SOLUTION
BASIC MATHEMATICS & LOGARITHM

EXERCISE-I (MHT CET LEVEL)

Q.1 (3)

A & B are two rational number then $\frac{A}{B}$ is

Also rational number if $B \neq 0$.

Q.2 (2)

Q.3 **(1)**

$$x^{x.x^{1/3}} = (x\,.\,x^{1/3})^x \Rightarrow x^{x^{1+\frac{1}{3}}} = \left(x^{1+\frac{1}{3}}\right)^x$$

$$\Rightarrow x^{x^{4/3}} = \left(x^{4/3}\right)^x = x^{x^{4/3}} = x^{\frac{4}{3}x} \Rightarrow x^{4/3} = \frac{4}{3}x\,;$$

Also is an obvious solution.

Q.4 **(4)**

$$x = 3 - \sqrt{5}$$

$$\sqrt{x} = \sqrt{3-\sqrt{5}} = \frac{1}{\sqrt{2}}.\sqrt{6-2\sqrt{5}} = \frac{1}{\sqrt{2}}(\sqrt{5}-1)$$

$$3x - 2 = 9 - 3\sqrt{5} - 2 = 7 - 3\sqrt{5} = \frac{14-6\sqrt{5}}{2}$$

$$= \frac{(3-\sqrt{5})^2}{2}\,; \quad \therefore \quad \sqrt{3x-2} = \frac{3-\sqrt{5}}{\sqrt{2}}$$

$$\sqrt{2} + \sqrt{3x-2} = \frac{5-\sqrt{5}}{\sqrt{2}} = \sqrt{5}\left(\frac{\sqrt{5}-1}{\sqrt{2}}\right)$$

$$\Rightarrow \sqrt{2} + \sqrt{3x-2} = \sqrt{5}\,.\sqrt{x}\,; \;\therefore\; \frac{\sqrt{x}}{\sqrt{2}+\sqrt{3x-2}} = \frac{1}{\sqrt{5}}$$

Q.5 **(3)**

Putting $x = 1$, remainder = 7

Q.6 **(1)**

$$4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$$

$$\Rightarrow \left(2^{(x^2+2)}\right)^2 - 9.2^{(x^2+2)} + 8 = 0$$

Put $2^{(x^2+2)^2} = y$. Then $y^2 - 9y + 8 = 0$, which gives

$y = 8, y = 1$.

when $y = 8 \Rightarrow 2^{x^2+2} = 8 \Rightarrow 2^{x^2+2} = 2^3 \Rightarrow x^2 + 2 = 3$

$\Rightarrow x^2 = 1 \Rightarrow x = 1, -1$.

when $y = 1 \Rightarrow 2^{x^2+2} = 1 \Rightarrow 2^{x^2+2} = 2^o$

$\Rightarrow x^2 + 2 = 0 \Rightarrow x^2 = -2$, which is not

Q.7 **(2)**

$$134 + \sqrt{6292} = [11^2 + (\sqrt{13})^2] + 2.11.\sqrt{13} = (11 + \sqrt{13})^2$$

$$\therefore \sqrt{134 + \sqrt{62\,92}} = 11 + \sqrt{13}\,.$$

Q.8 (2)

Taking log on both sides,

$(3x^2 - 10x + 3)\log|x - 3| = 0$

$\log|x - 3| = 0$ or $3x^2 - 10x + 3 = 0$

$x - 3\ {}^{1}\ 0; |x - 3| = 1$ or $(x - 3)(3x - 1) = 0$

$x \neq 3; (x - 3) = \pm 1$ or $x = 3; x = \frac{1}{3}$

$\therefore x = 2, 4$ or $x = \ \ [\because x \neq 3]$

Hence, three real solutions

Q.9 (a)

$xy - 12x - 12y = 0 \Rightarrow (x - 12)(y - 12) = 144$

Now 144 can be factorised into two factors x and y where $x \leq y$ and the factors are (1, 144), (2, 72), (3, 48), (4, 36), (6, 24), (8, 18), (9, 16), (12, 12).

Thus there are eight solutions.

Q.10 (a)

Q.11 (c)

Q.12 **(d)**

Q.13 **(1)**

Let *x* be the required logarithm , then by definition

$$\log_{2\sqrt{2}} 32\sqrt[5]{4} = x$$

$$(2\sqrt{2})^x = 32\sqrt[5]{4} \Rightarrow (2.2^{1/2})^x = 2^5.2^{2/5}\,; \therefore\; 2^{\frac{3x}{2}} = 2^{5+\frac{2}{5}}$$

Here, by equating the indices, $\frac{3}{2}x = \frac{27}{5}$

$$\therefore x = \frac{18}{5} = 3.6\,.$$

Q.14 (3)

$\log_2 7 \Rightarrow \log_2 4 < \log_2 7 < \log_2 8$

$\Rightarrow 2 < \log_2 7 < 3$ i.e. not integer

Let $\log_2 7 = \frac{p}{q}$ (where p and q are coprime)

$\Rightarrow 2^{p/q} = 7 \Rightarrow 2^p = 7^q$

which is not possible so $\log_2 7$ is an irratoinal number

Q.15 **(1)**

$$\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$$

$$= \frac{1}{2}\log_e(ab) = \log_e \sqrt{ab}$$

$$\Rightarrow \frac{a+b}{2} = \sqrt{ab} \Rightarrow a+b = 2\sqrt{ab}$$

$$\Rightarrow \left(\sqrt{a}-\sqrt{b}\right)^2 = 0 \Rightarrow \sqrt{a}-\sqrt{b} = 0 \Rightarrow a = b\ .$$

Q.16 **(2)**

$$\log_3 4.\log_4 5.\log_5 6.\log_6 7.\log_7 8.\log_8 9$$

$$= \frac{\log 4}{\log 3}.\frac{\log 5}{\log 4}.\frac{\log 6}{\log 5}.\frac{\log 7}{\log 6}.\frac{\log 8}{\log 7}.\frac{\log 9}{\log 8} = \frac{\log 9}{\log 3}$$

$$= \log_3 9 = \log_3 3^2 = 2\ .$$

Q.17 (c)

Q.18 (a)

Q.19 (d)

Q.20 **(3)**

$$\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = \log_7 \log_7 7^{7/8} = \log_7(7/8)$$

$$= \log_7 7 - \log_7 8 = 1 - \log_7 2^3 = 1 - 3\log_7 2\ .$$

Q.21 **(4)**

$$81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$$

$$= 3^{4\log_3 5} + 3^{3.\frac{1}{2}\log_3 36} + 3^{4\log_9 7}$$

$$= 3^{\log_3 5^4} + 3^{\log_3 36^{3/2}} + 3^{\log_3 7^{4/2}}$$

$$= 5^4 + 36^{3/2} + 7^2 = 890\ \cdot$$

Q.22 **(4)**

$$\log_{1000} x^2 = \log_{10^3} x^2 = 2\log_{10^3} x = \frac{2}{3}\log_{10} x = \frac{2}{3}y$$

Q.23 **(2)**

$$x = \log_a bc \Rightarrow 1 + x = \log_a a + \log_a bc = \log_a abc$$

$$\therefore (1+x)^{-1} = \log_{abc} a$$

$$\therefore (1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1} = \log_{abc} a + \log_{abc} b + \log_{abc} c$$

$$= \log_{abc} abc = 1\ .$$

Q.24 **(3)**

$$a = \log_{24} 12 = \frac{\log 12}{\log 24} = \frac{2\log 2 + \log 3}{3\log 2 + \log 3}$$

$$b = \log_{36} 24 = \frac{3\log 2 + \log 3}{2(\log 2 + \log 3)}$$

$$c = \log_{48} 36 = \frac{2(\log 2 + \log 3)}{4\log 2 + \log 3}$$

$$\therefore abc = \frac{2\ \log 2 + \log 3}{4\log 2 + \log 3}$$

$$\Rightarrow 1 + abc = \frac{6\log 2 + 2\log 3}{4\log 2 + \log 3} = 2.\frac{3\log 2 + \log 3}{4\log 2 + \log 3} = 2bc\ .$$

Q.25 **(3)**

$$\log_7 \log_5(\sqrt{x^2+5+x}) = 0 = \log_7 1$$

$$\Rightarrow \log_5(x^2+5+x)^{1/2} = 1 = \log_5 5$$

$$\Rightarrow .\ (x^2+5+x)^{1/2} = 5$$

$$\Rightarrow (x^2+x+5) = 25 \Rightarrow x^2+x-20 = 0$$

$$\Rightarrow (x-4)(x+5) = 0 \Rightarrow x = 4, -5 \Rightarrow x = 4$$

Q.26 **(4)**

$$x^{\frac{3}{4}(\log_3 x)^2 + \log_3 x - \frac{5}{4}} = \sqrt{3} = 3^{\frac{1}{2}}\ \cdot$$

There is a possibility of a solution $x = 3$

For this value, LHS = $3^{\frac{3}{4}.1^2+1-\left(\frac{5}{4}\right)} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \text{RHS}$.

$\therefore$ $x = 3$ is a solution, which is a +ve integer.

Next, $\left[\frac{3}{4}(\log_3 x)^2 + \log_3 x - \frac{5}{4}\right]\log_3 x = \frac{1}{2}$

$$\Rightarrow [3(\log_3 x)^2 + 4\log_3 x - 5]\log_3 x - 2 = 0$$

$$\Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0\ ,\ [\ t = \log_3 x\ ]$$

$$\Rightarrow 3t^3 - 3t^2 + 7t^2 - 7t + 2t - 2 = 0$$

$$\Rightarrow (3t^2 + 7t + 2)(t-1) = 0 \Rightarrow (3t+1)(t+2)(t-1) = 0$$

$$\Rightarrow t = 1, -2, -\frac{1}{3} \Rightarrow \log_3 x = 1, -2, -\frac{1}{3}$$

$$\Rightarrow x = 3^1, 3^{-2}, 3^{-1/3}\ ;\ \therefore x = 3, \frac{1}{9}, \frac{1}{\sqrt[3]{3}}$$

Q.27 **(2)**

$$\log_2 .\log_3 \ldots\ldots \log_{99}\ \log_{100} 100^{99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}}}$$

$$= \log_2 \log_3 \ldots\ldots \log_{99} 99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}} \quad [\log_{100} 100\ = 1]$$

$$= \log_2 \log_3 \ldots\ldots \log_{98} 98^{97^{\cdot^{\cdot^{\cdot^{2^1}}}}}$$

$$= \log_2 \log_3 \ldots\ldots \log_{97} 97^{96^{\cdot^{\cdot^{\cdot^{2^1}}}}} = \log_2 \log_3 3^{2^1}$$

$$= \log_2\ 2'\log_3 3 = \log_2 2 = 1\ .$$

Q.28 **(3)**

$$\left(\frac{2}{3}\right)^{x+2} = \left(\frac{3}{2}\right)^{2-2x} \Rightarrow \left(\frac{2}{3}\right)^{x+2} = \left(\frac{2}{3}\right)^{2x-2}.$$

Clearly $x + 2 = 2x - 2 \Rightarrow x = 4$

Q.29 **(4)**

$\log_2(x+5) = 6 - x \Rightarrow x + 5 = 2^{6-x} \Rightarrow x + 5 = 64.2^{-x}$

Let $y = x + 5$, $y = 64.2^{-x}$ will intersect at one point.

Number of solutions = 1.

Q.30 (3)

$\Rightarrow$ antilog$_{16}$ 0.75 = $(16)^{0.75}$

$= (16)^{3/4} = (2^4)^{3/4} = 2^3 = 8$

Q.31 **(3)**

$y = 3^{12} \times 2^8 \Rightarrow \log_{10} y = 12\log_{10} 3 + 8\log_{10} 2$

$= 12 \times 0.47712 + 8 \times 0.30103$

$= 5.72544 + 2.40824 = 8.13368$

$\therefore$ Number of digits in y = 8 + 1 = 9.

Q.32 **(2)**

$\log_{1/2}(x^2 - 6x + 12) \geq -2$ …..(i)

For log to be defined, $x^2 - 6x + 12 > 0$

$\Rightarrow (x-3)^2 + 3 > 0$, which is true $\forall x \in R$.

From (i), $x^2 - 6x + 12 \leq \left(\frac{1}{2}\right)^{-2}$

$\Rightarrow x^2 - 6x + 12 \leq 4 \Rightarrow x^2 - 6x + 8 \leq 0$

$\Rightarrow (x-2)(x-4) \leq 0 \Rightarrow 2 \leq x \leq 4 \therefore x \in [2, 4]$.

Q.33 **(3)**

$\log_{0.04}(x-1) \geq \log_{0.2}(x-1)$ …..(i)

For log to be defined $x - 1 > 0 \Rightarrow x > 1$

From (i), $\log_{(0.2)^2}(x-1) \geq \log_{0.2}(x-1)$

$\Rightarrow \frac{1}{2}\log_{0.2}(x-1) \geq \log_{0.2}(x-1) \Rightarrow \sqrt{x-1} \leq (x-1)$

$\Rightarrow \sqrt{x-1}(1 - \sqrt{x-1}) \leq 0 \Rightarrow 1 - \sqrt{x-1} \leq 0$

$\Rightarrow \sqrt{x-1} \geq 1 \Rightarrow x \geq 2 \therefore x \in [2, \infty)$

Q.34 **(1)**

$\log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$

$\log_{0.3}(x-1) < \frac{1}{2}\log_{(0.3)}(x-1)$

$\log_{0.3}(x-1) < \log_{(0.3)}(x-1)^{1/2}$

here base is less than 1, therefore the inequality is reversed

$(x-1) > (x-1)^{1/2}$

$(x-1)^2 > (x-1)$

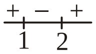
$x^2\ 1 - 2x - x + 1 > 0$

$x^2 - 3x + 2 > 0$

$x^2 - 2x - x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$(x-1)(x-2) > 0$



$x \in (-\infty, 1) \cup (2, \infty)$

$\because x - 1 > 0 \Rightarrow x > 1$

than $x \in (2, \infty)$

Q.35 **(a)**

Q.36 **(a)**

Q.37 **(a)**

Q.38 **(3,4)**

Q.39 **(d)**

Q.40 (4)

$|x|^2 - 3|x| + 2 = 0 \Rightarrow (|x| - 2)(|x| - 1) = 0$

$\Rightarrow |x| = 1, 2 \Rightarrow x = \pm 1, \pm 2$

$\therefore$ number of real roots is 4.

Q.41 $S = \frac{1}{3}\left[\log\frac{1}{2} + \log\frac{2}{3} + \log\frac{3}{4} + ... + \log\frac{99}{100}\right]$

$= \frac{1}{3}\left[\log\frac{1}{2}.\frac{2}{3}.\frac{3}{4}..........\frac{99}{100}\right]$

$S = \frac{1}{3}\log_{10}\frac{1}{100} = \frac{-2}{3}$. Ans.

Q.42 **(c)**

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)

It is a property

Q.2 (2)

x and y are rational and

$(x + y) + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$

Since real part of both sides are equal and coefficient of irrational parts should be zero in both sides

$\therefore$ x – 2y = 0 $\Rightarrow$ x = 2y

and x – y – 1 = 0 $\Rightarrow$ y = 1 $\therefore$ x = 2

x = 2, y = 1

Q.3 (4)

It is a property

Q.4 (3)

As $x^{\log_9 4} = x^{\log_3 (2^2)x} = x^{\log_3 2} = 2^{\log_3 x}$

$\therefore$ Equation becomes $2^{\log_3 x} + 8 = 3.2^{\log_3 x}$

$\Rightarrow 2^{\log_3 x} = 4$ or $x = 9$

Q.5 (1)

Here $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$

Here sum of three + ve number can not be equal to zero then all three number independently be zero.

$\therefore$ There are no any real root.

Q.6 (3)

Let $P(x) = x^3 - a^2x + x + 2$ be the given polynomial

Then by factor theorem, (x – a) is a factor of P(x) iff P(a) = 0

$\Rightarrow\ a^3 - a^2 . a + a + 2 = 0$
$\Rightarrow\ a + 2 = 0 \Rightarrow a = -2$

Q.7 (2)
$2x^2 - 4xy + xy - 2y^2 = 7$
$2x(x - 2y) + y(x - 2y) = 7$
$(x - 2y)(2x + y) = 7$
x, y are integers $\Rightarrow x - 2y, 2x + y$ are also integers
Four cases are possible
Case I: $x - 2y = 1, 2x + y = 7 \Rightarrow x = 3, y = 1$
$|x + y| = 4$

Case II : $x - 2y = 7, 2x + y = 1 \Rightarrow x = \frac{9}{5}$ rejected

Case III : $x - 2y = -1, 2x + y = -7$
$\Rightarrow x = -3, y = -1$
$|x + y| = 4$

Case IV : $x - 2y = -7, 2x + y = -1 \Rightarrow x = -\frac{9}{5}$ rejected

Hence $|x + y| = 4$

Q.8 (2)
$a(a - b) + b(b - c) + c(c - a) = 0$
$\Rightarrow\ a^2 + b^2 + c^2 - ab - bc - ca = 0$
Multiplying & deviding by 2,

$\frac{1}{2}\ [(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (a^2 + c^2 - 2ac)] = 0$

$\Rightarrow \frac{1}{2}\ [(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$
$\Rightarrow\ (a - b)^2 + (b - c)^2 + (c - a)^2 = 0 \Rightarrow a = b = c$

Q.9 **(A)**
$60^a = 3 \quad \Rightarrow\ a = \log_{60} 3$
$60^b = 5 \Rightarrow\ b = \log_{60} 5$

let $\quad x = 12^{\frac{1-a-b}{2(1-b)}}$

$\log_{12} x = \frac{1 - a - b}{2(1 - b)} = \frac{1 - (a + b)}{2(1 - b)}$

$= \frac{1 - (\log_{60} 3 + \log_{60} 5)}{2(\log_{60} 60 - \log_{60} 5)} = \frac{1 - (\log_{60} 15)}{2(1 - \log_{60} 5)} = \frac{\log_{60} 4}{2\log_{60} 12}$

$= \frac{1}{2}\ \log_{12} 4 = \log_{12} 2 \qquad (a + b = \log_{16} 15)$

$\therefore\ \ \log_{12} x = \log_{12} 2\ \Rightarrow\ x = 2$ Ans.

Q.10 **(4)**

$\frac{\log x}{b - c} = \frac{\log y}{c - a} = \frac{\log z}{a - b} = k\,(\text{say})$

$\Rightarrow\ \log x = k(b - c), \log y = k(c - a), \log z = k(a - b)$

$\Rightarrow\ x = e^{k(b-c)}, y = e^{k(c-a)}, z = e^{k(a-b)}$

$\therefore\ xyz = e^{k(b-c)+k(c-a)+k(a-b)} = e^0 = 1$

$x^a y^b z^c = e^{k(b-c)a+k(c-a)b+k(a-b)c} = e^0 = 1 = xyz$

$x^{b+c} y^{c+a} z^{a+b} = e^{k(b^2-c^2)+k(c^2-a^2)+k(a^2-b^2)} = e^0 = 1$

Q.11 (2)
Applying base change theorem,

$= \log_{abc} \sqrt{bc} + \log_{abc} \sqrt{ca} + \log_{abc} \sqrt{ab}$

$= \log_{abc} \sqrt{bc} \cdot \sqrt{ca} \cdot \sqrt{ab} = \log_{abc} abc = 1$

Q.12 (3)
$\log_2 15 . \log_{1/6} 2 . \log_3 1/6$

$= \frac{\log_e 15}{\log_e 2} \times \frac{\log_e 2}{\log_e 1/6} \times \frac{\log_e 1/6}{\log_e 3} = \frac{\log_e 15}{\log_e 3} =$

$\frac{\log_e(3 \times 5)}{\log_e 3} = 1 + \log_3 5$

$\therefore\ \ [1 + \log_3 5] = 2$

Q.13 (4)

$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2 + 1) - 2a}{a^2 - a - 1}$

$= \frac{(a^2)^2 - (a + 1)^2}{(a^2 - a - 1)} = a^2 + a + 1$

Q.14 **(3)**
$x < 0\ ;\ \ \sqrt{\log_{10}(-x)} = \log_{10} |x|$
squaring $\ \log_{10}(-x) = \log_{10}{}^2(-x) \quad |x| = -x;\ \text{for } x < 0$
$\log_{10}(-x) = 0\ \text{or}\ \log_{10}(-x) = 1$
$x = -1\ \ \text{or}\ \ x = -10$

Q.15 (1)
Domain $x^2 + 4x - 5 \geq 0$
$\Rightarrow\ x \in (-\infty, -5] \cup [1, \infty)$
Case I :
$x \in (-\infty, -5] \cup [1, 3)$
– ve < + ve alsways true
$\therefore\ \ x \in (-\infty, -5] \cup [1, 3)$
... (1)
Case II :
$x \in [3, \infty)$
.. (i)

$x - 3 < \sqrt{x^2 + 4x - 5}$
$\Rightarrow\ x^2 - 6x + 9 < x^2 + 4x - 5$

$\Rightarrow\ x > \frac{7}{5}$

... (ii)
(i) $\cap$ (ii) $x \in [3, \infty)$
... (2)

(1) $\cup$ (2) $x \in (-\infty, -5] \cup [1, \infty)$

Ans. (1)

Q.16 (2)

$$\log_{\sqrt{0.9}} \log_5 \left(\sqrt{x^2+5+x}\right) > 0$$

$$\log_5 \left(\sqrt{x^2+5+x}\right) < 1$$

$(x^2 + 5 + x)^{1/2} < 5$ and $x^2 + x + 5 > 0$

$\Rightarrow\ x^2 + 5 + x < 25$

$\Rightarrow\ x^2 + x - 20 < 0$

$\Rightarrow\ (x + 5)(x - 4) < 0$

$\Rightarrow\ x \in (-5, 4)$

$\therefore\ n = 8$

Q.17 (1)

$x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

$\log_5(x^2 - 4) > 0 \Rightarrow x^2 - 4 > 1 \Rightarrow x^2 - 5 > 0$

$\Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

Now

$\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1 \Rightarrow \log_5(x^2 - 4) < 1$

$x^2 - 4 < 5 \Rightarrow x^2 - 9 < 0$, $x \in (-3, 3)$

$\therefore$ **Ans. :** $\left(-3, -\sqrt{5}\right) \cup \left(\sqrt{5}, 3\right)$

Q.18 (4)

$\log_{1-x}(x - 2) \geq -1$

$1 - x > 0 \Rightarrow 1 > x \Rightarrow x \in (-\infty, 1) - \{0\}$

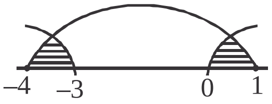
$x - 2 > 0 \Rightarrow x > 2$ No solution.

Q.19 (2)

$2 - \log_2 (x^2 + 3x) \geq 0$

$\log_2(x^2 + 3x) \leq 2$

$\Rightarrow x^2 + 3x \leq 4$



$\Rightarrow x^2 + 3x - 4 \leq 0$

$\Rightarrow (x + 4)(x - 1) \leq 0$

$\Rightarrow x \in [-4, 1]$

and $x^2 + 3x > 0 \Rightarrow x \in (-\infty, -3) \cup (0, \infty)$

Ans. : $[-4, -3) \cup (0, 1]$

Q.20 (D)

$$\text{Let } N = \left(\frac{5}{4}\right)^{-100}$$

$$\Rightarrow \log_{10} N = -100 \log_{10} \left(\frac{5}{4}\right) = -100 \left(\log_{10}\left(\frac{10}{2}\right) - \log_{10} 4\right)$$

$= -100\ (1 - 3\log_{10} 2)$

$= -100(1 - 3 \times 0.3010) = -100(-0.9030) = -100 \times 0.0970 = -9.7$

$= -10 + 0.3 = \overline{10}.03$

$\therefore$ Number of zeroes = 9 Ans.

Q.20 (2)

Q.21 **(3)**

$N = (75)^{-10}$

$\log_{10} N = -10 \log (75)$

$= -10[\log_{10} 25 + \log_{10} 3]$

$= -10\ [2 \log_{10} 5 + \log_{10} 3]$

$= -10\ [2\ \{\log_{10} 10 - \log_{10} 2\} + \log_{10} 3]$

$= -10[2\{1 - 0.301\} + 0.477]$

$= -10[0.699 \times 2 + 0.477]$

$= -10\ [1.398 + 0.477]$

$= -10[1.875]$

$= -18.75 \quad \Rightarrow$ characteristic of $N = -19 = p$

$\therefore$ Number of zeros after decimal

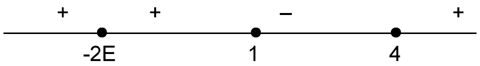
$= |p| - 1 = |-19| - 1 = 19 - 1 = 18.$

Q.22 (1)

Since $(x^2 + x - 2) - (x^2 - 2x - 8) = 3x + 6 = 3(x+2)$

$\therefore\ (x^2 - 2x - 8)(x^2 + x - 2) \leq 0$

i.e. $(x - 4)(x + 2)(x + 2)(x - 1) \leq 0$



$\therefore$ Solution set is $[1,4] \cup \{-2\}$

Q.23 (2)

$||x - 1| - 1| \leq 1$

$\Rightarrow -1 \leq |x - 1| - 1 \leq 1$

$\Rightarrow 0 \leq |x - 1| \leq 2$

$\Rightarrow -2 \leq x - 1 \leq 2$

$\Rightarrow -1 \leq x \leq 3$

$\therefore$ **Ans. :** $x \in [-1, 3]$

Q.24 (4)

$y = |2x - |x - 2|| = |\,2x - (2 - x)| = |3x - 2|$ as $x < 0$ hence $y = 2 - 3x$

Q.25 (2)

$$0 < \frac{2x - 2007}{x + 1} \leq 1$$

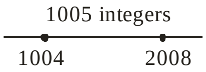
$0 < 2x - 2007 \leq x + 1$

$0 < x \leq 2008$ ....(1)

also $2x - 2007 > 0$

$x > 1003.5$ or $x \geq 1004$ ....(2)

from (1) and (2)



Q.26 (1)

$$\text{Given, } \frac{3 - |x|}{4 - |x|} \geq 0$$

$\Rightarrow 3 - |x| \leq 0$ and $4 - |x| < 0$

or $3 - |x| \geq 0$ and $4 - |x| < 0$

$\Rightarrow |x| \geq 3$ and $|x| > 4$

or $|x| \leq 3$ and $|x| < 4$

$\Rightarrow |x| > 4$ or $|x| \leq 3$

$\Rightarrow x \in (-\infty, -4) \cup [-3, 3] \cup (4, \infty)$

Q.27 (3)

Dividing R at $\frac{2}{3},\frac{4}{3}$ and 2, analyse 4 cases. When

$x \le \frac{2}{3}$, the inequality becomes

$$2-3x+4-3x+6-3x \ge 12$$

implying $-9x \ge 0 \Rightarrow x \le 0$

when $x \ge 2$ the ineqality becomes

$$3x-2+3x-4+3x-6 \ge 12$$

Implying $9x \ge 24 \Rightarrow x \ge 8/3$

The inequality in invalid in the other two sections.

$\therefore$ either $x \le 0$ or $x \ge 8/3$

Q.28 (3)

$2 \log_x b = \log_x a \log_x c$

$\Rightarrow 2 \log_x b = \log_x ac$

$\Rightarrow b^2 = ac$. An.s

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 0000

$$\log_x \frac{4x+5}{6-5x} < -1$$

We must have

$$\frac{4x+5}{6-5x} > 0 \ \Rightarrow \ \frac{4x+5}{5x-6} < 0$$

$$\Rightarrow \ x \in \left(\frac{-5}{4},\frac{6}{5}\right)$$

Also $x > 0$ and $x \ne 1$

$$\therefore \quad x \in \left(0,\frac{6}{5}\right)-\{1\} \qquad \text{...(i)}$$

Case I: $0 < x < 1$ ...(ii)

$$\log_x \left(\frac{4x+5}{6-5x}\right) < -1$$

$$\Rightarrow \ \frac{4x+5}{6-5x} > \frac{1}{x}$$

$$\Rightarrow \ \frac{4x+5}{6-5x} - \frac{1}{x} > 0$$

$$\Rightarrow \ \frac{4x^2+5x+5x-6}{x(6-5x)} > 0$$

$$\Rightarrow \ \frac{4x^2+10x-6}{x(5x-6)} < 0$$

$$\Rightarrow \ \frac{2(x+3)(2x-1)}{x(5x-6)} < 0$$

$$\Rightarrow \ x \in (-3,0) \cup \left(\frac{1}{2},\frac{6}{5}\right) \qquad \text{...(iii)}$$

From (i), (ii) and (iii), we get

$$\therefore \ x \in \left(\frac{1}{2},1\right)$$

Case II : $x > 1$ ...(iv)

$$\log_x \left(\frac{4x+5}{6-5x}\right) < -1$$

$$\Rightarrow \ \frac{4x+5}{6-5x} < \frac{1}{x}$$

$$\Rightarrow \ x \in (-\infty,-3) \cup \left(0,\frac{1}{2}\right) \cup \left(\frac{6}{5},\infty\right)$$

From (i), (iv) and (v), we get

$x \in \phi$

Thus, $x \in \left(\frac{1}{2},1\right)$

Q.2 **5625**

Let $\log_{10}x = a$; $\log_{10}y = b$ and $\log_{10}z = c$

Here $xyz = 10^{81}$

$\Rightarrow \ \log_{10}x + \log_{10}y + \log_{10}z = 81$

i.e. $a + b + c = 81$ ....(1)

Also $a(b + c) + bc = 468$

$ab + bc + ca = 468$ ....(2)

Now, $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\sum ab$

$= (81)^2 - (2)(468)$

$= 6561 - 936$

$= 5625$ **Ans.**

Q.3 **0001**

Let $x + |x - 2| = y$

$\therefore$ Equation becomes

$\log_x y^2 = \log_x(5y - 6)$

$\Rightarrow \ y^2 = 5y - 6$

$\Rightarrow \ y^2 - 5y + 6 = 0$

$\Rightarrow \ y = 2$ or 3

If $y = 2$

then $x + |x - 2| = 2$

$\Rightarrow \ 0 < x < 1 \ \cup \ 1 < x \le 2$

If $y = 3$

then $x + |x - 2| = 3$

$\Rightarrow \ x = \frac{5}{2}$ only

Hence number of integral solutions is 1.

Q.4 **0054**

$\therefore$ $\log_3 M = a_1 + b_1$ and $\log_5 M = a_2 + b_2$

$\therefore$ $M = 3^{a_1 + b_1}$ and $M = 5^{a_2 + b_2}$

$3^{a_1} \le M < 3^{a_1+1}$ and $5^{a_2} \le M < 5^{a_2+1}$

$\therefore\ a_1a_2 = 6 = 1 \times 6 = 2 \times 3 = 3 \times 2 = 6 \times 1$

Let $a_1 = 1$ and $a_2 = 6$

$3^1 \le M < 3^2$ and $5^6 \le M < 5^7$

So there is no common value of M for (1, 6) so only 3 × 2 is satisfy the both number $a_1 = 3, a_2 = 2$

$3^3 \le M < 3^4$ and $5^2 \le M < 5^3$

Number of integers = 54 and 100

So value is 54.

