} + \frac{b^2}{d}$$

Q.87 **(c)**
Given line is $x\cos\alpha + y\sin\alpha = P$ …(1)
Any tangent to the ellipse is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1 \quad \text{…(2)}$$

Comparing (1) and (2)

$$\frac{\cos\theta}{a\cos\alpha} = \frac{\sin\theta}{b\cos\alpha} = \frac{1}{P}$$

$$\Rightarrow \cos\theta = \frac{a\cos\alpha}{P} \text{ and } \sin\theta = \frac{b\sin\alpha}{P}$$

Eliminate θ, $\cos^2\theta + \sin^2\theta$

$$= \frac{a^2\cos^2\alpha}{P^2} + \frac{b^2\sin^2\alpha}{P^2},$$

or $a^2\cos^2\alpha + b^2\sin^2\alpha = P^2$

Q.88 (b)

Q.89 (4)
Equation of normal
$ax\sec\phi - by\,\mathrm{cosec}\phi = a^2 - b^2$ ...(1)
$x\cos\alpha + 4\sin\alpha = p$ ...(2)

$$\frac{a\sec\varphi}{\cos\alpha} = \frac{-by\,\mathrm{cosec}\,\varphi}{\sin\alpha} = \frac{a^2-b^2}{p}$$

$$\Rightarrow \cos\phi = \frac{ap}{(a^2-b^2)} \times \sec\alpha \quad \text{...(3)}$$

$$\Rightarrow \sin\phi = \frac{-bp}{(a^2-b^2)} \times \mathrm{cosec}\alpha \quad \text{...(4)}$$

squaring and adding

$$1 = \frac{p^2}{(a^2-b^2)^2}[a^2\sec^2\alpha + b^2\mathrm{cosec}^2\alpha]$$

Q.90 (4)
$3x^2 + 5x^2 = 15$

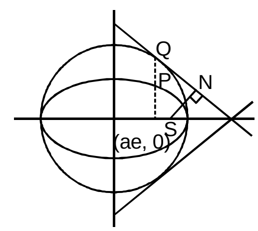
$$\frac{x^2}{5} + \frac{y^2}{3} = 1$$

Equation of director circle.
$x^2 + y^2 = 5+3 = 8$
clearly (2, 2) lies on it

here $\angle\theta = \frac{\pi}{2}$

Q.91 (2)
Ellipse $-2x^2 + 5y^2 = 20$, mid point (2, 1)
using $T = S_1$
$2x(2) + 5(y \times 1) - 20 = 2(2)^2 + 5(1)^2 - 20$
$4x + 5y = 13$

Q.92 (1)
$P(a\cos\alpha, b\sin\alpha)$
$Q(a\cos\alpha, a\sin\alpha)$
Tangent at Q point
$x\cos\alpha + y\sin\alpha = a$



$SN = |ae(\cos\alpha - a)|$

$$SP = \sqrt{(ae - a\cos\alpha)^2 + b^2\sin^2\alpha}$$

$$= \sqrt{a^2e^2 + a^2\cos^2\alpha - 2a^2e\cos\alpha + b^2 - b^2\cos^2\alpha}$$

$$= \sqrt{a^2 + \cos^2\alpha(a^2-b^2) - 2a^2e\cos\alpha}$$

$= |ae\cos\alpha - a|$
$\Rightarrow SP = SN$

Q.93 **(1)**
Same as Previous Question.
Ans.(1) Isosceles triangle

Q.94 (2)
$(S_1F_1)\cdot(S_2F_2) = b^2 = 3$

HYPERBOLA

Q.95 (2)
Given hyperbola
$(x-2)^2 - (y-2)^2 = -16$
Rectangular hyperbola
$\therefore\ e = \sqrt{2}$.

Q.96 **(b)**
$4x^2 - 9y^2 = 1$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} - \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$$

eccentricity, $e = \sqrt{1+\frac{\left(\frac{1}{3}\right)^2}{\left(\frac{1}{2}\right)^2}} = \frac{\sqrt{13}}{3}$

foci $= \left(\pm\frac{1}{2}\times\frac{\sqrt{13}}{3},0\right) = \left(\pm\frac{\sqrt{13}}{6},0\right)$

Q.97 (b)
Q.98 **(1)**

Q.99 **(1)**
Q.100 **(d)**

Q.101 **(1)**

Q.102 (3)
C(0, 0) $A_1(4, 0)$ F_1 (6, 0)
$CA_1 = 4$ $CF_1 = 6$
$\Rightarrow$ a = 4   ae = 6

$a^2e^2 = 36 \quad \Rightarrow a^2\left(1+\frac{b^2}{a^2}\right) = 36$

$\Rightarrow b^2 = 36 - 16 \qquad \Rightarrow b^2 = 20$

Hyp. $\frac{x^2}{16} - \frac{y^2}{20} = 1$ or $5x^2 - 4y^2 = 80$

Q.103 (1)

$F_1(6, 5)$ $F_2(-4, 5)$ $e = \frac{5}{4}$
$F_1F_2 = 2ae$ Centre of hyp. is the mid point of $F_1F_2 = (1, 5)$
2ae = 10

$\Rightarrow ae = 5 \quad \Rightarrow a^2e^2 = 25 \Rightarrow a^2\left(\frac{25}{16}\right) = 25$

$\Rightarrow a^2 = 16 \quad \Rightarrow b^2 = 9$

Hyp. $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$

Q.104 (1)
$\sqrt{2}^2 \sec^2\theta + \sqrt{2}^2 \tan^2\theta = 6$
$\Rightarrow 1 + 2\tan^2\theta = 3$
$\therefore \theta = \pi/4$ for first quadrant

Q.105 (c)

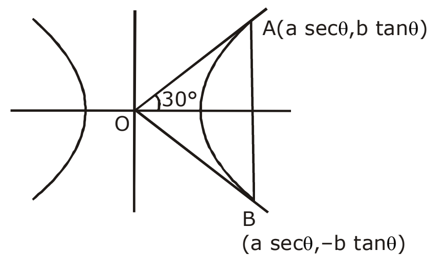
Q.106 **(d)**
Q.107 (4)

$\theta = 30°$

$\frac{b\tan\theta}{a\sec\theta} = \tan 30°$

$\frac{b}{a}\sin\theta = \frac{1}{\sqrt{3}}$

$\frac{b}{a} = \frac{1}{\sqrt{3}\sin\theta}$



$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{1}{3\sin^2\theta}$

$e^2 > 1 + \frac{1}{3}$

$e > \frac{2}{\sqrt{3}}$

Q.108 (1)

Q.109 (4)
$\left(1, 2\sqrt{2}\right)$ lies on director circle

of $\frac{x^2}{25} - \frac{y^2}{16} = 1$ i.e. $x^2 + y^2 = 9$
$\therefore$ Required angle $\pi/2$

Q.110 (4)
Locus of the feet of the $\perp^n$ drawn from any focus of the the hyp. upon any tangent is its auxilary circle

Hyp. $\frac{x^2}{\left(\frac{1}{16}\right)} - \frac{y^2}{\left(\frac{1}{9}\right)} = 1$

Auxiliary circle $x^2 + y^2 = \frac{1}{16}$

Q.111 (3)
by $T = S_1$ we get $5x + 3y = 16$

Q.112 (1)
by $T = S_1$

$3xh - 2yk + 2(x + h) - 3(y + k)$
$= 3h^2 - 2k^2 + 4h - 6k$
$\Rightarrow x(3h + 2) + y(-2k - 3) = 3h^2 - 2k^2 + 2h - 3k$
If is parallel to y = 2x

$\therefore \frac{(3h+2)}{(2k+3)} = 2$

$\Rightarrow 3x - 4y = 4$ Ans.

Q.113 (2)

Slope of the chord = $\frac{25}{16} \times \frac{x_1}{y_1}$

$= \frac{25}{16} \times \frac{6}{2} = \frac{75}{16}$

Equation of chord passing through (6, 2)

$y - 2 = \frac{75}{16}(x - 6)$

$16y - 32 = 75x - 450$
$75x - 16y = 418$

Q.114 (1)
Let pair of asymptotes be
$xy - xh - yk + \lambda = 0$ ...(1)
where λ : constant
$\therefore$ for (1) represents pair of straight line $\lambda = hk$
$\therefore$ Asymptotes $x - k = 0,\ y - h = 0$

Q.115 (1)
Hyp. $xy - 3x - 2y = 0$
$f(x, y) = xy - 3x - 2y$

$\frac{\delta f}{\delta x} = 0 \Rightarrow y = 3$

$\frac{\delta f}{\delta y} = 0 \Rightarrow x = 2$ Centre (2, 3)

Asy. $xy - 3x - 2y + C = 0$
will pass through (2, 3)
C = 6
$xy - 3x - 2y + 6 = 0$
$(y - 3)(x - 2) = 0$
$x - 2 = 0,\ y - 3 = 0$

Q.116 (4)
Let the circle on which
P, Q, R, S lie be
$x^2 + y^2 + 2gx + 2fy + C_1 = 0$

How let $\left(ct, \frac{c}{t}\right)$ lie on it

$\Rightarrow c^2t^4 + 2gct^3 + C_1t^2 + 2fct + c^2 = 0$
where $t_1, t_2, t_3\ t_4$ represents the parameters for P, Q, R, S
$\therefore t_1t_2t_3t_4 = 1$
also since orthocentre of ΔPQR be

$\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right) \Rightarrow (-x_4, -y_4)$

Q.117 (b)
We have $x^2 - y^2 - 4x + 4y + 16 = 0$
$\Rightarrow (x^2 - 4x) - (y^2 - 4y) = 16$
$\Rightarrow (x^2 - 4x + 4) - (y^2 - 4y + 4) = -16$
$\Rightarrow (x - 2)^2 - (y - 2)^2 = -16$

$\Rightarrow \frac{(x-2)^2}{4^2} - \frac{(y-2)^2}{4^2} = 1$

This is rectangular hyperbola, whose eccentricity is always $\sqrt{2}$.

Q.118 (d)

Q.119 (1)

Let $A\left(ct_1, \frac{c}{t_1}\right), B\left(ct_2, \frac{c}{t_2}\right), C\left(ct_3, \frac{c}{t_3}\right)$

then orthocentre be

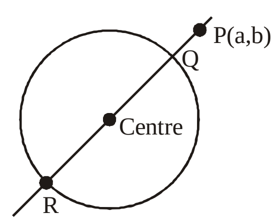
$H\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$ which lies on $xy = c^2$

EXERCISE-III

CIRCLE

Q.1 (0006)

The given cirlce is $(x+1)^2 + (y+2)^2 = 9$ has radius =3
The points on the circle which are nearest and farthest to the point P(a,b) are Q and R respectively



Thus, the circle centred at Q having radius PQ will be the smallest required circle while the circle centred at R having radius PR will be the largest required circle. hence, difference between their radii = PR-PQ=QR=6

Q.2 (0000)
The given circles are

$(x-1)^2 + y^2 = 4$ and $(x-1)^2 + y^2 = 16$

The Points $\left(a+1, \sqrt{3}a\right)$ lie on the line

$y=\sqrt{3}(x-1)$

whose slope = $\sqrt{3}$ hence makes angle 60° with x-axis.

$A=\left(1+2\cos 60^o, 2\sin 60^o\right)=\left(2,\sqrt{3}\right)$,

$B=\left(1+4\cos 60^o, 4\sin 60^o\right)=\left(3,2\sqrt{3}\right)$

Hence there is no point on the line segment AB

Q.3 **(0000)**

(1, 2) lies inside the circle

$\therefore$ no. of tangnet is zero.

Q.4 **(0010)**

Let the equation to the circle be

$x^2+y^2+2gx+2fy+c=0$ .....(i)

Since the three points lie on the circle, we have

$2g+4f+c=-5$ .....(ii)

$6g-8f+c=-25$ .....(iii)

$10g-12f+c=-61$ .....(iv)

Subtracting (ii) from (iii) and (iii) from (iv), we have

$4g-12f=-20$ and $4g-4f=-36$

Hence $f=-2$ and $g=-11$

Equation (ii) then gives c = 25.

Substituting these values in (i), the required equation is

$x^2+y^2-22x-4y+25=0$

Its centre is (11, 2) and radius is 10.

Q.5 **(0015)**

Since $S_1=10^2+7^2-4\times 10-2\times 7-20>0$.

So P lies outside the circle. Join P with centre C(2, 1) of the given circle. Suppose PC cuts the circle at A and B. Then PB is greatest distance of P from the circle.

$PC=\sqrt{(10-2)^2+(7-1)^2}=110$

CB = radius $=\sqrt{4+1+20}=5$

$\therefore$ $PB=PC+CB=10+5=15$

Q.6 **(20)**

The two diameters intersect at (8, –2) which is the centre of the circle. The circle passes through (6, 2). Therefore its radius $=\sqrt{20}$.

Hence the equation of the circle is $(x-8)^2+(y+2)^2=\left(\sqrt{20}\right)^2$

Q.7 **(0003)**

Two circles are $x^2+y^2-4x-6y-3=0$ and $x^2+y^2+2x+2y+1=0$

Centres : $C_1=(2, 3)$ $C_2=(-1,-1)$

radii : $r_1=4$ $r_2=1$

we have $C_1C_2=5=r_1+r_2$, therefore there are 3 common tangents to the given circles.