Q.5 **0002**

Let $\log_{3x}45 = \log_{4x} 40\sqrt{3} = k$

$45 = (3x)^k,\ 40\sqrt{3} = (4k)^k$

$$\frac{45}{40\sqrt{3}} = \left(\frac{3k}{4x}\right)^k \Rightarrow \frac{3\sqrt{3}}{8} = \left(\frac{3}{4}\right)^k$$

$$\Rightarrow \left(\frac{3}{4}\right)^{\frac{3}{2}} = \left(\frac{3}{4}\right)^k \Rightarrow k = \frac{3}{2}$$

$$\log_{3x} 45 = \frac{3}{2}$$

$$45 = (3x)^{\frac{3}{2}}$$

squaring both sides

$45 \times 45 = (3x)^3$

$x^3 = 75$

$\log_7 x^3 = \log_7 75$

$\Rightarrow\ \log_7 49 < \log_7 75 < \log_7 343$

$\Rightarrow\ 2 < \log_7 75 < 3$;

Characteristic = 2.

Q.6 **0006**

$y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$

$= \sqrt{\log_2 3 \cdot (2+\log_2 3)(4+\log_2 3)(6+\log_2 3)+16} - (2+\log_2 3)(4+\log_2 3)+10$

Let us put $\log_2 3 = x$

$= \sqrt{x(2+x)(4+x)(6+x)+16} - (2+x)(4+x)+10$

$= \sqrt{(x^2+6x)(x^2+6x+8)+16} - (x^2+6x+8)+10$

Put again $x^2 + 6x = \alpha$

$= \sqrt{\alpha(\alpha+8)+16} - (\alpha+8)+10$

$= \sqrt{\alpha^2+8\alpha+16} - (\alpha+8)+10$

$= \sqrt{(\alpha+4)^2} - (\alpha+8)+10$

$= (\alpha+4)-(\alpha+8)+10 = y = 6.$

Q.7 **0005**

Let $3^{2x-y} = t$

$$\Rightarrow 3t = \frac{3}{t} - 8 \Rightarrow 3t^2 + 8t - 3 = 0$$

$$\Rightarrow t = -3,\ \frac{1}{3}$$

But $t = -3$ (rejected)

So, $3^{2x-y} = t = 3^{-1}$

$\Rightarrow\ 2x - y = -1$ ......(1)

Again, $\log_6\left|2x^2y - xy^2\right| = 1 + \log_{36}(xy)$

$\Rightarrow\ \log_6|xy(2x-y)| = 1 + \log_6\sqrt{xy}$

$\Rightarrow\ \log_6|xy| = \log_6\left(6\sqrt{xy}\right)$

$\Rightarrow\ xy = 36$ ......(2)

$\therefore$ On solving (1) and (2), we get $x = 4$ and $y = 9$

$\Rightarrow\ |x-y| = 5$

Q.8 **0007**

As, $\dfrac{1}{\log_a\left(2-\sqrt{3}\right)} + \dfrac{1}{\log_b\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}$

$= \log_{2-\sqrt{3}} a + \log_{\frac{\sqrt{3}-1}{\sqrt{3}+1}} b$

$= \log_{2-\sqrt{3}} a + \log_{2-\sqrt{3}} b = \log_{2-\sqrt{3}}(ab)$

Now, $\left(2+\sqrt{3}\right)^{\log_{2-\sqrt{3}}(ab)} = \dfrac{1}{12}$

$\Rightarrow \left(2-\sqrt{3}\right)^{\log_{2-\sqrt{3}}\left(\frac{1}{ab}\right)} = \dfrac{1}{12}$

$\Rightarrow \dfrac{1}{ab} = \dfrac{1}{12} \Rightarrow ab = 12$

As a, b are co-prime numbers, so either a = 4, b = 3 or a = 3, b = 4.

Hence , (a + b) = 7. Ans.

Q.9 **0003**

$N = 10^p$; $p = \log_{10}8 - \log_{10}9 + 2\log_{10}6$

$$p = \log\left(\frac{8 \cdot 36}{9}\right) = \log_{10}32$$

$\therefore\ N = 10^{\log_{10} 32} = 32$

Hence characteristic of $\log_3 32$ is 3.

Q.10 **0009**

We have,

$2^{2x} - 8 \cdot 2^x + 15 = 0 \Rightarrow (2^x - 3)(2^x - 5) = 0 \Rightarrow 2^x = 3$ or $2^x = 5$

Hence smallest x is obtained by equating $2^x = 3$

$\Rightarrow\ x = \log_2 3$

So, $p = \log_2 3$

Hence, $4^{p} = 2^{2\log_2 3} = 2^{\log_2 9} = 9.$

PREVIOUS YEAR'S

MHT CET

Q.1 (1)

Q.2 (1)

JEE-MAIN

PREVIOUS YEAR'S

Q.1 [150]

$36 = 2^2 \times 3^2$

to get GCD (n, 36) = 2

Power of 2 n must be exactly 1

$\Rightarrow$ Number is divisible by 2 but not divisible by 4 and not divisible by 3 aslo.

Total 3 digits numbers.

= (divisible by 2) – (divisible by 4)

– (divisible by 3) + (divisible by 12)

$= 451 - 226 - 150 + 75$

$= 150$

Q.2 (4)

$$\alpha_n - 19^n - 12^n$$

$$\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} = \frac{31(19^9 - 12^9) - (19^{10} - 12^{10})}{57(19^8 - 12^8)}$$

$$= \frac{19^9(12) - 12^9(19)}{57(19^8 - 12^8)}$$

$$= \frac{19 \times 12(19^8 - 12^8)}{57(19^8 - 12^8)}$$

$$= 4$$

SETS

EXERCISE-I (MHT CET LEVEL)

Q.1 **(3)**
A collection of well defined objects which are distinct and distinguishable.

Q.2 **(4)**
Not a well defined collection

Q.3 **(3)**
$\therefore$ $x \in N$ and x is a prime number
where $3 < x < 5$ so void set.

Q.4 (3)
$A = \{\phi, \{\phi\}\}$
P(A) = set containing all subsets
$= \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}$
$= \{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

Q.5 (4)
conceptual
2^n

Q.6 (1)
Number of proper subsets of A = $2^n - 1$
Given : $A = \{1,2,3,4,5\}$
Here n=5
$\therefore$ no of proper subsets = $2^5 - 1$

Q.7 (2)

$$A = \left[x : x \in R, -1 < x < 1\right]$$

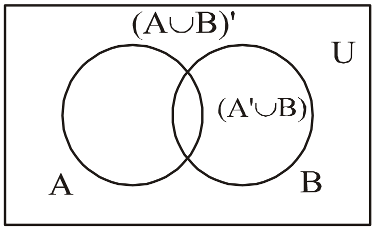
$$B = \left[x : x \in R : x - 1 \le -1 \text{ or } x - 1 \ge 1\right]$$

$$= \left[x : x \in R : x \le 0 \text{ or } x \ge 2\right]$$

$$\therefore A \cup B = R - D,$$

where $D = \left[x : x \in R, 1 \le x < 2\right]$

Q.8 (1)
From Venn-Euler's Diagram.



$\therefore$ $(A \cup B)' \cup (A' \cap B) = A'$

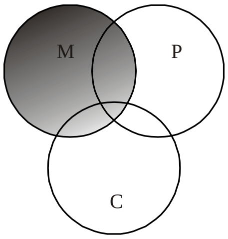
Q.9 (3)

n (M alone)

$$= n(M) - n(M \cap C) - n(M \cap P)$$

$$+ n(M \cap C \cap P)$$

$$= 100 - 28 - 30 + 18 = 60$$



Q.10 (2)
Given set can be written as

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

(By definition of symmetric difference)
Hence, $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

Q.11 (4)

Q.12 (3)

Q.13 (2)

Q.14 **(4)**
$A = \{x : x^2 = 4\} = \{+2, -2\}$
$B = \{x : x^2 - 5x + 6 = 0\} = \{2, 3\}$
$\therefore$ $A \cup B = \{-2, 2, 3\}$

Q.15 **(3)**
$N_a = \{a.n : n \in N\}$
$N_6 = \{6n : n \in N\} = \{6, 12, 18, 24, ...\}$
$N_8 = \{8n : n \in N\} = \{8, 16, 24, 32, ...\}$
$\therefore$ $N_6 \cap N_8 = \{24, 48, ...\} = \{24n : n \in N\}$
$\therefore$ $N_6 \cap N_8 = N_{24}$

Q.16 **[3]**
$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

Q.17 **[2]**
$A \cap B \subseteq A \subseteq A \cup B$, $\therefore$ $A \cap B \subseteq A \cup B$

Q.18 **(1)**
$A \cap B = \{2, 3, 4, 8, 10\} \cap \{3, 4, 5, 10, 12\}$
$= \{3, 4, 10\}$, $A \cap C = \{4\}$.
$\therefore (A \cap B) \cup (A \cap C) = \{3, 4, 10\}$.

Q.19 **(2)**
It is obvious.

Q.20 **(2)**
$A \cap B' = A = \{1, 2, 5\}$

Q.21 **(2)**
Given $n(A) = m$ & $n(B) = n$
$N_{SA} - N_{SB} = 56 \Rightarrow 2^m - 2^n = 56$
Checking options : $m = 6$, $n = 3$

Q.22 **(1)**

$$A = \left\{x : \frac{x}{2} \in z, 0 \le x \le 10\right\} = \{0, 2, 4, 6, 8, 10\}$$

$B = \{x : x$ is one digit prime number$\} = \{1, 2, 3, 5, 7\}$

$$C = \left\{x : \frac{x}{3} \in N, x \le 12\right\} = \{3, 6, 9, 12\}$$

$\therefore A \cap (B \cup C)$
$= \{0, 2, 4, 6, 8, 10\} \cap \{1, 2, 3, 5, 6, 7, 9, 12\} = \{2, 6\}$

Q.23 **(1)**
Minimum value of
$n = 100 - (30 + 20 + 25 + 15)) = 100 - 90 = 10$.

Q.24 **(3)**
Q.25 **(1)**
Q.26 **(3)**
Q.27 **(3)**
Q.28 (3)
$n(A \cap B) = n(A) + n(B) - n(A' \cap B')$
$= 200 + 300 - 100$
$n(A \cap B) = 400$
Now $n(A' \cap B') = U - n(A \cup B)$ (De marganistans)
$= 700 - 400 = 300$

Q.29 **[3]**

$n(A^c \cap B^c) = n[(A \cup B)^c] = n(U) - n(A \cup B)$

$= n(U) - [n(A) + n(B) - n(A \cap B)]$
$= 700 - [200 + 300 - 100] = 300.$

Q.30 **[3]**
$n(3) = 20, n(2) = 50, n(C \cap B) = 10$
Now $n(C \cup B) = n(3) + n(2) - n(C \cap B)$
$= 20 + 50 - 10 = 60.$
Hence, required number of persons = 60%.

Q.31 **(3)**
Since $A \subseteq B$, $\therefore A \cup B = B$.
So, $n(A \cup B) = n(B) = 6$.

Q.32 **(1)**
$n(A) = 10, n(B) = 6, n(C) = 5$
& $n(A \cap B) = n(B \cap C) = n(C \cap A) = 0$
$\therefore$ $n(A \cap B \cap C)$ $= n(A) + n(B) + n(C) = 21$

Q.33 **(3)**
$X \cap (Y \cup X)'$ Clearly $Y \cup X$ is superset of X
$\therefore$ $(Y \cup X')$ & X have no common element.
$\therefore$ $= \phi$

Q.34 **(3)**
$n(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B)$
$= n(U) - [n(A) + n(B) - n(A \cap B)]$
$= 700 - [200 + 300 - 100] = 300$

Q.35 **(2)**
$n(A) = 3$; $n(B) = 6$
$\because$ $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 3 + 6 - n(A \cap B)$
$\therefore$ $n(A \cup B) = 9 - n(A \cap B)$
$\therefore$ $[n(A \cup B)]_{min} = 9 - [n(A \cap B)]_{max} = 9 - 3 = 6$

Q.36 **(3)**
Clearly $(A \times B) \cap (B \times A)$ contains those elements of A & B only, which one common i.e. b, c, d

$$\therefore (A \times B) \cap (B \times A) = \left\{\begin{matrix} (b \times b), & (b \times c), & (b \times d), \\ (c \times b), & (c \times c), & (c \times d), \\ (d \times b), & (d \times c), & (d \times d), \end{matrix}\right\}$$

Hence 9 elements.

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)
$A = \{2, 3, 4 ....\}$
$B = \{0, 1, 2, 3 ....\}$
$A \cap B = \{2, 3\}$
Then $A \cap B$ is $\{x : x \in R, 2 \le x < 4\}$

Q.2 **(4)**

$A \cap (A \cap B)^c = A \cap (A^c \cup B^c)$

$= (A \cap A^c) \cup (A \cap B^c)$ $\phi \cup (A \cap B^c) = A \cap B^c$

Q.3 **(3)**
Since, $y = e^x$ and $y = x$ do not meet for any $x \in R$
$\therefore$ $A \cap B = \phi$.

Q.4 (2)

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} \forall a_i \in \{0, 1\}$$

Q.5 **[2]**
Since, $4^n - 3n - 1 = (3 + 1)^n - 3n - 1$

$= 3^n + {}^nC_1 3^{n-1} + {}^nC_2 3^{n-2} + ... + {}^nC_{n-1} 3 + {}^nC_n - 3n - 1$

$={}^n C_2 3^2 + {}^n C_3 .3^3 + ... + {}^nC_n 3^n$, $({}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}$ etc.)

$= 9[{}^nC_2 + {}^nC_3(3) + ... + {}^nC_n 3^{n-1}]$

$\therefore$ $4^n - 3n - 1$ is a multiple of 9 for $n \ge 2$.

For $n = 1$, $4^n - 3n - 1 = 4 - 3 - 1 = 0$,

For $= n = 2$, $4^n - 3n - 1 = 16 - 6 - 1 = 9$

Q.6 **(1)**

Let B, H, F denote the sets of members who are on the basketball team, hockey team and football team respectively.

Then we are given $n(B) = 21, n(H) = 26, n(F) = 29$

$n(H \cap B) = 14$, $n(H \cap F) = 15$, $n(F \cap B) = 12$

and $n(B \cap H \cap F) = 8$.

We have to find $n(B \cup H \cup F)$.

To find this, we use the formula

$n(B \cup H \cup F) = n(B) + n(H) + n(F)$

$-n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

Hence, $n(B \cup H \cup F) = (21 + 26 + 29) - (14 + 15 + 12) + 8 = 43$

Thus these are 43 members in all.

$X \subseteq Y$ *i.e.*, $X \cup Y = Y$.

Q.7 (3)

$U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

Solving for values ofx, we get

U = {0, 1, 2, 3}

A = {x : $x^2 - 5x + 6 = 0$}

Solving for values of x, we get

A = {2, 3}

and B = {x : $x^2 - 3x + 2 = 0$}

Solving for values of x, we get

B = {2, 1}

$A \cap B = \{2\}$

$\therefore (A \cap B)' = U - (A \cap B)$

$= \{0,1,2,3\} - \{2\} = \{0,1,3\}$

Q.8 (2)

$2^m - 2^n = 112 \Rightarrow 2^n(2^{m-n}) = 16.7$

$\therefore 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$

comparing we get n = 4 and m – n = 3

$\Rightarrow n = 4$ and m = 7

Q.9 **(2)**

n(1) = 40% of 10,000 = 4,000

n(2) = 20% of 10,000 = 2,000

n(3) = 10% of 10,000 = 1,000

$n(A \cap B)$ = 5% of 10,000 = 500

$n(B \cap C)$ = 3% of 10,000 = 300

$n(C \cap A)$ = 4% of 10,000 = 400

$n(A \cap B \cap C)$ = 2% of 10,000 = 200

We want to find $n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$

$= n(1) - n[A \cap (B \cup C)] = n(1) - n[(A \cap B) \cup (A \cap C)]$

$= n(1) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$

$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300.$

Q.10 **[2]**

$\because y = e^x$, $y = e^{-x}$ will meet, when $e^x = e^{-x}$

$\Rightarrow e^{2x} = 1, \therefore x = 0, y = 1$

$\therefore$ A and B meet on (0, 1), $\therefore$

Q.11 **(1)**

$3N = \{x \in N : x$ is a multiple of 3$\}$

$7N = \{x \in N : x$ is a multiple of 7$\}$

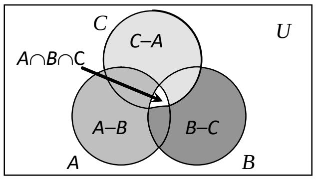
$\therefore 3N \cap 7N = \{x \in$ is a multiple of 3 and 7$\}$

$= \{x \in N : x$ is a multiple of 3 and 7$\}$

$= \{x \in N : x$ is a multiple of 21$\}$=21N.

Q.12 **(3)**

From *Venn-Euler's* Diagram,



Clearly, $\{(A - B) \cup (B - C) \cup (C - A)\}' = A \cap B \cap C$.

Q.13 **(3)**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$0.25 = 0.16 + 0.14 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 0.30 - 0.25 = 0.05$.

Q.14 **(3)**

Let A denote the set of Americans who like cheese and let B denote the set of Americans who like apples.

Let Population of American be 100.

Then $n(A) = 63, n(B) = 76$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 63 + 76 - n(A \cap B)$

$\therefore n(A \cup B) + n(A \cap B) = 139$

$\Rightarrow n(A \cap B) = 139 - n(A \cup B)$

But $n(A \cup B) \le 100$

$\therefore -n(A \cup B) \ge -100$

$\therefore 139 - n(A \cup B) \ge 139 - 100 = 39$

$\therefore n(A \cap B) \ge 39$ *i.e.*, $39 \le n(A \cap B)$

.....(i)

Again, $A \cap B \subseteq A, A \cap B \subseteq B$

$\therefore n(A \cap B) \le n(A) = 63$ and $n(A \cap B) \le n(B) = 76$

$\therefore n(A \cap B) \le 63$ …..(ii)

Then, $39 \le n(A \cap B) \le 63 \Rightarrow 39 \le x \le 63$.

Q.15 **(4)**

$n(C) = 224, n(H) = 240, n(B) = 336$

$n(H \cap B) = 64,\ n(B \cap C) = 80$

$n(H \cap C) = 40\ ,\ n(C \cap H \cap B) = 24$

$n(C^c \cap H^c \cap B^C) = n[(C \cup H \cup B)^c]$

$= n(\cup) - n(C \cup H \cup B)$

$= 800\ - [n(C) + n(H) + n(B) - n(H \cap C)$

$-n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$

$= 800\ - [224\ + 240\ + 336\ - 64\ - 80\ - 40\ + 24]$

$= 800\ - 640\ = 160\ \cdot$

Q.16 **(1)**

Let $n(P)$ = Number of teachers in Physics.

$n(M)$ = Number of teachers in Maths

$n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$20\ = n(P) + 12 - 4\ \Rightarrow\ n(P) = 12$.

Q.17 (4)

We have

$n\ (A \cup B \cup C) = n\ (A) + n\ (B) + n\ (C)$

$-\ n\ (A \cap B) - n(B \cap C) - n\ (C \cap A)$

$+\ n\ (A \cap B \cap C)$

$= 10\ +15 + 20 - 8 - 9 - n\ (C \cap A)$

$+\ n(A \cap B \cap C)$

$= 28 - \{n(C \cap A) - n\ (A \cap B \cap C)\}$ ...(i)

Since $n\ (C \cap A) \geq n\ (A \cap B \cap C)$

We have $n\ (C \cap A) - n\ (A \cap B \cap C) \geq 0$ ...(ii)

From (i) and (ii)

$n\ (A \cup B \cup C) \leq 28$ ...(iii)

Now, $n(A \cup B) = n\ (A) + n\ (B) - n\ (A \cap B)$

$= 10 + 15 - 8 = 17$

and $n\ (B \cup C) = n\ (B) + n\ (C) - n\ (B \cap C)$

$= 15 + 20 - 9 = 26$

Since, $n\ (A \cup B \cup C) \geq n\ (A \cup C)$ and

$n\ (A \cup B \cup C) \geq n\ (B \cup C)$, we have

$n\ (A \cup B \cup C) \geq 17$ and $n\ (A \cup B \cup C) \geq 26$

Hence $n\ (A \cup B \cup C) \geq 26$

...(iv)

From (iii) and (iv) we obtain

$26 \leq n\ (A \cup B \cup C) \leq 28$

Also $n\ (A \cup B \cup C)$ is a positive integer

$\therefore\ n(A \cup B \cup C) = 26$ or 27 or 28.

Q.18 **[3]**

$n(A \times B) = pq$

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 (0012)

$X \cap Y = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200\}$

$\therefore n(X \cap Y) = 12$

Q.2 (0060)

Given, $n(M) = 100, n(P) = 70\ (C) = 40$

$n(M \cap P) = 30, n(M \cap C), 28,$

$n(P \cap C) = 23$ and $n(M \cap P \cap C) = 18$

$\therefore\ n(M \cap P' \cap C') = n[M \cap (P \cap C')]$

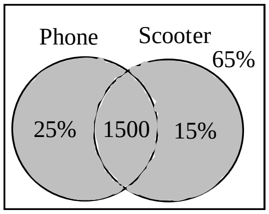
$= n(M) - n[M \cap (P \cap C)]$

$= n(M) - [n\ (M \cap P) + n(M \cap C) - n(M \cap P \cap C)]$

$= 100 - [30 + 28 - 18] = 60$

Q.3 30000

Let the total population of town be x.



$$\therefore\ \frac{25x}{100} + \frac{15x}{100} - 1500 + \frac{65x}{100} = x$$

$$\Rightarrow\ \frac{105x}{100} - x = 1500$$

$$\Rightarrow\ \frac{5x}{100} = 1500\ \Rightarrow\ x = 30000$$

Q.4 (0009)

Given, $n(A) = 4, n(B) = 5$ and $n(A \cap B) = 3$

$\therefore\ n[(A \times B) \cap (B \times A)] = 3^2 = 9$

Q.5 (0008)

$A \cap B = \{2, 4\}$

$\{A \cap B\} \subseteq \{1, 2, 4\} \{3, 2, 4\}, \{6, 2, 4\}, \{1, 3, 2, 4\}$

$\{1, 6, 2, 4\}, \{6, 3, 2, 4\}, \{2, 4\}, \{1, 3, 6, 2, 4\} \subseteq A \cup B$

$\Rightarrow\ n(C) = 8$

PREVIOUS YEAR'S

MHT CET

Q.1 (1)

Q.2 (2)

Q.3 (2)

Q.4 (2)

JEE-MAIN PREVIOUS YEAR'S

Q.1 **(2)**

$A : x \in (-3,1)$ $B : x \in (-\infty, 1) \cup [3, \infty)$

(A) $A - B = (-1,1)$

(B) $B - A = (-\infty, -3) \cup [3, \infty) = \mathbb{R} - (-3,3)$

(C) $A \cap B = (-3, -1)$

(D) $A \cup B = (-\infty, 1) \cup [3, \infty]$

$= \mathbb{R} - [1,3]$

So, option B is incorrect

Q.2 [11]

$S = \{4, 6, 9\}, T = \{9, 10, 11, ..., 1000\}$

$A\{a_1 + a_2 + ... + a_k : k \in N\}$ & $a_i \in S$

Here by the definition of set 'A'

$A = \{a : a = 4x + 6y + 9z\}$

Except the element 11, every element of set T is of the form $4x + 6y + 9z$ for some $x, y, z \in W$

$\therefore T - A = \{11\}$

Q.3 [7073]

Required no. = Total – no character from {1,2,3,4,5}

$= (10^6 - 5^6) + (10^7 - 5^7) + (10^8 - 5^8)$

$= 10^6 (1 + 10 + 100) - 5^6 (1 + 5 + 25)$

$= 10^6 \times 111 - 5^6 \times 31$

$= 26 \times 5^6 \times 111 - 5^6 \times 31$

$= 5^6 (2^6 \times 111 - 31)$

$= 5^6 \times 7073$

$\therefore \alpha = 7073$

Q.4 [112]

Total subsets of A

$= 2^7 = 128$

number of subsets of A when $C \cap B = \phi$

$= 2^4 = 16$ (C is subset of {1,2,4,5})

required answer $= 128 - 16$

$= 112$

Q.5 [107]

Let $A_1 = \{T \subseteq A : 1 \notin T\}$

$B_1 = \{T \subseteq A : 2 \in T\}$

So, $B = A_1 \cup B_1$

$n(B) = n(A_1 \cup B_1)$

$= n(A_1) + n(B_1) - n(A_1 \cap B_1)$

$= 2^6 + 2^6 - 2^5 = 96$

C = {T ⊆ A : sum of all the elements of T is a prime number}

$2 \rightarrow \{2\}$

$3 \rightarrow \{1, 2\}\{3\}$

$5 \rightarrow \{5\}, \{4, 1\}, \{3, 2\}$

$7 \rightarrow \{7\}, \{6, 1\}, \{5, 2\}, \{4, 3\}, \{1, 2, 4\}$

$11 \rightarrow \{4,7\},\{5,6\},\{1,3,7\},\{1,4,6\},\{2, 3, 6\},\{2,4,5\},\{1, 2,3,5\}$

$13 \rightarrow \{6,7\},\{1,5,7\},\{7,4,2\},\{6,5,2\},\{1,2,3,7\},\{1,2,4,6\},\{1,3,4,5\}$

$17 \rightarrow \{4,6,7\},\{1,3,6,7\},\{1,4,5,7\},\{2,3,5,7\},\{2,4,5,6\},\{1,2,3,4,7\},\{1,2,3,5,6\}$

$19 \rightarrow \{7,6,5,1\}\{1,2,3,6,7\},\{7,6,4,2\}\{1,2,4,5,7\},\{7,5,4,3\}\{1,3,4,5,6\}$

$23 \rightarrow \{7,6,5,4,1\}\{7,6,4,3,2,1\},\{7,6,5,3,2\}$

So, $n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$= 96 + 42 - 31$

$n(B \cup C) = 107$

RELATIONS AND FUNCTIONS

EXERCISE-I (MHT CET LEVEL)

RELATIONS

Q.1 (4)

A relation from P to Q is a subset of P × Q.

Q.2 (3)

R = A × B

Q.3 (4)

$n(A \times A) = n(A).n(A) = 3^2 = 9$

So, the total number of subsets of $A \times A$ is 2^9 and a subset of is a relation over the set A.

Q.4 (1)

$A = \{2,4,6\}; \ B = \{2,3,5\}$

$\therefore A \times B$ contains $3 \times 3 = 9$ elements.

Hence, number of relations from A to B $= 2^9$.

Q.5 (1)

It is obvious.

Q.6 (1)

$A = \{2, 4, 6\}$; $B = \{2, 3, 5\}$

$\therefore A \times B$ contains $3 \times 3 = 9$ elements.

Hence, number of relations from A to B $= 2^9$.

Q.7 (1)

Since R is reflexive relation on A, therefore $(a, a) \in R$ for all $a \in A$.

The minimum number of ordered pairs in R is n.

Q.8 (1)

It is obvious.

Q.9 (4)

Given, $xRy \Rightarrow x$ is relatively prime to y.

$\therefore$ Domain of $R = \{2, 3, 4, 5\}$.

Q.10 (1)

since, $|a-a| = 0 \le 1,$ so $aRa, \forall a \in R$

$\therefore R$ is reflexive. Now,

$aRb \Rightarrow |a-b| \le 1 \Rightarrow |b-a| \le 1 \Rightarrow bRa$

$\therefore R$ is symmetric. But R is not transitive as

$1R2, 2R3$ but $1 \not R 3$

$[\because |1-3| = 2 > 1]$

Q.11 (4)

$(3,3), (6,6), (9,9), (12,12), \in R$

R is transitive as the only pair which needs verification is (3,6) and (6,12) in R.

$\Rightarrow \quad (3,12) \in R$

Q.12 (2)

Obviously, the relation is not reflexive and transitive, but it is symmetric, because

$$x^2 + x^2 = 2x^2 \ne 1$$

and $x^2 + x^2 = 1, \ y^2 + z^2 = 1$

$$\Rightarrow x^2 + z^2 = 1$$

But $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$

Q.13 (4)

Q.14 (2)

Q.15 (3)

We have, $R = \{(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4, 5)\}$

$R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 2); (5, 3); (5, 4)\}$

Hence $RoR^{-1} = \{(3, 3); (3, 5); (5, 3); (5, 5)\}$.

Q.16 (2)

$R = \{(2, 1), (4, 2), (6, 3), ...\}$.

So, $R^{-1} = \{(1, 2), (2, 4), (3, 6), ...\}$.

Q.17 (1)

Q.18 **(3)**

Q.19 **(4)**

Q.20 **(3)**

Q.21 **(1)**

We first find R^{-1}, we have

$R^{-1} = \{(5, 4); (4, 1); (6, 4); (6, 7); (7, 3)\}$. We now obtain the elements of $R^{-1}oR$ we first pick the element of R and then of R^{-1} . Since $(4, 5) \in R$ and $(5, 4) \in R^{-1}$, we have $(4, 4) \in R^{-1}oR$

Similarly, $(1, 4) \in R, (4, 1) \in R^{-1} \Rightarrow (1, 1) \in R^{-1}oR$

$(4, 6) \in R, (6, 4) \in R^{-1} \Rightarrow (4, 4) \in R^{-1}oR,$

$(4, 6) \in R, (6, 7) \in R^{-1} \Rightarrow (4, 7) \in R^{-1}oR$

$(7, 6) \in R, (6, 4) \in R^{-1} \Rightarrow (7, 4) \in R^{-1}oR,$

$(7, 6) \in R, (6, 7) \in R^{-1} \Rightarrow (7, 7) \in R^{-1}oR$

$(3, 7) \in R, (7, 3) \in R^{-1} \Rightarrow (3, 3) \in R^{-1}oR,$

Hence, $R^{-1}oR = \{(1, 1); (4, 4); (4, 7); (7, 4), (7, 7); (3, 3)\}$.

Q.22 (2)

Domain of $f(x) = R - \{3\}$, and range {1, -1}.

Q.23 (4)

$[x] = I$

(Integers only).

Q.24 (2)

$-1 \le 5x \le 1 \Rightarrow \frac{-1}{5} \le x \le \frac{1}{5}$

Hence domain is $\left[\frac{-1}{5}, \frac{1}{5}\right]$.