Q.8 (–48)

Given, $x^2+y^2-2x-6y-\frac{7}{3}=0$

The centre of this circle is (1, 3)

Also, two diameter of this circle are along the lines $3x+y=c_1$ and $x-3y=c_2$

These two diameters should be passed from (1, 3)

$\therefore$ $c_1=6$ and $c_2=-8$ Hence, $c_1c_2=6\times(-8)=-48$

Q.9 (8)

Equation of circle is

$(x-4)(x+2)+(y-7)(y+1)=0$

$\Rightarrow x^2-2x-8+y^2+y-7y-7=0$

$\Rightarrow x^2+y^2-2x-6y-15=0$

Here, $g=-1, c=-15$

$\therefore AB=2\sqrt{g^2-c}$

$=2\sqrt{1+15}$

$=8$

Q.10 [4]

Given, $x^2+y^2=6x$ ....(i)

and $x^2+y^2+6x+2y+1=0$ ....(ii)

From Eq. (i), $x^2-6x+y^2=0$

$\Rightarrow (x-3)^2+y^2=3^2$

$\therefore$ Centre (3, 0), r = 3

From Eq. (ii),

$x^2+6x+y^2+2y+1+3^2=3^2$

$\Rightarrow (x+3)^2+(y+1)^2=3^2$

$\therefore$ Centre (–3, –1) radius = 3

Now, distance between centres

$=\sqrt{(3+3)^2+1}$

$=\sqrt{37}>r_1+r_2=6$

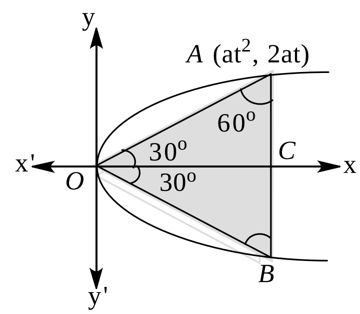
$\therefore$ Circles do not cut each other

$\Rightarrow$ 4 tangents (two direct and two transversal) are possible

PARABOLA

Q.11 [8]

ΔOAC, $\tan 30^o=\frac{AC}{OC}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2at}{at^2}, t = 2\sqrt{3}$

Again in ΔOCA,

$OA = \sqrt{OC^2 + AC^2} = \sqrt{(at^2)^2 + (2at)^2}$

$= \sqrt{\left[(2\sqrt{3})^2\right]^2 a^2 + 4a^2(2\sqrt{3})^2} = \sqrt{192a^2} = 8a\sqrt{3}$

Q.12 **[0]**

Given curve is $y^2 = 4x$ …(i)

Let the equation of line be $y = mx + c$

Since, $\frac{dy}{dx} = m = 1$ and above line is passing through the point (0, 1)

$1 = 1(0) + c \Rightarrow c = 1$

$y = x + 1$ …(ii)

On solving Eqs. (i) and (ii), we get

$x = 1$ and $y = 2$

This shows that line touch the curve at one point. So, length of intercept is zero.

Q.13 **(6)**

Given parabola is $y^2 = 12x$

Here, a = 3 For point P(x, y), y = 6

This point lie on the parabola

$\therefore (6)^2 = 12x \Rightarrow x = 3$

Thus , focal distance of point P is 6

Q.14 **[1.5]**

Any point on the parabola $y^2 = 4ax$ is $(at^2, 2at)$

$\therefore at^2 = \frac{9}{2}$

and $2at = 6 \Rightarrow t = \frac{3}{a}$ .....(1)

$\therefore a\left(\frac{3}{a}\right)^2 = \frac{9}{2} \Rightarrow a = 2$

On putting the value of a in Eq. (i), we get

$t = \frac{3}{2}$

$\therefore$ Parameter of the point P is $\frac{3}{2}$

Q.15 **[1.5]**

The equation of parabola can be written as

$(y + 2)^2 = -4\left(x - \frac{1}{2}\right)$

$\Rightarrow y^2 = -4x$ where $X = x - \frac{1}{2}$, $Y = y + 2$

An equation of its directrix is X = 1

$\therefore$ Required directrix is $x = \frac{3}{2}$

Q.16 [64]

Let k = 64

$\therefore y = x^2 - 2 \times 8x + 64$

$\Rightarrow y = (x - 8)^2$

$\Rightarrow$ It has vertex on x –axis

Q.17 **[4.8]**

Since, the semi latusrectum of a parabola is the HM of segments of a focal chord.

$\therefore$ Semilatusrectum $= \frac{2SP.SQ}{SP + SQ}$

$= \frac{2 \times 3 \times 2}{3 + 2} = \frac{12}{5}$

$\therefore$ Latusrectum of the parabola $= \frac{24}{5}$

Q.18 **[8]**

Given curve is $y^2 = 16h$ Let any point be (h, k) But 2h = k, then $k^2 = 16h$

$\Rightarrow 4h^2 = 16h$

$\Rightarrow h = 0, h = 4$

$\Rightarrow k = 0, k = 8$

$\therefore$ Points are (0, 0), (4, 8)

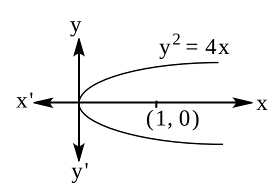
Hence, focal distance are respectively

0 + 4 = 4, 4 + 4= 8 [$\because$ focal distance = h + a]

Q.19 **[1]**

Given curve is $y^2 = 4x$

Also, point (1, 0) is the focus of the parabola. It is clear from the graph that only normal is possible



Q.20 **[0.5]**

We know that, if three normals to the parabola $y^2 = 4ax$ through point(h, k), then h > 2a

Here, h = a and $a = \frac{1}{4}$

$\therefore a > 2.\frac{1}{4} \Rightarrow a > \frac{1}{2}$

ELLIPSE

Q.21 (0007)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(1)$$

$\frac{x}{7}+\frac{y}{2}=1$ meets x-axis at A (7, 0), line $\frac{x}{3}-\frac{y}{5}=1$ meets y-axis at B (0, –5).

(1) passes through A & B

$\Rightarrow \frac{49}{a^2}+0=1,\ 0+\frac{25}{b^2}=1$

$\Rightarrow a^2 = 49, b^2 = 25; b^2 = a^2 (1– e^2)$

$\Rightarrow 25 = 49(1 – e^2)$

$\Rightarrow e^2=\frac{24}{49} \Rightarrow e=\frac{2\sqrt{6}}{7}$

Q.22 [0004]

Four**.** For example from the centre of ellipse, axes of ellipse are normals.

Q.23 [6.67]

$(x-2)^2+(y+3)^2=\left(\frac{1}{2}\right)^2\left(\frac{3x-4y+7}{5}\right)^2$ is an ellipse, whose focus is (2, –3), directrix 3x – 4y + 7 = 0 and eccentricity $\frac{1}{2}$.

Length of the perpendicular from the focus to the directrix is $\frac{3\times2-4\times(-3)+7}{5}=5$

so that $\frac{a}{e}-ae=5 \Rightarrow 2a-\frac{a}{2}=5 \Rightarrow a=\frac{10}{3}$

So length of the major axis is $\frac{20}{3}$

Q.24 [0002]

Since tangent from $(\lambda, 3)$ are at right angles.

So, this point lies on director circle.

i.e. $x^2 + y^2 = a^2 + b^2$

$\therefore \quad \lambda^2+9=9+4 \quad \Rightarrow \quad \lambda=\pm 2$

Q.25 (3.6)

$4 = 9 (1 – e^2) \Rightarrow e = \sqrt{5}/3$

Distance between the directrices = $\frac{2a}{e}=\frac{2\times3\times3}{\sqrt{5}}=\frac{18}{\sqrt{5}}$

Q.26 (10)

The sum of distances of P from the foci = 2a = 2 × 5 = 10.

Q.27 [0003]

Since $3.3^2+5.5^2–32>0$, the point (3,5) lies outside the first ellipse. Also $25.3^2+ 9.5^2 – 450 = 0$, the point (3, 5) lies on the second ellipse. Hence the number of tangents that can be drawn

= 2 + 1 = 3.

Q.28 [0.89]

Equation of any tangent to $y^2 = 4ax$ is

$y = mx+\frac{a}{m} \Rightarrow m^2x – my + a = 0$

Comparing it with the given tangent 2x + 3y – 1 = 0, we find

$\frac{m^2}{2}=\frac{-m}{3}=\frac{a}{-1} \Rightarrow m=\frac{-2}{3}$ and $a=\frac{m}{3}=-\frac{2}{9}$

Hence the length of the latus rectum = 4a = $\frac{8}{9}$ ignoring the negative sign for length.

Q.29 [0512]

$y^2 = 8x$

Let $P(t_1^2, 4t)$ & $a(t_2^2, 4t_1)$

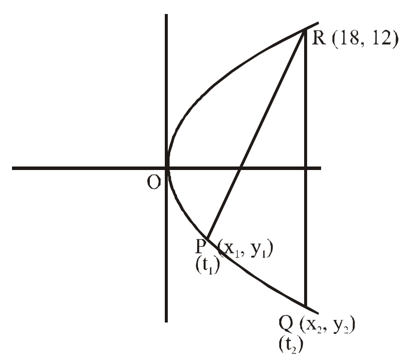
& Normal at P & Q intersect R(18, 12)

Let $R(18, 12) = (2t_3^2, 4t_3)$

$\Rightarrow t_3 = 3$

$\therefore \quad t_3=-t-\frac{2}{t}$ [Point of again intersection by normal to the parabola]

$3=-t-\frac{2}{t}$



$t^2 + 3t + 2 = 0$

$\Rightarrow t_1 = –1 ;\ t_2 = –2$

Hence P(2, –4) & Q(8, –8)

$\therefore$ a = 2; b = –4; c = 8; d = 8

abcd = 512 **Ans.**

Q.30 [0001]

$y^2 = 4x \qquad ……(1)$

$y = mx – x^3 – 2m$

Let P(h, k) is on this normal

$\Rightarrow k = mh – m^3 – 2m$

$\Rightarrow m^3 + m(2-h) + k = 0$ → m_1, m_2, m_3

……(2)

If three normals at (h, k)

$m_1 + m_2 + m_3 = 0$

$m_1 m_2 m_3 = -k$

& $m_1 m_2 = \alpha \Rightarrow m_3 = \frac{-k}{\alpha}$

$m_3 = \frac{-k}{\alpha}$ is a root of the eq. (2),

$\therefore \frac{-k^3}{\alpha^3} - \frac{k}{\alpha}(2-h) + k = 0$

$\Rightarrow k = 0; \frac{-k^2}{\alpha^3} - \frac{(2-h)}{\alpha} + 1 = 0$

$\Rightarrow \frac{k^2}{\alpha} = \frac{h+2-\alpha}{\alpha}$

$\Rightarrow k^2 = \alpha^2(h+2-\alpha)$ ……(4)

Eq. (1) & (4) are identical

$\therefore \alpha = 2$

HYPERBOLA

Q.31 [0002]

Product of perpendiuclars drawn from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to asymptotes is $\frac{a^2b^2}{a^2+b^2}$.

Given hyperbola $\frac{x^2}{2} - \frac{y^2}{1} = 1$

$\therefore$ Product $= \frac{2}{3}$

Hence k = 2.

Q.32 **(0002)**

$\frac{9}{a^2} - \frac{4}{b^2} = 1$

and $\frac{289}{a^2} - \frac{144}{b^2} = 1$….. (3). Solving (2) and (3), we get $a^2 = 1, b^2 = \frac{1}{2} \Rightarrow 2a = 2$.

$\Rightarrow$ Length of transverse axis is 2a = 2.

Q.33 **(0001)**

The eccentricity e_1 of the given hyperbola is obtained from

$b^2 = a^2(e_1^2 - 1)$ .....(i)

The eccentricity e_2 of the conjugate hyperbola is given by

$a^2 = b^2(e_2^2 - 1)$ .....(ii)

Multiply (i) and (ii), we get

$1 = (e_1^2 - 1)(e_2^2 - 1) \Rightarrow 0 = e_1^2e_2^2 - e_1^2 - e_2^2$

$\Rightarrow e_1^{-2} + e_2^{-2} = 1$

Q.34 (0004)

On eliminating ybetween the line and hyperbola we get

$$25x^2 - 9\left(\frac{45-25x}{12}\right)^2 = 225$$

which, on simplifying becomes

$x^2 - 10x + 25 = 0$

$\Rightarrow x = 5$

Hence $y = -\frac{20}{3}$

Q.35 [0009]

The hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Let P be $(4\sec\theta, 3\tan\theta)$.

Now the line x = 4 $\sec\theta$ intersects the asymptote $y = \frac{3}{4}x$ at $Q(4\sec\theta, 3\sec\theta)$ and the asymptote $y = -\frac{3}{4}x$ at $R(4\sec\theta, -3\sec\theta)$. So,

$PQ = 3|\sec\theta - \tan\theta|$ and $PR = 3|\sec\theta + \tan\theta|$

$\therefore PQ.PR = 9$

Q.36 (0000)

Equation of tangent $\perp r$ to 5x+2y-3=0 is 2x-5y+k=0

By using $c^2=a^2m^2-b^2$ then we get k^2 is negative which is not possible. so total no of tangents to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is zero.

Q.37 [1.5]

The hyperbola is $\frac{x^2}{2} - \frac{y^2}{3} = 1$

Its tangent $y - mx = \pm\sqrt{2m^2 - 3}$ passes through P(h, k) then $k - mh = \pm\sqrt{2m^2 - 3}$

$\Rightarrow (h^2-2)m^2 - 2hkm + k^2 + 3 = 0$

If slope of these tangents be m_1 and m_2 then $m_1m_2 = 1$

$\Rightarrow \frac{k^2+3}{h^2-2} = 1$ or $h^2 - k^2 = 5p$

So locus of P is $x^2 - y^2 = 5$

Q.38 [0001]

Equation of normal at point t i.e., (ct, c/t) is

$y - xt^2 = \frac{c}{t}(1 - t^4)$ ..... (1)

It meets the curve again at t_1 then $(ct_1, c/t_1)$ must satisfy (1)

$$\Rightarrow \frac{c}{t_1} - ct_1t^2 = \frac{c}{t}(1-t^4) \Rightarrow \frac{1}{t_1} - t_1t^2 = \frac{1}{t} - t^3$$

$$\Rightarrow \frac{1}{t_1} - \frac{1}{t} + t^2(t - t_1) = 0$$

$$\Rightarrow \frac{(t-t_1)}{tt_1}(1 + t^3t_1) = 0$$

Clearly $t \neq t_1 \Rightarrow t^3t_1 + 1 = 0$.

Q.39 (1.5)

Lines joining origin to the points of intersecting of the line $x\sqrt{3} + y = 2$ and the curve $y^2 - x^2 = 4$ are given by

$$y^2 - x^2 = 4\left(\frac{x\sqrt{3}+y}{2}\right)^2 \Rightarrow y^2 - x^2 = (x\sqrt{3}+y)^2$$

$$\Rightarrow 4x^2 + 2xy\sqrt{3} = 0 \Rightarrow 4x^2 + 2xy\sqrt{3} + 0.y^2 = 0$$

Comparing with $ax^2 + 2hxy + by^2 = 0$, we get a = 4,

$$b = 0, h = \sqrt{3};\ \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2h}{a} \text{ as } b = 0$$

$$\tan\theta = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Q.40 [0007]

We must have ae = a'e'

$$\Rightarrow \quad 4e = \frac{12}{5}e'$$

Here $b^2 = 16(1 - e^2)$

and $$\frac{81}{25} = \frac{144}{25}\left[(e')^2 - 1\right] \Rightarrow e' = \frac{15}{12} \text{ and } e = \frac{3}{4}$$

PREVIOUS YEAR'S

MHT CET
CIRCLE

Q.1 (3)

Q.2 (2)

Q.3 (2)

Q.4 (1)

Q.5 (3)

Q.6 (2)

Q.7 (2)

Q.8 (3)

Q.9 (3)

Q.10 (1)

Q.11 (4)

Q.12 (1)

Q.13 (3)

Q.14 (3)

Q.15 (2)

Q.16 (1)

Q.17 (2)

Q.18 (2)

Q.19 (2)

Q.20 (4)

Since, $\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{2}{3}$

$$\therefore \quad \frac{\sqrt{x_1^2 + y_1^2 + 4x_1 + 3}}{\sqrt{x_1^2 + y_1^2 - 6x_1 + 5}} = \frac{2}{3}$$

$$\Rightarrow 9x_1^2 + 9y_1^2 + 36x_1 + 27 - 4x_1^2 - 4y_1^2 + 24x_1 - 20 = 0$$

$$\Rightarrow 5x_1^2 + 5y_1^2 + 60x_1 + 7 = 0$$

$\therefore$ Locus of point (x,y) is

$5x^2 + 5y^2 + 60x + 7 = 0$

Q.21 (4)

The centres of given circles are $C_1(-3, -3)$, $C_2(6,6)$ and radii are

$$r_1 = \sqrt{9+9+0} = 3\sqrt{2}, \quad r_2 = \sqrt{36+36+0} = 6\sqrt{2}$$

respectively.

Now $C_1C_2 = \sqrt{(6+3)^2 + (6+3)^2} = 9\sqrt{2}$

and $r_1 + r_2 = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$

Here $C_1C_2 = r_1 + r_2$

So, both circles touch each other externally.

Q.22 (1)

Centres adn radii of the given circles are $C_1(0,0)$, $r_1 = 3$ and C_2 (–a, – 1)

$$r_2 = \sqrt{\alpha^2 + 1 - 1} = |\alpha|$$

Since, two circles touch internally.

$\therefore \quad C_1C_2 = r_1 - r_2$

$\Rightarrow \quad \sqrt{\alpha^2 + 1^2} = 3 - |\alpha|$

$\Rightarrow \quad \alpha^2 + 1 = 9 + \alpha^2 - 6|\alpha|$

$\Rightarrow \quad 6|\alpha| = 8 \Rightarrow |\alpha| = \frac{4}{3}$

$\therefore \quad \alpha = \pm \frac{4}{3}$

Q.23 (4)

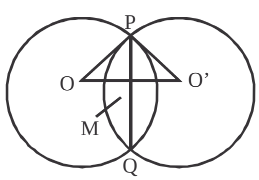
Given, equation of circles are $x^2 + y^2 - 4y = 0$

and $x^2 + y^2 - 8x - 4y + 11 = 0$

$\therefore$ Equation of chords is

$x^2 + y^2 - 4y - (x^2 + y^2 - 8x - 4y + 11) = 0$

$\Rightarrow \; 8x - 11 = 0$



So, centre and radius of first circle are O (0,2) and OP = r = 2.

Now, perpendicular distance from O(0,2) to the line 8x –11 is

$$d = OM = \frac{|8 \times 0 - 11|}{\sqrt{8^2}} = \frac{11}{8}$$

In ΔOMP, $PM = \sqrt{OP^2 - OM^2}$

$$= \sqrt{2^2 - \left(\frac{11}{8}\right)^2} = \sqrt{4 - \frac{121}{64}}$$

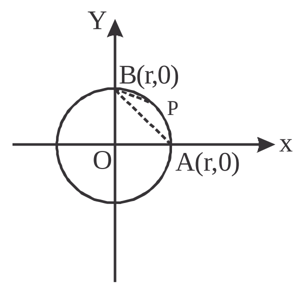
$$= \sqrt{\frac{256-121}{64}} = \frac{\sqrt{135}}{8}\text{cm}$$

$\therefore$ Length of chord PQ = 2PM

$$= 2 \times \frac{\sqrt{135}}{8} = \frac{\sqrt{135}}{4}\text{cm}$$

Q.24 (2)

Let OA as X-axis , A = (r,0) and any point P on the circle is (r cos θ, r sin θ) . If (x,y) is the centroid of ΔPAB, then



$3x = r\cos\theta + r + 0$ ...(i)

and $3y = r\sin\theta + 0 + r$ ...(ii)

From Eqs. (i) and (ii),

$\therefore (3x - r)^2 + (3y - r)^2 = r^2$

Hence, locus of P is a circle.

Q.25 (2)

Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

$\therefore$ Coordinates of centre of the circle = (–g, –f)

As, the circle passes through the origin,

$0^2 + 0^2 + 2g(0) + 2f(0) + c = 0$

$\Rightarrow c = 0$

Given, centre lies on y = x

$\Rightarrow$ Coordintes of the centre are (–g, –g).

Given, two circles, $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 - 4x - 6y + 10 = 0$ are orthogonal.

Therefore, $2 \times g \times (-2) + 2 \times f \times (-3) = c + 10$

$[\because 2g_1g_2 + 2f_1f_2 = c_1 + c_2]$

$\Rightarrow -4g - 6f = c + 10$

$\Rightarrow -10g = c + 10 \quad [\because g = f]$

$\Rightarrow -10g = 10 \quad [\because c = 0]$

$\Rightarrow g = -1 \therefore f = -1$

Hence, equation of circle is

$x^2 + y^2 - 2x - 2y = 0.$

PARABOLA

Q.26 (4)

Q.27 (2)

Q.28 (2)

Q.29 (2)

Q.30 (Bouns)

Q.31 (4)

Q.32 (3)

Q.33 (3)

Q.34 (1)

Q.35 (2)

Q.36 (3)

Q.37 (1)

Q.38 (3)

Q.39 (4)

Q.40 (Bonus)

Q.41 (2)

Q.42 (1)

Q.43 (4)
Q.44 (2)

Q.45 (1)

ELLIPSE
Q.46 (2)

Q.47 (4)

Q.48 (1)

Q.49 (3)

Q.50 (2)

Q.51 (1)

Q.52 (4)
Q.53 (2)

Q.54 (3)

HYPERBOLA
Q.55 (2)

Q.56 (3)

Q.57 (1)

Q.55 (1)

Q.59 (2)

Q.60 (2)

Q.61 (3)

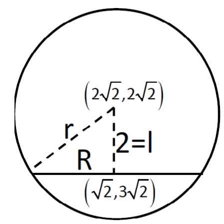
Q.62 (4)

Q.63 (1)

Q.64 (2)

JEE-MAIN
CIRCLE
Q.1 (10)

Radius of circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$

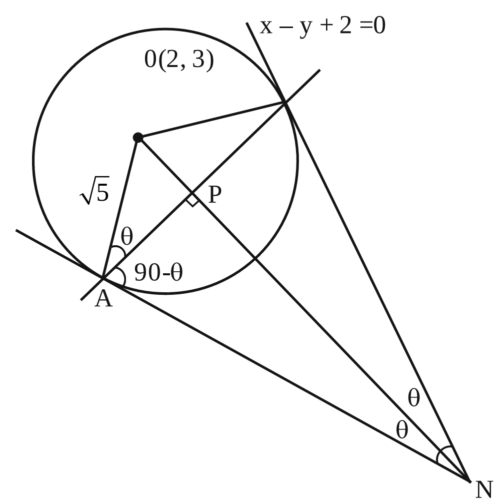


$l = \sqrt{2+2} = 2$

$R = \sqrt{2+18-14} = \sqrt{6}$

$\therefore r^2 = R^2 + 4$
$= 6 + 4$
$= 10$

Q.2 (3)



$OP = \dfrac{3}{\sqrt{2}}$

$AP = \sqrt{OA^2 - OP^2}$

$= \frac{1}{\sqrt{2}}$

$\tan\theta = 3$

$\therefore \sin\theta = \frac{3}{\sqrt{10}} = \frac{AP}{AN}$

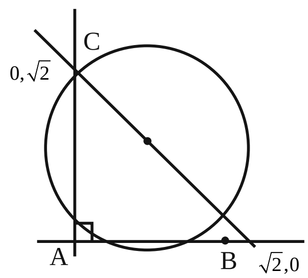
$\Rightarrow AN = \frac{\sqrt{5}}{3} = BN$

Area of Δ ANB $= \frac{1}{2} . (AN^2)\sin 2\theta = \frac{1}{6}$

Q.3 (0)

$x^2 - \sqrt{2}(x+y) + y^2 = 0$

As C = 0, Circle P.T. origin



Centre $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, r = 1

$\therefore$ BC = 2

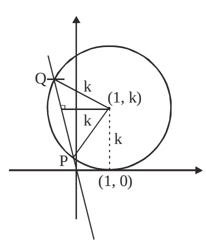
Given $AB = \sqrt{2}$

From right angle ΔABC

$AC = \sqrt{2}$

area of $\Delta ABC = \frac{1}{2} \cdot AB \cdot AC = \frac{1}{2} \cdot \sqrt{2} \cdot \sqrt{2} = 1$

Q.4 **(7)**



$2\sqrt{r^2 - p^2} = 2$

$\sqrt{r^2 - \left(\frac{1+k}{\sqrt{2}}\right)^2} = 1$

$K^2 - \left(\frac{1+k}{\sqrt{2}}\right)^2 = 1 \qquad (\because r = k)$

$2k^2 - (k^2 + 2k + 1) = 2$

$k^2 - 2k - 3 = 0$

$(k-3)(k+1) = 0$

$k = 3, -1$

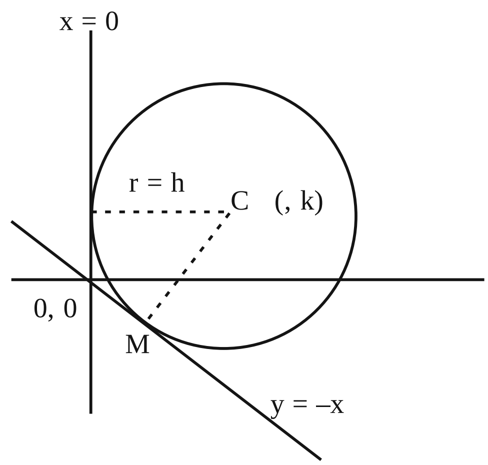
$k = 3$

$r = 3$

$h = 1$

$h + k + r = 7$

Q.5 **(4)**



CM = r = h

$\frac{|h+k|}{\sqrt{2}} = h$

$h^2 + k^2 + 2hk = 2h^2$

$h^2 - k^2 - 2hk = 0$

$x^2 - y^2 = 2xy$

Q.6 (2)

Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-(2x+2g)}{(2y+2f)}$

Comparing with $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$

$\Rightarrow$ b = 0, a = –2 , c = 2

$\Rightarrow -2g = -2 \Rightarrow g = 1 \qquad 2f = -2$

$\qquad f = -1$

Now circle will be

$x^2 + y^2 + 2x - 2y + c = 0$

its passes through (2, 5)

which will give c = –23

so circle will be $x^2 + y^2 + 2x - 2y - 23 = 0$

centre C = (–1, 1)

and radius 5

Now P is (11, 6)

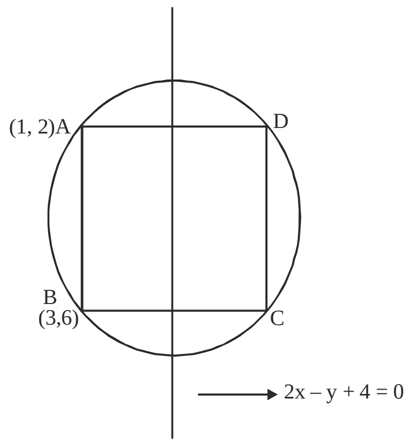
So minimum distance of P from circle will be