Q.25 $f(x) = \frac{\sin^{-1}(3-x)}{\log[|x|-2]}$

Let $g(x) = \sin^{-1}(3-x) \Rightarrow -1 \le 3-x \le 1$

Domain of $g(x)$ is [2, 4]

and let $h(x) = \log[|x|-2] \Rightarrow |x|-2 > 0$

$\Rightarrow |x| > 2 \Rightarrow x < -2$ or $x > 2 \Rightarrow (-\infty, -2) \cup (2, \infty)$

We know that

$(f/g)(x) = \frac{f(x)}{g(x)} \forall x \in D_1 \cap D_2 - \{x \in R : g(x) = 0\}$

Domain of $f(x) = (2, 4] - \{3\} = (2, 3) \cup (3, 4]$.

Q.26 (4)

f(x) is defined if

$x^2 - \left[x^2\right] \ge 0 \Rightarrow x^2 \ge [x]^2$

which is true for all positive real x and all negative integers x.

Q.27 **(2)**

Q.28 (2)

$f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is defined

(i) $-1 \le x-3 \le 1 \Rightarrow 2 \le x \le 4$ and

(ii) $9 - x^2 > 0 \Rightarrow -3 < x < 3$

Taking common solution of (i) and (ii),

we get $2 \le x < 3$

$\therefore$ Domain = [2, 3)

Q.29 **(2)**

Q.30 (1)

$y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right] \Rightarrow -1 \le \log_3\left(\frac{x}{3}\right) \le 1$

$\Rightarrow \frac{1}{3} \le \frac{x}{3} \le 3 \Rightarrow 1 \le x \le 9 \Rightarrow x \in [1, 9]$

Q.31 (3)

For $x = -3, 3, |x^2 - 9| = 0$

Therefore $\log|x^2 - 9|$ does not exist at $x = -3, 3$.

Hence domain of function is $R - \{-3, 3\}$.

Q.32 (3)

$f(x) = \log|\log x|$, f(x) is defined if $|\log x| > 0$ and $x > 0$ i.e., if $x > 0$ and $x \ne 1$ $(\because |\log x| > 0$ if $x \ne 1)$

$\Rightarrow x \in (0, 1) \cup (1, \infty)$.

Q.33 (1)

$f(x) = \sin^{-1}[\log_2(x/2)]$,

Domain of $\sin^{-1} x$ is $x \in [-1, 1]$

$\Rightarrow -1 \le \log_2(x/2) \le 1 \Rightarrow \frac{1}{2} \le \frac{x}{2} \le 2 \Rightarrow 1 \le x \le 4$

$\therefore x \in [1, 4]$

Q.34 (4)

Here $x + 3 > 0$ and $x^2 + 3x + 2 \ne 0$

$\therefore x > -3$ and $(x + 1)(x + 2) \ne 0$ i.e. $x \ne -1, -2$

Domain $= (-3, \infty) - \{-1, -2\}$

Q.35 (2)

The function $\sec^{-1} x$ is defined for all $x \in R - (-1, 1)$

and the function $\frac{1}{\sqrt{x - [x]}}$ is defined for all $x \in R - Z$.

So the given function is defined for all $x \in R - \{(-1, 1) \cup (n \mid n \in Z)\}$.

Q.36 (2)

$x^2 - 6x + 7 = (x-3)^2 - 2$

Obviously, minimum value is – 2 and maximum ∞. Hence range of function is $[-2, \infty]$.

Q.37 (2)

$f(x) = \sqrt{\log \frac{1}{|\sin x|}} \Rightarrow 3 + x > 0$

$\Rightarrow x \ne n\pi + (-1)^n 0 \Rightarrow x \ne n\pi$.

Domain of $f(x) = R - \{n\pi, n \in I\}$

Q.38 (3)

$f(x) = \log(\sqrt{x-4} + \sqrt{6-x})$

$\Rightarrow y = x^x \Rightarrow \log y = x \log x$ and $6 - x \ge 0$

$\Rightarrow x \ge 4$ and $x \le 6$

Domain of $f(x) = [4, 6]$.

Q.39 (3)

f(x) is to be defined when $x^2 - 1 > 0$

$\Rightarrow x^2 > 1, \Rightarrow x < -1$ or $x > 1$ and $3 + x > 0$

$\therefore x > -3$ and $x \ne -2$

$\therefore D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$

Q.40 (2)

According to question, as $\sqrt{\sin 2x}$ can't be negative. So the option (2) is correct

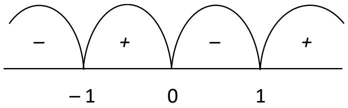
Domain of function $\sqrt{\sin 2x}$ is $[n\pi, n\pi+\pi/2]$.

Q.41 (4)

$f(x)=\frac{3}{4-x^2}+\log_{10}(x^3-x)$. So, $4-x^2\neq 0$

$\Rightarrow \quad x\neq \pm\sqrt{4}$

and $x^3-x>0\Rightarrow x(x^2-1)>0 \Rightarrow \quad x>0, x>1$



$\therefore\ D=(-1,0)\cup(1,\infty)-\{\sqrt{4}\}$

i.e., $D=(-1,0)\cup(1,2)\cup(2,\infty)$

Q.42 (2)
The quantity under root is positive, when

$-1-\sqrt{3}\le x\le -1+\sqrt{3}.$

Q.43 (2)
Obviously, here $|x|>2$ and $x\neq 1$

i.e., $x\in(-\infty,-2)\cup(2,\infty)$

Q.44 (2)

$\log\left\{\frac{5x-x^2}{6}\right\}\ge 0\Rightarrow\frac{5x-x^2}{6}\ge 1$

or $x^2-5x+6\le 0$ or $(x-2)(x-3)\le 0$.

Hence $2\le x\le 3.$

Q.45 (3)

(i) $x\le 2$ (ii) $\sqrt{9-x^2}>0\Rightarrow |x|<3$

or $-3<x<3.$

Hence domain is $(-3, 2].$

Q.46 (4)

$1+x\ge 0\Rightarrow x\ge -1$; $1-x\ge 0\Rightarrow x\le 1,\ x\neq 0$

Hence domain is $[-1,1]-\{0\}$.

Q.47 (4)

$f(x)=\sqrt{x-x^2}+\sqrt{4+x}+\sqrt{4-x}$

Clearly $f(x)$ is defined, if $4+x\ge 0\Rightarrow x\ge -4$

$4-x\ge 0 \Rightarrow x\le 4$

$x(1-x)\ge 0\Rightarrow x\ge 0$ and $x\le 1$

Domain of $f=(-\infty,4]\cap[-4,\infty)\cap[0,1]=[0,1]$.

Q.48 (1)

Clearly $-1\le\frac{1}{1+e^x}\le 1$

But $2<e^x<3\Rightarrow 3<(e^x+1)<4$

$\Rightarrow\frac{1}{4}<\frac{1}{1+e^x}<\frac{1}{3}$

Domain of $f(x)=\left(\frac{1}{4},\frac{1}{3}\right)$

Q.49 (3)

The function $f(x)=\sqrt{\log(x^2-6x+6)}$ is defined when $\log(x^2-6x+6)\ge 0$

$\Rightarrow x^2-6x+6\ge 1 \Rightarrow (x-5)(x-1)\ge 0$

This inequality holds if $x\le 1$ or $x\ge 5$. Hence, the domain of the function will be $(-\infty,1]\cup[5,\infty)$.

Q.50 (1)

Here $|x|>1,$ therefore $x\in(-\infty,-1)\cup(1,\infty).$

Q.51 (1)

For it must $|x|-x>0$

$|x|>x$ but $|x|=x$ for x positive and $|x|>x$ for x negative. So, domain will be $(-\infty, 0)$.

Q.52 (4)

$f(x)=\sqrt{x^2-1}+\sqrt{x^2+1}\Rightarrow f(x)=y_1+y_2$

Domain of $y_1=\sqrt{x^2-1}\Rightarrow x^2-1\ge 0\Rightarrow x^2\ge 1$

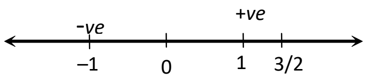
$x\in(-\infty,\infty)-(-1,1)$ and Domain of y_2 is real number,

$\therefore$ Domain of $f(x)=(-\infty,\infty)-(-1,1)$.

Q.53 (4)

$f(x)=e^{\sqrt{5x-3-2x^2}}$

$\Rightarrow 5x-3-2x^2\ge 0$ or $(x-1)\left(x-\frac{3}{2}\right)\ge 0$



$\therefore\ D\in[1,3/2]$.

Q.54 (2)

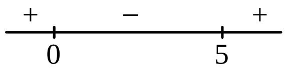
To define $f(x)$, $9-x^2>3\Rightarrow -3<x<3$ .....(i)

$-1\le(x-3)\le 1\Rightarrow 2\le x\le 4$ .....(ii)

From (i) and (ii), $2\le x<3$ i.e., [2, 3).

Q.55 (1)

$$\frac{5x-x^2}{4}>0 \Rightarrow x^2-5x<0 \Rightarrow x(x-5)<0$$



Hence $x\in(0,5)$

Q.56 (4)

$\sec^{-1}\left(\frac{2-|x|}{4}\right)\ge 0$ and domain of $\sec^{-1}x$

is $x\in R-(-1,1)$

$$\frac{2-|x|}{4}\ge 1 \qquad \text{or} \qquad \frac{2-|x|}{4}\le -1$$

$$\begin{array}{l|l} 2-|x|\ge 4 & 2-|x|\le -4 \\ -|x|\ge 2 & -|x|\le -6 \\ |x|\le -2 & |x|\ge 6 \end{array}$$

Not possible $\Rightarrow x\in(-\infty,-6]\cup[6,\infty)$

$\therefore x\in R-(-6,6)$

Q.57 (4)

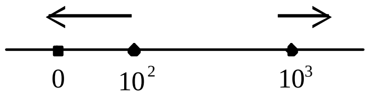
f(x) to be defined if

$\{(\log_{10}x)^2-5\log_{10}x+6\}>0$ and $x>0$

i.e. $(\log_{10}x-2)(\log_{10}x-3)>0$ and $x>0$

i.e. $\log_{10}x<2$ or $\log_{10}x>3$ and $x>0$

i.e. $x<10^2$ or $x>10^3$ and $x>0$



Domain $\in(0,10^2)\cup(10^3,\infty)$

Q.58 (1)

The domain of function $\log_e\{x-[x]\}$ is R, because [x] is a greatest integer whose value is equal to or less than zero.

Q.59 (4)

$$-1\le\log_3 x\le 1;\ 3^{-1}\le x\le 3 \Rightarrow \frac{1}{3}\le x\le 3$$

$\therefore$ Domain of function $=\left[\frac{1}{3},3\right]$.

Q.60 (3)

(i) $x\le 2$

(ii) $\sqrt{9-x^2}>0 \Rightarrow |x|<3$

or $-3<x<3$

Hence domain is $(-3,\ 2]$.

Q.63 (4)

$1-\frac{1}{x}>0 \Rightarrow x>1$ Also, $x\ne 0$.

$\therefore$ Required interval $=(-\infty,0)\cup(1,\infty)$.

Q.66 (2)

$$f(x)=\frac{\sin^{-1}(3-x)}{\log[|x|-2]}$$

Let $g(x)=\sin^{-1}(3-x) \Rightarrow -1\le 3-x\le 1$

Domain of $g(x)$ is $[2,4]$

and let $h(x)=\log[|x|-2] \Rightarrow |x|-2>0$

$\Rightarrow |x|>2 \Rightarrow x<-2$ or $x>2$

$\Rightarrow (-\infty,-2)\cup(2,\infty)$

we know that

$$(f/g)(x)=\frac{f(x)}{g(x)}\forall x\in D_1\cap D_2-\{x\in R: g(x)=0\}$$

$\therefore$ Domain of $f(x)=(2,4]-\{3\}=(2,3)\cup(3,4]$.

Q.67 (1)

$$y=\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right] \Rightarrow -1\le\log_3\left(\frac{x}{3}\right)\le 1$$

$$\Rightarrow \frac{1}{3}\le\frac{x}{3}\le 3 \Rightarrow 1\le x\le 9 \Rightarrow x\in[1,9].$$

Q.68 (4) Here $x+3>0$ and $x^2+3x+2\ne 0$

$\therefore\ x>-3$ and $(x+1)(x+2)\ne 0$, i.e. $x\ne -1,-2$

$\therefore$ Domain $=(-3,\infty)-\{-1,-2\}$.

Q.69 (2)

$x^2-6x+7=(x-3)^2-2$

Obviously, minimum value is -2 and maximum ∞.

Hence range of function is $[-2,\infty]$.

Q.70 (2)

$$f(x)=\sqrt{\log\frac{1}{|\sin x|}} \Rightarrow \sin x\ne 0$$

$\Rightarrow x\ne n\pi+(-1)^n 0$

$\Rightarrow x\ne n\pi$ Domain of $f(x)=R-\{n\pi,\ n\in I\}$.

Q.71 (2)

According to question, as $\sqrt{\sin 2x}$ can’t be negative.

So the option (2) is correct

Domain of function $\sqrt{\sin 2x}$ is $[n\pi, n\pi+\pi/2]$.

Q.72 (2)

$$\log\left\{\frac{5x-x^2}{6}\right\}\ge 0 \Rightarrow \frac{5x-x^2}{6}\ge 1$$

or $x^2-5x+6\le 0$ or $(x-2)(x-3)$

Hence $2 \le x \le 3$.

Q.73 $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right)$

We know that, $0 \le \cos^2 x \le 1$ at $\cos x = 0,\ f(x) = 1$ and at $\cos x = 1,\ = \alpha.1 - \beta.1 = \alpha - \beta.$;

$\therefore\ 1 \le x \le \sqrt{2}\quad x \in [1, \sqrt{2}]$

$\therefore\ 1 \le x \le \sqrt{2} \Rightarrow x \in [1, \sqrt{2}]$

Q.74 (3)

$$f(x) = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$\Rightarrow$ Range $= (1, 7/3]$.

Q.75 **(3)**

Q.76 **(2)**

Q.77 **(3)**

Q.78 (4)

$f(x) = a\cos(bx + c) + d$

…..(i)

For minimum $\cos(bx + c) = -1$

from (i), $f(x) = -a + d = (d - a)$

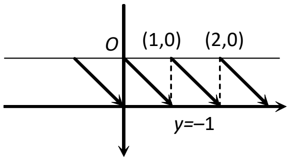
For maximum $\cos(bx + c) = 1$

from (i), $f(x) = a + d = (d + a)$

Range of $f(x) = [d - a,\ d + a]$.

Q.79 (2)

As shown in graph



Range is (–1, 0].

Q.80 (2)

$f(x) = \cos(x/3)$

We know that $-1 \le \cos(x/3) \le 1$.

Q.81 (2)

$$f(x) = \frac{x + 2}{|x + 2|}$$

$$f(x) = \begin{cases} -1, & x < -2 \\ 1, & x > -2 \end{cases}$$

Range of $f(x)$ is $\{-1, 1\}$.

Q.82 (4)

Since maximum and minimum values of $\cos x - \sin x$ are $\sqrt{2}$ and $-\sqrt{2}$ respectively, therefore range of $f(x)$ is $[-\sqrt{2}, \sqrt{2}]$.

Q.83 (2)

R^+ {as y is always positive $\forall\ x \in R$}

Q.84 (2)

Let $\frac{x^2 + 34x - 71}{x^2 + 2x - 7} = y$

$\Rightarrow x^2(1 - y) + 2(17 - y)x + (7y - 71) = 0$

For real value of x, $B^2 - 4AC \ge 0$

$\Rightarrow y^2 - 14y + 45 \ge 0 \Rightarrow y \ge 9,\ y \le 5$

Q.85 **(3)**

$y = \frac{1}{2 - \sin 3x}\quad -1 \le \sin\theta \le 1$

$-1 \le \sin 3x \le 1$

$1 \ge -\sin 3x \ge -1$

$3 \ge 2 - \sin 3x \ge 1$

$\frac{1}{3} \le \frac{1}{2 - \sin 3x} \le 1$

$\therefore$ range $\in \left[\frac{1}{3}, 1\right]$

Q.86 **(4)**

$y = \log_{\sqrt{7}}(\sqrt{2}(\sin x - \cos x) + 5)$

$-\sqrt{2 + 2} \le \sqrt{2}\ (\sin x - \cos x) \le \sqrt{2 + 2}$

$-2 + 5 \le \sqrt{2}\ (\sin x - \cos x) + 5 \le 2 + 5$

$\log_{\sqrt{7}} 3 \le \log_{\sqrt{7}}[\sqrt{2}(\sin x - \cos x) + 5] \le \log_{\sqrt{7}} 7$

$2\log_7 3 \le y \le 2$

$\therefore$ Range $\in [2\log_7 3, 2]$

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)

$A = \{2, 3, 4 \ldots\ldots\ldots.\}$

$B = \{0, 1, 2, 3 \ldots\ldots\ldots.\}$

$A \cap B = \{2, 3\}$

Then $A \cap B$ is $\{x : x \in R, 2 \le x < 4\}$

Q.2 **(4)**

$A \cap (A \cap B)^c = A \cap (A^c \cup B^c)$

$= (A \cap A^c) \cup (A \cap B^c)\ \phi \cup (A \cap B^c) = A \cap B^c$

Q.3 **(3)**

Since, $y = e^x$ and $y = x$ do not meet for any $x \in R$

$\therefore\ A \cap B = \phi$.

Q.4 (2)

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} \quad \forall\, a_i \in \{0, 1\}$$

Q.5 **[2]**

Since, $4^n - 3n - 1 = (3+1)^n - 3n - 1$

$= 3^n + {}^nC_1 3^{n-1} + {}^nC_2 3^{n-2} + ... + {}^nC_{n-1} 3 + {}^nC_n - 3n - 1$

$= {}^n C_2 3^2 + {}^n C_3 .3^3 + ... + {}^nC_n 3^n$, $({}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}$ etc.)

$= 9[{}^nC_2 + {}^nC_3(3) + ... + {}^nC_n 3^{n-1}]$

$\therefore\ 4^n - 3n - 1$ is a multiple of 9 for $n \ge 2$.

For $n = 1,\ 4^n - 3n - 1 = 4 - 3 - 1 = 0$,

For $= n = 2,\ 4^n - 3n - 1 = 16 - 6 - 1 = 9$

Q.6 **(1)**

Let *B*, *H*, *F* denote the sets of members who are on the basketball team, hockey team and football team respectively.

Then we are given $n(B) = 21, n(H) = 26, n(F) = 29$

$n(H \cap B) = 14$, $n(H \cap F) = 15$, $n(F \cap B) = 12$

and $n(B \cap H \cap F) = 8$.

We have to find $n(B \cup H \cup F)$.

To find this, we use the formula

$n(B \cup H \cup F) = n(B) + n(H) + n(F)$

$-n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

Hence, $n(B \cup H \cup F) = (21 + 26 + 29) - (14 + 15 + 12) + 8 = 43$

Thus these are 43 members in all.

$X \subseteq Y$ *i.e.*, $X \cup Y = Y$.

Q.7 (3)

$$U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$$

Solving for values ofx, we get

U = {0, 1, 2, 3}

$A = \{x : x^2 - 5x + 6 = 0\}$

Solving for values of x, we get

A = {2, 3}

and $B = \{x : x^2 - 3x + 2 = 0\}$

Solving for values of x, we get

B = {2, 1}

$$A \cap B = \{2\}$$

$$\therefore (A \cap B)' = U - (A \cap B)$$

$$= \{0,1,2,3\} - \{2\} = \{0,1,3\}$$

Q.8 (2)

$$2^m - 2^n = 112 \Rightarrow 2^n(2^{m-n}) = 16.7$$

$$\therefore 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$$

comparing we get n = 4 and m – n = 3

$$\Rightarrow n = 4 \text{ and } m = 7$$

Q.9 **(2)**

n(1) = 40% of 10,000 = 4,000

n(2) = 20% of 10,000 = 2,000

n(3) = 10% of 10,000 = 1,000

$n(A \cap B)$ = 5% of 10,000 = 500

$n(B \cap C)$ = 3% of 10,000 = 300

$n(C \cap A)$ = 4% of 10,000 = 400

$n(A \cap B \cap C)$ = 2% of 10,000 = 200

We want to find $n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$

$= n(1) - n[A \cap (B \cup C)] = n(1) - n[(A \cap B) \cup (A \cap C)]$

$= n(1) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$

$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300.$

Q.10 **[2]**

$\because y = e^x,\ y = e^{-x}$ will meet, when $e^x = e^{-x}$

$\Rightarrow e^{2x} = 1,\ \therefore x = 0, y = 1$

$\therefore$ *A* and *B* meet on (0, 1), $\therefore$.

Q.11 **(1)**

$3N = \{x \in N : x$ is a multiple of 3}

$7N = \{x \in N : x$ is a multiple of 7}

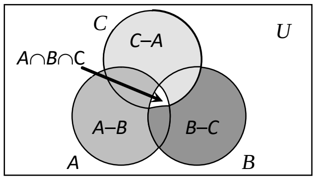
$\therefore 3N \cap 7N = \{x \in$ is a multiple of 3 and 7}

$= \{x \in N : x$ is a multiple of 3 and 7}

$= \{x \in N : x$ is a multiple of 21}=21*N*.

Q.12 **(3)**

From *Venn-Euler's* Diagram,



Clearly, $\{(A - B) \cup (B - C) \cup (C - A)\}' = A \cap B \cap C$.

Q.13 **(3)**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$0.25 = 0.16 + 0.14 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 0.30 - 0.25 = 0.05$.

Q.14 **(3)**

Let *A* denote the set of Americans who like cheese and let *B* denote the set of Americans who like apples.

Let Population of American be 100.

Then $n(A) = 63, n(B) = 76$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 63 + 76 - n(A \cap B)$

$\therefore\ n(A \cup B) + n(A \cap B) = 139$

$\Rightarrow\ n(A \cap B) = 139 - n(A \cup B)$

But $n(A \cup B) \le 100$

$\therefore -n(A \cup B) \ge -100$

$\therefore\ 139 - n(A \cup B) \ge 139 - 100 = 39$

$\therefore\ n(A \cap B) \geq 39$ *i.e.*, $39 \leq n(A \cap B)$
.....(i)

Again, $A \cap B \subseteq A, A \cap B \subseteq B$

$\therefore\ n(A \cap B) \leq n(A) = 63$ and $n(A \cap B) \leq n(B) = 76$

$\therefore n(A \cap B) \leq 63$ .....(ii)

Then, $39 \leq n(A \cap B) \leq 63 \Rightarrow 39 \leq x \leq 63 \cdot$

Q.15 **(4)**

$n(C) = 224, n(H) = 240, n(B) = 336$

$n(H \cap B) = 64,\ n(B \cap C) = 80$

$n(H \cap C) = 40\ ,\ n(C \cap H \cap B) = 24$

$n(C^c \cap H^c \cap B^C) = n[(C \cup H \cup B)^c]$

$= n(\cup) - n(C \cup H \cup B)$

$= 800\ - [n(C) + n(H) + n(B) - n(H \cap C)$

$-n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$

$= 800\ - [224\ + 240\ + 336\ - 64\ - 80\ - 40\ + 24]$

$= 800\ - 640\ = 160\ \cdot$

Q.16 **(1)**

Let $n(P)$ = Number of teachers in Physics.

$n(M)$ = Number of teachers in Maths

$n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$20\ = n(P) + 12\ - 4\ \Rightarrow n(P) = 12$.

Q.17 (4)

We have

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(C \cap A)$
$+ n(A \cap B \cap C)$
$= 10 + 15 + 20 - 8 - 9 - n(C \cap A)$
$+ n(A \cap B \cap C)$
$= 28 - \{n(C \cap A) - n(A \cap B \cap C)\}$ ...(i)

Since $n(C \cap A) \geq n(A \cap B \cap C)$

We have $n(C \cap A) - n(A \cap B \cap C) \geq 0$ ...(ii)

From (i) and (ii)

$n(A \cup B \cup C) \leq 28$ ...(iii)

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 10 + 15 - 8 = 17$

and $n(B \cup C) = n(B) + n(C) - n(B \cap C)$
$= 15 + 20 - 9 = 26$

Since, $n(A \cup B \cup C) \geq n(A \cup C)$ and
$n(A \cup B \cup C) \geq n(B \cup C)$, we have
$n(A \cup B \cup C) \geq 17$ and $n(A \cup B \cup C) \geq 26$

Hence $n(A \cup B \cup C) \geq 26$
...(iv)

From (iii) and (iv) we obtain
$26 \leq n(A \cup B \cup C) \leq 28$

Also $n(A \cup B \cup C)$ is a positive integer
$\therefore n(A \cup B \cup C) = 26$ or 27 or 28.

Q.18 **[3]**

$n(A \times B) = pq$

EXERCISE-III

Q.1 (0012)

$X \cap Y = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200\}$

$\therefore n(X \cap Y) = 12$

Q.2 (0060)

Given, $n(M) = 100, n(P) = 70\ (C) = 40$

$n(M \cap P) = 30, n(M \cap C), 28,$

$n(P \cap C) = 23$ and $n(M \cap P \cap C) = 18$

$\therefore\ n(M \cap P' \cap C') = n[M \cap (P \cap C')]$

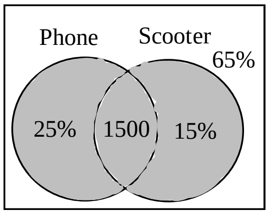
$= n(M) - n[M \cap (P \cap C)]$

$= n(M) - [n(M \cap P) + n(M \cap C) - n(M \cap P \cap C)]$

$= 100 - [30 + 28 - 18] = 60$

Q.3 30000

Let the total population of town be x.



$$\therefore\ \frac{25x}{100} + \frac{15x}{100} - 1500 + \frac{65x}{100} = x$$

$$\Rightarrow\ \frac{105x}{100} - x = 1500$$

$$\Rightarrow\ \frac{5x}{100} = 1500 \quad \Rightarrow \quad x = 30000$$

Q.4 (0009)

Given, $n(A) = 4, n(B) = 5$ and $n(A \cap B) = 3$

$\therefore\ n[(A \times B) \cap (B \times A)] = 3^2 = 9$

Q.5 (0008)

$A \cap B = \{2, 4\}$

$\{A \cap B\} \subseteq \{1, 2, 4\}\{3, 2, 4\}, \{6, 2, 4\}, \{1, 3, 2, 4\}$

$\{1, 6, 2, 4\}, \{6, 3, 2, 4\}, \{2, 4\}, \{1, 3, 6, 2, 4\} \subseteq A \cup B$

$\Rightarrow\ n(C) = 8$

PREVIOUS YEAR'S

MHT CET

Q.1 (3)
Q.2 (2)
Q.3 (4)
Q.4 (2)
Q.5 (2)

Trigonometric Ratio and Identities

EXERCISE-I (MHT CET LEVEL)

Q.1 (1)
It is obvious.

Q.2 (2)
The true relation is $\sin 1 > \sin 1^{o}$

Since value of $\sin\theta$ is increasing $\left[0 \to \frac{\pi}{2}\right]$.

Q.3 **(1)**

Q.4 (2)

$$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$$

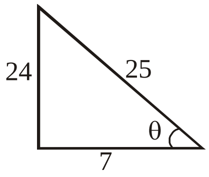
$$=\frac{\cos^2\theta}{\cos\theta-\sin\theta}-\frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$=\cos\theta+\sin\theta$$

Q.5 (1)
$\tan 1^{o}\tan 2^{o}$ .... $\tan 89^{o}$
$= (\tan 1^{o}\tan 89^{o})(\tan 2^{o}\tan 88^{o})$ ....$= 1 \times 1 \times 1$.... $= 1$.

Q.6 (4)
We have,
$\sin\theta + \operatorname{cosec}\theta = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$
$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$
$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$
Required value of $\sin^{10}\theta + \operatorname{cosec}^{10}\theta = (1)^{10} + \frac{1}{(1)^{10}} = 2$.

Q.7 (3)



$$\text{Sec}\theta = \frac{25}{7}$$

$$\tan\theta = \frac{24}{7}$$

$$\text{Sec}\theta + \tan\theta = \frac{25}{7}+\frac{24}{7}=\frac{25+24}{7}=\frac{49}{7}$$

Life in second Quadrent than $\sec\theta + \tan\theta = -7$

Q.8 (c)

Q.9 **(d)**

Q.10 **(d)**

Q.11 **(a)**

Q.12 **(a)**

Q.13 (4)

$$3\tan A + 4 = 0 \Rightarrow \tan A = -\frac{4}{3}$$

$$\Rightarrow \sin A = \pm\frac{\tan A}{\sqrt{1+\tan^2 A}} = \pm\frac{-4/3}{\sqrt{1+16/9}} = \frac{4}{5}$$

(Q A is in 2nd quadrant)

and $\cos A = -\frac{3}{5}$, Thus, $2\cot A - 5\cos A + \sin A$

$$=2\left(-\frac{3}{4}\right)-5\left(-\frac{3}{5}\right)+\frac{4}{5}=\frac{23}{10}$$

Q.14 (4)

$\tan A + \cot A = 4$

$\Rightarrow \tan^2 A + \cot^2 A + 2\tan A\cot A = 16$

$\Rightarrow \tan^2 A + \cot^2 A = 14 \Rightarrow \tan^4 A + \cot^4 A + 2 = 196$

$\Rightarrow \tan^4 A + \cot^4 A = 194.$

Q.15 (3)

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$$=\frac{\tan^2\alpha}{1+\tan^2\alpha}+\frac{\tan^2\beta}{1+\tan^2\beta}+\frac{\tan^2\gamma}{1+\tan^2\gamma}$$

$$=\frac{x}{1+x}+\frac{y}{1+y}+\frac{z}{1+z}\ (x=\tan^2\alpha, y=\tan^2\beta, z=\tan^2\gamma)$$

$$=\frac{(x+y+z)+(xy+yz+zx+2xyz)+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}$$

$$=\frac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}=1$$

$(\because xy+yz+zx+2xyz=1)$

Q.16 (2)

Since $\sin 190° = -\sin 10°, \sin 200° = -\sin 20°,$

$\sin 210° = -\sin 30°, \sin 360° = \sin 180° = 0$ etc.

Q.17 (4)

The expression is equal to

$$\sin(x-y)+\cos(x-y)=\sqrt{2}\left\{\sin\left(\frac{\pi}{4}+x-y\right)\right\},$$

which is zero, if $\sin\left(\frac{\pi}{4}+x-y\right)=0$

i.e., $\frac{\pi}{4}+x-y=n\pi(n\in I)\Rightarrow x=n\pi-\frac{\pi}{4}+y$.