$\sqrt{(11+1)^2+(6-1)^2}-5$

$= 13 - 5$

$= 8$

Q.7 (16)



Eq. of line AB

$y = 2x$

Slope of AB = 2

Slope of given diameter = 2

So the diameter is paralled to AB

Distance between diameter and line AB

$-\left(\dfrac{4}{\sqrt{2^2+1^2}}\right)=\dfrac{4}{\sqrt{5}}$

Thus $BC = 2 \times \dfrac{4}{\sqrt{5}} = \dfrac{8}{\sqrt{5}}$

$AB = \sqrt{(1-3)^2+(2-6)^2} = \sqrt{20} = 2\sqrt{5}$

$\text{Area} = AB \times BC = \dfrac{8}{\sqrt{5}} \times 2\sqrt{5} = 16$

Q.8 **(4)**

$C : 4x^2 + 4y^2 - 12x + 8y + k = 0$

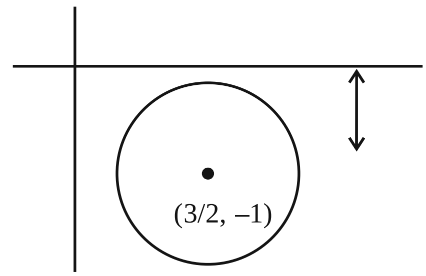
$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\dfrac{k}{4}\right) = 0$

Center $\left(\dfrac{3}{2},-1\right)$; $r = \dfrac{\sqrt{13-k}}{2} \Rightarrow k \le 13$ .... (1)

(i) Point lies on or inside circle C

$\Rightarrow S_1 \le 0 \Rightarrow k \le \dfrac{92}{9}$ .... (2)

(ii) C lies in 4th quadrant



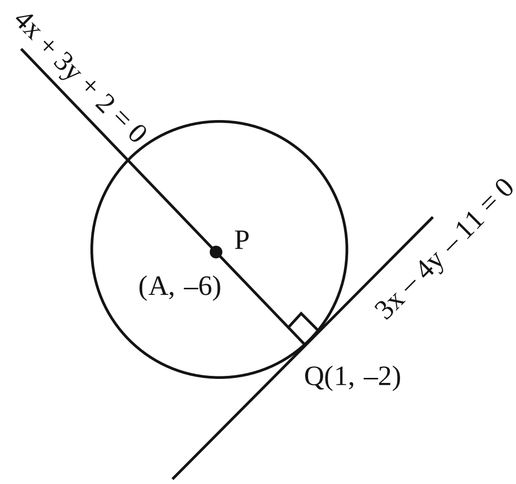
$r < 1$

$\Rightarrow \dfrac{\sqrt{13-k}}{2} < 1$

$\Rightarrow k > 9$ ... (3)

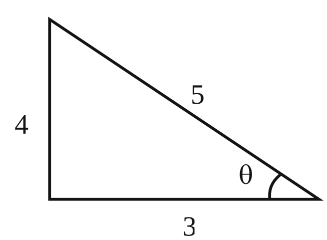
Hence $(1) \cap (2) \cap (3) \Rightarrow k \in \left(9, \dfrac{92}{9}\right]$

Q.9 **(11)**



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$\dfrac{x}{-25} = \dfrac{y}{50} = \dfrac{1}{-25}$

$\dfrac{x-1}{\cos\theta} = \dfrac{y+2}{\sin\theta} = \pm 5$

$$y = -2 + 5\left(-\frac{4}{5}\right) = -6$$

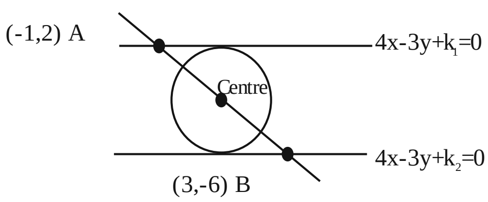
$$x = 1 + 5\left(\frac{3}{5}\right) = 4$$

Req. distance

$$= \left|\frac{5(4) - 12(-6) + 51}{13}\right|$$

$$= \left|\frac{20 + 72 + 51}{13}\right| = \frac{143}{13} = 11$$

Q.10 (3)



L_1 : $4x-3y + k_1 = 0$ (put A in L_1)
$-4 - 6 + k_1 = 0$
$k_1 = 10$

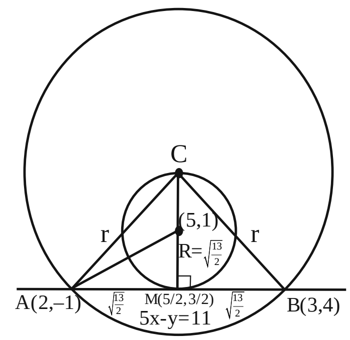
L_2: $4x-3y+k_2=0$
Put B in L_2
$12 + 18 + k_2 = 0$
$k_2 = -30$

distance between L_1 and L_2 = diameter $= \left|\frac{40}{\sqrt{4^2+3^2}}\right| = 8$

$\therefore$ radius = 4
Centre is mid point of AB $\Rightarrow$ center is (1, -2)
$\therefore$ circle is $(x-1)^2 + (y+2)^2 = 16$ Ans.

Q.11 (7)
Equation of circle will be
$(2x^2 - rx + p) + (2y^2 - 2sy - 2q) = 0$
$= 2(x^2 + y^2) - rx - 2sy + p - 2q = 0$ ...(1)
Compairing with
$2(x^2 + y^2) - 11x - 14y - 22 = 0$ ...(2)
$r = 11, \quad s = 7, \quad p - 2q = -22$
$\therefore 2r + s - 2q + p = 22 + 7 - 22 = 7$

Q. 12 (2)



$$AB = \sqrt{26}$$

$$r^2 = CM^2 + AM^2$$

$$= \left(2 \times \sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2$$

$$r^2 = \frac{65}{2}$$

Q.13 (3)
Tangent at O(0, 0)
$-(x + 0) - 2(y + 0) = 0$
$\Rightarrow x + 2y = 0$
Tangent at $P(1+\sqrt{5},\ 2)$

$$x(1+\sqrt{5}) + y \cdot 2 - (x + 1 + \sqrt{5}) - 2(y + 2) = 0$$

Put $x = -2y$

$$-2y(1+\sqrt{5}) + 2y + 2y - 1 - \sqrt{5} - 2y - 4 = 0$$

$$-2\sqrt{5}y = 5 + \sqrt{5} \Rightarrow y = \left(\frac{\sqrt{5}+1}{2}\right)$$

$$Q\left(\sqrt{5}+1, -\frac{\sqrt{5}+1}{2}\right)$$

Length of tangent OQ $= \dfrac{5+\sqrt{5}}{2}$

Area $= \dfrac{RL^3}{R^2 + L^2}$

$R = \sqrt{5}$

$$= \frac{\sqrt{5} \times \left(\frac{5+\sqrt{5}}{2}\right)^3}{5 + \left(\frac{5+\sqrt{5}}{2}\right)^2}$$

$$= \frac{\sqrt{5}}{2} \times \frac{(125+75+75\sqrt{5}+5\sqrt{5})}{(20+25+10\sqrt{5}+5)}$$

$$= \frac{5+3\sqrt{5}}{2}$$

Q.14 (816)

Normals are

$$y+2x = \sqrt{11}+7\sqrt{7}$$

$$2y+x = 2\sqrt{11}+6\sqrt{7}$$

Centre of the circle is point of intersection of normals i.e.,

$$\left(\frac{8\sqrt{7}}{3}, \sqrt{11}+\frac{5\sqrt{7}}{3}\right)$$

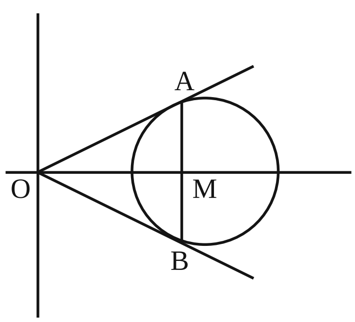
Tangent is $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$

Radius will be $\perp$ distance of tangent from centre i.e.,

$$4\sqrt{\frac{7}{5}}$$

Now, $(5h-8k)^2 + 5r^2 = 816$

Q.15 (2)



$C : (x-2)^2 + y^2 = 1$

Equation of chord AB : $2x = 3$

$OA = OB = \sqrt{3}$

$$AM = \frac{\sqrt{3}}{2}$$

Area of triangle OAB $= \frac{1}{2}(2AM)(OM)$

$$= \frac{3\sqrt{3}}{4} \text{ sq. units}$$

Q.16 (4)

Sn: $|z-3+2i| = \frac{n}{4}$

Tn: $|z-2+3i| = \frac{1}{n}$

$$S_n : (x-3)^2 + (y+2)^2 = \left(\frac{n}{4}\right)^2 \quad \&$$

$$T_n : (x-2)^2 + (y+3)^2 = \left(\frac{1}{n}\right)^2$$

Now $S_1 \cap S_2 = \phi$

$C_1C_2 > r_1 + r_2$

$$\sqrt{(3-2)^2 + (-2+3)^2} > \frac{n}{4} + \frac{1}{n}$$

$$\left(\frac{n}{4}+\frac{1}{n}\right)^2 < 2$$

Case - II $C_1C_2 < |r_1 - r_2|$

$$\sqrt{(3-2)^2 + (-2+3)^2} < \left|\frac{n}{4} - \frac{1}{n}\right|$$

$$\sqrt{2} < \left|\frac{n}{4} - \frac{1}{n}\right|$$

$\Rightarrow$ n has a solution

$\Rightarrow N \in \{1,2,3,4\}$

Q.17 (3)

C: (α,β) & radius = r.

S: $(x-\alpha)^2 + (y-\beta)^2 = r^2$

S touches externally S_1

$\Rightarrow CC_1 = r + r_1$

$\alpha^2 + (\beta - 1)^2 = (1 + r)^2$ ...(1)

S touches x-axis

$\Rightarrow$ y coordinates of centre = radius of circle

$\Rightarrow \beta = r$

Put in (1)

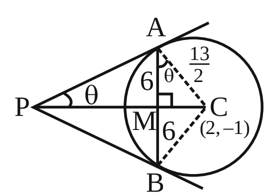
$$\alpha^2 + \beta^2 - 2\beta + x = \alpha + \beta^2 + 2\beta$$

$$\alpha^2 = 4\beta$$

$\Rightarrow$ Locus in $\boxed{x^2 = 4y}$

$$\text{Area} = \frac{2}{3}[PQRS] = \frac{2}{3}[8\times4] = \frac{64}{3}$$

Q.18 (72)



$$\cos\theta = \frac{6}{13} = \frac{12}{13}$$

$$\sin\theta = \frac{5}{13}$$

PM = AMcotθ

$PM = 6\left(\frac{12}{5}\right)$

$\therefore 5(PM) = 72$

Q.19 (12)

Image of centre $c_1 \equiv (1, 3)$ in $x - y + 1 = 0$ is given by

$\frac{x_1 - 1}{1} = \frac{y_1 - 3}{-1} = \frac{-2(1-3+1)}{1^2 + 1^2}$

$\Rightarrow x_1 = 2, y_1 = 2$

$\therefore$ Centre of circle $c_2 \equiv (2, 2)$

$\therefore$ Equation of c_2 be $x^2 + y^2 - 4x - 4y + \frac{38}{5} = 0$

Now radius of c_2 is $\sqrt{4+4-\frac{38}{5}} = \sqrt{\frac{2}{5}} = r$

(radius of $c_1)^2$ = (radius of $c_2)^2$

$\Rightarrow 10 - \alpha = \frac{2}{5} \Rightarrow \alpha = \frac{48}{5}$

$\therefore \Rightarrow \alpha + 6r^2 = \frac{48}{5} + \frac{12}{5} = 12$

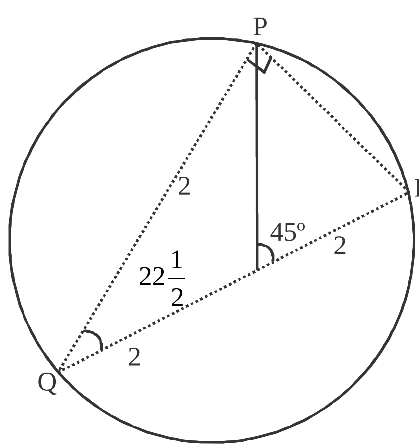
Q.20 (2)

$x^2 + y^2 - x + 2y = \frac{11}{4}$

$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = (2)^2$

Or ΔPQR

$PR = QR \sin 22\frac{1}{2}$



$= 4\sin\frac{\pi}{8}$

$PQ = QR \cos 22\frac{1}{2}$

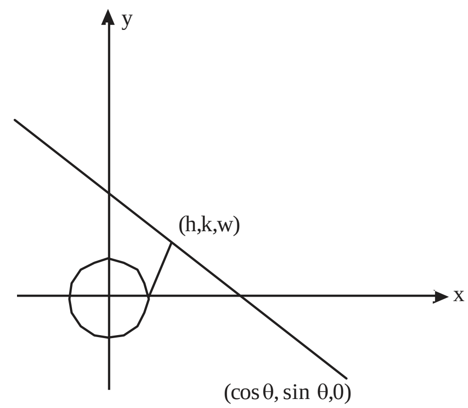
$= 4\cos\frac{\pi}{8}$

As $\Delta PQR = \frac{1}{2} PR \times PQ$

$= \frac{1}{2}\left(4\sin\frac{\pi}{8}\right)\left(4\cos\frac{\pi}{8}\right)$

$= 4\sin\frac{\pi}{4} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Q.21 (2)



$\frac{h - \cos\theta}{2} = \frac{k - \sin\theta}{3} = \frac{w - 0}{1}$

$= \frac{-1(2\cos\theta + 3\sin\theta - 6)}{14}$

$h = \frac{-2(2\cos\theta + 3\sin\theta - 6)}{14} + \cos\theta$

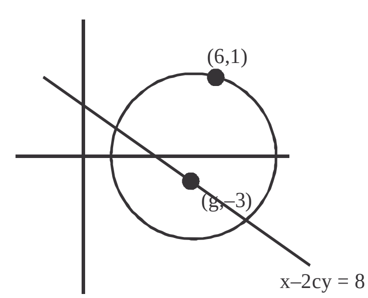
$= \frac{10\cos\theta - 6\sin\theta + 12}{14}$

$k = \frac{5\sin\theta - 6\cos\theta + 18}{14}$

Elementry $\sin\theta$ and $\cos\theta$

$(5h + 6k - 12)^2 + 4(3h + 5k - 9)^2 = 1$

Q.22 (4)



$\because$ Centre (g-3) lies on $x - 2cy = 8$

$\Rightarrow g - 2c(-3) = 8$

$g + 6c = 8$ ....(1)

$\because$ (6,1) lies on circle
$\Rightarrow (6)^2 + (1)^2 - 2g(6) + 6(1) - 19c = 0$
$\Rightarrow 37 + 6 - 12g - 19c = 0$
$\Rightarrow 12g + 19c = 43$ ....(2)
On solving (1) & (2), we get
c = 1, g = 2
Now equation of circle becomes
$x^2 + y^2 - 4x + 6y - 19 = 0$ ....(3)
Intercept on x-axis, put y = 0 in (3)
$\Rightarrow x^2 - 4x - 19 = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{16+76}}{2} = \frac{4 \pm \sqrt{92}}{2} = \frac{4 \pm 4\sqrt{23}}{2}$$

$$= 2 \pm 2\sqrt{23}$$

Q.23 (1)

$x^2 - 4x - 6 = 0$ ⟨ x_1, x_2 $\quad y^2 + 2y - 7 = 0$ ⟨ y_1, y_2

equation of circle
$x^2 + y^2 - 4x + 2y - 13 = 0$
$a = -2, b = 1, c = -13$
$\Rightarrow a + b - c = -2 + 1 + 13 = 12$ Ans.

Q.24 **[1]**
$C1 : (x-2)^2 + y^2 = 4$
& $y = 2x$
for A
$(x-2)^2 + 4x^2 = 4$
$x^2 + 4 - 4x + 4x^2 = 4$

$$x = 0, \frac{4}{5} \Rightarrow y = 0, \frac{8}{5}$$

$$A = \left(\frac{4}{5}, \frac{8}{5}\right)$$

$$m_{OA} = \frac{8/5}{4/5} = 2$$

$$m_{PQ} = \frac{-1}{2}$$

trngent at A

$$y - \frac{8}{5} = -\frac{1}{2}\left(x - \frac{4}{5}\right)$$

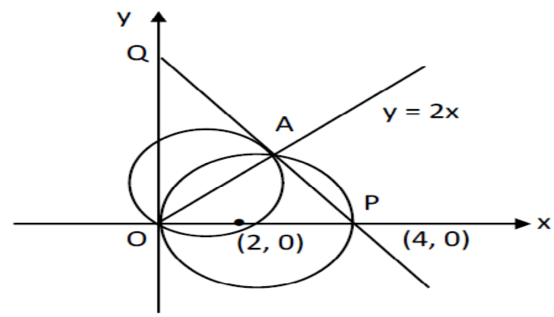
$$2y - \frac{16}{5} = -x + \frac{4}{5}$$

$$\boxed{2y + x = 4}$$

$p \equiv (4,0) \qquad Q \equiv (0,2)$

$$Ap = \sqrt{\frac{320}{25}} \qquad AQ = \sqrt{\frac{20}{25}}$$

$$\frac{AQ}{AP} = \sqrt{\frac{20}{320}} = \frac{1}{4}$$



Q.25 [25]

$C_1(-3,-4) \quad C_2(\sqrt{3}-3, \sqrt{6}-4)$

$$r_1 = \sqrt{9+16-16} = 3$$

$$r_2 = \sqrt{12 - 6\sqrt{3} + 22 - 8\sqrt{6} + K + 6\sqrt{3} + 8\sqrt{6}}$$

$$r_2 = \sqrt{34+K}$$

$$C_1C_2 = \sqrt{3+6} = 3 = \sqrt{34+K} - 3$$

$34 + K = 36$
$K = 2$
$\therefore r_2 = 6$

$$m_{C_1C_2} = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$$

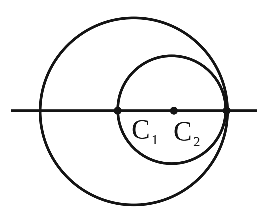
$$\tan\theta = \sqrt{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \text{ and } \sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$$

$\alpha = x_1 + r\cos\theta$
$\beta = y_1 + r\sin\theta$

$$\alpha = -3 - 3\cdot\frac{1}{\sqrt{3}} \Rightarrow \alpha + \sqrt{3} = -3$$

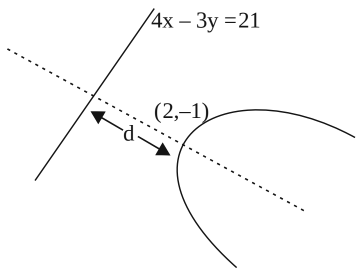
$$\beta = -4 - 3\cdot\frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \beta + \sqrt{6} = -4$$

$$(\alpha + \sqrt{3})^2 + (\beta + \sqrt{6})^2 = 25$$



PARABOLA

Q.26 (2)



$$d = \frac{|8+3-2|}{5} = 2$$

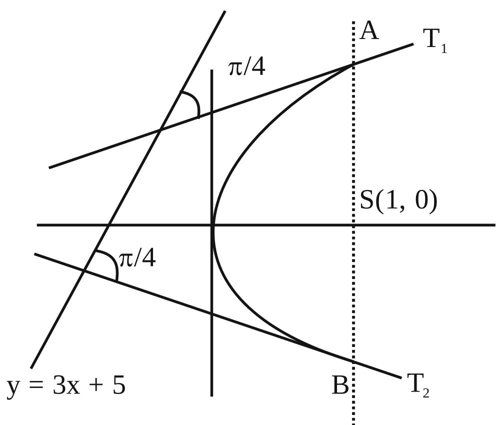
$\therefore$ Length of Latus rectum = 4d

= 8 units

Q.27 (4)

$y^2 = 4ax,\ a > 0$

Line : $4 = 3x + 5$



T_1, T_2 are tangents on parabola

Let slope of tangent = m

$$\tan\frac{\pi}{4} = \left|\frac{3-m}{1+3m}\right|$$

$$\frac{3-m}{1+3m} = 1 \quad \text{and} \quad \frac{3-m}{1+3m} = -1$$

$3 - m = 1 + 3m \qquad \Rightarrow 3 - m = -1 - 3m$

$4m = 2 \qquad \Rightarrow m = -2$

$$m = \frac{1}{2}$$

points $A \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$, $B \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$A \equiv (4a,\ 4a)$, $B\left(\frac{a}{4}, -a\right)$

Given $A(4a, 4a)$, $B\left(\frac{a}{4}, -a\right)$, $S(a, 0)$ are collinear

$$\begin{vmatrix} 4a & 4a & 1 \\ a/4 & -a & 1 \\ a & 0 & 1 \end{vmatrix} = 0$$

$C_1 \to C_1 – C_2$

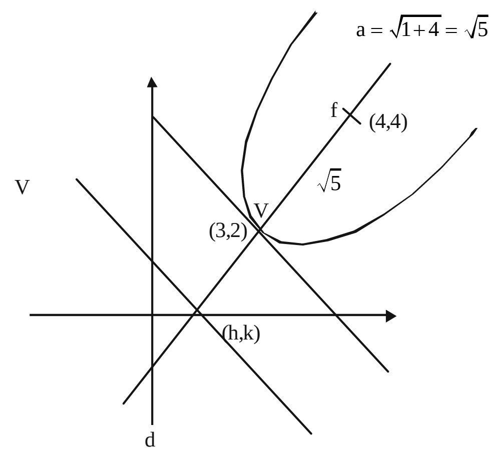
$$\begin{vmatrix} 0 & 4a & 1 \\ 5a/4 & -a & 1 \\ a & 0 & 1 \end{vmatrix} = 0$$

$$-4a\left(\frac{5a}{4} - 0\right) + 1(a^2) = 0$$

$-a^2 + a^2 = 0$

Which is always true for $a \in R$

Q.28 **(10)**



$$3 = \frac{h+4}{2} \qquad \frac{k+4}{2} = 2$$

$6 = h + 4 \qquad k + 4 = 4$

$h = 2 \qquad k = 0$

$$m_{v_f} = \frac{4-2}{4-3} = 2$$

$$m_d = \frac{-1}{2}$$

$$(y–0) = \frac{-1}{2}(x–2)$$

$2y = –x + 2$

$x + 2y – 2 = 0$

$x + 2y = 6$

$$\frac{x-3}{1} = \frac{y-2}{2} = -2\frac{(3+4-6)}{1+4}$$

$$x - 3 = \frac{y-2}{2} = \frac{-2}{5}$$

$$x = 3 - \frac{2}{5} = \frac{13}{5}$$

$y - 2 = \frac{-4}{5}$

$y = 2 - \frac{4}{5} = \frac{6}{5}$

$x + 2y - \lambda = 0$

$$\frac{\left|\frac{13}{5} + \frac{12}{5} - \lambda\right|}{\sqrt{5}} = \sqrt{5}$$

$|5 - \lambda| = 5$

$5 - \lambda = \pm 5$

$(\lambda = 10)$

Q.29 **(3)**

$y = x - x^2$

$$v\left(-\frac{b}{2a}, \frac{-D}{4a}\right) \equiv \left(\frac{-1}{-2}, \frac{-(1)}{4(-1)}\right) = \left(\frac{1}{2}, \frac{1}{4}\right)$$

$y = x - x^2$

$x^2 - x + kx + 4 = 0$

$x^2 + x(k-1) + 4 = 0$

$D = 0 \quad \Rightarrow (k-1)^2 - 4^2 = 0$

$\Rightarrow k - 1 = 4, -4$

$\Rightarrow k = 5, -3$

$\Rightarrow k = 5 \qquad (\because k > 0)$

Now, equation of tangent is $y = 4 + 5x$

$5x + 4 = x - x^2$

$x^2 + 4x + 4 = 0 \Rightarrow x = -2$

So, $P(-2, -6)$, $v\left(\frac{1}{2}, \frac{1}{4}\right)$

$$m_{PV} = \frac{\frac{1}{4} + 6}{\frac{1}{2} + 2} = \frac{25}{4} \times \frac{2}{5} = \frac{5}{2}$$

Q.30 (63)

Vertex and focus of parabola $y^2 = 2x$

are V (0, 0) and S $\left(\frac{1}{2}, 0\right)$ respectively

Let equation of circle be

$(x - h)^2 + (y - k)^2 = 4$

$\because$ circle passes through (0, 0) ...(1)

$h^2 + k^2 - h = \frac{15}{4}$

$\because$ Circle passes through $\left(\frac{1}{2}, 0\right)$

$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$

$\Rightarrow h^2 + k^2 - h = \frac{15}{4}$ ....(2)

On solving (1) and (2)

$4 - h = \frac{15}{4}$

$h = 4 - \frac{15}{4} = \frac{1}{4}$

$K = +\frac{\sqrt{63}}{4}$

$K = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre

$\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + \left(y - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure

$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$

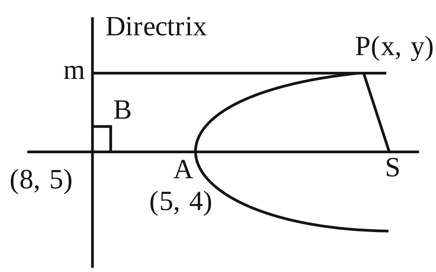
$4\alpha - 8 = \sqrt{63}$

$(4\alpha - 8)^2 = \sqrt{63}$

Q.31 **(4)**

Vertex (5, 4)

Directrix : $3x + y - 29 = 0$



Co-ordinates of B (foot of directrix)

$$\frac{x-5}{3} = \frac{y-4}{1} = -\left(\frac{15+4-29}{10}\right) = 1$$

$x = 8, y = 5$

S = (2, 3) (focus)

Equation of parabola

PS = PM

so equation is

$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

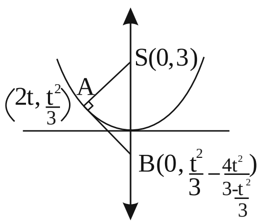
$a + b + c + d + k = 9 - 6 + 134 - 2 - 711 = -576$

Q.32 (4)