Q.18 (3)

$$\sin\frac{\pi}{10}\sin\frac{3\pi}{10}=\sin 18°.\sin 54°$$

$$=\sin 18°.\cos 36°=\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4}=\frac{1}{4}.$$

Q.19 (2)

$$\cos A+\sin(270°+A)-\sin(270°-A)+\cos(180°+A)$$

$$\cos A-\cos A+\cos A-\cos A=0$$

Q.20 (4)

$$\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}$$

$$=\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{\pi}{8}$$

$$=2\left(\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}\right)=2\times 1=2.$$

Q.21 (4)

We know that $\sin(A+B)=\sin A\cos B+\cos A\sin B$

$$=\frac{1}{\sqrt{10}}\sqrt{1-\frac{1}{5}}+\frac{1}{\sqrt{5}}\sqrt{1-\frac{1}{10}}$$

$$=\frac{1}{\sqrt{10}}\sqrt{\frac{4}{5}}+\frac{1}{\sqrt{5}}\sqrt{\frac{9}{10}}=\frac{1}{\sqrt{50}}(2+3)=\frac{5}{\sqrt{50}}=\frac{1}{\sqrt{2}}$$

$$\sin(A+B)=\sin\frac{\pi}{4}$$

Hence, $A+B=\frac{\pi}{4}$.

Q.22 (2)

Q.23 (3)

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}}=1$$

$$\therefore A+B=45°=\frac{\pi}{4}$$

Q.24 (1)

$$\cos^2\left(\frac{\pi}{6}+\theta\right)-\sin^2\left(\frac{\pi}{6}-\theta\right)$$

$$=\cos\left(\frac{\pi}{6}+\theta+\frac{\pi}{6}-\theta\right)\cos\left(\frac{\pi}{6}+\theta-\frac{\pi}{6}+\theta\right)a$$

$$=\cos\frac{2\pi}{6}\cos 2\theta=\frac{1}{2}\cos 2\theta$$

Q.25 (c)

$2e^{iB}=e^{iA}+e^{iC}$

$\Rightarrow 2\cos B=\cos A+\cos C$ .....(i)

& $2\sin B=\sin A+\sin C$ ....(ii)

Squaring and adding we get

$\cos(A-C)=1\Rightarrow A-C=0$

$\therefore A=C$, From (i) and (ii) $\cos B=\cos A$

and $\sin B=\sin A$

So, $A=B\Rightarrow A=B=C$

Q.26 (3)

$$2\tan(A-B)=2\left(\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$$

$$=2\frac{(2\tan B+\cot B-\tan B)}{1+(2\tan B+\cot B)\tan B}=2\frac{\tan B+\cot B}{2(1+\tan^2 B)}$$

$$=\frac{\cot B(\tan^2 B+1)}{(1+\tan^2 B)}=\cot B$$

Q.27 (4)

As given $\frac{\sin A+\sin B}{\cos A+\cos B}=\frac{C}{D}$

$$\Rightarrow\frac{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}}=\frac{C}{D}\Rightarrow\tan\frac{A+B}{2}=\frac{C}{D}$$

Thus, $\sin(A+B)=\frac{2\tan\frac{A+B}{2}}{1+\tan^2\frac{A+B}{2}}$

$$=\frac{2\frac{C}{D}}{1+\frac{C^2}{D^2}}=\frac{2CD}{(C^2+D^2)}.$$

Q.28 (2)

$$\sin 50^{o}-\sin 70^{o}+\sin 10^{o}$$

$$=-2\cos 60^{o}\sin 10^{o}+\sin 10^{o}$$

$$=\sin 10^{o}(1-2\cos 60^{o})=0.$$

Q.29 (2)

$$\cos^2 A-\sin^2 B=\cos(A+B).\cos(A-B)$$

$$\therefore\cos^2 48^{o}-\sin^2 12^{o}=\cos 60^{o}.\cos 36^{o}$$

$$=\frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right)=\frac{\sqrt{5}+1}{8}.$$

Q.30 (3) $\dfrac{\cos 12^o-\sin 12^o}{\cos 12^o+\sin 12^o}+\dfrac{\sin 147^o}{\cos 147^o}$

$$=\frac{1-\tan 12^o}{1+\tan 12^o}+\tan 147^o=\tan 33^o-\tan 33^o=0$$

Q.31 (1)

Since $\tan 3A=\dfrac{\tan A+\tan 2A}{1-\tan A\tan 2A}$

$\Rightarrow \tan 3A-\tan 2A-\tan A=\tan 3A\tan 2A\tan A$

Q.32 (3) $\sin^4\dfrac{\pi}{8}+\sin^4\dfrac{3\pi}{8}+\sin^4\dfrac{5\pi}{8}+\sin^4\dfrac{7\pi}{8}$

$$=\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$$

$$+\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$$

$$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$$

$$+\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$$

$$=\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]+\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$$

$$=\frac{1}{4}(3)+\frac{1}{4}(3)=\frac{3}{2}$$

Q.33 **(a)**

Q.34 (4)

$$\cos 20^o\cos 40^o\cos 80^o=\frac{\sin 2^3 20^o}{2^3\sin 20^o}=\frac{\sin 160^o}{8\sin 20^o}=\frac{1}{8}.$$

Q.35 (4) $\sin 36^o\ \sin 72^o \sin 108^o\ \sin 144^o$

$$=\sin^2 36^o\ \sin^2 72^o=\frac{1}{4}\left\{(2\sin^2 36^o)\ (2\sin^2\ 72^o)\right\}$$

$$=\frac{1}{4}\left\{(1-\cos\ 72^o)\ (1-\cos\ 144^o)\right\}$$

$$=\frac{1}{4}\left\{(1-\sin\ 18^o)\ (1+\cos\ 36^o)\right\}$$

$$=\frac{1}{4}\left[\left(1-\frac{\sqrt{5}-1}{4}\right)\ \left(1+\frac{\sqrt{5}+1}{4}\right)\right]=\frac{20}{16}\times\frac{1}{4}=\frac{5}{16}\cdot$$

Q.36 (1)

Given that $\sec\theta=\dfrac{5}{4}$

$$\sec\theta=\frac{1+\tan^2(\theta/2)}{1-\tan^2(\theta/2)}\Rightarrow\frac{5}{4}=\frac{1+\tan^2(\theta/2)}{1-\tan^2(\theta/2)}$$

$$\Rightarrow 5-5\tan^2(\theta/2)=4+4\tan^2(\theta/2)$$

$$\Rightarrow 9\tan^2(\theta/2)=1\Rightarrow\tan(\theta/2)=\frac{1}{3}$$

Q.37 (1) $\sin 4\theta=2\sin 2\theta\cos 2\theta$

$$=2.2\sin\theta\cos\theta(1-2\sin^2\theta)$$

$$=4\sin\theta(1-2\sin^2\theta)\sqrt{1-\sin^2\theta}$$

Q.38 (3) $\sqrt{2+\sqrt{2+2\cos 4\theta}}=\sqrt{2+\sqrt{2.2\cos^2 2\theta}}$

$$=\sqrt{2+2\cos 2\theta}=\sqrt{4\cos^2\theta}=2\cos\theta\cdot$$

Q.39 (2)

Given that, $\tan x=\dfrac{b}{a}$

Now $\sqrt{\dfrac{a+b}{a-b}}+\sqrt{\dfrac{a-b}{a+b}}=\sqrt{\dfrac{1+b/a}{1-b/a}}+\sqrt{\dfrac{1-b/a}{1+b/a}}$

$$=\frac{2}{\sqrt{1-\frac{b^2}{a^2}}}=\frac{2}{\sqrt{1-\tan^2 x}}=\frac{2}{\sqrt{1-\frac{\sin^2 x}{\cos^2 x}}}=\frac{2\cos x}{\sqrt{\cos 2x}}.$$

Q.40 (4)

$$\frac{\sin 3A-\cos\left(\frac{\pi}{2}-A\right)}{\cos A+\cos(\pi+3A)}=\frac{\sin 3A-\sin A}{\cos A-\cos 3A}$$

$$\frac{2\cos 2A\sin A}{2\sin 2A\sin A}=\frac{\cos 2A}{\sin 2A}=\cot 2A$$

Q.41 (2)

We have $\tan A=\dfrac{1}{2}$

$$\Rightarrow\tan 3A=\frac{3\tan A-\tan^3 A}{1-\tan^2 A}=\frac{3.\frac{1}{2}-\frac{1}{8}}{1-3.\frac{1}{4}}=\frac{12-1}{2}=\frac{11}{2}$$

Q.42 (3)

L.H.S. $= \frac{1}{2}\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A \sin B \sin C} = 2$.

Q.43 (1)

We know that $A + C = 180°$, since $ABCD$ is a cyclic quadrilateral. $\Rightarrow A = 180° - C$

$\Rightarrow \cos A = \cos(180° - C) = -\cos C$

$\Rightarrow \cos A + \cos C = 0$ .....(i)

Now $B + D = 180°$, then $\cos B + \cos D = 0$ ....(ii)

Subtracting (ii) from (i), we get

$\cos A - \cos B + \cos C - \cos D = 0$.

Q.44 (3)

Trick: For $A = B = C = 60^o$ only option (3) satisfies the condition.

Q.45 (2)

$A + B + C = 270^o \Rightarrow A = B = C = 90^o$,

then $\cos 2A + \cos 2B + \cos 2C + 4\sin A \sin B \sin C$

$= \cos 180^o + \cos 180^o + \cos 180^o + 4\sin 90^o \sin 90^o \sin 90^o$

$= -1 - 1 - 1 + 4.1.1.1 = -3 + 4 = 1$

Q.46 (2)

We have $x + \frac{1}{x} = 2\cos\theta$,

Now $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (2\cos\theta)^3 - 3(2\cos\theta) = 8\cos^3\theta - 6\cos\theta$

$= 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$.

Trick : Put $x = 1 \Rightarrow \theta = 0^\circ$

Then $x^3 + \frac{1}{x^3} = 2 = 2\cos 3\theta$

Q.47 (4)

Maximum value of $f(x) = \sqrt{1^2 + 1^2} = \sqrt{2}$

Q.48 (2)

Maximum distance $= \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = 2$.

Hence, in the graph of function $\sqrt{3}\sin x + \cos x$, maximum distance of a point from *x*-axis is 2.

Q.49 (2)

$\cos^2\theta = \frac{3}{4} = \cos^2\left(\frac{\pi}{6}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$

Q.50 (2)

We have $(2\cos x - 1)(3 + 2\cos x) = 0$

If $2\cos x - 1 = 0$, then $\cos x = \frac{1}{2}$

$\therefore \quad x = \pi/3, 5\pi/3$

If $3 + 2\cos x = 0$, the $\cos x = -3/2$ which is not possible.

Q.51 (1)

$2\cos^2 x + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$

$\Rightarrow x = \frac{p}{6}, \frac{5p}{6}, \frac{p}{2}$, i.e. 30°, 150°, 90°.

Q.52 (3)

$2\tan^2\theta = \sec^2\theta \Rightarrow 2\tan^2\theta = \tan^2\theta + 1$

$\Rightarrow \tan^2\theta = 1 = \tan^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$.

Q.53 (2)

$\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta \tan 3\theta = 1$

$\Rightarrow \frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan 5\theta = \tan\frac{\pi}{6}$

$\Rightarrow 5\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \left(n + \frac{1}{6}\right)\frac{\pi}{5}$

Q.54 (1)

$\tan 2\theta = \cot\theta \Rightarrow \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{6}$

Q.55 (3)

$\frac{1}{\sin\theta} = 1 + \frac{\cos\theta}{\sin\theta} \Rightarrow \sin\theta + \cos\theta = 1$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} \Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

Hence $\theta = 2n\pi$ or $\theta = 2n\pi + \frac{\pi}{2}$

But $\theta = 2n\pi$ is ruled out.

Q.56 (4)

$\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = 3 \Rightarrow \frac{1 - (1 - 2\sin^2\theta)}{1 + (2\cos^2\theta - 1)} = 3$

$\Rightarrow \tan^2\theta = 3 \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

Q.57 (2)

$(2\cos x-1)\,(3+2\cos x)=0$

Then, $\cos x=\frac{1}{2}$ as $\cos x\neq\frac{-3}{2}$

$$\Rightarrow x=2n\pi\pm\frac{\pi}{3};\left\{\begin{array}{l}\text{for } n=0,\ x=\frac{\pi}{3},\frac{5\pi}{3}\\ \text{for } n=1,\ x=\frac{5\pi}{3}\end{array}\right\}$$

Q.58 (2)

$\cos\theta=-\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{3\pi}{4},\frac{5\pi}{4}$; $\tan\theta=1\Rightarrow\theta=\frac{\pi}{4},\frac{5\pi}{4}$

∴ The general value is $2n\pi+\frac{5\pi}{4}$ or $(2n+1)\pi+\frac{\pi}{4}$

Q.59 (2)

$\tan\theta=\frac{-1}{\sqrt{3}}=\tan\left(\pi-\frac{\pi}{6}\right)$, $\sin\theta=\frac{1}{2}=\sin\left(\pi-\frac{\pi}{6}\right)$

and $\cos\theta=\frac{-\sqrt{3}}{2}=\cos\left(\pi-\frac{\pi}{6}\right)$

Hence principal value is $\theta=\frac{5\pi}{6}$

Q.60 (3)

$\sec^2\theta+\tan^2\theta=\frac{5}{3}$, also $\sec^2\theta-\tan^2\theta=1$

$\Rightarrow\ \tan^2\theta=\frac{1}{3}=\tan^2\left(\frac{\pi}{6}\right)\Rightarrow\theta=n\pi\pm\frac{\pi}{6}$.

Q.61 (3)

$\cot\theta+\tan\theta=2\ \text{cosec}\theta\Rightarrow\frac{2}{\sin\theta}=\frac{1}{\sin\theta\cos\theta}$

$\Rightarrow\ \cos\theta=\frac{1}{2}$ or $\sin\theta=0\Rightarrow\theta=2n\pi\pm\frac{\pi}{3}\Rightarrow\theta=n\pi$

Q.62 **(2)**

Q.63 (2)

$3\sin^2x+10\cos x-6=0$

$3(1-\cos^2x)+10\cos x-6=0$

On solving, $(\cos x-3)(3\cos x-1)=0$

Either $\cos x=3$, (which is not possible) or $\cos x=\frac{1}{3}$

Q.64 (1)

$\tan(3x-2x)=\tan x=1\ \Rightarrow\ x=n\pi+\frac{\pi}{4}$

Q.65 (2)

We have $1-\cos\theta=\sin\theta.\sin\frac{\theta}{2}$

$\Rightarrow\ 2\sin^2\frac{\theta}{2}=2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\sin\frac{\theta}{2}$

$\Rightarrow 2\sin^2\frac{\theta}{2}\left[1-\cos\frac{\theta}{2}\right]=0\Rightarrow\ \sin\frac{\theta}{2}=0$ or $2\sin^2\frac{\theta}{4}=0$

$\Rightarrow\sin\frac{\theta}{2}=0$ or $\sin\frac{\theta}{4}=0\Rightarrow\ \frac{\theta}{2}=k\pi$ or $\frac{\theta}{4}=k\pi$

Hence $\theta=2k\pi$ or $\theta=4k\pi\ \ k\in I$

Q.66 (2)

$5-5\sin^2\theta+7\sin^2\theta=6\Rightarrow2\sin^2\theta=1$

$\Rightarrow\sin^2\theta=\frac{1}{2}=\sin^2\left(\frac{\pi}{4}\right)\Rightarrow\theta=n\pi\pm\frac{\pi}{4}$

Q.67 (3)

$\sin4\theta=\cos\theta-\cos7\theta\Rightarrow\sin4\theta=2\sin(4\theta)\sin(3\theta)$

$\Rightarrow\ \sin4\theta=0\Rightarrow\ 4\theta=n\pi$ or $\sin3\theta=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow\ 3\theta=n\pi+(-1)^n\frac{\pi}{6}\Rightarrow\theta=\frac{n\pi}{4},\frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$

Q.68 (1,2)

$\cos\theta+\cos2\theta+\cos3\theta=0$

$\Rightarrow\ (\cos\theta+\cos3\theta)+\cos2\theta=0$

$\Rightarrow\ 2\cos2\theta\cos\theta+\cos2\theta=0\ \Rightarrow$

$\cos2\theta(2\cos\theta+1)=0$

$\Rightarrow\ \cos2\theta=0=\cos\frac{\pi}{2}\ \Rightarrow\ \ \theta=\frac{\pi}{4}\ \Rightarrow\ \theta=2m\pi\pm\frac{\pi}{4}$

or $\ \cos\theta=\frac{-1}{2}=\cos\frac{2\pi}{3}\ \Rightarrow\ \ \theta=2m\pi\pm\frac{2\pi}{3}$.

Q.69 **(1)**

Q.70 (3)

$\sin5x+\sin3x+\sin x=0$

$\Rightarrow\ \ -\sin3x=\sin5x+\sin x=2\sin3x\cos2x$

$\Rightarrow\ \ \sin3x=0\Rightarrow\ x=0$

or $\cos2x=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)$

$\Rightarrow\ 2x=2n\pi\pm\left(\pi-\frac{\pi}{3}\right)\Rightarrow x=n\pi\pm\left(\frac{\pi}{3}\right)$

For *x* lying between 0 and $\frac{\pi}{2}$, we get $x=\frac{\pi}{3}$.

Q.71 (4)

$\cos6\theta+\cos4\theta+\cos2\theta+1=0$

$\Rightarrow2\cos^2 3\theta+2\cos3\theta.\cos\theta=0$

$\Rightarrow4\cos3\theta\cos2\theta\cos\theta=0$

$\Rightarrow3\theta=(2n+1)\frac{\pi}{2};\ 2\theta=(2n+1)\frac{\pi}{2}$ and $\theta=(2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = 30^o, 90^o, 150^o, 45^o, 135^o$

Q.72 (1)

$\sin 7\theta + \sin\theta - \sin 4\theta = 0$

$\Rightarrow\ 2\sin 4\theta \cos 3\theta - \sin 4\theta = 0$

$\sin 4\theta(2\cos 3\theta - 1) = 0 \Rightarrow \sin 4\theta = 0,\ \cos 3\theta = \frac{1}{2}$

Now sin $4\theta = 0 \Rightarrow 4\theta = \pi\ \Rightarrow\ \theta = \frac{\pi}{4}$

and $\cos 3\theta = \frac{1}{2} \Rightarrow 3\theta = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{9}$

Q.73 (2)

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow 2\sin 2x\cos x - 3\sin 2x - 2\cos 2x\cos x + 3\cos 2x = 0$

$\Rightarrow \sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$

$\Rightarrow (\sin 2x - \cos 2x)\ (2\cos x - 3) = 0 \Rightarrow \sin 2x = \cos 2x$

$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$ i.e., $x = \frac{n\pi}{2} + \frac{\pi}{8}$

Q.74 (4)

$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{\sqrt{2}}{2}$ {dividing by $\sqrt{(\sqrt{3})^2 + 1^2} = 2$}

$\Rightarrow\ \sin\left(\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4}\right)$

$\Rightarrow\ \theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}$

Q.75 (1)

Let $\sqrt{3} + 1 = r\cos\alpha$ and $\sqrt{3} - 1 = r\sin\alpha$

Then $r = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} = 2\sqrt{2}$

tan $\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{1-(1/\sqrt{3})}{1+(1/\sqrt{3})} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) \Rightarrow \alpha = \frac{\pi}{12}$

The given equation reduces to

$2\sqrt{2}\cos(\theta - \alpha) = 2 \Rightarrow \cos\left(\theta - \frac{\pi}{12}\right) = \cos\frac{\pi}{4}$

$\Rightarrow\ \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$

Q.76 (4)

Given equation is $\sqrt{3}\sin x + \cos x = 4$

which is of the form $a\sin x + b\cos x = c$ with $a = \sqrt{3}, b = 1, c = 4$.

Here $a^2 + b^2 = 3 + 1 = 4 < c^2$, therefore the given equation has no solution.

Q.77 (4)

$3\cos x + 4\sin x = 6$

$\Rightarrow \frac{3}{5}\cos x + \frac{4}{5}\sin x = \frac{6}{5} \Rightarrow \cos(x - \theta) = \frac{6}{5}$

[where $\theta = \cos^{-1}(3/5)$]

So, that equation has no solution.

Q.78 (4)

$\cos^2\theta - \frac{5}{2}\cos\theta + 1 = 0$

$\Rightarrow\ \cos\theta = \frac{(5/2) \pm \sqrt{(25/4) - 4}}{2} = \frac{5 \pm 3}{4}$

Rejecting (+) sign,

$\Rightarrow\ \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

Q.79 (1)

$f(x) = \cos x - x + \frac{1}{2},\ f(0) = \frac{3}{2} > 0$

$f\left(\frac{\pi}{2}\right) = 0 - \frac{\pi}{2} + \frac{1}{2} = \frac{1-\pi}{2} < 0\ \left(\because \pi = \frac{22}{7}\text{ nearly}\right)$

$\therefore$ One root lies in the interval $\left[0, \frac{\pi}{2}\right]$.

Q.80 (3)

$\sec x\cos 5x = -1 \Rightarrow \cos 5x = -\cos x$

$\Rightarrow 5x = 2n\pi \pm (\pi - x) \Rightarrow x = \frac{(2n+1)\pi}{6}$ or $\frac{(2n-1)\pi}{4}$

Hence, $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{9\pi}{6}, \frac{11\pi}{6}$

Q.81 (1)

We have $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

Now check by options, put $x = \frac{\pi}{6}$

then $(81)^{\sin^2\pi/6} + (81)^{\cos^2\pi/6} = 30$

$\Rightarrow\ (81)^{1/4} + (81)^{3/4} = 30 \Rightarrow 30 = 30$

Hence (1) is the correct answer.

Q.82 (3)

We shall first consider values of θ between

θ and 2π $\sin\theta = -\frac{1}{2} = -\sin\frac{\pi}{6} - \sin\left(\pi + \frac{\pi}{6}\right)$

or $\sin\left(2\pi - \frac{\pi}{6}\right)$

$$\therefore \theta = \frac{7\pi}{6}; \frac{11\pi}{6}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right)$$

or $\tan\left(\pi + \frac{\pi}{6}\right)$ $\therefore \theta - \frac{\pi}{6}, \frac{7\pi}{6}$

The value of θ which satisfies both the equations is $\frac{7\pi}{6}$

Hence the general value of is $2n\pi + \frac{7\pi}{6}$

Where $n \in I$

Q.83 (4)

$$\sin\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right)$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right) = \tan\left(\pi + \frac{\pi}{6}\right) \Rightarrow \theta = \left(\pi + \frac{\pi}{6}\right)$$

Hence general value of θ is $2n\pi + \frac{7\pi}{6}$

Q.84 (1)

Since A.M. ≥ G.M. $\frac{1}{2}(2^{\sin x} + 2^{\cos x}) \geq \sqrt{2^{\sin x}.2^{\cos x}}$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2.2^{\frac{\sin x + \cos x}{2}}$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1 + \frac{\sin x + \cos x}{2}}$$

and we know that $\sin x + \cos x \geq -\sqrt{2}$

$$\therefore 2^{\sin x} + 2^{\cos x} > 2^{1 - (1/\sqrt{2})} \text{ for } x = \frac{5\pi}{4}$$

Q.85 (1)

We know $\frac{5^x + 5^{-x}}{2} \geq 1$, (using A.M. ≥ G.M.)

But since $\cos(e^x) \leq 1$

So, there does not exist any solution.

Q.86 (2)

A, *B*, *C* are in A. P. then angle $B = 60^o$,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac},$$

$$\left\{\begin{matrix} \text{since } A + B + C = 180^o \text{ and} \\ A + C = 2B \Rightarrow B = 60^o \end{matrix}\right\}$$

$$\Rightarrow \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow a^2 + c^2 - b^2 = ac$$

$$\Rightarrow b^2 = a^2 + c^2 - ac$$

Q.87 (2)

$A = 180° - 60° - 75° = 180° - 135° = 45°$

Now. $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\Rightarrow \frac{2}{\sin 45°} = \frac{b}{\sin 60°}$$

$$\Rightarrow b = \frac{2.(\sqrt{3}/2)}{1/\sqrt{2}} = \sqrt{6}$$

Q.88 (1)

Q.89 (1)

$$a\sin(B - C) + b\sin(C - A) + c\sin(A - B)$$

$$= k(\Sigma \sin A \sin(B - C) = k\{\Sigma \sin(B + C)\sin(B - C)\}$$

$$= k\left\{\Sigma \frac{1}{2}(\cos 2C - \cos 2B)\right\} = 0$$

Q.90 (3)

$$\cos C = \frac{\pi}{3} \Rightarrow a^2 + b^2 - c^2 = ab$$

$$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$$

$$\Rightarrow b(b + c) + a(a + c) = (a + c)(b + c)$$

Divide by $(a + c)$ $(b + c)$ and add 2 on both sides ⇒

$$1 + \frac{b}{a + c} + 1 + \frac{a}{b + c} = 3 \Rightarrow \frac{1}{a + c} + \frac{1}{b + c} = \frac{3}{a + b + c}.$$

Q.91 (2)

$$\cos\theta = \frac{4+6-(\sqrt{3}+1)^2}{2.2.\sqrt{6}} \Rightarrow \theta = 75^o$$

Q.92 (2)

$\angle C = 90^o, \angle A = 30^o, c = 20,$

then $a = \frac{c\sin A}{\sin C} = 10$ and $b = \frac{c\sin B}{\sin C} = 10\sqrt{3}$.

Trick : Since the angles are $30^o, 60^o, 90^o$, therefore sides must be $1 : \sqrt{3} : 2$. Hence $a = 10, b = 10\sqrt{3}$.

Q.93 (1)

$\Sigma a^2(\cos^2 B - \cos^2 C) = \Sigma a^2(\sin^2 C - \sin^2 B)$

$= k^2\Sigma a^2(c^2 - b^2) = 0$

Q.94 (2)

$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \lambda$ (Let)

$\therefore b + c = 11\lambda$ ….(i)

$c + a = 12\lambda$ ….(ii)

and $a + b = 13\lambda$ ….(iii)

From (i) + (ii) + (iii), $2(a + b + c) = 36\lambda$

$\therefore a + b + c = 18\lambda$

Now, (iv) – (i) gives, $a = 7\lambda$

(iv) – (ii) gives, $b = 6\lambda$

(iv) – (iii) gives, $c = 5\lambda$

Q.95 (4)

We have, $b = \sqrt{3}, c = 1, \angle A = 30^o$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \frac{\sqrt{3}}{2} = \frac{(\sqrt{3})^2 + 1^2 - a^2}{2.\sqrt{3}.1}$$

$\therefore$ $a = 1$, $b = \sqrt{3}$, $c = 1$

b is the largest side. Therefore, the largest angle *B* is given by $\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{1+1-3}{2.1.1} = -\frac{1}{2}$

$\therefore$ $B = 120°$

Q.96 (3)

$$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \sin B = \frac{b\sin A}{a} = \frac{8\sin 30^o}{6} = \frac{2}{3}$$

Q.97 (2)

$$2ac\ \sin\frac{A - B + C}{2} = 2ac\ \sin\frac{\pi - 2B}{2} = 2ac\ \cos B$$

$$2ac\frac{c^2 + a^2 - b^2}{2ca} = c^2 + a^2 - b^2$$

Q.98 (1)

$a^4 + b^4 + c^4 - 2a^2c^2 - 2b^2c^2 + 2a^2b^2 = 2a^2b^2$

$\Rightarrow$

$(a^2 + b^2 - c^2)^2 = (\sqrt{2}ab)^2 \Rightarrow a^2 + b^2 - c^2 = \pm\sqrt{2}ab$

$$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \pm\frac{\sqrt{2}ab}{2ab} = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow$ $\cos C = \cos 45^o$ or $\cos 135^o \Rightarrow C = 45^o$ or 135^o

Q.99 (1)

$$\tan\left(\frac{B - C}{2}\right) = \frac{b - c}{b + c}\cot\frac{A}{2} \quad \Rightarrow$$

$$\tan\left(\frac{90°}{2}\right) = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}\cot\frac{A}{2}$$

$$\Rightarrow \tan\left(\frac{A}{2}\right) = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{3 + 1 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

$$\Rightarrow \frac{A}{2} = 15^o \Rightarrow A = 30^o .$$

Q.100 (3)

$(b + c)\cos A + (c + a)\cos B + (a + b)\cos C = a + b + c$

From expanding and collecting terms using projection rule, $a = b\cos C + c\cos B$

Q.101 **(c)**

Q.102 (3)

$\sin^2 B - \sin^2 A = \sin^2 C - \sin^2 B$

$\therefore$ $\sin(B + A)\sin(B - A) = \sin(C + B)\sin(C - B)$

or $\sin C(\sin B\cos A - \cos B\sin A)$

$= \sin A(\sin C\cos B - \cos C\sin B)$

Divide by $\sin A\sin B\sin C$

$\therefore$ $\cot A - \cot B = \cot B - \cot C$. Hence the result.

Q.103 (1)

From the given relation $\sin C = \frac{1 - \cos A\cos B}{\sin A\sin B} \le 1$

…..(i)

$\Rightarrow 1 \le \cos A\cos B + \sin A\sin B$

$\Rightarrow \cos(A - B) \ge 1$; $\because \cos\theta \not> 1$ …..(ii)

$\therefore$ $A - B = 0$ or $A = B$

Hence from (i), $\sin C = \frac{1 - \cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1$

$\therefore$ $C = 90^o \Rightarrow A + B = 90^o$ or $A = B = 45^o$ {by (ii)}

Hence,

$a : b : c = \sin A : \sin B : \sin C = 1 : 1 : \sqrt{2}$.

Q.104 (2)

$$\Delta=\frac{1}{2}\frac{a^2\sin B\sin C}{\sin(B+C)}=\frac{1}{2}\frac{(\sqrt{3}+1)^2.\frac{1}{2}\times\frac{1}{\sqrt{2}}}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}}=\frac{\sqrt{3}+1}{2}$$

Q.105 (1)

$\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}=1=\tan\left(\frac{\pi}{4}\right)$ from given data.

Hence $C=90^o$.

Q.106 (1)

$$2s=a+b+c\ \cos\frac{B}{2}=\sqrt{\frac{30\times 6}{320}}=\frac{3}{4}$$

Q.107 (2)

$$\cos^2\frac{A}{2}-\sin^2\frac{A}{2}=\cos\frac{2A}{2}=\cos A.$$

Q.108 (2)

It is obvious

Q.109 (3)

$$ab^2\cos A+ba^2\cos B+ac^2\cos A+ca^2\cos C$$
$$+bc^2\cos B+b^2c\cos C$$
$$=ab(b\cos A+a\cos B)+ac\ c(\cos A+a\cos C)$$
$$+bc(c\cos B+b\cos C)$$
$$=abc+abc+abc=3abc$$

Q.110 (4)

Δ is right angled, $\angle C=90^o$

$$\therefore 4\Delta^2=4\left(\frac{1}{2}ab\right)^2=a^2b^2$$

Q.111 (3)

$$\frac{b-c}{a}=\frac{\sin B-\sin C}{\sin A}=\frac{2\sin\frac{B-C}{2}\cos\frac{B+C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}=\frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}}$$

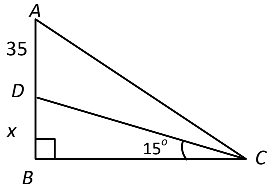
$$\Rightarrow(b-c)\cos\frac{A}{2}=a\sin\frac{B-C}{2}$$

Q.112 (1)

$\angle DCB=15^o$

$\angle CAB=45^o$ and $\angle CDB=75^o$

Let $BD=x$ and $AD=35$ cm.