$x = 2t, y = \frac{t^2}{3}$

$y = \frac{x^2}{12} \Rightarrow x^2 = 12y$

$$m_{AS}=\frac{\frac{t^2}{3}-3}{2t} \qquad m_{AB}=\frac{2t}{3-\frac{t^2}{3}}$$



Equation of AB: $y-\frac{t^2}{3}-\frac{2t}{3-\frac{t^2}{3}}(x-2t)$

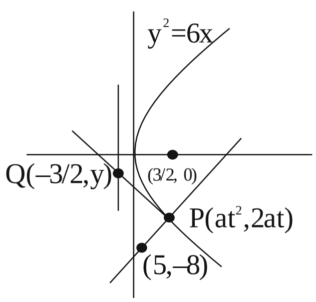
For B, put x=0

$$\therefore y=\frac{t^2}{3}-\frac{4t^2}{3-\frac{t^2}{3}}$$

$$\therefore k=\frac{3+\frac{t^2}{3}+\frac{t^2}{3}-\frac{4t^2}{3-\frac{t^2}{3}}}{3}$$

$$\lim_{t\to 1}k=\frac{3+\frac{2}{3}-\frac{3}{2}}{3}=\frac{13}{18}$$

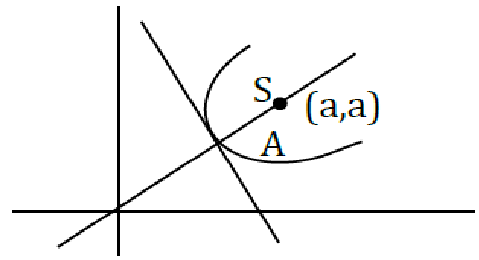
Q. 33 (2)



Equation of normal: $y=-tx+2at+at^2$
since passing through (5,–8), we get t = –2
Co-ordinate of Q : (6, –6)
Eqution of tangent at Q: x+2y+6=0

Put $x=\frac{-3}{2}$ to get $R\left(\frac{-3}{2},\frac{-9}{4}\right)$

Q.34 (3)



Distance from focus to target = A (let)

$$A=\left(\frac{a+a-a}{\sqrt{2}}\right)=\frac{a}{\sqrt{2}}$$

Length of latus secution = 4 A = $\frac{4a}{\sqrt{2}}$=16 (given)

a = $4\sqrt{2}$

Q.35 (9)

$$y^2=-\frac{x}{2}$$

$$y=mx-\frac{1}{8m}$$

This tangent pass through (2, 0)

$m=\pm\frac{1}{4}$ i.e., one tangent is x – 4y – 2 = 0

17r = 9

Q.36 [10]
Circle touches (ii) y-axis
$\Rightarrow$ S: $(x-r)^2+(y-B)^2=r^2$
Circle also touches Parabola

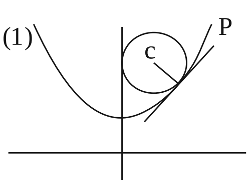
$\Rightarrow$ P: $x^2=\frac{64}{75}(5y-3)$ at A $\left(\frac{8}{5},\frac{6}{5}\right)$

Now a Lies on s = 0

$$\left(\frac{8}{5}-r\right)^2+\left(\frac{8}{5}-\beta\right)^2=r^2 \qquad ...(1)$$

acc. to figure

(1)



$$mT\Big|_A^P.m_{AC}=-1$$

$$\left(2.\frac{8}{5}.\frac{75}{64}.\frac{1}{5}\right)=\left(\frac{\frac{6}{5}-\beta}{\frac{8}{5}-\gamma}\right)=-1$$

$$\left(\frac{6}{5}-\beta\right)=\left(\frac{8}{5}-r\right)\left(\frac{-4}{3}\right)$$

Put in (1)

$$\left(\frac{8}{5}-r\right)^2+\left(-\frac{4}{3}\left(\frac{8}{5}-r\right)\right)^2=r^2$$

$$\left(\frac{8}{5}-r\right)^2+\left[1+\frac{16}{9}\right]=r^2$$

$$\left(\frac{\frac{8}{5}-r}{r}\right)^2+\left(\frac{25}{9}\right)=1$$

$\frac{8-5r}{5r}=\frac{3}{5}$ & $\frac{8-5r}{5r}=\frac{-3}{5}$

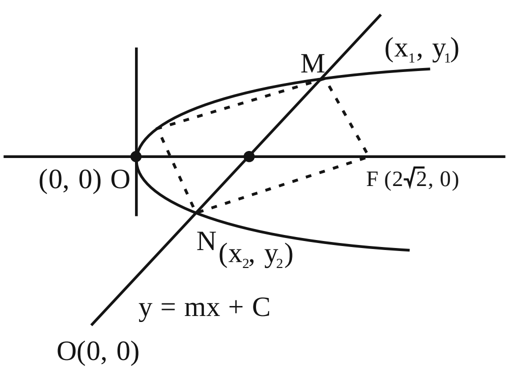
$40-25r=15r$ & $40-25r=-15r$

$r_1=1$ $\quad 40 = 10r$

$\Rightarrow \boxed{r_2=4}$

sum of diameter $=2r_1+2r_2=10$ Ans.

Q.37 (2)



$H:\frac{x^2}{4}-\frac{y^2}{4}=1$

Focus (ae, 0)

$F(2\sqrt{2},0)$

Line L : $y=mx+c$ pass (1, 0)

$o=m+c$ ... (1)

Line L is tangent to Hyperbola. $\frac{x^2}{4}-\frac{y^2}{4}=1$

$c=\pm\sqrt{a^2m^2-b^2}$

$c=\pm\sqrt{4m^2-4}$

From (1)

$-m=\pm\sqrt{4m^2-4}$

Squaring

$m^2=4m^2-4$

$4=3m^2$

$\boxed{\frac{2}{\sqrt{3}}=m}$ (as $m>0$)

$c=-m$

$c=\frac{-2}{\sqrt{3}}$

$y=\frac{2x}{\sqrt{3}}-\frac{2}{\sqrt{3}}$

$y^2=4x$

$\Rightarrow\left(\frac{2x-2}{\sqrt{3}}\right)^2=4x$

$\Rightarrow x^2+1-2x=3x$

$\Rightarrow x^2-5x+1=0$

$y^2=4\left(\frac{\sqrt{3}y+2}{2}\right)$

$y^2=2\sqrt{3}y+4$

$\Rightarrow y^2-2\sqrt{3}y-4=0$

Area

$$\left|\frac{1}{2}\begin{vmatrix}0 & x_1 & 2\sqrt{2} & x_2 & 0\\ 0 & y_1 & 0 & y_2 & 0\end{vmatrix}\right|$$

$$=\left|\frac{1}{2}\left[-2\sqrt{2}y_1+2\sqrt{2}y_2\right]\right|$$

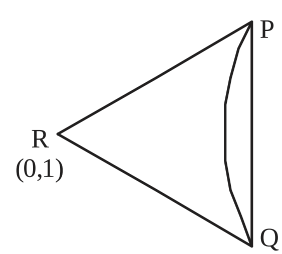
$$=\sqrt{2}\,|y_2-y_1|=(\sqrt{2})\sqrt{12+16}$$

$$=\sqrt{56}$$

$$=2\sqrt{14}$$

Q.38 (2)

$y^2=2x-3$ ...(1)



Equation of chord of contact

PQ : T = 0

$y \times 1 = (x + 0) - 3$

$y = x - 3$ ...(2)

from (1) and (2)

$(x-3)^2 = 2x - 3$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

x = 2 or 6

y = – 1 or 3

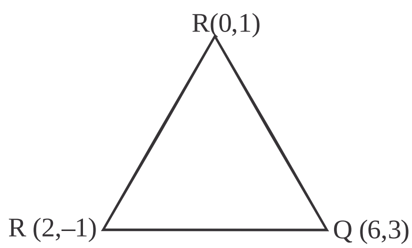
$$m_{PQ} = \frac{4}{4} = 1$$

$$m_{QR} = \frac{2}{6} = \frac{1}{3}$$

$$m_{PR} = \frac{2}{-2} = -1$$

$m_{PQ} \times m_{PR} = -1 \quad \Rightarrow PQ \perp PR$

Orthocentre = P (2,–1)



Q.39 (4)

p(a,b) on $y^2 = 8x$

$\Rightarrow b^2 = 8a$ .....(1)

Tangent at p(a,b) on $y^2 = 8x$ is given by

$yb = 4(x+a)$ .....(2)

(2) Passes through centre of the circle $x^2 + y^2 - 10x - 14y - 65 = 0$

(2) Passes through (5,7)

$\Rightarrow 7b = 4(a + 5)$

$\Rightarrow 7b - 4a = 20$

Putting (1) in (3), we get

$$7b - 4\frac{b^2}{8} = 20$$

$\Rightarrow b^2 - 14b + 40 = 0$

$\Rightarrow b^2 - 4b - 10b + 40 = 0$

$\Rightarrow (b - 4)(b - 10) = 0$

$\Rightarrow b = 4, 10$

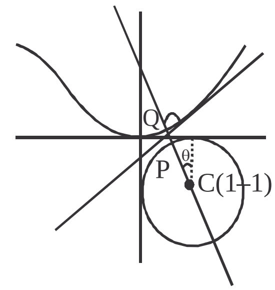
And $a = \frac{b^2}{8} \Rightarrow a = \frac{16}{8}, \frac{100}{8} = 2, \frac{25}{2}$

$\therefore A = 4 \times 10 = 40$

and $B = 2 \times \frac{25}{2} = 25$

A + B = 40 + 25 = 65

Q.40 (3)



$Q = (t, t^2)$

$m_{CQ} = m_{normal}$

$$\frac{t^2 + 1}{t - 1} = -\frac{1}{2t}$$

Let $f(t) = 2t_3 + 3t - 1$

$$f\left(\frac{1}{4}\right) f\left(\frac{1}{3}\right) < 0 \Rightarrow t \in \left(\frac{1}{4}, \frac{1}{3}\right)$$

$P \equiv (1 + \cos(90 + \theta), -1 + \sin(90 + \theta))$

$P = (1 - \sin\theta, -1 + \cos\theta)$

$$m_{normal} = m_{CP} \Rightarrow -\frac{1}{2t} = \frac{\cos\theta}{-\sin\theta} \Rightarrow \tan\theta = 2t$$

$$x = 1 - \sin\theta = 1 - \frac{2t}{\sqrt{1 + 4t^2}} = g(t)(\text{let})$$

$\Rightarrow g'(t) < 0$

g(t) ↓ function

$$t \in \left(\frac{1}{4}, \frac{1}{3}\right)$$

$$\Rightarrow g(t) \in (0.44, 0.485) \in \left(\frac{1}{4}, \frac{1}{2}\right)$$

Q.41 (2)

$y = x^2 \Rightarrow y = mx - am^2$

$$y = mx - \frac{m^2}{4} \qquad ...(1)$$

put in $= -(x-2)^2$

$$mx - \frac{m^2}{4} = -(x - 2)^2$$

$4mx - m^2 = -4(x^2 - 4x + 4)$

$4x^2 + 4x(m - 4) + (16 - m^2) = 0$

D = 0

$16(m - 4)^2 - 16(16 - m^2) = 0$

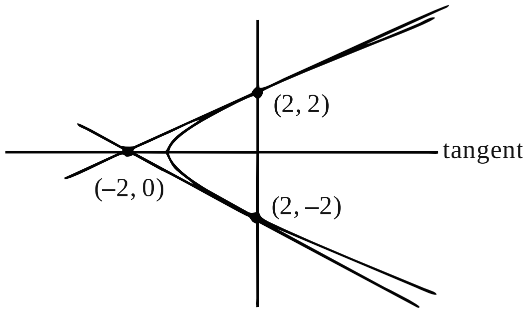
$m^2 - 8m + 16 - 16 + m^2 = 0$

$2m^2 = 8m \Rightarrow m = 0, 4$

put m = 4 in (1)

y = 4x – 4

Q.42 (4)



$y^2 - 2x - 2y = 1$ $\quad X = x + 1$

$(y-1)^2 = 2x + 2$ $\quad Y = y - 1$

$(y-1)^2 = 2(x+1)$ $\quad x = 1, y = 3 \Rightarrow X = 2, Y = 2$

$x = 1, y = -1 \Rightarrow X = 2, Y = -2$

$Y^2 = 2X$

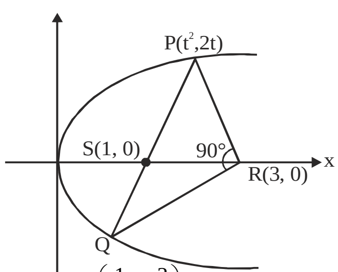
Tangents are $2Y = X + 2$ and $-2Y = X + 2$

$\therefore$ Area $= \frac{1}{2}(4)(4) = 8$

ELLIPSE

Q.43 (2)

PQ is focal chord



$\left(\frac{1}{t^2}, -\frac{2}{t}\right)$

$m_{PR} . m_{PQ} = -1$

$$\frac{2t}{t^2-3} \times \frac{-2/t}{\frac{1}{t^2}-3} = -1$$

$(t^2-1)^2 = 0$

$\Rightarrow t = 1$

P & Q must be end point of latus rectum : P(1, 2) & Q(1, –2)

$$\therefore \frac{2b^2}{a} = 4 \ \& \ ae = 1; b^2 = a^2(1-e^2)$$

$$\therefore a = 1 + \sqrt{2} \ ; e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{e^2} = 3 + 2\sqrt{2}$$

$3 + 2\sqrt{2}$

Q.44 **[2929]**

$P(\alpha, \beta)$ lies on the ellipse $25x^2 + 4y^2 = 1$

$\therefore 25\alpha^2 + 4\beta^2 = 1$

....(1)