Q.113 (3)

$$\frac{1}{a}\cos^2\frac{A}{2}+\frac{1}{b}\cos^2\frac{B}{2}+\frac{1}{c}\cos^2\frac{C}{2}$$

$$=\sum\frac{s(s-a)}{abc}=\frac{s(s-a+s-b+s-c)}{abc}=\frac{s^2}{abc}$$

Q.114 (2)

$$\cot\frac{B}{2}.\cot\frac{C}{2}=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}.\frac{s(s-c)}{(s-a)(s-b)}}$$

$=\frac{s}{s-a}$ {Since $3a=b+c$ or $a+b+c=2s=4a$}

$=2a/a=2$

Q.115 (3)

$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=\frac{1}{2R}\Rightarrow\sin A=\frac{a}{2R}$ ETC.

Therefore $2R^2\sin A\sin B\sin C=2R^2\frac{a}{2R}.\frac{b}{2R}.\frac{c}{2R}$

$$=\frac{abc}{4R}=\Delta$$

Q.116 (3)

$$\cos A=0\Rightarrow 36+64-a^2=0\Rightarrow a=10\Rightarrow R=\frac{a}{2\sin A}=\frac{5}{1}$$

Q.117 (3)

$$s=\frac{1}{2}(a+b+c)=21$$

$$\Delta=\sqrt{[s(s-a)(s-b)(s-c)]}=84\ ;\ \therefore\ r=\frac{\Delta}{s}=4$$

Q.118 (3)

$a=b=c=2\sqrt{3}$

$$\Delta=\left(\frac{\sqrt{3}a^2}{4}\right)=3\sqrt{3}\text{sq.cm},\ \ \therefore R=\frac{abc}{4\Delta}=2\text{cm}$$

Q.119 (2)

$$r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$\Rightarrow r=4R\sin^3 30^o$, $\{\because A=B=C=60^o\}$

$$\Rightarrow r=\frac{R}{2}$$

Q.120 (1)

$a=5k, b=6k$ and $c=5k$

$$s=\frac{a+b+c}{2}=\frac{5k+6k+5k}{2}=8k$$

$$r=\frac{\Delta}{s}=\sqrt{\frac{s(s-a)(s-b)(s-c)}{s}}$$

$$r=\sqrt{\frac{8k(8k-5k)(8k-6k)(8k-5k)}{8k}}$$

$$r=\frac{3k}{2}\Rightarrow k=\frac{2r}{3}=\frac{2\times6}{3}=4$$

Q.121 (3)

Radius of circum-circle (*R*)

$$=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}$$

$$R=\frac{b}{2\sin B}=\frac{2}{2\sin 30^\circ}=2$$

Now, area of circle $=\pi R^2=4\pi$

Q.122 (2)

$$R=\frac{abc}{4\Delta}\text{, where }\Delta=\sqrt{s(s-a)(s-b)(s-c)}$$

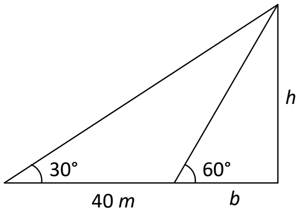
$$a=13, b=12, c=5, s=\frac{30}{2}=15$$

$$\Delta=\sqrt{15(2)(3)10}=3\times2\times5=30$$

$$\therefore R=\frac{13\times12\times5}{4\times30}=\frac{13}{2}$$

Q.123 (1)

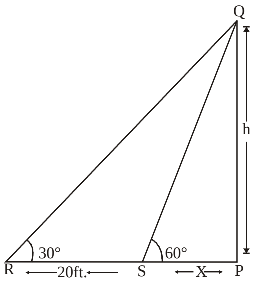
$$b=h\cot 60^o, b+40=h\cot 30^o$$



$$\Rightarrow\ \frac{b}{b+40}=\frac{\cot 60^o}{\cot 30^o}\Rightarrow\ b=20m$$

Q.124 (4)

let h be the height of tree PQ and breadth of river. PS be x ft. angle of elevation subtended by a tree is 60^o. also when he retreats 20 feet, the angle j Becomes 30^o also in ΔPQS

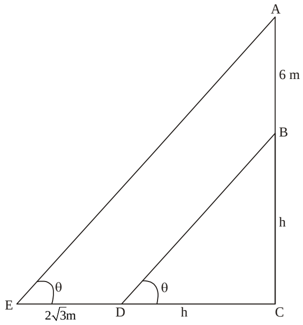


Q.125 (1)

Accordingly,

$$\tan\theta=\frac{h}{x}=\frac{h+6}{x+2\sqrt{3}}=\frac{6}{2\sqrt{3}}\Rightarrow\theta=60^o$$

[Since the triangles AEC and BDC are similar]



$$h\cot\alpha=(h-100)\cot\beta$$

$$\therefore h=\frac{100\cot\beta}{\cot\beta-\cot\alpha}$$

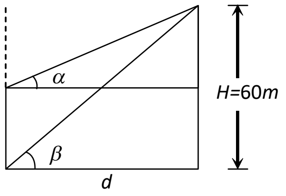
Q.126 (3)

$$\text{Length of ladder }=\frac{6\sqrt{3}}{\sin 60^\circ}=12m$$

Q.127 (4)

$H=d\tan\beta$ and $H-h=d\tan\alpha$

$$\Rightarrow\frac{60}{60-h}=\frac{\tan\beta}{\tan\alpha}\Rightarrow\ -h=\frac{60\tan\alpha-60\tan\beta}{\tan\beta}$$



$$\Rightarrow h=\frac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta\frac{\sin\beta}{\cos\beta}}\Rightarrow x=\cos\alpha\sin\beta$$

Q.128 (1)

64 cot θ = d

Also (100 – 64) tan θ=d

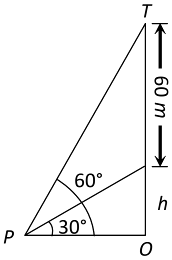
or (64)(36) = d^2 $\therefore$ d= 8 × 6 = 48 m.

Q.129 (2)

$$\text{Required distance }=60\cot 15^o=60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

Q.130 (1)

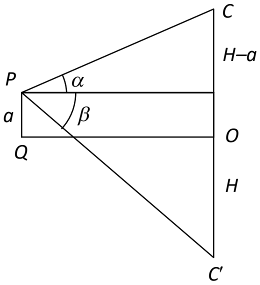
$(60+h)\cot 60^{o} = h\cot 30^{o} \Rightarrow h = 30\text{m}$



Q.131 (2)

$(H+a)\cot\beta = (H-a)\cot\alpha$

$$H = \frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$$

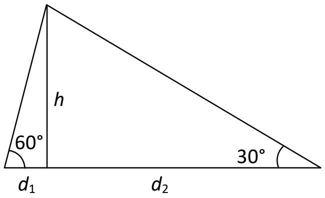


Using $\cot\alpha + \cot\beta = \dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$

and $\cot\alpha - \cot\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

Q.132 (2)

$d_2 = h\cot 30^{o} = 500\sqrt{3}, d_1 = \dfrac{500}{\sqrt{3}}$

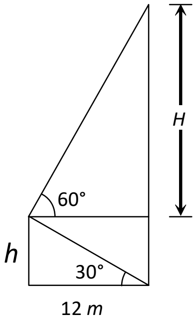


Diameter $D = 500\sqrt{3} + \dfrac{500}{3}\sqrt{3} = \dfrac{2000}{\sqrt{3}}\text{m}$

Q.133 (2)

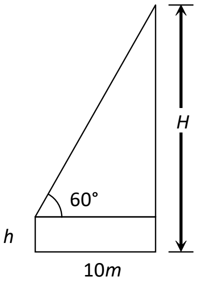
$h = 12\tan 30^{o} = \dfrac{12}{\sqrt{3}}$ and $H = 12\tan 60^{o} + \dfrac{12}{\sqrt{3}}$

$12\sqrt{3} + \dfrac{12}{\sqrt{3}} = 16\sqrt{3}\text{m}$



Q.134 (1)

$H = (10\tan 60^{o} + 1.5) = (10\sqrt{3} + 1.5)\text{m}$



Q.135 (1)

Trick: From $H = l\tan\alpha.\tan\beta$, the height of tower is

$h\tan 30^{o}\cot 60^{o}$ *or* $\dfrac{h}{3}$

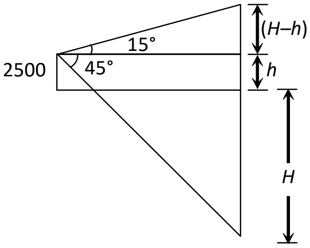
Q.136 (3)

Obviously, the length of the tree is equal to $10 + 10\sqrt{2} = 10\ (1+\sqrt{2})\text{m}$

Q.137 (1)

$(H-h)\cot 15^{o} = (H+h)\cot 45^{o}$

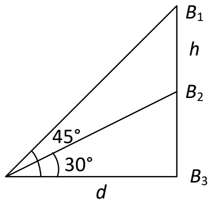
or $H = \dfrac{h(\cot 15^{o} + 1)}{(\cot 15^{o} - 1)}$



Since $h = 2500$ and substitute

$\cot 15^{o} = 2+\sqrt{3}$, we get, $H = 2500\sqrt{3}$

Q.138 (2)



$B_1B_2 = h = (d\tan 45^o - d\tan 30^o)$

Time taken = 10 *min*

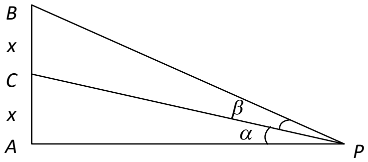
Rate = $4 = \frac{d}{10}\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$

$\Rightarrow\ d = \frac{40\sqrt{3}}{\sqrt{3}-1} = 20(3+\sqrt{3})\,m$.

Q.139 (2)

Let $AC = x = CB, AP = 3AB = 6x$. Let $\angle CPA = \alpha$

In $\Delta ACP, \tan\alpha = \frac{x}{6x} = \frac{1}{6}$



In ΔABP , $\tan(\alpha+\beta) = \frac{2x}{6x} = \frac{1}{3}$

Now $\tan\beta = \tan\{(\alpha+\beta)-\alpha\} = \frac{\tan(\alpha+\beta)-\tan\alpha}{1+\tan(\alpha+\beta)\tan\alpha}$

$$= \frac{\frac{1}{3}-\frac{1}{6}}{1+\frac{1}{3}.\frac{1}{6}} = \frac{1}{6}\times\frac{18}{19} = \frac{3}{19}$$

Q.140 (4)

Let the two roads intersect at A. If the bus and the car are at B and C on the two roads respectively, then c ,= AB = 2km, b = AC = 3 km. The distance between the two vehicles = BC = a km

Now $\cos A = \cos 60^o = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow\ \frac{1}{2} = \frac{3^2+2^2-a^2}{2.3.2} \Rightarrow a = \sqrt{7}$ km.

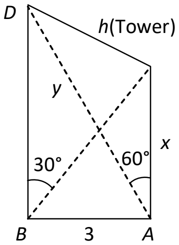
Q.141 (4)

From Δ CDA, $x = h\cot 60^o = \frac{h}{\sqrt{3}}$

From ΔCDB, y=h cot30^o= $\sqrt{3}$h

From ΔABC, by Pythagoras theorem

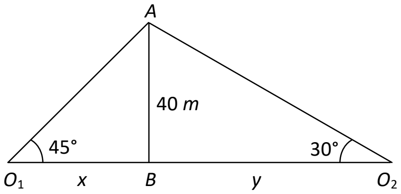
$x^2 + 3^2 = y^2$



$\Rightarrow\left(\frac{h}{\sqrt{3}}\right)+3^2 = (\sqrt{3}h)^2 \Rightarrow h = \frac{3\sqrt{6}}{4}$ km.

Q.142 (4)

From Δ O_1AB, $\tan 45^o = \frac{40}{x} \Rightarrow x = 40m$

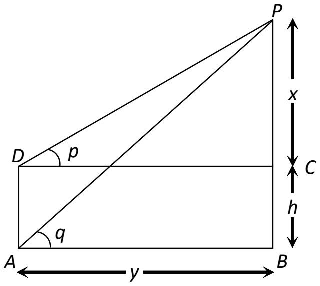


From ΔAO_2B, $\cot 30^o = \frac{y}{40}$

$\Rightarrow$ $y = 40\ \cot\ 30^o = 40\sqrt{3}$

Distance between the men = $40+40\sqrt{3} = 109.28$ *m*.

Q.143 (2)



Let *AD* be the building of height *h* and *BP* be the hill

then $\tan q = \frac{h+x}{y}$ and $\tan p = \frac{x}{y}$

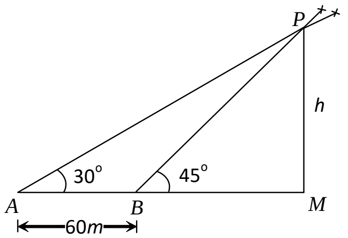
$\Rightarrow$ $\tan q = \frac{h+x}{x\cot p}$

$\Rightarrow$ $x\cot p = (h+x)\cot q$

$\Rightarrow$ $x = \frac{h\cot q}{\cot p - \cot q}$

$\Rightarrow$ $h + x = \frac{h\cot p}{\cot p - \cot q}$

Q.144 (4)

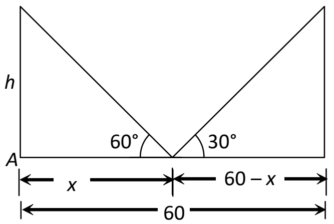


$$\because AB = AM - BM \Rightarrow \frac{AB}{h} = \frac{AM}{h} - \frac{BM}{h}$$

$$\frac{AB}{h} = \cot 30^o - \cot 45^o \Rightarrow h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{3-1}$$

$$\Rightarrow h = 30(\sqrt{3}+1)\,m$$

Q.145 (1)



$$\tan 60^\circ = \frac{h}{x} \Rightarrow \frac{\sqrt{3}}{1} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \qquad .....(i)$$

$$\tan 30^\circ = \frac{h}{60-x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{60-x} \Rightarrow 60 - x = \sqrt{3}h$$

.....(ii)

From equation (i) and (ii), $60 - x = \sqrt{3}(\sqrt{3}x)$

$$\frac{60}{4} = x \Rightarrow x = 15$$

Then $h = \sqrt{3}x \Rightarrow h = 15\sqrt{3}$ *metre.*

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (2)

$\tan\alpha + \cot\alpha = a$

By squaring both side, we get

$\Rightarrow \tan^2\alpha + \cot^2\alpha = a^2 - 2$

By squaring both sides again, we get

$\Rightarrow \tan^4 a + \cot^4 a + 2 = a^4 - 4a^2 + 4$

$\Rightarrow \tan^4\alpha + \cot^4\alpha = a^4 - 4a^2 + 2$

Q.2 (1)

$a \cos\theta + b \sin \theta = 3$ & $a \sin \theta - b \cos \theta = 4$

By squaring both side and adding, we get

$a^2 + b^2 = 3^2 + 4^2 = 25$

Q.3 (2)

Q.4 (3)

Q.5 (3)

$$12\cot^2\theta - 31\,\mathrm{cosec}\,\theta + 32 = 0$$

$$\Rightarrow 12\left(\mathrm{cosec}^2\theta - 1\right) - 31\,\mathrm{cosec}\,\theta + 32 = 0$$

$$\Rightarrow 12\mathrm{cosec}^2\theta - 31\,\mathrm{cosec}\,\theta + 20 = 0$$

$$\Rightarrow 12\mathrm{cosec}^2\theta - 16\,\mathrm{cosec}$$

$$\theta - 15\ \mathrm{cosec}\,\theta + 20 = 0$$

$$\Rightarrow \left(4\,\mathrm{cosec}\,\theta - 5\right)\left(3\,\mathrm{cosec}\,\theta - 4\right) = 0$$

$$\Rightarrow \mathrm{cosec}\theta = \frac{5}{4}, \frac{4}{3}; \therefore \sin\theta = \frac{4}{5}, \frac{3}{4}$$

Q.6 (a)

$$x \sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \ ....(i)$$

and $x\sin\theta = y\cos\theta$ .....(ii)

Equation (i) may be written as

$$x\sin\theta \cdot \sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$\Rightarrow y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$\Rightarrow y\cos\theta\left(\sin^2\theta + \cos^2\theta\right) = \sin\theta\cos\theta$$

$$\Rightarrow y\cos\theta = \sin\theta\cos\theta \therefore y = \sin\theta \ ....(iii)$$

Putting the value of y from (iii) in (ii), we get

$$x\sin\theta = \sin\theta\cdot\cos\theta \Rightarrow x = \cos\theta \quad \ldots(iv)$$

Squaring (iii) and (iv) and adding, we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$$

Q.7 (b)

cos A = n cos B and sin A = m sin B squaring and adding, we get $1 = n^2\cos^2 B + m^2\sin^2 B$

$$\Rightarrow 1 = n^2(1 - \sin^2 B) + m^2\sin^2 B$$

$$\therefore (m^2 - n^2)\sin^2 B = 1 - n^2$$

Q.8 (4)

$$\frac{\tan\left(x - \frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2} + x\right) - \sin^3\left(\frac{7\pi}{2} - x\right)}{\cos\left(x - \frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2} + x\right)}$$

$$= \frac{(-\cot x)(\sin x) - (-\cos^3 x)}{\sin x(-\cot x)}$$

$$= \frac{-\cos x + \cos^3 x}{-\cos x} = \frac{-\cos x(1 - \cos^2 x)}{-\cos x} = \sin^2 x$$

Q.9 (a)

$$Since, \sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$$

$$\therefore \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1 \Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2}$$

$$\therefore \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$$

Q.10 (1)

$\tan A = \frac{4}{3} \Rightarrow A \rightarrow \text{III}^{rd}$ quadrant

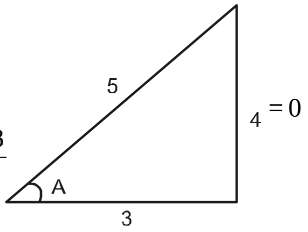
5 sin 2A + 3 sinA + 4 cosA

= 10 sinA cosA + 3 sinA + 4 cosA

= 10 sinA cosA + 3 sinA + 4 cosA

$\sin A = \frac{-4}{5}$

$\cos A = \frac{-3}{5}$



$$= 10 \times \frac{-4}{5} \times \frac{-3}{5} + 3 \times \frac{-4}{5} + 4 \times \frac{-3}{5}$$

$$= \frac{+120}{25} - \frac{12}{5} - \frac{12}{5} = 0$$

Q.11 (1)

$$\frac{\sin 24^o \cos 6^o - \sin 6^o \cos 4^o}{\sin 21^o \cos 39^o - \sin 39^o \cos 21^o} = \frac{\sin(24^o - 6^o)}{\sin(21^o - 39^o)} = \frac{\sin 18^o}{\sin(-18^o)} =$$

-1

Q.12 (b)

Q.13 **(c)**

Q.14 **(*)**

Q.15 **(c)**

Q.16 (2)

$$\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$$

$$= \frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}, \frac{1 + \cos\alpha \sin\alpha}{1 + \cos a + \sin\alpha}$$

$$= \frac{(1 + \sin\alpha)^2 - \cos^2\alpha}{(1 + \sin\alpha)(1 + \cos + \sin\alpha)}$$

$$= \frac{2\sin\alpha(1 + \sin\alpha)}{(1 + \sin\alpha)(1 + \cos\alpha + \sin\alpha)}$$

$$= \frac{2\sin\alpha}{1 + \cos + \sin\alpha} = y$$

Q.17 (3)

$$\sqrt{3} = \tan 60^o = \tan\left(40^o + 20^o\right)$$

$$= \frac{\tan 40^o + \tan 20^o}{1 - \tan 40^o \tan 20^o}$$

$$\therefore \sqrt{3} - \sqrt{3} \tan 40^o \tan 20^o$$

$$= \tan 40^o + \tan 20^o$$

Hence $\tan 40^o + \tan 20^o + \sqrt{3}$

$$\tan 40^o \tan 20^o = \sqrt{3}$$

Q.18 (2)

Using cosine formula

$2 \sin(\theta + \phi) \cos(\theta - \phi) = 1/2$ ....(i)

$2 \cos(\theta + \phi) \cos(\theta - \phi) = 3/2$ ....(ii)

Squation (1) and (2) and then addition

$$4\cos^2(\theta - \phi) = \frac{1}{4} + \frac{9}{4} = \frac{5}{2} \Rightarrow \cos^2(\theta - \phi)$$

$$= \frac{5}{8}$$

Q.19 (1)

$$\frac{m}{n} = \frac{\sin(\theta + 2\alpha)}{\sin\theta}$$

$$\Rightarrow \frac{m+1}{m-1} = \frac{\sin(\theta + 2\alpha) + \sin\theta}{\sin(\theta + 2\alpha) - \sin\theta}$$

$$\therefore \frac{m+1}{m-n} = \frac{2\sin(\theta + \alpha)\cos\theta}{2\sin(\theta + \alpha)\sin\theta} = \tan(\theta + \alpha)\cot\alpha$$

Q.20 (1)

$$A + B + C = \pi \therefore \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\Rightarrow \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}.\tan\frac{B}{2}} = \cot\frac{C}{2} \Rightarrow \frac{\frac{1}{3} + \frac{2}{3}}{1 - \frac{1}{3}.\frac{2}{3}} = \frac{9}{7} = \cot\frac{C}{2}$$

$$\therefore \tan\frac{C}{2} = \frac{7}{9}.$$

Q.21 (3)

$$\frac{1 - \tan^2 15^o}{1 + \tan^2 15^o} = \cos(2 \times 15^o) = \cos 30^o$$

Q.22 (4)

$\tan^2\theta = 2\tan^2\phi + 1$ ... (i)

$$\cos 2\theta + \sin^2\phi = \frac{1-\tan^2\theta}{1+\tan^2\theta} + \sin^2\phi$$

$$= \frac{1-2\tan^2\phi-1}{1+2\tan^2\phi+1} + \sin^2\phi = \frac{-2\tan^2\phi}{2\,(1+\tan^2\phi)} + \sin^2\phi$$

$= -\sin^2\phi + \sin^2\phi = 0$

which is independent of ϕ

Q.23 (3)

$\cos A = \frac{3}{4}$

$16\cos^2\frac{A}{2} - 32\sin\frac{A}{2}\sin\frac{5A}{2}$

$= \frac{16(1+\cos A)}{2} - 16\,(\cos 2A - \cos 3A)$

$= \frac{16(1+\cos A)}{2} - 16\ \{(2\cos^2 A - 1) - (4\cos^3 A - 3\cos A)\}$

$$= 8\left(1+\frac{3}{4}\right) - 16\left\{2\times\frac{9}{16} - 1 - 4\times\frac{27}{64} + 3\times\frac{3}{4}\right\} = 3$$

Q.24 (2)
Q.25 (1)
Q.26 (3)
Q.27 (3)
Q.28 (2)
Q.29 **(1)**
Q.30 **(c)**
Q.31 (2)

Given, $3\cos^2 A + 2\cos^2 B = 4$

$\Rightarrow 2\cos^2 B - 1 = 4 - 3\cos^2 A - 1$

$\Rightarrow \cos 2B = 3(1-\cos^2 A) = 3\sin^2 A$ .....(1)

and $2\cos B\sin B = 3\sin A\cos A$

$\sin 2B = 3\sin A\cos A$

Now, cos (A+2B) = cos A cos 2B – sihn A sin 2B

$= \cos A\,(3\sin^2 A) - \sin A\,(3\sin A\cos A) = 0$

[using eqs. (1) and (2)]

$$\Rightarrow A + 2B = \frac{\pi}{2}$$

Q.32 **(3)**

$$\frac{1+\sin A-\cos A}{1+\sin A+\cos A}$$

$$\frac{2\sin^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}}{2\sin^2\frac{A}{2} + 2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\frac{2\sin\frac{A}{2}\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)}{2\cos\frac{A}{2}\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)} = \tan\frac{A}{2}.$$

Q.33 (3)

$\sin t + \cos t = \frac{1}{5}$

$$\Rightarrow \frac{2\tan\frac{t}{2} + 1 - \tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}} = \frac{1}{5}$$

$\Rightarrow 10\tan^2\frac{t}{2} + 5 - 5\tan^2\frac{t}{2} = 1 + \tan^2\frac{t}{2}$

$\Rightarrow 6\tan^2\frac{t}{2} - 10\tan\frac{t}{2} - 4 = 0$

$\Rightarrow 3\tan^2\frac{t}{2} - 6\tan\frac{t}{2} + \tan\frac{t}{2} - 2 = 0$

$\Rightarrow 3\tan\frac{t}{2}\left(\tan\frac{t}{2} - 2\right) + 1\left(\tan\frac{t}{2} - 2\right) = 0$

$\Rightarrow \tan\frac{t}{2} = 2,\ \tan\frac{t}{2} = -\frac{1}{3}$

Q.34 (4)

If $A + B + C = \frac{3\pi}{2}$ then

$2\cos(A+B)\cos(A-B) + \cos 2C$

$= -2\sin C\cos(A-B) + \cos 2C$

$= -2\sin C\cos(A-B) + 1 - 2\sin^2 C$

$= 1 - 2\sin C\,[\cos(A-B) + \sin C]$

$= -2\sin C\,[\cos(A-B) + \sin[3\pi/2 - (A+B)]]$

$= 1 - 2\sin C\,[\cos(A-B) - \cos(A+B)]$

$= 1 - 2\sin C\,.\,2\sin A\sin B$

$= 1 - 4\sin A\sin B\sin C$

Q.35 (3)

$f(\theta) = \sin^4\theta + \cos^2\theta$

$= \sin^2\theta\,(1-\cos^2\theta) + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta - \sin^2\theta\cos^2\theta$

$f(\theta) = 1 - \frac{1}{4}\sin^2 2\theta$

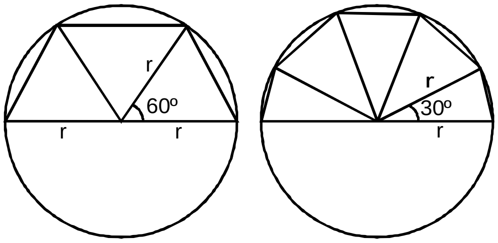
$\because\ 0 \le \sin^2 2\theta \le 1$

$f(\theta)_{\max} = 1$ $\qquad$ $f(\theta)_{\min} = 1 - \frac{1}{4} = 3/4$

$\therefore$ Range is $\left[\frac{3}{4}, 1\right]$

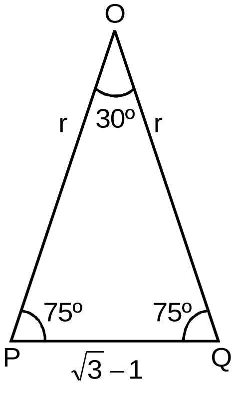
Q.36 (4)

Each side of hexagon (inscribe the circle)



is equal to radius of circle
each side of dodecagon subtends
an angle of 30° at centre of circle
and other two angles are 75º & 75º ∴ PQ = OP cos 75º+OQ cos 75º

$\sqrt{3}-1$ = r cos 75º+r cos 75º



$\Rightarrow$ 2r $\frac{(\sqrt{3}-1)}{2\sqrt{2}} = (\sqrt{3}-1)$

$\Rightarrow$ r = $\sqrt{2}$

Which is the side of hexagon

Q.37 (4)

Q.38 (4)

4sinθ.cosθ – 2cosθ – 2 $\sqrt{3}$ sinθ + $\sqrt{3}$ = 0

$\Rightarrow$ 2cosθ (2sinθ –1) – $\sqrt{3}$ (2sinθ –1) = 0 $\Rightarrow$

(2sinθ –1) (2cosθ – $\sqrt{3}$) = 0

$\Rightarrow$ sinθ = $\frac{1}{2}$, cos θ = $\frac{\sqrt{3}}{2}$ $\Rightarrow$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

Q.39 (2)

$$\sin 9\theta = \sin\theta \Rightarrow 9\theta = n\pi + (-1)^n\,\theta$$

If $n = 2m$ then $9\theta = 2m\pi + \theta \Rightarrow \theta = \frac{m\pi}{4}$

If $n = 2m+1$ then $9\theta = (2m+1)\pi - \theta$

$$\Rightarrow \theta = (2m+1)\frac{\pi}{10}$$

The values belonging to [0, π] are

$$\theta = 0, \frac{\pi}{10}, \frac{\pi}{4}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{3\pi}{4}, \frac{9\pi}{10},$$

$$\pi, \frac{11\pi}{10}, \frac{5\pi}{4}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}, \frac{7\pi}{4}, \frac{19\pi}{10}, 2\pi$$

Q.40 (4)

Q.41 (3)

Q.42 **(2)**

Q.43 (4)

Given

sin 3θ = 4 sin θ . sin 2θ sin 4θ, where $0 \le \theta \le \pi$

$\Rightarrow$ 3 sinθ – 4sin^{3}θ – 4 sinθ sin 2θ sin 4θ = 0

$\Rightarrow$ sin θ [3 – 4 sin^{2}θ – 4 sin2θ sin 4θ]

$\Rightarrow$ sin θ [3 – 2 + 2 cos 2θ – 2 cos2θ + 2 cos 6θ]

$\Rightarrow$ sinθ [1 + 2 cos 6θ] = 0

$\Rightarrow$ sinθ = 0 or cos 6θ = – $\frac{1}{2}$

θ = 0, π or $0 \le \theta \le \pi$

0, π ∈ [0, π] or $0 \le 6\theta \le 6\pi$ (3 rounds)

Number of solutions in 1 round = 2

total number of solutions in θ ∈ [0, π]

= 2 + 6 = 8

Q.44 (2)

Given 4 cos^{3}x – 4 cos^{2}x – cos (π + x) – 1 = 0

$\Rightarrow$ 4 cos^{3}x – 4cos^{2}x + cos x – 1 = 0

A.M. of roots x ∈ [0, 315]

$\Rightarrow$ 4 cos^{2}x (cosx – 1) + (cos x – 1) = 0

$\Rightarrow$ (cosx – 1) (4 cos^{2}x + 1) = 0

$\Rightarrow$ cos x = 1 or cos^2 x = – $\frac{1}{4}$ (not possible)

x = 2nπ, n ∈ I

0 ≤ 2n ≤ 100x ∈ [0, 315]

0 ≤ n ≤ 50 or [0, 100π]

x = 0, 2π, 4π, 6π, ... , 100 π

A.M. = $\frac{2\pi[0+1+2+3+...+50]}{51}$

= $\frac{2\pi}{51}$ $\frac{50 \times 51}{2}$ = 50 π

Q.45 (1)

2 cos^2 (π + x) + 3 sin (π + x) = 0

$\Rightarrow$ 2 cos^2 x – 3 sin x = 0 $\Rightarrow$ 2 – 2 sin^2 x – 3 sin x = 0

$\Rightarrow$ 2 sin^2 x + 3 sin x – 2 = 0 $\Rightarrow$ sin x = –2, $\frac{1}{2}$

$\Rightarrow$ sin x = $\frac{1}{2}$ $\Rightarrow$ x = $\frac{\pi}{6}$, $\frac{5\pi}{6}$

Q.46 (3)

We have $8\tan^2\frac{x}{2} = 1 + \sec x$

$$\Rightarrow 8\left(\frac{1-\cos x}{1+\cos x}\right)=1+\frac{1}{\cos x}=\frac{1+\cos x}{\cos x}$$

$$\Rightarrow 8\cos x-8\cos^2 x=(1+\cos x)^2$$

$$\Rightarrow 9\cos^2 x-6\cos x+1=0$$

$$\Rightarrow (3\cos x-1)^2=0\Rightarrow 3\cos x-1=0$$

$$\Rightarrow \cos x=\frac{1}{3}=\cos\alpha(say)\Rightarrow x=2n\pi\pm\alpha$$

$$\therefore x=2n\pi\pm\cos^{-1}\left(\frac{1}{3}\right), where\, n\in 1$$

Q.47 (4)

Q.48 (4)

Q.49 (3)

Given, $\frac{\sin 3\theta}{2\cos 2\theta+1}=\frac{1}{2}$

$$\Rightarrow \frac{3\sin\theta-4\sin^3\theta}{2-4\sin^2\theta+1}=\frac{1}{2}$$

$\Rightarrow$ $2\sin\theta\,[3-4\sin^2\theta]=(3-4\sin^2\theta)$

$\Rightarrow$ $(2\sin\theta-1)(3-4\sin^2\theta)=0$

$\therefore$ $\sin\theta=\frac{1}{2}$ $\Rightarrow$ $\theta=n\pi+(-1)^n\frac{\pi}{6}, n\in I$

or $\sin^2\theta=\frac{3}{4}$ $\Rightarrow$ $\theta=n\pi\pm\frac{\pi}{3}, m\in I$

which does not satisfy the given equation

Q.50 (3)

$$\tan x+\tan\left(x+\frac{\pi}{3}\right)+\tan\left(x+\frac{2\pi}{3}\right)=3$$

$\Rightarrow$ $3\tan 3x=3$

$\Rightarrow$ $\tan 3x=1$

$\Rightarrow$ $x=\frac{n\pi}{3}+\frac{\pi}{12}, n\in I$

Q.51 (4)

$$\cos 7\theta=\cos\theta-\sin 4\theta$$

$$\Rightarrow \sin 4\theta=\cos\theta-\cos 7\theta$$

$$\Rightarrow \sin 4\theta=2\sin 4\theta\sin 3\theta$$

$$\Rightarrow \sin 4\theta(1-2\sin 3\theta)=0$$

$$\therefore \sin 4\theta=0 \text{ or } \sin 3\theta=\frac{1}{2}$$

$$\Rightarrow \quad 4\theta=n\pi \text{ or } 3\theta=n\pi+(-1)^n\frac{\pi}{6}$$

$$\Rightarrow \quad \theta=\frac{n\pi}{4} \text{ or } \frac{n\pi}{3}+(-1)^n\frac{\pi}{18}$$

Q.52 (2)

Given,

$$\tan(\cot x)=\cot(\tan x)=\tan\left(\frac{\pi}{2}-\tan x\right)$$

$$\Rightarrow \cot x=n\pi+\frac{\pi}{2}-\tan x$$

$$\Rightarrow \cot x+\tan x=n\pi+\frac{\pi}{2}$$

$$\Rightarrow \frac{1}{\sin x\cos x}=n\pi+\frac{\pi}{2}\Rightarrow\frac{1}{\sin 2x}=\frac{n\pi}{2}+\frac{\pi}{4}$$

$$\Rightarrow \sin 2x=\frac{1}{\frac{n\pi}{2}+\frac{\pi}{4}}=\frac{4}{(2n+1)\pi}$$

Q.53 (3)

We have cos x + cos 2x + cos 3x = 0

or (cos 3x + cos x) + cos 2x = 0

or 2cos 2x.cos x + cos 2x = 0

or cos 2x(2cos x + 1) = 0

We have, either cos 2x = 0 or 2cos x + 1 = 0

If cos 2x = 0, then $2x=(2m+1)\frac{\pi}{2}$

or $2x=(2m+1)\frac{\pi}{2}$, $m\in I$

It 2 cos x + 1 = 0, then $\text{cox}=-\frac{1}{2}=\cos\frac{2\pi}{3}$

$\Rightarrow$

$$x=2n\pi\pm\frac{2\pi}{3}, n\in I$$

Hence the required general solution are

$$x=(2m+1)\frac{\pi}{4} \text{ and } x=2n\pi\pm\frac{2\pi}{3}, m, n\in I$$

Q.54 (1)

Given $\frac{\tan 3x-\tan 2x}{1+\tan 3x\times\tan 2x}=1$

$\Rightarrow$ $\tan(3x-2x)=1$

$\Rightarrow$ $\tan x=1=\tan\frac{\pi}{4}$

$\Rightarrow$ $x=n\pi+\frac{\pi}{4}$ $n\in I$

But at this value of x, tan 2x = ∞

which is not acceptable

$\therefore$ $x\in\phi$

Q.55 (4)

$$\Rightarrow 1+\sin 2x+2(\text{sinx}+\text{cosx})=0$$

$$\Rightarrow (\sin\ \ x+\cos)^2+2(\sin\ x+\cos)=0$$

$$\Rightarrow (\sin\ x+\cos x)(\sin\ x+2)=0$$

$\therefore$ sin x + cos x =–2 is indmissible.

$$Since, |\sin\ | \le 1, |\cos x|\ \le 1$$

$$\therefore \sin\ x+\cos x=0 \Rightarrow \sin\left(x+\frac{\pi}{4}\right)=0$$

$$\Rightarrow x+\frac{\pi}{4}=n\pi \Rightarrow x=n\pi-\frac{\pi 4}{}$$

Q.56 (2)

Q.57 (4)

Given

sinθ + 2sin2θ + 3 sinθ + 4sin4θ = 10 in (0, π)

Using boundary of SM

It's only possible if ⇒ 1 + 2 + 3 + 4 = 10

(Using boundary values)

⇒ sin θ = 1 & sin 2θ = 1 & sin 3θ = 1 & sin 4θ = 1

$$\theta=2m\pi+\frac{\pi}{2}, 2\theta=2n\pi+\frac{\pi}{2}, 3\theta=2k\pi+\frac{\pi}{2}, 4\theta=2p\pi+\frac{\pi}{2}$$

$m \in I\ n \in I\ k \in I\ p \in I$

$$\theta=\frac{\pi}{2}\ \ \theta=\frac{\pi}{4}\ \ \theta=\frac{\pi}{6},\frac{5\pi}{6}\ \ \ \theta=\frac{\pi}{8},\frac{5\pi}{8}$$

$$\therefore \quad x \in \left\{\frac{\pi}{2}\right\} \cap \left\{\frac{\pi}{4}\right\} \cap \left\{\frac{\pi}{6},\frac{5\pi}{6}\right\} \cap \left\{\frac{\pi}{8},\frac{5\pi}{8}\right\}$$

x ∈ { } number of solutoins is zero.

Q.58 (3)

Q.59 (4)

Given $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$

$a_1 + a_2 (1 – 2\sin^2 x) + a_3 \sin^2 x = 0$

$(a_1 + a_2) = (2a_2 – a_3) \sin^2 x$

$$\sin^2 x=\frac{a_1+a_2}{2a_2-a_3}\ \ \because\ \ 0 \le \sin^2 x \le 1$$

$$0 \le \frac{a_1+a_2}{2a_2-a_3} \le 1$$

$0 \le a_1 + a_2 \le 2a_2 – a_3$

$a_1 + a_2 \ge 0$ ....(i)

$2a_2 – a_3 \ge 0$ ....(ii)

$–a_1 + a_2 – a_3 \ge 0$ ...(iii)

$N^r \le D^r$

homogenous system of equations

$$\begin{bmatrix}1 & 1 & 0\\ 0 & 2 & -1\\ -1 & 1 & -1\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix} \ge \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

|A| = 1(–2 + 1) – 1 (0 – 1) + 0 = –1 + 1= 0

So number of solution is infinite

Q.60 (3)

Q.61 **(4)**

Q.62 **(3)**

∵ A : B : C = 3 : 5 : 4

∴ A = 45° , B = 75° , C = 60°

∵ from Sine - rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

$$\Rightarrow \frac{a}{\frac{1}{\sqrt{2}}}=\frac{b}{\frac{\sqrt{3}+1}{2\sqrt{2}}}=\frac{c}{\frac{\sqrt{3}}{2}}=k\ (\because \sin 75° = \sin(45° + 30°))$$

$$\therefore\ a=\frac{k}{\sqrt{2}},\ b=\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k \text{ and } c=\frac{k\sqrt{3}}{2}$$

$$\therefore\ a+b+c\sqrt{2}=\frac{k}{\sqrt{2}}+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k+\left(\frac{k\sqrt{3}}{2}\right)\sqrt{2}$$

$$=\frac{k}{2\sqrt{2}}\left[2+(\sqrt{3}+1)+2\sqrt{3}\right]=\frac{3k(\sqrt{3}+1)}{2\sqrt{2}}=3b$$

Q.63 (2)

A + B = 180° - C = 90°

a = 2R sin A, b = 2R sin B, c = 2R sinC

$$\therefore \frac{a^2-b^2}{a^2+b^2}=\frac{\sin^2 A-\sin^2 B}{\sin^2 A+\sin^2 B}$$

$$=\frac{\sin(A+B)\sin(A-B)}{\sin^2 A+\sin^2(90^\circ-A)}$$

$$[\therefore A+B=90^\circ]$$

$$=\frac{\sin 90^\circ \sin(A-B)}{\sin^2 A+\cos^2 A}=\sin(A-B)$$

Q.64 (1)

Let A, B and C be the three angles of ΔABC and It is givne that the angles are in AP.

∴ 2B = A + C on adding B both the sides, we get

3B = A + B +C

$$\Rightarrow 3B=180^\circ \Rightarrow B=60^\circ$$

Now, we know cos $B=\dfrac{a^2+c^2-b^2}{2ac}$

$$\Rightarrow \cos 60^\circ=\frac{10^2+c^2-9^2}{2\times 10\times c}$$

$$\Rightarrow \frac{1}{2} = \frac{100 + c^2 - 81}{20c}$$

$$\Rightarrow c^2 - 10c + 19 = 0 \Rightarrow c = 5 \pm \sqrt{6}$$

Q.65 (1)

Q.66 (3)

Q.67 **(3)**

$$\because \frac{bc\sin^2 A}{\cos A + \cos B\cos C}$$

$$= \frac{k^2 \sin B \sin C \sin^2 A}{-\cos(B+C) + \cos B \cos C}$$

$$= \frac{k^2 \sin B \sin C \sin^2 A}{\sin B \sin C} = k^2 \sin^2 A = a^2.$$

Q.68 **(3)**

$(a + b + c)\,(b + c - a) = k\,bc$

$\Rightarrow (b + c + a)\,(b + c - a) = kbc$

$\Rightarrow (b + c)^2 - a^2 = kbc$

$\Rightarrow b^2 + c^2 - a^2 + 2bc = kbc$

$\Rightarrow b^2 + c^2 - a^2 = (k - 2)\,bc$

$$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{k-2}{2}$$

$$\because -1 < \frac{k-2}{2} < 1$$

$\Rightarrow -2 < k - 2 < 2 \;\Rightarrow\; 0 < k < 4$

Q.69 **(3)**

$$AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

$\Rightarrow 4AD^2 = 2b^2 + 2c^2 - a^2$

$\Rightarrow 4BE^2 = 2c^2 + 2a^2 - b^2$

$\Rightarrow 4CF^2 = 2a^2 + 2b^2 - c^2$

$\therefore 4(AD^2 + BE^2 + CF^2) = 3(a^2 + b^2 + c^2)$

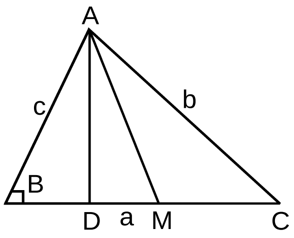
$$\Rightarrow \frac{AD^2 + DE^2 + CF^2}{BC^2 + CA^2 + AB^2} = \frac{3}{4} = 3:4$$

Q.70 **(2)**

$BM = a/2$

$BD = c \cos B$

$$DM = \frac{a}{2} - c\cos B$$

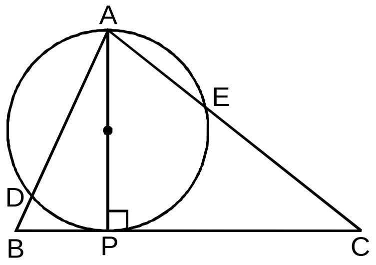


$$= \frac{a}{2} - c\,\frac{(c^2 + a^2 - b^2)}{2ca}$$

$$= \frac{a^2 - c^2 - a^2 + b^2}{2a} = \frac{b^2 - c^2}{2a}$$

Q.71 **(4)**

Sine Rule in ΔADE



$$\frac{DE}{\sin A} = AP$$

$$\Rightarrow DE = \frac{2\Delta}{a}\sin A = 2\Delta \times \frac{1}{R} = \frac{\Delta}{R}\left\{\frac{\sin A}{a} = \frac{1}{2R}\right.$$

Q.72 (4)

Q.73 **(2)**

$$A.M. = \frac{a+b+c}{3}$$

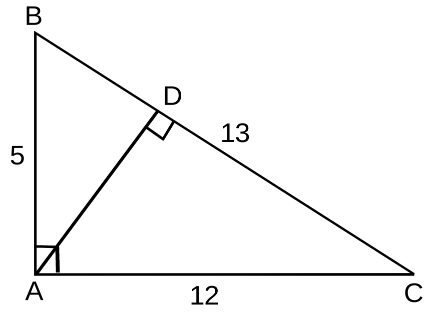
Altitudes are $\frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$

$$H.M. = \frac{3}{\frac{a}{2\Delta} + \frac{b}{2\Delta} + \frac{c}{2\Delta}} = \frac{6\Delta}{(a+b+c)}$$

$$(A.M.) \times (H.M.) = \frac{(a+b+c)}{3} \times \frac{3 \times 2\Delta}{(a+b+c)} = 2\Delta$$

Q.74 **(2)**

Given $c = 5$, $b = 12$, $a = 13$



$$\cos A = \frac{12^2 + 5^2 - 13^2}{2(12)(5)} = 0$$

i.e. $A = 90^\circ$

Altitude AD = $\frac{2\Delta}{a} = \frac{2\times30}{13} = \frac{60}{13}$

$\{\Delta = \frac{1}{2}\times5\times12 = 30\}$

Q.75 **(2)**

$\because$ s – a = 3 ...(1)

and s – c = 2 ...(2)

by (1) – (2), we get

c – a = 1

(1) + (2), we get 2s – a – c = 5

$\Rightarrow$ b = 5

$\because$ ΔABC is right angled at B

$\therefore$ $a^2 + c^2 = 25$ ...(3)

$\therefore$ $(c-a)^2 + 2ac = 25$

ac = 12 ...(4)

$\therefore$ $a(1+a) = 12 \Rightarrow a^2 + a - 12 = 0$

$\Rightarrow (a+4)(a-3) = 0$

$\Rightarrow$ a = 3 and c = 4.

Q.76 **(1)**

$$\because\ b\cos^2\frac{A}{2} + a\cos^2\frac{B}{2} = \frac{3}{2}c.$$

$$\Rightarrow b\frac{s(s-a)}{bc} + a\frac{s(s-b)}{ac} = \frac{3}{2}c.$$

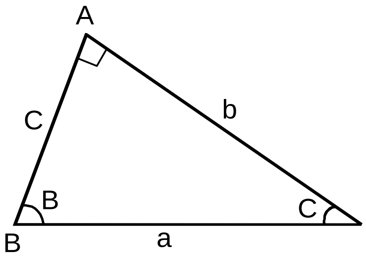
$$\Rightarrow \frac{s}{c}[s-a+s-b] = \frac{3}{2}c \Rightarrow \frac{s}{c}\times c = \frac{3}{2}c$$

$$\Rightarrow \frac{a+b+c}{2} = \frac{3c}{2} \qquad \Rightarrow a + b = 2c$$

$\Rightarrow$ a, c, b are in A.P.

Q.77 **(4)**

$\Delta ABC, \angle A = \frac{\pi}{2}$



$$\tan\frac{C}{2} = \frac{r}{s-c} = \frac{\Delta}{s(s-c)}\ \{\because \Delta = \frac{1}{2}bc\}$$

$$\tan\frac{C}{2} = \frac{\frac{1}{2}bc}{\frac{(a+b+c)}{2}\frac{(a+b-c)}{2}} = \frac{2bc}{2b^2+2ab}$$

$\{\because a^2 = b^2 + c^2\}$

$$= \frac{2bc}{2b(b+a)} = \frac{c}{a+b}\times\frac{a-b}{a-b}$$

$$= \frac{c(a-b)}{a^2-b^2} = \frac{c(a-b)}{c^2} = \frac{a-b}{c}$$

Q.78 **(2)**

$$\because\ \frac{b^2-c^2}{2aR} = \frac{4R^2\left(\sin^2 B - \sin^2 C\right)}{2.2R\sin A.R}$$

$$= \frac{\sin(B+C).\sin(B-C)}{\sin A} = \sin(B-C)$$

Q.79 **(2)**

$$\cos\frac{\pi}{6} = \frac{b^2+c^2-a^2}{2bc} = \frac{\sqrt{3}}{2} \Rightarrow a = 1$$

$$2R = \frac{a}{\sin A} \Rightarrow R = 1$$

Q.80 (2)

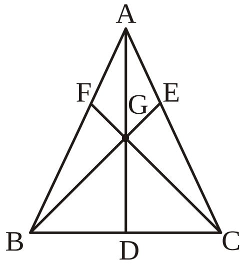
We have $\Delta = \frac{\sqrt{3}}{4}a^2, s = \frac{3a}{2}$

$$\therefore r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}, R = \frac{abc}{4\Delta} = \frac{a^3}{\sqrt{3a^2}} = \frac{a}{\sqrt{3}}$$

$$\text{and } r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}/4a^2}{a/2} = \frac{\sqrt{3}}{2}a$$

Hence, $r:R:r_1 = \frac{a}{2\sqrt{3}}:\frac{a}{\sqrt{3}}:\frac{\sqrt{3}}{2}a = 1:2:3$

Q.81 (1)



The foot of the pole is at the centroid is the point of intersection of medians AD, BE and CF, which are the lines joining a vertex with the mid point of opposite side.

Q.82 (3)

Q.83 **(1)**

$$\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$$

$$= \frac{R(\sin 2A + \sin 2B + \sin 2C)}{2R(\sin A + \sin B + \sin C)}$$

$$= \frac{4\sin A \sin B \sin C}{8\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= \frac{r}{R}.$$

Q.84 **(2)**

Given a : b : c = 3 : 7 : 8

We know $R = \frac{a}{2\sin A}$ & $r = \frac{\Delta}{s}$

$$\Rightarrow \frac{R}{r} = \frac{as}{2\Delta \sin A} = \frac{a(a+b+c)}{4\times\frac{1}{2}bc\sin^2 A}$$

$= \frac{a(a+b+c)}{2bc\sin^2 A}$ $\{\Delta = \frac{1}{2}$ bc sin A$\}$ $\{\because$ a = 3k, b= 7k, c = 8k$\}$

$$\because \cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{k^2}{k^2}\ \frac{49+64-9}{2.7.8} = \frac{104}{2.7.8} = \frac{13}{14}$$

$$\therefore \sin^2 A = 1 - \frac{13^2}{14^2} = \frac{27}{14^2}$$

$$\Rightarrow \frac{R}{r} = \frac{3(3+7+8)}{2\times 7\times 8\sin^2 A}\ \frac{k^2}{k^2} = \frac{3\times 18\times 14\times 14}{2\times 7\times 8\times 27} \Rightarrow$$

$$\frac{R}{r} = \frac{14}{4} = \frac{7}{2} \Rightarrow R : r = 7 : 2$$

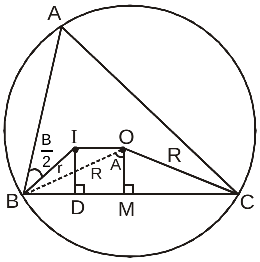
Q.85 **(2)**

In ΔOBM, $\cos A = \frac{r}{R}$

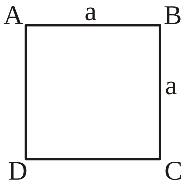
$\Rightarrow \cos A + \cos B + \cos C = 1 + 4\,\pi \sin\frac{A}{2}$

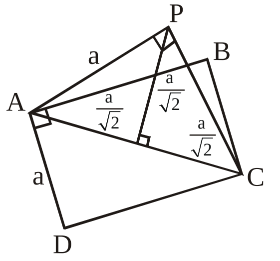
$\Rightarrow \frac{r}{R} + \cos B + \cos C = 1 + \frac{r}{R}$

$\Rightarrow \cos B + \cos C = 1$



Q.86 (3)





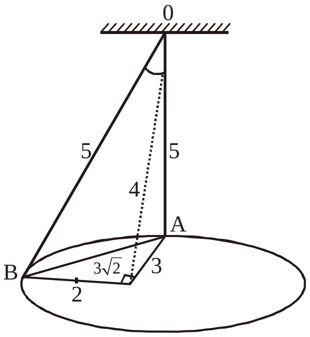
$\Rightarrow$ AP = AD = PD = a

$\Rightarrow$ angle subtended

by aside = $\frac{\pi}{3}$

($\because$ equilateral Δu)

Q.87 **(3)**



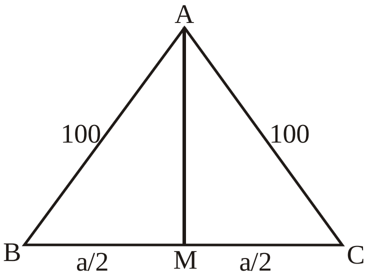
$$\cos\theta = \frac{5^2+5^2-(3\sqrt{2})^2}{2(5)(5)} = \frac{25+25-18}{50}$$

$$\cos\theta = \frac{32}{50} = \frac{16}{25}$$

Q.88 (1)

tan 30º = h $\Rightarrow$ h = $^{1}/_{\sqrt{3}}$ km

Q.89 **(2)**



$$AM = \sqrt{2(100)^2 + 2(100)^2 - a^2}$$

given $\frac{AM}{h} = \frac{16}{5}$ ......(1)

$\frac{h}{a/2} = \frac{5}{12}$ ......(2)

Q.90 (3)

$$\text{Let } ED = h, \angle EAB = \alpha$$

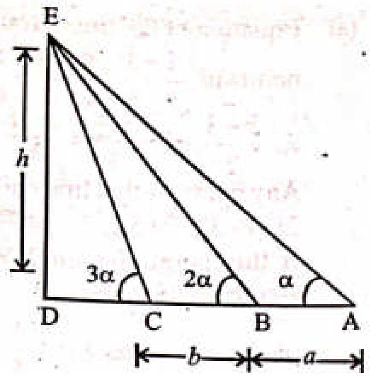
$$\therefore \quad \angle EBD = 2\alpha, \angle ECD = 3\alpha$$

$$\text{Now, } \angle DBE = \angle EAB + \angle BEA$$

$$\Rightarrow \quad 2\alpha = \alpha + \angle BEA$$

$$\Rightarrow \angle BEA = \alpha = \angle EAB$$

$$\Rightarrow AB = EB = a$$



$$\text{Similarly, } \angle EBC = \alpha$$

$$\text{In } \Delta EBC, \frac{BC}{\sin\alpha} = \frac{EB}{\sin\left(180^\circ - 3\alpha\right)}$$

$$\Rightarrow \quad \frac{b}{\sin\alpha} = \frac{a}{\sin 3\alpha} \Rightarrow \frac{a}{b} = \frac{\sin 3\alpha}{\sin 3\alpha}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{3\sin\alpha - 4\sin^3\alpha}{\sin\alpha} = 3 - 4\sin^2\alpha$$

$$\Rightarrow \quad 4\sin^2\alpha = 3 - \frac{a}{b} = \frac{3b-a}{b}$$

$$\Rightarrow \sin\alpha = \sqrt{\frac{3b-a}{4b}}$$

$$\text{In } \Delta EBD, \sin 2\alpha = \frac{ED}{EB}$$

$$\Rightarrow \quad ED = a \cdot 2\sin\alpha \cdot \cos\alpha$$

$$\Rightarrow \quad h = 2a\sqrt{\frac{3b-a}{4b}}, \sqrt{1 - \frac{3b-a}{4b}}$$

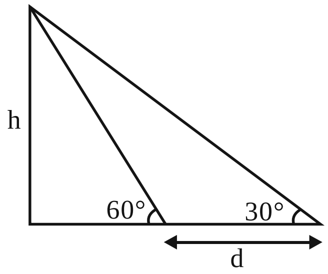
$$= 2a\sqrt{\frac{3b-a}{4b}}\sqrt{\frac{b-a}{4b}}$$

$$= \frac{a}{2b}\sqrt{(a+b)(3b-a)}$$

Q.91 (3)

d = h cot 30° – h cot 60° and time = 3 min.

$$\therefore \text{ Speed} = \frac{h(\cot 30^\circ - \cot 60^\circ)}{3} \text{per minute}$$

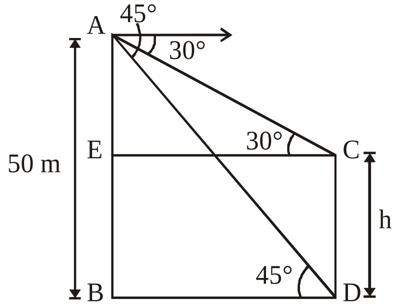


It will travel distane h cot 60° in

$$\frac{h\cot 60^\circ \times 3}{h(\cot 30^\circ - \cot 60^\circ)} = 1.5 \min ute$$

Q.92 **(4)**

Let height of the tower be *h* m and distance between tower and cliff be x m.



$$\therefore CD = h, BD = x$$

$$\text{In } \Delta ABD, \tan 45^\circ = \frac{AB}{BD} \quad \text{or} \quad 1 = \frac{50}{x}$$

$x = 50$ …(i)

$$\tan 30^\circ = \frac{AE}{EC} = \frac{AB - EB}{EC} = \frac{AB - DC}{BD}$$

$(\because EB = DC, EC = BD)$

$$\frac{1}{\sqrt{3}} = \frac{50-h}{x} \quad or \quad x = 50\sqrt{3} - h\sqrt{3}$$

$$\text{or } 50 = 50\sqrt{3} - h\sqrt{3} \qquad \left[\text{From}(i), x = 50\right]$$

$$\text{or} \quad h\sqrt{3} = 50\sqrt{3} - 50$$

$$\text{or } h = \frac{50\left(\sqrt{3}-1\right)}{\sqrt{3}} = 50\left(1 - \frac{1}{\sqrt{3}}\right)$$

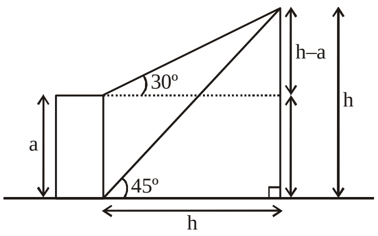
$$\therefore h = 50\left(1 - \frac{\sqrt{3}}{3}\right)$$

Q.93 (1)

Q.94 (3)

Q.95 (1)

Q.96 **(3)**



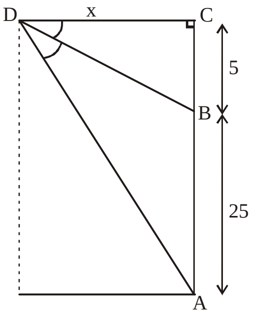
$$\frac{1}{\sqrt{3}} = \frac{h-a}{h}$$

$h = h\sqrt{3} - a\sqrt{3} \Rightarrow a\sqrt{3} - h(\sqrt{3} - 1)$

$$\Rightarrow h = \frac{a\sqrt{3}}{(\sqrt{3}-1)} = a(3-\sqrt{3})$$

$$\boxed{h = \frac{a\sqrt{3}}{(\sqrt{3}-1)}}$$

Q.97 **(2)**



In ΔBCD In ΔACD

$$\tan\alpha = \frac{5}{x} \;;\; \tan 2\alpha = \frac{30}{x}$$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{30}{x}$$

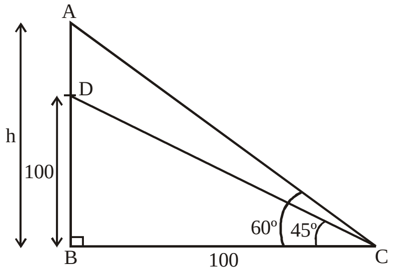
$$\frac{2\times\frac{5}{x}}{1-\frac{25}{x^2}} = \frac{30}{x}$$

$$\frac{10}{x^2}.\frac{x^2}{(x^2-25)} = \frac{30}{x} \Rightarrow x^2 = 3c^2 - 75$$

$75 = 2x^2$

$$x = 5\sqrt{\frac{3}{2}}$$

Q.98 **(3)**

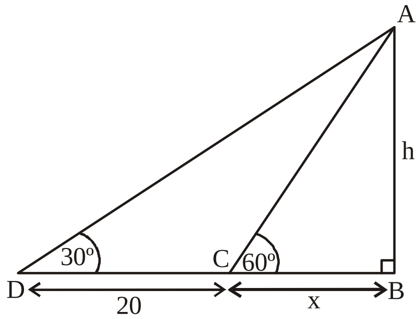


$$\frac{h}{100} = \tan 60^\circ \quad \Rightarrow \quad h = 100\sqrt{3}$$

Height increased by $(100\sqrt{3} - 100)$

$= 100(\sqrt{3} - 1)$

Q.99 **(3)**



In ΔABC

$$\frac{h}{x} = \tan 60^\circ$$

$\boxed{h = \sqrt{3}x}$ ....(1)

2 ΔABD

$$\frac{h}{x+20} = \tan 30^\circ \Rightarrow \sqrt{3}g = x+20$$

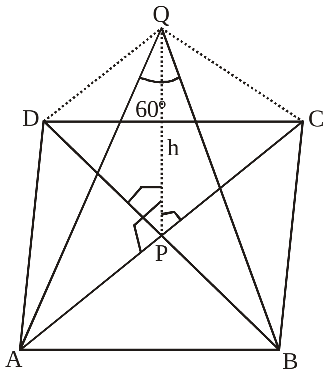
......(2)

from (1) & (2)