Given parabola $y^2=4x$

Equation of tangent in slope form is $y=mx+a/m$

It passes from (α, β)

$$\therefore \beta = \alpha m + \frac{a}{m}$$

$$\beta m = \alpha m^2 + a$$

$$m^2\alpha - \beta m + a = 0$$

from $y^2 = 4x \Rightarrow a=1$ $\quad \therefore m^2\alpha - \beta m + 1 = 0$

m_1 & m_2 are roots & $m_1=m$ & $m_2=4m$

$$m + 4m = \frac{\beta}{\alpha}$$

$5m = \frac{\beta}{\alpha}$ & $4m^2 = \frac{1}{\alpha}$

$m = \frac{\beta}{5\alpha}$ ....(2) $\quad m^2 = \frac{1}{4\alpha}$

....(3)

$\therefore$ from (2) & (3)

$$\left(\frac{\beta}{5\alpha}\right)^2 = \frac{1}{4\alpha}$$

$$\frac{\beta^2}{25\alpha^2} = \frac{1}{4\alpha} \Rightarrow 4\beta^2\alpha = 25\alpha^2, \ \Rightarrow 4\beta^2 = 25\alpha .....(4)$$

From (1) & (4)

$$25\alpha^2 + 25\alpha = 1, \quad \Rightarrow \alpha^2 + \alpha = \frac{1}{25} ....(5)$$

Now $(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = 25(2\alpha + 1)^2 + (4(25\alpha) + 50)^2$

$= 25(2\alpha+1)^2 + (50)^2(2\alpha+1)^2$

$= (2\alpha+1)^2(25+(50)^2) = (4\alpha^2+4\alpha+1)(25+(50)^2)$

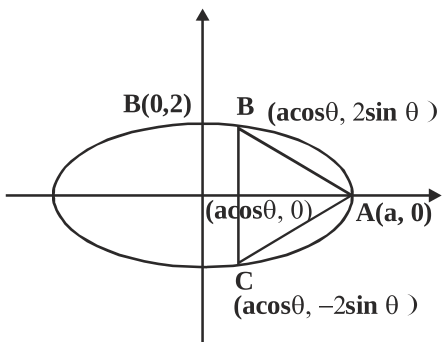
$= (4(\alpha^2+\alpha)+1)(25+(50)^2)$ [from (5)]

$$= \left(4\left(\frac{1}{25}\right)+1\right)\left(25+(50)^2\right)$$

$$= \frac{(4+25)}{25}(25)(1+100)$$

$= (29)(101) = 2929$

Q.45 (1)



Areas of $\Delta = \frac{1}{2}$ [4sinθ] [a – acosθ]

$A = 2a(\sin\theta)(1-\cos\theta)$

$A = 2a[\sin\theta - \frac{1}{2}\sin 2\theta]$

$\frac{dA}{d\theta} = 2a[\cos\theta - \cos 2\theta] = 0$

$\Rightarrow \cos\theta - (2\cos^2\theta - 1) = 0$

$\Rightarrow 2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}, 1$

$\theta = 0, \frac{2\pi}{3}$

$\frac{d^2A}{d\theta^2}\Big|_{\theta=\frac{2\pi}{3}} = 2a[-\frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}]$

$\theta = \frac{2\pi}{3} = -ve$

at $\theta = \frac{2\pi}{3}$ Area in maximum

$A = 2a[\sin\theta][1-\cos\theta]$

$6\sqrt{3} = 2a \cdot \left[\frac{\sqrt{3}}{2}\right]\left[\frac{3}{2}\right]$

$12\sqrt{3} = 3\sqrt{3}\,a$

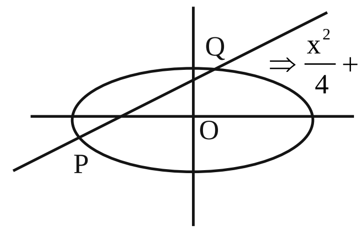
$a = 4$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

$4 = 16(1-e^2)\ 1-e^2 = \frac{1}{4}$

$= e = \frac{\sqrt{3}}{2}$

Q.46 **(1)**



$\Rightarrow \frac{x^2}{4} + \frac{y^2}{2} = 1$

$\frac{x^2}{4} + \frac{(x+1)^2}{2} = 1$

$x^2 + 2x^2 + 4x + 2 = 4$

$3x^2 + 4x - 2 = 0$

$x = \frac{-4 \pm \sqrt{16+24}}{6} = \frac{-4 \pm 2\sqrt{10}}{6}$

$x = \frac{-2+\sqrt{10}}{3}, \frac{-2-\sqrt{10}}{3}$

at $= \frac{-2+\sqrt{10}}{3}, y = \frac{-2+\sqrt{10}}{3} + 1 = \frac{1+\sqrt{10}}{3}$

at $x = \frac{-2-\sqrt{10}}{3}, y = \frac{1-\sqrt{10}}{3}$

$PQ = \sqrt{\left(\frac{2\sqrt{10}}{3}\right)^2 + \left(\frac{2\sqrt{10}}{3}\right)^2}$

$= \sqrt{2 \times \frac{40}{9}} = \frac{2}{3}\sqrt{20}$

$r = \frac{PQ}{2} = \frac{\sqrt{20}}{3}$

$(3r)^2 = 20$

Q.47 **(2)**

tangent line to circle $x^2+y^2 = 12$

$y = mx \pm \sqrt{12+12m^2}$ ...(i)

tangent line to ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$y = mx \pm \sqrt{16m^2 + 9}$ ...(ii)

equation (i) and (ii) are identical

$mx \pm \sqrt{12+12m^2} = mx \pm \sqrt{16m^2+9}$

$\Rightarrow 12 + 12m^2 = 16m^2 + 9$

$\Rightarrow m^2 = \frac{3}{4}$

$12m^2 = \frac{3}{4} \times 12 = 9$

Q.48 **(3)**

Let point on ellipse Q(2cosθ, $\sqrt{2}$ sin θ)

given point P (4, 3)
mid point of P and Q

$$(h, k) = \left(\frac{2\cos\theta+4}{2}, \frac{\sqrt{2}\sin\theta+3}{2}\right)$$

$$\cos\theta = \frac{2h-4}{2}, \sin\theta = \frac{2k-3}{\sqrt{2}}$$

squaring and adding

$$(h-2)^2 + \left(\frac{2k-3}{\sqrt{2}}\right)^2 = 1$$

$$\frac{(x-2)^2}{1} + \frac{\left(y-\frac{3}{2}\right)}{\frac{1}{2}} = 1$$

$$e^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$e = \frac{1}{\sqrt{2}}$$

Q.49 (2)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16}a^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{16\times\frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$$

$$\frac{80}{5a^2} = 1$$

$16 = a^2$
$b^2 = 15$

Q. 50 (4)

$$x^2 + y^2 = \frac{9}{4} \qquad y=4x$$

Equation of tangent in slope form

$$y = mx \pm \frac{3}{2}\sqrt{(1+m^2)} \quad .....(1)$$

$$y = mx + \frac{1}{m} \quad ....(2)$$

compare (1) & (2)

$$\pm\frac{3}{2}\sqrt{(1+m2)} = \frac{1}{m^2}$$

$9m^2(1+m^2) = 4$
$9m^4+9m^2-4=0$
$9m^4+12m^2-3m^2-4=0$
$3m^2(3m^2+4)-(3m^2+4)=0$

$$m^2 = \frac{-4}{3}\text{(Rejected)}$$

$$m^2 = \frac{1}{3} \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

Equation of common tangent

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$$

on X axis y=0
OQ = –3

$$B = |OQ| = 3$$

a=6

$$b^2 = a^2(1-e^2) \Rightarrow e^2 = 1 - \frac{9}{36} = \frac{3}{4}$$

$$e = \frac{2b^2}{a} = \frac{2\times 9}{6} = 3$$

$$\frac{e}{e^2} = \frac{3}{3/4} = 4$$

Q.51 (13)

Ellipse is

$$\frac{x^2}{2} + \frac{y^2}{4} = 1;\ e = \frac{1}{\sqrt{2}};\ S \equiv \left(0, -\sqrt{2}\right)$$

Chord of contact is

$$\frac{x}{\sqrt{2}} + \frac{\left(2\sqrt{2}-2\right)}{4} = 1$$

$\Rightarrow \frac{x}{\sqrt{2}} = 1 - \frac{(\sqrt{2}-1)}{2}$ solving with ellipse

$\Rightarrow y = 0, \sqrt{2}$

$\therefore x = \sqrt{2}, 1$

$P \equiv (1, \sqrt{2}), Q \equiv (\sqrt{2}, 0)$

$\therefore (SP)^2 + (SQ)^2 = 13$

Q.52 (27)

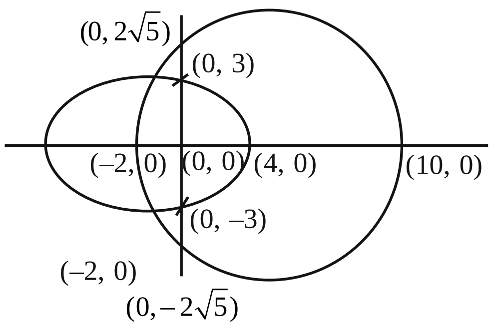
$S : \frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \le 1;\ x,\ y \in \{1, 2, 3, ...\}$

$T : (x-7)^2 + (y-4)^2 \le 36\ x, y \in R$

Let $x - 3 = X : y - 4 = Y$

$S : \frac{X^2}{16} + \frac{Y^2}{9} \le 1;\ x \in \{-2, -1, 0, 1, ...\}$

$T : (X-4)^2 + Y^2 \le 36;\ Y \in \{-3, -2, -1, 0, ...\}$



$S \cap T = (-2, 0), (-1, 0), ... (4, 0) \to (7)$

$(-1, 1), (0, 1), ... (3, 1) \to (5)$

$(-1, -1), (0, -1), ... (3, -1) \to (5)$

$(-1, 2), (0, 2), (1, 2), (2, 2) \to (4)$

$(-1, -2), (0, -2), (1, -2), (2, -2) \to (4)$

$(0, 3), (0, -3) \to (2)$

Q.53 (2)

Line is passing through intersection of $bx + 10y - 8 = 0$ and $2x - 3y = 0$ is $(bx + 10y - 8) + \lambda(2x - 3y) = 0$. As line is passing through (1, 1). So $\lambda = b + 2$

Now line $(3b + 4)x - (3b - 4)y - 8 = 0$ is tangent to circle $17(x^2 + y^2) = 16$

So, $\frac{8}{\sqrt{(3b+4)^2 + (3b-4)^2}} = \frac{4}{\sqrt{17}}$

$\Rightarrow b^2 = 2 \Rightarrow e = \sqrt{\frac{3}{5}}$

Q.54 [75]

$x^2 + 4y^2 + 2x + 8y - \lambda = 0$

$(x+1)^2 - 1 + 4(y^2 + 2y) - \lambda = 0$

$(x+1)^2 - 1 + 4(y+1)^2 - 4 - \lambda = 0$

$(x+1)^2 + 4(y+1)^2 - 5 - \lambda = 0$

$(x+1)^2 + 4(y+1)^2 = 5 + \lambda$

$\frac{(x+1)^2}{(s+\lambda)} + \frac{(y+1)^2}{\left(\frac{s+\lambda}{4}\right)} = 1$

Length of Latus Rectum $= \frac{2\left(\frac{5+\lambda}{4}\right)}{\sqrt{(5+\lambda)}} = 4$

$\Rightarrow \frac{\sqrt{(5+\lambda)}}{2} = 4$

$\Rightarrow 5 + \lambda = 64$

$\Rightarrow \lambda = 59$

Major axis $= \ell$

$\Rightarrow 2\sqrt{(5+\lambda)} = \ell$

$\ell = 2\sqrt{5+59}$

$\ell = 2\sqrt{64}$

$\Rightarrow = \lambda = 16$

$\Rightarrow \lambda + \ell = 59 + 16$

$= 75$

Q.55 (2)

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$(y - mx)^2 = \frac{5}{2}m^2 + \frac{5}{3}$

$\Downarrow (1,3)$

$(3 - m)^2 = \frac{5}{6}(3m^2 + 2)$

$6(9 + m^2 - 6m) = 15m^2 + 10$

$9m^2 + 36m - 44 = 0 \quad \langle\ m_1,\ m_2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \left|\frac{\sqrt{(m_1 + m_2)^2 - 4m_1m_2}}{1 + m_1m_2}\right|$

$\Rightarrow \left|\frac{\sqrt{16 + 4 \times \frac{44}{9}}}{1 - \frac{44}{9}}\right|$

$$\Rightarrow\left(\frac{9\times 4\sqrt{1+\frac{11}{9}}}{35}\right)$$

$$\Rightarrow\left(\frac{12\sqrt{20}}{35}\right)\Rightarrow\theta=\tan^{-1}\left(\frac{24\sqrt{5}}{35}\right)$$

$$\Rightarrow\theta=\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$$

Q.56 (1)

x-intercept of $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ is 7

y-intercept of $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ is $-2\sqrt{6}$

$\therefore\ a=7,\ b=2\sqrt{6}$

$\therefore\ e^2=1-\frac{24}{49}\Rightarrow e=\frac{5}{7}$

HYPERBOLA

Q.57 (4)

$e=\sqrt{1+\frac{b^2}{a^2}}$, $\ell=\frac{2b^2}{a}$

Given $(e)^2=\frac{11}{14}\ell$

$1+\frac{b^2}{a^2}=\frac{11}{14}\cdot\frac{2b^2}{a}$

$\frac{a^2+b^2}{a^2}=\frac{11}{7}\cdot\frac{b^2}{a}$............ (1)

Also $e'=\sqrt{1+\frac{a^2}{b^2}},\ \ell'=\frac{2a^2}{b}$

Given $(e')^2=\frac{11}{8}\ell'$

$1+\frac{a^2}{b^2}=\frac{11}{8}\cdot\frac{2a^2}{b}$

$\frac{a^2+b^2}{b^2}=\frac{11}{4}\cdot\frac{a^2}{b}$.............(2)

Now (1) $\div$ (2)

$\frac{b^2}{a^2}=\frac{4}{7}\cdot\frac{b^3}{a^3}$

$\therefore\ 7a=4b$ .... (3)

From (2)

$\frac{\frac{16b^2}{49}+b^2}{b^2}=\frac{11}{4}\cdot\frac{16b^2}{49b}$

$\frac{65}{49}=\frac{11}{4}\cdot\frac{16}{49}\cdot b$

$\therefore\ b=\frac{4\times 65}{11\times 16}$ ....(4)

We have to find value of

$77a+44b$

$11(7a+4b)=11(4b+4b)=11\times 8b$

Value of $11\times 8b=11\times 8\times\frac{4\times 65}{16\times 11}=130$

Q.58 (88)

$e^2=1+\frac{b^2}{a^2}=\frac{11}{4}$

$7a^2=4b^2$

$b^2=\frac{7}{4}a^2$

So hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{\left(\frac{\sqrt{7}}{2}a\right)^2}=1$$

Sum of length of transverse axis and conjugate axis

$2a+\sqrt{7}\,a=(2\sqrt{2}+\sqrt{14})4$

$(2+\sqrt{7})a=4\sqrt{2}\,(2+\sqrt{7})$

$\Rightarrow a=4\sqrt{2}$

$\Rightarrow b^2=56$

$\therefore\ a^2+b^2=32+56=88$

Q.59 **(4)**

For hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

The equation of tangent in slope form is

$y=mx\pm\sqrt{a^2m^2-b^2}$ & condition of tangency is $c^2=a^2m^2-b^2$

$\therefore$ Given hyperbola $a^2x^2-y^2=b^2$

$$\frac{x^2}{\left(\frac{b^2}{a^2}\right)}-\frac{y^2}{b^2}=1$$

$\therefore$ Given line $\lambda x-2y=\mu$

$2y=\lambda x-\mu$

$y=\left(\frac{\lambda}{2}\right)x+\left(\frac{-\mu}{2}\right)$ is tangent

$\uparrow$ m $\qquad\qquad$ $\uparrow$ c

$$\therefore \left(\frac{-\mu}{2}\right)^2 = \left(\frac{b^2}{a^2}\right)\left(\frac{\lambda}{2}\right)^2 - b^2$$

$$\frac{\mu^2}{4} = \frac{\lambda^2 b^2}{4a^2} - b^2$$

$$\frac{\mu^2}{4b^2} = \frac{\lambda^2}{4a^2} - 1$$

$$\frac{\lambda^2}{4a^2} - \frac{\mu^2}{4b^2} = 1$$

$$\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2 = 4$$

Q.60 (42)

$$\frac{x^2}{a^2} - \frac{y^2}{1} = 1 \qquad\qquad \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\frac{2(1)}{a} = \frac{2(3)}{2}$$

$$a = \frac{2}{3}$$

$$1 = \frac{4}{9}(e_H^2 - 1) \Rightarrow e_H^2 = \frac{13}{4}$$

$$3 = 4(1 - e_E^2) \Rightarrow e_E^2 = 1 - \frac{3}{4} = \frac{1}{4}$$

$$e_H^2 + e_E^2 = \frac{13}{4} + \frac{1}{4} = \frac{14}{4}$$

$$12(e_H^2 + e_E^2) = 12 \times \frac{14}{4} = 42$$

Q.61 **[85]**

$$b^2 = a^2\left(\frac{25}{16} - 1\right) = a^2 \times \frac{9}{16}$$

$$\frac{x^2}{a^2} - \frac{y^2 \times 16}{9a^2} = 1$$

It passes through $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$

$$\frac{64}{5a^2} - \frac{144 \times 16}{25 \times 9a^2} = 1$$

$320 - 256 = 25a^2$

$64 = 25a^2$

$$a = \frac{8}{5} \text{ and } b^2 = \frac{9a^2}{16}$$

$$\Rightarrow b = \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$$

Now, $\dfrac{x^2}{\left(\frac{8}{5}\right)^2} - \dfrac{y^2}{\left(\frac{6}{5}\right)^2} = 1$

Equation of normal

$$\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$$

$$\frac{64}{25} \times \frac{x\sqrt{5}}{8} + \frac{36}{25} \times \frac{y \times 5}{12} = \frac{64 + 36}{25}$$

$$\frac{8x\sqrt{5}}{25} + \frac{15y}{25} = \frac{100}{25}$$

$\Rightarrow \beta = 15, \lambda = 100$

$\Rightarrow \lambda - \beta = 85$

Q.62 **(3)**

$\dfrac{x^2}{a^2} - \dfrac{y^2}{9} = 1$, point $(8, 3\sqrt{3})$ will satisfy given equation.

$$\frac{64}{a^2} - \frac{27}{9} = 1$$

$\Rightarrow a^2 = 16 \Rightarrow a = 4$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Equation of normal

$$\frac{x - x_1}{\frac{x_1}{a^2}} = \frac{y - y_1}{-\frac{y_1}{b^2}}$$

Put $(x_1, y_1) = (8, 3\sqrt{3})$

$$\Rightarrow \frac{x - 8}{\left(\frac{8}{16}\right)} = -\frac{y - 3\sqrt{3}}{\left(\frac{3\sqrt{3}}{9}\right)}$$

$\Rightarrow 2(x - 8) = -\sqrt{3}(4 - 3\sqrt{3})$

$\Rightarrow 2x + \sqrt{3}y - 25 = 0$

$(-1, 9\sqrt{3})$ satisfies equation.

Q.63 **[12]**

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

equation of tangent to hyperbola

$y = mx \pm \sqrt{a^2m^2 - b^2}$

$\Rightarrow y = mx \pm \sqrt{16m^2 - 4}$

equation of line perpendicular to tangent line and passing through origin

$$y = \frac{-x}{m}$$

Put $m = \frac{-x}{y}$

to get locus of point of intersection

$$y = \frac{-x^2}{y} \pm \sqrt{\frac{16x^2}{y^2} - 4}$$

$$\Rightarrow \left(y + \frac{x^2}{y}\right)^2 = \frac{16x^2 - 4y^2}{y^2}$$

$\Rightarrow (x^2 + y^2)^2 = 16x^2 - 4y^2$

$(\alpha, \beta) = (16, -4)$

$\alpha + \beta = 12$

Q.64 (2)

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$m = 2, c^2 = a^2m^2 - b^2$

$c^2 = 4a^2 - b^2$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$\frac{5}{2} = 1 + \frac{b^2}{a^2}$$

$$\frac{3}{2} = \frac{b^2}{a^2} \Rightarrow b^2 = \frac{3a^2}{2}$$

$$\frac{2b^2}{a} = 6\sqrt{2}$$

$$\frac{2}{a} \times \frac{3a^2}{2} = 6\sqrt{2}$$

$3a = 6\sqrt{2}$

$a = 2\sqrt{2} \Rightarrow a^2 = 8$

$$b^2 = \frac{3}{2} \times 8 = 12$$

$\therefore c^2 = 4 \times 8 - 12$

$c^2 = 20$

Q.65 (2)

$$H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Foci : S(ae, 0), S'(–ae, 0)

Foot of directrix of parabola is (–ae, 0)

Focus of parabola is (ae, 0)

Now, semi latus rectum of parabola = SS' = |2ae|

Given, $4ae = e\left(\frac{2b^2}{a}\right)$

$\Rightarrow b^2 = 2a^2$ ... (1)

Given, $(2\sqrt{2}, -2\sqrt{2})$ lies on H

$$\Rightarrow \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{8} \quad \text{... (2)}$$

From (1) and (2)

$a^2 = 4, b^2 = 8$

$\because b^2 = a^2(e^2 - 1)$

$\therefore e = \sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2 = 8\sqrt{3}x$

Q.66 [2]

$S : x^2 + y^2 - 2x + 2fy + 1 = 0$

$d_1 : 2px - y = 1$

$d_2 : 2x + py = 4p$

Center : (1, –f) lies on

$d_1 \Rightarrow 2p + f = 1$ $\quad\Rightarrow\quad$ $2p^2 + pf = p$

$d_2 \Rightarrow 2 - pf = 4p$ $\quad\quad$ $\underline{2 - pf = 4p}$

$2p^2 + 2 = 5p$

$2p^2 - 5p + 2 = 0$

$2p^2 - 4p - p + 2 = 0$

$(2p - 1)(p - 2) = 0$

$P = \frac{1}{2}$ & $p = 2$

$\Downarrow \quad \Downarrow$

$f = 0 \quad f = -3$

$H : \frac{x^2}{1} - \frac{y^2}{3} = 1$ & Centre are

$$\begin{cases} C_1 : (1, 0) \\ C_2 : (1, 3) \end{cases}$$

Now tangent of slope m & passes centre

$T : y = mx \pm \sqrt{m^2 - 3}$

Pass (1,0) &

$\Rightarrow m \pm \sqrt{m^2 - 3} = 0$

$m^2 - 3 = m^2$

Not possible

Pass (1,3)

$3 - m = \pm\sqrt{m^2 - 3}$

$(m - 3)^2 = (m^2 - 3)$

$m^2 + 9 - 6m = m^2 - 3$

$6m = 12$

$\boxed{m = 2}$ Ans.

Q.67 (3)

$$e_E = \sqrt{1 - \frac{b^2}{a^2}}, e_H = \sqrt{2}$$

$\Rightarrow eE = \frac{1}{e_H}$

$\Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{2}$

$2a^2 - 2b^2 = a^2$

$a^2 = 2b^2$

And $y = \sqrt{\frac{5}{2}}x + K$ is tangent to ellipse then

$K^2 = a^2 \times \frac{5}{2} + b^2 = \frac{3}{2}$

$6b^2 = \frac{3}{2} \Rightarrow b^2 = \frac{1}{4}$ and $a^2 = \frac{1}{2}$

$\therefore 4.(a^2 + b^2) = 3$

Q.68 [1552]

Hyp : $\frac{y^2}{64} - \frac{x^2}{49} = 1$

An ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola H: $\frac{x^2}{49} - \frac{y^2}{64} = -1$.

So, $b^2 = 64$

$e_H = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{49}{64}}$

Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$e_E = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{a^2}{64}}$

$= \sqrt{\frac{64 - a^2}{64}} \times \frac{\sqrt{113}}{8} = \frac{1}{2} \Rightarrow \sqrt{64 - a^2} \times \sqrt{113} = 32$

$(64 - a^2) = \frac{32^2}{113}$

$\Rightarrow a^2 = 64 - \frac{32^2}{113}$

$l = \frac{2a^2}{b} = \frac{2}{8}\left(64 - \frac{32^2}{113}\right) = \frac{1552}{113}$

$113l = 1552$

Q.69 (3)

$\frac{x^2}{\frac{6}{k}} - \frac{y^2}{6} = 1$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{6 \times k}{6}$

$e^2 = \sqrt{1 + k}$

equation of directrix is $x = \pm\frac{a}{e} = 1$

$a^2 = e^2$

$\frac{6}{k} = k + 1$

$k^2 + k - 6 = 0 \Rightarrow k = 2$

$\Rightarrow$ equation is $2x^2 - y^2 = 6$

Q.70 (4)

$\beta^2 = 24\alpha$ ... (1)

$\frac{dy}{dx} = \frac{12}{y}$

$\left(\frac{dy}{dx}\right)_{\alpha,\beta} = \frac{12}{\beta}$

$\left(\frac{12}{\beta}\right)(-1) = -1$

$\beta = 12$

$\alpha = 6$

Now point = (10, 16)

equation of hyperbola $\frac{x^2}{36} - \frac{y^2}{144} = 1$

equation of normal $2x + 5y = 100$

which not passes through (15, 13)

Q.71 **[20]**

$T_1: y = mx \pm\sqrt{(4m^2 + 9)}$

$T_2: y = mx \pm\sqrt{(42m^2 - 143}$

So $4m^2 + 9 = 42m^2 - 142$

$\Rightarrow 38m^2 = 152$

$\Rightarrow m = \pm 2$ & $c = \pm 5$

For this tangent not to pass through 4th quadrant

T : y = 2 x+ 5

Now, compare with $\frac{xx_1}{4} + \frac{yy_1}{9} = 1$

we get, $\frac{x_1}{8} = \frac{-1}{5} \Rightarrow x_1 = -\frac{8}{5}$

$\frac{xx_2}{42} - \frac{yy_2}{143} = 1$

$2x - y = -5$

$$\Rightarrow \frac{x_2}{84} = -\frac{1}{4} \Rightarrow x_2 = -\frac{84}{5}$$

So $|2x_1 + x_2| = \left|\frac{-100}{5}\right| = 20$

Q.72 (4)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ have same foci, then

$a^2 - b^2 = l^2 + m^2$

$$16 - 7 = \frac{144}{25} + \frac{\alpha}{25}$$

$9 \times 25 - 144 = \alpha$

$\alpha = 81$

$$\text{L.R.} = \frac{2b^2}{a} = \frac{2 \times \left(\frac{\alpha}{25}\right)}{\frac{12}{5}}$$

$$= \frac{2 \times 81 \times 5}{12 \times 25} = \frac{27}{10}$$