$h = \sqrt{3}(\sqrt{3}h - 20) \Rightarrow 2h = 20\sqrt{3}$

$\Rightarrow \boxed{h = 10\sqrt{3}}$

Q.100 **(2)**



$\angle$ AQB = 60° and AQ = BQ

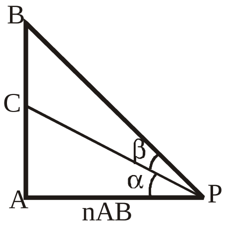
$\Rightarrow$ ABQ is equilateral triangle

In Δ DPQ

$AQ^2 = h^2 + AP^2$

$$Q^2 = h^2 + \frac{a^2}{2} \Rightarrow \boxed{2h^2 = a^2}$$

Q.101 (1)



In ΔACP

$$\tan\alpha = \frac{AC}{AP} = \frac{AB/2}{nAP} = \frac{1}{2n}$$

In ΔABP

$$\tan(\alpha+\beta) = \frac{AB}{AP} = \frac{AB}{nAB} = \frac{1}{n}$$

$$\tan(\alpha+\beta) = \frac{1}{n}$$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{1}{n}$$

$$\frac{\frac{1}{2n}+\tan\beta}{1-\frac{1}{2n}\tan\beta} = \frac{1}{4}$$

$$\frac{\frac{1+2n\tan\beta}{2n}}{\frac{2n-\tan\beta}{2n}} = \frac{1}{n}$$

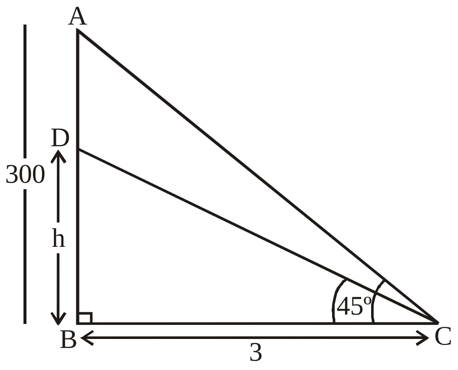
$n(1+2n\tan\beta) = 2n - \tan\beta$

$n + 2n^2\tan\beta = 2n - \tan\beta$

$\tan\beta(2n^2+1) = 2n - n$

$$\tan\beta = \frac{n}{2n^2+1}$$

Q.102 **(1)**



$$BC = 300\cot 60^o = \frac{100}{\sqrt{3}}$$

$$\text{If } \Delta BCD \boxed{h = \frac{300}{\sqrt{3}}} = 100\sqrt{3}$$

EXERCISE-III

NUMERICAL VALUE BASED
TRIGONOMETRIC RATIOS AND IDENTITIES

Q.1 **0006**

$$\frac{\cos 5A}{\cos A} + \frac{\sin 5A}{\sin A}$$

$$= \frac{\sin A\cos 5A + \cos A\sin 5A}{\sin A\cos A}$$

$$= \frac{2\sin 6A}{\sin 2A} = \frac{2[3\sin 2A - 4\sin^3 2A]}{\sin 2A}$$

$$= 6 - 8\sin^2 2A = 6 - 8\cdot\left[\frac{1-\cos 4A}{2}\right]$$

$= 2 + 4\cos 4A = a + b\cos 4A$

$a + b = 6$ **Ans.**

Q.2 **0002**

$\cos\theta$

$$= \frac{2\cos(\theta-\phi).\cos(\theta+\phi)}{\cos(\theta-\phi)+\cos(\theta+\phi)} = \frac{2(\cos^2\theta-\sin^2\phi)}{2\cos\theta.\cos\phi}$$

$\Rightarrow \cos^2\theta . \cos\phi = \cos^2\theta - \sin^2\phi$

$\Rightarrow \sin^2\phi = \cos^2\theta\,(1 - \cos\phi)$

$$\Rightarrow 4\sin^2\frac{\phi}{2} . \cos^2\frac{\phi}{2} = 2\cos^2\theta . \sin^2\frac{\phi}{2}$$

$$\Rightarrow \cos\theta\sec\frac{\phi}{2} = \sqrt{2}$$

Q.3 **0001**

Since sin A. cos A and tan A are G.P.

we have $\cos^2 A = \sin A\tan A$

$\Rightarrow \cot^2 A = \sec A$

$\Rightarrow \cot^4 A = 1 + \tan^2 A$

$\Rightarrow \cot^6 A - \cot^2 A = 1$

Q.4 0001

We have $\text{cosec}\,\theta - \sin\theta = a^3$

$\Rightarrow \cos^2\theta = a^3\sin\theta$ & $\sec\theta - \cos\theta = b^3$

$\Rightarrow \sin^2\theta = b^3\cos\theta$ on simplifying

$\Rightarrow a^2 b^2 (a^2 + b^2) = 1$

Q.5 0004

$\sin\theta + \sin^2\theta + \sin^2\theta = 1$

$\sin\theta\,(1 + \sin^2\theta) = 1 - \sin^2\theta$

$\Rightarrow \sin\theta\,(2 - \cos^2\theta) = \cos^2\theta \Rightarrow$ on squaring

$\Rightarrow \sin^2\theta\,(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

Q.6 **0002**

Explanation:

The value of $(1 - \cot 23^o)\,(1 - \cot 22^o)$

$$\Rightarrow \frac{(\sin 23^o - \cos 23^o)(\sin 22^o - \cos 22^o)}{\sin 23^o \sin 22^o}$$

$$\Rightarrow \frac{\sqrt{2}\sin(23^o - 45^o).\sqrt{2}\sin(22^o - 45^o)}{\sin 23^o \sin 22^o}$$

$\Rightarrow 2$

Q.7 **0003**

Explanation:

$\tan 20^o + \tan 40^o + \sqrt{3}\tan 20^o \tan 40^o$

$$= \frac{\sin 60^o}{\cos 20^o \cos 40^o} + \frac{\sqrt{3}\sin 20^o \sin 40^o}{\cos 20^o \cos 40^o}$$

$$= \frac{\sqrt{3}\left(\frac{1}{2} + \sin 20^o \sin 40^o\right)}{\cos 20^o \sin 40^o} = \sqrt{3}$$

Q.8 1.5

$$\cos^4\frac{\pi}{8} + \cos^4\frac{7\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8}$$

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right) + \cos^4\left(\frac{\pi}{2} + \frac{\pi}{8}\right)$$

$$= 2\left[\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}\right]$$

$$= 2\left[(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8})^2 - 2\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right]$$

$$= 2\left[1 - \frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right]$$

$$= 2\left[1 - \frac{1}{4}\right] = \frac{3}{2}$$

Q.9 0.0625

$$\cos\frac{\pi}{15}.\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}$$

$$= \frac{1}{4}\left(2\cos\frac{4\pi}{15}\cos\frac{\pi}{15}\right)\left(2\cos\frac{8\pi}{15}\cos\frac{2\pi}{15}\right)$$

$$= \frac{1}{4}\,(\cos 60^o + \cos 36^o)\,(\cos 120^o + \cos 72^o)$$

$$= \frac{1}{4}\left(\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right)\left(-\frac{1}{2} + \frac{\sqrt{5}-1}{4}\right)$$

$$= \frac{1}{4}\left[-\frac{1}{4} + \frac{1}{2}\left(\frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4}\right) + \frac{5-1}{16}\right] = -\frac{1}{16}$$

Q.10 **10**

$$5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$$

$$= 5\cos\theta + 3\,(\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}) + 3$$

$$= 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$$\therefore \text{maximum value} = 3 + \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2}$$

$$= 3 + \sqrt{\frac{196}{4}} = 3 + 7 = 10$$

TRIGONOMETRIC EQUATION

Q.11 0006

Apply sin C + sin D and then solve

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6} \;\&\; \pi$$

Q.12 0000

Given $\sin\theta = 3\sin(\theta + 2\alpha)$

$\Rightarrow \sin(\theta + \alpha - \alpha) = 3\sin(\theta + \alpha + \alpha)$

$\Rightarrow \sin(\theta + \alpha)\cos\alpha - \cos(\ + \alpha)\sin\alpha$

$= 3\sin(\theta + \alpha)\cos\alpha + 3\cos(\theta + \alpha)\sin\alpha$

$\Rightarrow -2\sin(\theta + \alpha)\cos\alpha = 4\cos(\theta + \alpha)\sin\alpha$

$$\Rightarrow \frac{-\sin(\theta+\alpha)}{\cos(\theta+\alpha)} = \frac{2\sin\alpha}{\cos\alpha}$$

$\Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$

Q.13 **0000**

$$1 + \sin x\left(\frac{1 - \cos x}{2}\right) = 0$$

$\Rightarrow 4 + 2\sin x = \sin 2x \Rightarrow$ No. sol

Q.14 **0004**

$$1 - \sin^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$$

$$\sin^2x - \frac{\sqrt{3}+1}{2}\sin x + \frac{\sqrt{3}}{4} = 0;$$

$$4\sin^2x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} = 0$$

On solving we get

$$\sin x = 1/2\,;\ \frac{\sqrt{3}}{2} = (\pi/6\,,\ 5\pi/6\,;\ \pi/3\,,\ 2\pi/3\,]$$

Q.15 **0000**

$\sin x \cos x\,[\sin^2x + \sin x \cos x + \cos^2 x] = 1$

$\Rightarrow \sin x \cos x + (\sin x \cos x)^2 = 1$

$\sin^2 2x + 2\sin 2x - 4 = 0$

$$\Rightarrow \sin 2x = \frac{-2 \pm \sqrt{4+16}}{2} = -1 \pm \sqrt{5}\,,$$

which is not possible.

Q.16 **0007**

Given equation is

$\sin x\,[4\,(1 - \sin^2 x) - 2\sin x - 3] = 0$

$\therefore\ \sin x = 0$ or $4\sin^2 x + 2\sin x - 1 = 0$

$$\sin x = 0 \quad \text{or} \quad \sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin x = 0 \quad \text{or} \quad \sin x = \frac{\sqrt{5}-1}{4}$$

$$\text{or} \quad \sin x = \frac{\sqrt{5}+1}{4}$$

$\therefore$ General solution is

$$x = n\pi \quad \text{or} \quad x = n\pi + (-1)^n \frac{\pi}{10}$$

$$\text{or} \quad x = n\pi + (-1)^n\left(-\frac{3\pi}{10}\right),\ n \in Z.$$

For $n = 1, 2$

$$x = \frac{13\pi}{10}, \frac{17\pi}{10}$$

Q.17 0004

apply sin C + sin D & cos C + cos D then solve

Q.18 **0008**

given equation can be written as

$3\sin\theta - 4\sin^3\theta = 4\sin\theta \sin 2\theta \sin 4\theta$

hence either $\sin\theta = 0 \ \Rightarrow\ \theta = n\pi$

or $3 - 4\sin^2\theta = 4\sin 2\theta \sin 4\theta$

$3 - 2\,(1 - \cos 2\theta) = 2\,(\cos 2\theta - \cos 6\theta)$

or $1 = -2\cos 6\theta$

$$\cos 6\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$$

$$6\theta = 2n\pi \pm \frac{2\pi}{3}$$

if $0 \le \theta \le \pi$ then total solution are

$$0, \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \pi$$

is 8 real solutions.

Q.19 0005

Apply cos C + cos D and then solve x = 30°, 45°, 90°, 135°, 150°

Q.20 0005

Apply cos C + cos D and sin C + sin D , then solve x =

$$\frac{\pi}{7}, \frac{5\pi}{7}, \pi, \frac{9\pi}{7}, \frac{13\pi}{7}$$

Q.21 **0000**

Take, $a\sin(B - C) = a\sin B\cos C - a\cos B\sin C$

$$= \frac{ab}{k}\left(\frac{b^2 + a^2 - c^2}{2ab}\right) - \frac{ac}{k}\left(\frac{c^2 + a^2 - b^2}{2ac}\right)$$

$$= \frac{b^2 + a^2 - c^2 - c^2 - a^2 + b^2}{2k} = \frac{b^2 - c^2}{k}$$

Similarly, $b\sin(C - A) = \dfrac{c^2 - a^2}{k}$

and $c\sin(A - B) = \dfrac{a^2 - b^2}{k}$

Adding them, we get zero.

Q.22 **0003**

We have $B + C = 180° - A$, $B - C = 60°$

$$\therefore\ B = 120 - \frac{A}{2} \ \text{and}\ C = 60° - \frac{A}{2}$$

$$\sin B = \sin(180° - B) = \sin\left(60 + \frac{A}{2}\right)$$

$$\sin C = \sin\left(60° - \frac{A}{2}\right)$$

Also area of the $\Delta = \dfrac{1}{2}ca\sin B$

$$= \frac{1}{2}a\,2R\sin C\sin B = \frac{1}{2}a\left(\frac{a}{\sin A}\right)\sin B\sin C$$

$$= \frac{a^2\sin B\sin C}{2\sin A}$$

$$\therefore\quad 12 = \frac{1}{2}\,\frac{(36)\sin(60° - \frac{A}{2})\sin(60° + \frac{A}{2})}{\sin A}$$

$$4\sin A = 3\left(\cos A - \cos 120°\right) = 3\cos A + \frac{3}{2}$$

$\therefore \quad 8\sin A - 6\cos A = 3.$

Q.23 **0009**

$a : b : c = 4 : 5 : 6$

Let $a = 4k$, $b = 5k$, $c = 6k$

$$S = \frac{15k}{2}$$

$$\Delta = \sqrt{s(s-a)\,(s-b)\,(s-c)}$$

$$= \sqrt{\left(\frac{15k}{2}\right)\left(\frac{15k}{2} - 4k\right)\left(\frac{15k}{2} - 5k\right) \times \left(\frac{15k}{2} - 6k\right)}$$

$$= \sqrt{\left(\frac{15k}{2}\right)\left(\frac{7k}{2}\right)\left(\frac{5k}{2}\right)\left(\frac{3k}{2}\right)} = \frac{15\sqrt{7}}{4}\,k^2$$

$$r = \frac{\Delta}{s} = \frac{\frac{15\sqrt{7}}{4}k^2}{\left(\frac{15k}{2}\right)} = \frac{\sqrt{7}}{2}k.$$

$$R = \frac{abc}{4\Delta} = \frac{(4k)\,(5k)\,(6k)}{4 \cdot \frac{15\sqrt{7}}{4}k^2} = \frac{8}{\sqrt{7}}k$$

$$\frac{R}{s} = \frac{\frac{8}{\sqrt{7}}}{\frac{\sqrt{7}}{2}} = \frac{16}{7}$$

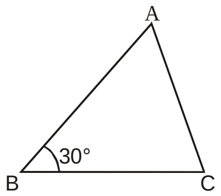
$m = 16, n = 7, m - n = 9.$ **Ans.**

Q.24 0004

$$\cos B = \frac{a^2 + 16 - 8}{2 \times a \times 4}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a^2 + 8}{8a}$$

$$\Rightarrow a^2 - 4\sqrt{3}a + 8 = 0$$



$$\Rightarrow a_1 + a_2 = 4\sqrt{3}, a_2 a_2 = 8$$

$$\Rightarrow |a_1 - a_2| = 4$$

$$\Rightarrow |\Delta_1 - \Delta_2| = \frac{1}{2} \times \sin 30° \times 4 = 4$$

Q.25 0000

$\because$ a, b, c are in A.P. as well as in G.P.

$b^2 = ac$

$a + c = 2b$

$(a + c)^2 = 4ac$

$(a - c)^2 = 0$

$a = c$

$\Rightarrow \; 2a = 2b, a = b$

$$r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$

$$\therefore r_1 = r_2 = r_3 (\because a = b = c)$$

$$\therefore \frac{r_1}{r_2} = \frac{r_2}{r_3}\cdot \text{ Hence } \frac{r_1}{r_2} - \frac{r_2}{r_3} = 0$$

Q.26 0002

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25 + 36 - 16}{60} = \frac{3}{4}$$

Similarly, $\cos B = \dfrac{9}{16}$ and $\cos C = \dfrac{1}{8}$

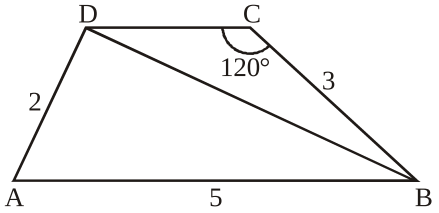
$$\therefore \quad \cos A + \cos B + \cos C = \frac{23}{16} = \frac{K}{M} \text{ then } \left(\frac{K}{M}\right) = 2$$

(where $(.)$ denotes least integer)

Q.27 0002

$$\cos 60° = \frac{2^2 + 5^2 - BD^2}{2(2)(5)}, BD^2 = 19$$

$$\text{Now } \cos 120° = \frac{x^2 + 3^2 - 19}{(2)(3)(x)}$$



$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow x = -5, 2$

PREVIOUS YEAR'S

MHT CET

Q.1	(2)
Q.2	(3)
Q.3	(1)
Q.4	(2)
Q.5	(2)
Q.6	(2)

Q.7 (4)

Q.8 (4)

Q.9 (2)

Q.10 (1)

Q.11 (2)

Q.12 (1)

Q.13 (2)

Q.14 (1)

Q.15 (2)

Q.16 (2)

Q.17 (1)

Q.18 (1)

Given, $\tan\theta = \frac{4}{5}$

$\therefore\ \sin\theta = \frac{4}{\sqrt{41}}$ and $\cos\theta = \frac{5}{\sqrt{41}}$

Now, $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} = \frac{5\times\frac{4}{\sqrt{41}} - 3\times\frac{5}{\sqrt{14}}}{\frac{4}{\sqrt{14}} + 2\times\frac{5}{\sqrt{14}}} = \frac{5}{14}$

Q.19 (1)

Let 3A = A + 2A

tan 3 A = tan (A + 2A)

$\tan 3A = \frac{\tan A + \tan 2A}{1 - \tan A.\tan 2A}$

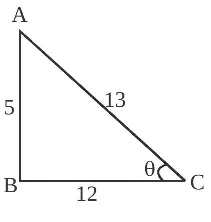
$\Rightarrow$ tan A + tan 2A = tan 3A – tan 3A tan 2A. tan A

$\Rightarrow$ tan 3A – tan 2A – tan A = tan 3A tan 2A. tan A

Q.20 (4)

Given $\tan x = \frac{5}{12}$ and x lies in III quadrant.

$\therefore\ \sin x = \frac{-5}{13}$ and $\cos x = \frac{-12}{13}$



Now, $\cos x = 2\cos^2\frac{x}{2} - 1 \Rightarrow \cos^2\frac{x}{2} = \frac{1}{2}(\cos x + 1)$

$$= \frac{1}{2}\left(\frac{-12}{13} + 1\right) = \frac{1}{2}\left(\frac{1}{13}\right) = \frac{1}{26}$$

$\therefore\ \cos\frac{x}{2} = \sqrt{\frac{1}{26}}$

Q.21 (2)

Given, $\sin A + \cos A = \sqrt{2}$

$\therefore\ \frac{1}{\sqrt{2}}\sin A + \frac{1}{\sqrt{2}}\cos A = 1$

$\Rightarrow\ \sin\frac{\pi}{4}\sin A + \cos A\cos\frac{\pi}{4} = 0$

$\Rightarrow\ \cos\left(a - \frac{\pi}{4}\right) = 1$

$\Rightarrow\ A - \frac{\pi}{4} = 0$

$\Rightarrow\ A = \frac{\pi}{4}$

Now, $\cos^2 A = \cos^2\frac{\pi}{4} = \left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

Q.22 (1)

tan 75º – cot 75º

$$= \frac{\sin 75^o}{\cos 75^o} - \frac{\cos 75^\circ}{\sin 75^\circ}$$

$$= \frac{\sin^2 - 75^\circ - \cos^2 75^\circ}{\sin 75^\circ.\cos 75^\circ} = \frac{-2\cos 150^\circ}{\sin 150^\circ}$$

$$= \frac{-2\cos(90^o + 60^o)}{\sin(90 + 60^o)} = \frac{2.\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}$$

Q.23 (1)

Given , a = sin θ + cos θ

and b = $\sin^3\theta + \cos^3\theta$

So, $a^3 = \sin\theta + \cos^3\theta + 3\sin\theta\cos\theta(\sin\theta + \cos\theta)$

and 3a = 3 (sinθ + cos θ)

$a^3 - 3a = \sin^3\theta + \cos^3\theta + 3\sin\theta\cos\theta$ $(\sin\theta + \cos\theta) - 3(\sin\theta + \cos\theta)$

$= \sin^3\theta + \cos^3\theta + 3(\sin\theta + \cos\theta)(\sin\theta\cos\theta - 1)$

$= \sin^3\theta + \cos^3\theta - 3(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$

$= \sin^3\theta + \cos^3\theta - 3(\sin\theta + \cos\theta)$ $(\sin^2\theta + \cos^2\theta - \sin\theta.\cos\theta)$

$= \sin^3\theta + \cos^3\theta - 3(\sin^3\theta + \cos^3\theta)$

[using identity, $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$]

$= -2(\sin^3\theta + \cos^3\theta)$

$\Rightarrow a^3 - 3a = -2b$ or $a^3 - 3a + 2b = 0$

Q.24 (2)

We have, sin x–3 sin 2x + sin 3x = cos x – 3 cos 2x + cos x

$\Rightarrow$ (sin x + sin 3x) – 3 sin 2x

= (cos x + cos 3x) –3 cos 2x

$\Rightarrow$ 2 sin 2x cos x –3 sin 2x = 2cos 2x cos x – 3 cos 2x

$$\left[\begin{array}{l}\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\ \text{and}\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$\Rightarrow$ (2cos x –3) sin 2x = (2cos x –3) cos 2x

$\Rightarrow$ sin2x = cos 2x $\Rightarrow$ tan 2x = 1

$$\Rightarrow\ \tan 2x \tan\left(n\pi+\frac{\pi}{4}\right), n\in Z$$

$$\Rightarrow\ 2x = np + \frac{\pi}{4}$$

$$\Rightarrow\ x=\frac{n\pi}{2}+\frac{\pi}{8}$$

Q.25 (3)

We have,

cos12º + cos84º + cos156º + cos 132º

= (cos12º + cos132º) + (cos84º + cos 156º)

= 2 cos 72º cos60º + 2cos120º cos 36º

$$\left[\because \sin 18^\circ=\frac{\sqrt{5}-1}{4}, \cos 36^\circ=\frac{\sqrt{5}+1}{4}\right]$$

$$=\frac{\sqrt{5}-1-\sqrt{5}-1}{4}=\frac{-2}{4}=\frac{-1}{2}$$

Q.26 (2)

We have,

$\cos^2 x + \cos^2 y$ –2 cos x cos y cos (x +y)

= $\cos^2 x + \cos^2 y$ – 2 cos x cos y

[cos x cos y – sin x sin y]

[$\because$ cos (x + y) = cos x cos y – sin x sin y]

= $\cos^2 x + \cos^2 y - 2\cos^2 x \cos^2 y$ + 2 cos x cos y sinx siny

= $\cos^2 x + \cos^2 y - \cos^2 x \cos^2 y$

– $\cos^2 x \cos^2 y$ + 2 cosx cos y sinx siny

= $(\cos^2 x - \cos^2 x \cos^2 y) + (\cos^2 y - \cos^2 x \cos^2 y)$

+ 2 cos x cosy sin x siny

= $\cos^2 x\,(1 - \cos^2 y) + \cos^2 y\,(1 - \cos^2 x)$

+ 2 cos x cos y sin x sin y

= $\sin^2 x \cos^2 y + \sin^2 y \cos^2 x$

+2 cos x cos y sin x siny

= $(\sin x \cos y)^2 + (\sin y \cos x)^2$

+ 2 cos x cos y sin x sin y

= $(\sin x \cos y + \sin y \cos x)^2$

= $\{\sin(x+y)\}^2 = \sin^2(x+y)$

Q.27 (3)

We have, sin (270º – θ) sin (90º – θ)

– cos (270º – θ) cos (90º + θ)

= – cosθ. cosθ – (–sin θ) (–sin θ)

= $-\cos^2\theta - \sin^2\theta = -(\cos^2\theta + \sin^2\theta) = -1$

[$\because \sin^2\theta + \cos^2\theta = 1$]

Q.28 (2)

We have, tan 57º – tan12º – tan57º tan12º

We know that,

$$\because\ \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$\Rightarrow$ tan A – tanB = (1+ tanA. tanB) × tan (A–B)

Put A = 57º and B = 12º

$\Rightarrow$ tan57º – tan12º = (1+ tan 57º tan 12º) tan (57º – 12º)

$\Rightarrow$ tan57º – tan 12º – tan 57º tan 12º =1

$\Rightarrow$ tan57º – tan12º – tan57º tan12º = tan 45º

[$\because$ 1 = 45º]

Q.29 (4)

$$\text{We have } \frac{1}{3}\left(\sqrt{3}\cos 23^\circ-\sin 23^\circ\right)$$

$$=\frac{2}{3}\left(\frac{\sqrt{3}}{2}\cos 23^\circ-\frac{1}{2}\sin 23^\circ\right)$$

$$=\frac{2}{3}(\cos 30^\circ \cos 23^\circ - \sin 30^\circ \sin 23^\circ)$$

$$\left[\because \cos 30^\circ=\frac{\sqrt{3}}{2}, \sin 30^\circ=\frac{1}{2}\right]$$

$$=\frac{2}{3}\cos(30^\circ+23^\circ)$$

[$\because$ cos(A+B) = cosAcosB – sinAsinB]

$$=\frac{2}{3}\cos 53^\circ$$

Q.30 (2)

We have, $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

= $\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta$

$+ \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$

= $(\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta)$

+ 2 [cos α cos β + sin α sin β]

= 1 + 1 + 2 [cos (α – β)]

= 2 [1+ cos (α + β)]

$$=2\left[2\cos^2\left(\frac{\alpha+\beta}{2}\right)\right]$$

$$=4\cos^2\left(\frac{\alpha+\beta}{2}\right)$$

Q.31 (3)

Q.32 (3)

Q.33 (3)
Q.34 (2)
Q.35 (2)
Q.36 (1)
Q.37 (3)

tanx + secx = 2cos x

$$\frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$$

$\Rightarrow$ $1+\sin x = 2\cos^2 x$
$\Rightarrow$ $1+\sin x = 2(1-\sin^2 x)$
$\Rightarrow$ $1+\sin x = 2-2\sin^2 x$
$\Rightarrow$ $2\sin^2 x+\sin x-1=0$
$\Rightarrow$ $2\sin^2 x+2\sin x-\sin x-1=0$
$\Rightarrow$ $2\sin x(\sin x+1)-1(\sin x+1)=0$
$\Rightarrow$ $(\sin x+1=0 \Rightarrow 2\sin x-1=0$

$$\Rightarrow \sin x = 1, \sin x = \frac{1}{2}$$

$$\Rightarrow x=\frac{3\pi}{2}, x=\frac{\pi}{6}$$

Q.38 (2)
Q.39 (c)
Q.40 (1)
Q.41 (3)
Q.42 (2)
Q.43 (4)
Q.44 (3)
Q.45 (3)
Q.46 (1)
Q.47 (1)
Q.48 (2)
Q.49 (1)

Using sine rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=K$$

We have, $\frac{\cos A}{a}=\frac{\cos B}{b}$

$$\Rightarrow \frac{\cos A}{K\sin A}=\frac{\cos B}{K\sin B}$$

$\Rightarrow$ KsinBcosA = Ksin A cosB
$\Rightarrow$ cosA sin B– cos B sin A = 0
$\Rightarrow$ sin(A–B) = 0
A = B
$\therefore$ Triangle in an isoscles triangle.

JEE-MAIN
PREVIOUS YEAR'S
TRIGONIMETRIC RATION AND IDENTITIES

Q.1 **(4)**

2sin12° – sin72°
sin12° – sin72° + sin12°
sin12° – {sin72° – sin12°}
sin12° – {2sin30° cos24}
cos78 – cos42° = –2sin18°sin60°

$$=-\sqrt{2}\times\frac{\sqrt{5}-1}{4}\times\frac{\sqrt{3}}{2}$$

$$=\frac{\sqrt{3}(1-\sqrt{5})}{4}$$

Q.2 **(2)**

16sin20° sin40°sin80°
= 16sin20°. sin(60°– 20°).sin(60°+ 20°)

$$(\because \sin\theta\cdot\sin(60-\theta)\cdot\sin(60+\theta)=\frac{1}{4}\sin 3\theta)$$

$$=16\left(\frac{1}{4}\sin 60^\circ\right)=4\left(\frac{\sqrt{3}}{2}\right)=2\sqrt{3}$$

Q.3 (2)

$$\text{COS}\frac{2\pi}{7}+\text{COS}\frac{4\pi}{7}+\text{COS}\frac{6\pi}{7}$$

$$=\frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}}\times\cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right)$$

$$=\frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}}\times\text{COS}\left(\frac{4\pi}{7}\right)$$

$$=\frac{\sin\left(\frac{7\pi}{7}\right)+\sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$$

$$=\frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}$$

Q.4 **(3)**

$$\cos 72^\circ=\frac{\sqrt{5}-1}{4}$$

$$\Rightarrow 1-2\sin^2 36^\circ=\frac{\sqrt{5}-1}{4}$$

$\Rightarrow$ $4-8\alpha^2=\sqrt{5}-1$
$\Rightarrow$ $5-8\alpha^2=\sqrt{5}$
$\Rightarrow$ $(5-8\alpha^2)^2=5$
$\Rightarrow$ $25+64\alpha^4-80\alpha^2=5$

$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0$
$\Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$
$\Rightarrow 16x^4 - 20x^2 + 5 = 0$

Q.5 (1)

$\cot\alpha = 1$, $\sec\beta = \frac{-5}{3}$, $\cos\beta = \frac{-3}{5}$, $\tan\beta = \frac{-4}{3}$

$$\tan(\alpha+\beta) = \frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1} = \frac{-1}{7}$$

Q. 6 [80]

$$\sin 10^\circ\left(\frac{1}{2}.2\sin 20^\circ \sin 40^\circ\right).\sin 10^\circ \sin(60^\circ - 10^\circ)\sin(60^\circ + 10^\circ)$$

$$= \sin 10^\circ \frac{1}{2}(\cos 20^\circ - \cos 60^\circ).\frac{1}{4}\sin 30^\circ$$

$$= \frac{1}{2}.\frac{1}{4}.\frac{1}{2}.\sin 10^\circ\left(\cos 20^\circ - \frac{1}{2}\right)$$

$$= \frac{1}{32}(\sin 30^\circ - \sin 10^\circ - \sin 10^\circ)$$

$$= \frac{1}{32}\left(\frac{1}{2} - 2\sin 10^\circ\right)$$

$$= \frac{1}{64}\left(1 - 4\sin 10^\circ\right)$$

$$= \frac{1}{64} - \frac{1}{16}\sin 10^\circ$$

Hence $\alpha = \frac{1}{64}$

$\Rightarrow 16 + \alpha^{-1} = 16 + 64 = 80$

Q.7 (2)

Let $S = 2\cos\frac{5\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{\pi}{11}$

Now, $\cos\frac{3\pi}{11} = -\cos\frac{8\pi}{11}$

And $\cos\frac{5\pi}{11} = -\cos\frac{16\pi}{11}$

$\therefore\ S = 2\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{8\pi}{11}\cdot\cos\frac{16\pi}{11}$

$$= \frac{2\sin\left(32\frac{\pi}{11}\right)}{2^5\cdot\sin\frac{\pi}{11}} = \frac{1}{16}$$

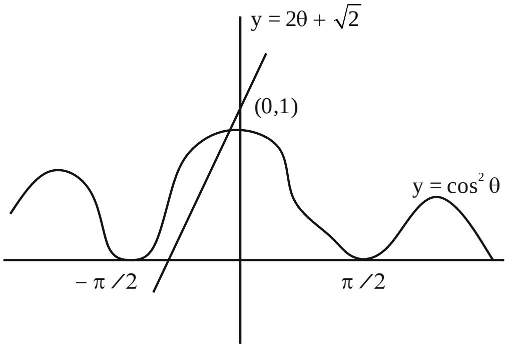
TRIGONOMETRIC EQUATION

Q.8 [1]

$2\theta - \cos^2\theta + \sqrt{2} = 0$

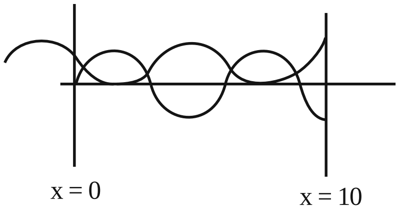
$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

$y = 2\theta + \sqrt{2}$



Both graphs intersect at one point.

Q.9 (4)
$\sin x = \cos^2 x$
Let $y = \sin x$ and $y = \cos^2 x$



Total number of solutions = 4

Q.10 **(2)**

$\sin\theta\tan\theta + \tan\theta = \sin 2\theta$, $\theta \in [-\pi, \pi] - \left\{\frac{\pi}{2}, \frac{-\pi}{2}\right\}$

$\tan\theta(\sin\theta + 1) - \sin 2\theta = 0$

$$\frac{\sin\theta}{\cos\theta}\left(\sin\theta + 1\right) - 2\sin\theta\cos\theta = 0$$

$$\sin\left[\frac{\sin\theta +}{\cos\theta} - 2\cos\theta\right] = 0$$

$\sin\theta = 0$ & $\sin\theta + 1 - 2\cos^2\theta = 0$, $\cos\theta \neq 0$
$\theta = -\pi, 0, \pi$

$\sin\theta + 1 - 2(1 - \sin^2\theta) = 0$
$\sin\theta + 1 - 2 + 2\sin^2\theta = 0$
$2\sin^2\theta + \sin\theta - 1 = 0$

$2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

$$2\sin\theta(\sin\theta+1)-(\sin\theta+1)=0$$

$$(2\sin\theta-1)(\sin\theta+1)=0$$

$$\boxed{\sin\theta=-1} \qquad \sin\theta=\frac{1}{2}$$

$$\uparrow \qquad\qquad \downarrow$$

No solution $\quad \theta=\frac{\pi}{6},\frac{5\pi}{6}$

$\therefore \quad n(S)=5$

$T=\Sigma_{\theta} =_{s}\cos 2\theta$

$= \cos2(-\pi)+\cos2(0)+\cos(2\pi)+\cos2(\pi/6)+\cos2(5\pi/6)$

$$=1+1+1+\frac{1}{2}+\frac{1}{2}$$

$T = 4$

$T + n(S) = 4 + 5 = 9$

Q.11 (d)

$$\text{Cos}^2\frac{\pi}{3}-\sin^2x=\frac{1}{4}\cos^2 2x$$

$$\Rightarrow \frac{1}{4}-\sin^2x=\frac{1}{4}[1-2\sin^2x]^2$$

$\Rightarrow 1-4[=1+4t^2-4t]$

$4t^2=0 \Rightarrow \sin^2x=0 \Rightarrow x=-3\pi,-2\pi,-\pi,0,\pi,2\pi,3\pi$

$=7$

Q.12 (4)

$14\text{cosec}^2x-2\sin^2x=21-4\cos^2x$

$14\text{cosec}^2x=2\sin^2x+21-4+4\sin^2x$

$$\frac{14}{\sin^2 x}=6\sin^2 x+17$$

Let $\sin^2x=t$

$14=6t^2+17t$

$6t^2+17t-14=0$

$6t^2+21t-4t-14=0$

$(3t-2)(2t+7)=0$

$$\sin^2 x=\frac{2}{3}$$

$$\sin x=\pm\sqrt{\frac{2}{3}}$$

$$\text{If } x\in\left(\frac{\pi}{4},\frac{7\pi}{4}\right)$$

$$\sin x=\sqrt{\frac{2}{3}}\rightarrow 2 \text{ values of x}$$

$$\sin x=-\sqrt{\frac{2}{3}}\rightarrow 2 \text{ values of x}$$

Ans.= 4 values of x

Q.13 (3)

$$8^{2\sin^2\theta}+\frac{8^2}{8^{2\sin^2\theta}}=16$$

$$t+\frac{6^4}{t}=16$$

$t=8$

$\Rightarrow 8^{2\sin^2\theta}=8$

$$\Rightarrow \theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$$

$n(5)=4$

$$\sum_{\theta\in S}\frac{2}{2\sin\left(\frac{\pi}{4}+2\theta\right)\cos\left(\frac{\pi}{4}+2\theta\right)}$$

$$\sum_{\theta\in S}\frac{2}{\sin\left(\frac{\pi}{2}+4\theta\right)}$$

$$\sum_{\theta\in S}2\sec 4\theta$$

$=2[\sec\pi+\sec3\pi+\sec5\pi+\sec7\pi]$

$=-8$

$$n(5)+\sum_{\theta\in S}\sec\left(\frac{\pi}{4}+2\theta\right)\text{cosec}\left(\frac{\pi}{4}+2\theta\right)=4-8=-4$$

Q.14 (3)

$$\sum_{m=1}^{9}\frac{1}{\cos(\theta+(m-1)\frac{\pi}{6})\cos(\theta+\frac{m\pi}{6})}=\frac{-8}{\sqrt{3}}$$

$$\Rightarrow 2\sum_{m=1}^{9}\frac{\sin\left((\theta+\frac{m\pi}{6})\right)-\left(\theta+(m-1)\frac{\pi}{6}\right)}{\cos(\theta+(m-1)\frac{\pi}{6})\cos(\theta+\frac{m\pi}{6})}=\frac{-8}{\sqrt{3}}$$

$$\Rightarrow\sum_{m=1}^{9}\frac{\sin\left(\theta+\frac{m\pi}{6}\right)\cos\left(\theta+(m-1)\frac{\pi}{6}\right)-\cos\left(\theta+\frac{m\pi}{6}\right)\sin\left(\theta+(m-1)\frac{\pi}{6}\right)}{\cos\left(\theta+(m-1)\frac{\pi}{6}\right)\cos\left(\theta+\frac{m\pi}{6}\right)}=\frac{-4}{\sqrt{3}}$$

$$\Rightarrow\sum_{m=1}^{9}\tan\left(\theta\frac{m\pi}{6}\right)-\tan\left(\theta+(m-1)\frac{\pi}{6}\right)=\frac{-4}{\sqrt{3}}$$

$$\Rightarrow\tan\left(\theta+\frac{\pi}{6}\right)-\tan\theta$$

$$\tan\left(\theta+\frac{2\pi}{6}\right)-\tan\left(\theta+\frac{\pi}{6}\right)$$

$\vdots$

$$\tan\left(\theta+\frac{9\pi}{6}\right)-\tan\left(\theta+\frac{8\pi}{6}\right)$$

$$\Rightarrow \tan\left(\frac{3\pi}{2}+\theta\right)-\tan\theta=\frac{-4}{\sqrt{3}}$$

$$-\cot\theta-\tan\theta=\frac{-4}{\sqrt{3}}$$

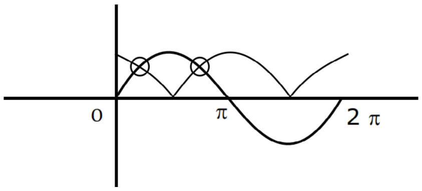
$$\theta=\frac{\pi}{3},\frac{\pi}{6}$$

$$\boxed{\sum_{\theta\in s}\theta=\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}}$$

Q.15 [5]

Q.16 (3)

|cos x| = sin x



Total solu. : 8

Q.17 (1)

$$2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}$$

L.H.S. ≤ 2. & R.H.S. ≥ 2

Hence, L.H.S. = 2 & R.H.S. = 2

$$2\cos\left(\frac{x^2+x}{6}\right)=2;\ 4^x+4^{-x}=2$$

Check x = 0

Possible hence only one solution

Q.18 [16]

$7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$4\cos^2\theta + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2(1 + \cos 2\theta) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2\cos^2 2\theta - 5\cos 2\theta = 0$

$\cos 2\theta(2\cos 2\theta - 5) = 0$

$\cos 2\theta = 0$

$$\theta=(2n+1)\frac{\pi}{4}$$

$$S=\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$$

For all four values of θ

$x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

Sum of roots of all four equations = 4 × 4 = 16

Q.19 [3]

$2\sin^2\theta - \cos 2\theta = 0$

$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$

$$\Rightarrow \sin^2\theta=\frac{1}{4}$$

$$\Rightarrow \sin^2\theta=\pm\frac{1}{2}$$

$$\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) + 3\sin\theta = 0$

$2\sin^2 2\theta - 3\sin\theta - 2 = 0$

$$\Rightarrow \sin^2\theta=\frac{-1}{2},2(\text{rejected})$$

$$\theta=\frac{7\pi}{6},\frac{11\pi}{6}$$

sum of solutions

$$\frac{7\pi}{6}+\frac{11\pi}{6}\Rightarrow 3\pi=k\pi\Rightarrow k=3$$

SOLUTION OF A TRIANGLE

Q.20 (1)

a+b = 7k

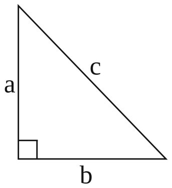
b+c = 8k

c+a = 9k

⇒a+b+c = 12k

∴ a = 4k, b = 3k, c = 5k

ΔABC will be Right angled triangle



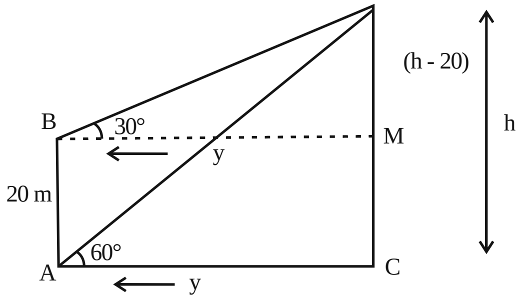
∴ 2R = c

$$R=\frac{5k}{2}$$

$$r=\frac{\Delta}{s}=\frac{\left(\frac{1}{2}\times4\times3\right)k^2}{6k}\quad r=k\qquad\Rightarrow\frac{R}{r}=\frac{5}{2}$$

HEIGHTS AND DISTANCES

Q.21 (4)



Let height of the tower DC = h

from ΔDCA $\frac{h}{y} = \tan 60° = \sqrt{3} \Rightarrow y = \frac{h}{\sqrt{3}}$ ... (1)

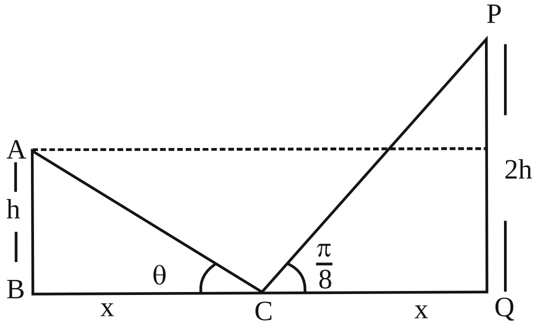
from ΔDMB

$$\frac{h-20}{y} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$h-20 = \frac{y}{\sqrt{3}} = \frac{h}{\sqrt{3} \cdot \sqrt{3}}$$

$3h - 60 = h$
$h = 30$ m

Q.22 (3)



Let BC = CQ = x & AB = h and PQ = 2h

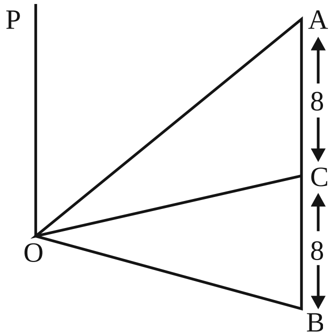
$$\tan\theta = \frac{h}{x},\ \tan\frac{\pi}{8} = \frac{2h}{x}$$

$$\frac{\tan\theta}{\tan\left(\frac{\pi}{8}\right)} = \frac{1}{2}$$

$$\tan\theta = \frac{1}{2}\tan\left(\frac{\pi}{8}\right) = \frac{1}{2}(\sqrt{2}-1)$$

$$\tan^2\theta = \frac{1}{4}(3-2\sqrt{2})$$

Q.23 (2)



$$\frac{OP}{OA} = \tan 15°$$

$\Rightarrow OA = OP\cot 15°$

$$\frac{OP}{OC} = \tan 45° \Rightarrow OP = OC$$

Now, $OP = \sqrt{OA^2 - 8^2}$

$\Rightarrow OP^2 = (OP)^2\cot^2 15° - 64$

$$\Rightarrow OP^2 = \frac{32}{\sqrt{3}}(2-\sqrt{3})$$

Q.24 (1)

In ΔPQA

$$\text{Tan } 45^o = \frac{PQ}{QA}$$

$QA = (h + 15) \cot 75^o$ ...(1)

Now In ΔARQ $\tan 60^o = \frac{RQ}{QA}$

$QA = 15. \cot 60^o$ ...(2)

From Eq. (1) & (2)

$(h + 15) \cot 75^o = 15 \cot 60^o$

$$h = \frac{15\left(\cot 60^o - \cot 75^o\right)}{\cot 75^o}$$

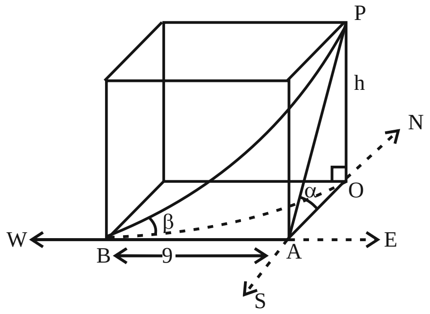
$$h = 15\ \frac{\left(\frac{1}{\sqrt{3}} - \left(2-\sqrt{3}\right)\right)}{\left(2-\sqrt{3}\right)}$$

$$h = \frac{15\left(1-2\sqrt{3}+3\right)}{\left(2-\sqrt{3}\right)\sqrt{3}}$$

$h = \frac{15}{\sqrt{3}} \times 2$ $PQ = h + 15$

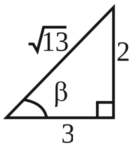
$h = 10\sqrt{3}$ $PQ = 5\left(2\sqrt{3}+3\right)$

Q.25 (1)

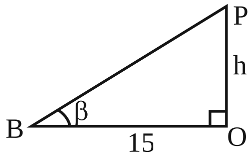


Given OB = 15

$$\cos\beta = \frac{3}{\sqrt{13}}$$



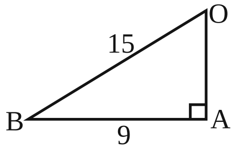
$$\tan\beta = \frac{2}{3}$$



$$\tan\beta = \frac{h}{15}$$

$$\frac{2}{3} = \frac{h}{15}$$

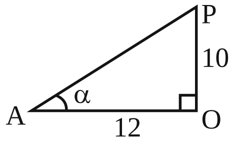
$$10 = h$$



$$OA^2 + AB^2 = 225$$

$$OA^2 + 81 = 225$$

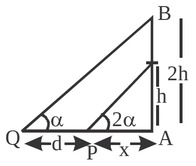
$$OA = 12$$



$$\tan\alpha = \frac{10}{12}$$

$$\cot\alpha = \frac{12}{10} = \frac{6}{5}$$

Q.26 (3)



$$d = \sqrt{7}h$$

$$\tan 2\alpha = \frac{h}{x}, \tan\alpha = \frac{2h}{d+x} = \frac{2h}{x+\sqrt{7}h}$$

$$\Rightarrow \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{\frac{4h}{x+\sqrt{7}h}}{1-\frac{4h^2}{\left(x+\sqrt{7}h\right)^2}}$$

$$= \frac{4h}{x+\sqrt{7}h} \times \frac{\left(x+\sqrt{7}h\right)^2}{x^2+7h^2+2\sqrt{7}xh-4h^2}$$

$$\frac{h}{x} = \frac{4h\left(x+\sqrt{7}h\right)}{x^2+3h^2+2\sqrt{7}xh}$$

$$x^2+3h^2+2\sqrt{7}xh = 4x^2+\sqrt[4]{7}xh$$

$$3x^2+2\sqrt{7}xh-3h^2 = 0$$

$$3\left(\frac{x}{h}\right)^2+2\sqrt{7}\left(\frac{x}{h}\right)-3 = 0$$

$$\frac{x}{h} = \frac{-2\sqrt{7}\pm\sqrt{28+36}}{6}$$

$$\frac{x}{h} = \frac{-2\sqrt{7}\pm 8}{6} = \frac{-\sqrt{7}\pm 4}{3}$$

$$\frac{x}{h} = \frac{4-\sqrt{7}}{3}$$

$$\because \tan\alpha = \frac{2}{\frac{x}{h}+\sqrt{7}}$$

$$= \frac{2}{\frac{4-\sqrt{7}}{3}+\sqrt{7}} = \frac{2\times 3}{4+2\sqrt{7}} = \frac{3}{2+\sqrt{7}} \times \frac{2-\sqrt{7}}{2-\sqrt{7}}$$

$$= \frac{3\left(2-\sqrt{7}\right)}{4-7} \qquad \Rightarrow \tan\alpha = \sqrt{7}-2$$

Q.27 [10620]

$$–800 \le f(n) \le 800$$

$$– 800 \le 2n^2 – n–1 \le 800$$

$$2n^2 – n + 799 \ge 0$$

$$a > 0$$

$$D = 1 – 4(2)(799) < 0$$

Always true

$n \in R$

$$2n^2 – n – 801 \le 0$$

$$n = \frac{1\pm\sqrt{1+4(2)(801)}}{4}$$

$$n = \frac{1\pm\sqrt{6408}}{4}$$

$$n = \frac{1\pm 80}{4}$$

$$n = \frac{-79}{4}, \frac{81}{4}$$

$$n \in [–19.75, 20.25]$$

$$n \in \{–19,–18,–17 .... 1,0,1,........20\}$$

$f(n) = 2n^2 - n - 1$

$f(-19) = 2(-19)^2 - (-19) - 1$

. . . .

. . . .

. . . .

. . . .

$f(19) = 2(19)^2$ - 19 - 1

$f(20) =$ $2(20)^2$ - (20) -

1

$2[(-19)^2 + (-18)^2 + \ldots \ldots + (20)^2]$

$- [(-19) + (-18) + \ldots \ldots + (-1) +$

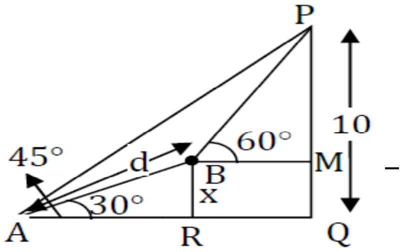
$(1) \ldots \ldots . +$

$(19) + 20] - 40$

$$= 2\left[400 + 2\left(\frac{19x20x39}{6}\right)\right] - 20 - 40$$

$= 10620$

Q.28 **(1)**



Let BR = X

In Δ ARB

$\sin 30° = \frac{x}{d} = \frac{1}{2}$ $\quad x = \frac{d}{2}$

PM = 10 - X Δ ARB $\tan 30° = \frac{x}{AR} \Rightarrow AR = \sqrt{3}x$

$\therefore$ BM = RQ = AQ – AR = 10 – $x\sqrt{3}$

In ,ΔBMP, $\tan 60° = \frac{10 - x}{10 - x\sqrt{3}} \Rightarrow x = 5\left(\sqrt{3} - 1\right)$

$\therefore d = 2x = 10\left(\sqrt{3} - 1\right)$

Now let area of trapezium PQRB = $\frac{1}{2}$ (x + 10)

$(10 - x\sqrt{3}\ )$

$= \frac{1}{2}(5\sqrt{3} - 5 + 10)\ (10 - 5\left(\sqrt{3}(\sqrt{3 - 1})\right)\ )$

$= \frac{1}{2}(5\sqrt{3} + 5)\ (10 - 15 + 5\sqrt{3}\ )$

$= \frac{1}{2}\ (75 - 25) = 0$

MATHEMATICAL INDUCTION

EXERCISE-I (MHT CET LEVEL)

Q.1 (1)

$P(n) :\ a^{2n-1} + b^{2n-1}$

$P(1) : a^1 + b^1 = a + b$, which is divisible by itself, i.e. by $(a + b)$.

$\therefore$ $P(n) : a^{2n-1} + b^{2n-1}$ is divisible by $(a + b)$, and is true for $n = 1$

Let P(k) be true, i.e. $P(k) : a^{2k-1} + b^{2k-1}$ is divisible by $(a + b)$

i.e. $a^{2k-1} + b^{2k-1} = m(a + b)$

Now,

$$P(k + 1) = a^{2k+1} + b^{2k+1} = a^{2k-1} \cdot a^2 + b^{2k+1}$$
$$= a^2 [m (a + b) - b^{2k-1}] + b^{2k+1}$$
$$= m (a + b)a^2 - a^2b^{2k-1} + b^{2k+1}$$
$$= m(a + b)a^2 - b^{2k-1} (a^2 - b^2)$$
$$= m(a + b)a^2 - (a + b) (a - b) b^{2k-1}$$
$$= (a + b) [ma^2 - (a - b) b^{2k-1}]$$

$\therefore$ P(k + 1) is divisible by (a + b) whenever P(k) is divisible by (a + b).

Hence P(n) is divisible by (a + b) for all $n \in N$. **Ans.**

Q.2 (2)

$P(n) : (n + 1) (n + 2) .... (n + r)$

$P(1) : (2) (3) .... (r + 1) = r ! (r + 1)$, which is divisible by r !

Let $P(k) : (k + 1) (k + 2) .... (k + r) = r! (m)$

$\therefore$ $P(k + 1) : (k + 2) (k + 3) .... (k + 1 + r) = r!(\lambda)$

L.H.S. of P(k + 1)

$$= (k + 2) (k + 3) ... (k + r + 1)$$

$$= \frac{(k+1)\ (k+2)(k+3)....(k+r+1)}{k+1}$$

$$= \frac{r!(m)\ (k+r+1)}{k+1} = r\,!\ (\lambda).$$

Thus, P(k + 1) is divisible by r! whenever P(k) is divisible by r!

Hence P(n) is divisible by r! for all $n \in N$. **Ans.**

Q.3 (2)

$P(n) : 49^n + 16n - 1$

$P(1) : 49 + 16 - 1 = 64$, which is divsible by 64

Let $P(k) : 49^k + 16k - 1 = 64\,m$

$\therefore$ $P(k + 1) : 49^{k+1} + 16(k + 1) - 1 = 64\lambda$

L.H.S. of $P(k + 1) = 49^{k+1} + 16(k + 1) - 1$

$= 49 (64m - 16k + 1) + 16k + 16 - 1$

[Assuming P(k) to be true]

$= 64(49m) - 48(16k) + 64$

$= 64(49\,m - 12k + 1) = 64\lambda$

Thus, P(k + 1) is divisible by 64 whenever P(k) is divisible by 64.

Hence, P(n) is divisible by 64. **Ans.**

Q.4 (3)

By Induction, P(n) is true for all $n \in N$.

Q.5 (b)

When k = 1, LHS = 1 but RHS = 1 + 10 = 11

$\therefore$ T(1) is not true

Let T(k) is true. That is

$1 + 3 + 5 + ...+ (2k - 1) = k^2 + 10$

Now, $1 + 3 + 5 + ...+(2k - 1) + (2k + 1)$

$= k^2 + 10 + 2k + 1 = (k + 1)^2 + 10$

$\therefore$ T(k + 1) is true.

That is T(k) is true. $\Rightarrow$ T(k + 1) is true.

But T(n) is not true for all $n\ \delta \in N$, as T(1) is not true.

Q.6 (2)

$P(n) : \cos\alpha \cos 2\alpha \cos 4\alpha ..... \cos 2^{n-1}\alpha$

$$P(1) : \cos\alpha = \frac{\sin 2\alpha}{2\sin\alpha}$$

$$P(2) : \cos\alpha \cos 2\alpha = \frac{\sin 4\alpha}{4\sin\alpha}$$

$$\text{Let } P(k) : \cos\alpha \cos 2\alpha \cos 4\alpha ..... \cos 2^{k-1}\alpha = \frac{\sin 2^k\alpha}{2^k \sin\alpha}$$

$\therefore$ $P(k + 1) : \cos\alpha \cos 2\alpha \cos 4\alpha .... \cos 2^k\alpha = \dfrac{\sin 2^{k+1}\alpha}{2^{k+1}\sin\alpha}$

L.H.S. of P(k + 1)

$= \cos\alpha \cos 2\alpha \cos 4\alpha ... \cos 2^k \alpha$

$$= \frac{\sin 2^k\alpha}{2^k \sin\alpha} \times \cos 2^k \alpha$$

[Assuming P(k) to be true]

$$= \frac{2\sin 2^k\alpha \cos 2^k\alpha}{2^{k+1}\sin\alpha} = \frac{\sin 2^{k+1}\alpha}{2^{k+1}\sin\alpha}$$

= R.H.S of P(k + 1)

Hence P(n) holds true for all $n \in N$,. That is,

$$\cos\alpha \cos 2\alpha \cos 4\alpha .... \cos 2^{n-1}\alpha = \frac{\sin 2^n\alpha}{2^n \sin\alpha}.$$ **Ans.**

Q.7 (3)

For n = 1, $2^{3n} - 7n - 1 = 2^3 - 7 - 1 = 0$

For n = 2, $2^{3n} - 7n - 1 = 2^6 - 14 - 1 = 64 - 15 = 49$

which is divisible by 49. **Ans.**

Q.8 (1)

$f(n) = 10^n + 3 \cdot 4^{n+2} + k$

$f(1) = 10 + 3 \cdot 4^2 + k = 10 + 48 + k = 58 + k$

$= 9 \times 7 - 5 + k$

If f(1) is to be divisible by 9, then the least positive integral value of k has to be 5. **Ans.**

Q.9 (2)

$f(n) = 10^n + 3 \cdot 4^{n+2} + 5$

$f(1) = 10 + 48 + 5 = 63$, which is divisible by 7 and 3

$f(2) = 100 + 3(256) + 5 = 105 + 768 = 873$, which is divisible by 3.

So, $f(n) = 10^n + 3 \cdot 4^{n+2} + 5$ is divisible by 3. **Ans.**

Q.10 (1)

Let $P(n) : x^n - 1 = \lambda(x - k)$

Now $P(1) : x - 1 = \lambda_1 (x - k)$

Also, $P(2) : x^2 - 1 = \lambda_2(x - k)$

$\Rightarrow P(2) : (x - 1)(x + 1) = \lambda_2(x - k)$

$\therefore$ The least value of k for which the proposition P(n) is true is k = 1. **Ans.**

Q.11 (2)

Let $P(n) : \frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots.$ (n terms)

$$\Rightarrow P(n) : \sum \frac{1^3+2^3+\ldots\ldots+n^3}{1+3+5+\ldots\ldots+(2n-1)} \quad \Rightarrow$$

$$P(n) : \sum \left(\frac{\sum n^3}{n^2} \right)$$

$$\Rightarrow P(n) : \sum \left[\frac{1}{4} \frac{n^2(n+1)^2}{n^2} \right]$$

$$\Rightarrow P(n) : \frac{1}{4} \sum (n^2 + 2n + 1)$$

$$\Rightarrow P(n) : \frac{1}{4} \left[\sum n^2 + 2\sum n + \sum (1) \right]$$

$$\Rightarrow P(n) : \frac{1}{4} \left[\frac{n(n+1)}{2} + \frac{1}{3} n(n+1)(2n+1) + n \right]$$

$$\Rightarrow P(n) : \frac{1}{24} n \left[3(n+1) + 2(n+1)(2n+1) + 6 \right]$$

$\therefore P(n) : \frac{1}{24} n \left(2n^2 + 9n + 13\right).$ **Ans.**

Q.12 (2)

$$\text{Let } P(n) = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$$

$$P(1) = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x} dx$$

$$= \int_0^{\pi/2} \sin x \, dx = \left[-\cos x \right]_0^{\pi/2} = 1$$

$$P(2) = \int_0^{\pi/2} \frac{\sin^2 2x}{\sin x} dx = \int_0^{\pi/2} \frac{(2 \sin x \cos x)^2}{\sin x} dx$$

$$\Rightarrow P(2) = \int_0^{\pi/2} 4 \sin x \cos^2 x \, dx$$

$$\Rightarrow P(2) = 4 \left[\frac{-\cos^2 x}{3} \right]_0^{\pi/2} = \frac{4}{3} = 1 + \frac{1}{3}$$

$\therefore$ For any $n \in N$,

$$P(n) = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$$

$= 1 + \frac{1}{3} + \frac{1}{5} + \ldots.. + \frac{1}{2n-1}$. **Ans.**

Q.13 (d)

$$\text{Let } P(n) : \frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$$

For n = 2,

$$P(2n) : \frac{4^2}{2+1} < \frac{4!}{(2)^2} \Rightarrow \frac{16}{3} < \frac{24}{4}$$

which is true.

Let for $n = m \le 2$, P(m) is true.

$$\text{i. e. } \frac{4^m}{m+1} < \frac{(2m)!}{(m!)^2}$$

$$\text{Now, } \frac{4^{m+1}}{m+1} = \frac{4^m}{m+1} \cdot \frac{4(m+1)}{m+2}$$

$$< \frac{(2m)!}{(m!)^2} \cdot \frac{4(m+1)}{(m+2)}$$

$$= \frac{(2m)!(2m+1)(2m+2)4(m+1)(m+1)^2}{(2m+1)(2m+2)(m!)^2(m+1)^2(m+2)}$$

$$= \frac{[2(m+1)]!}{[(m+1)!]^2}\cdot\frac{2(m+1)^2}{(2m+1)(m+2)}$$

$$< \frac{[2(m+1)!]}{[(m+1)!]^2}$$

Hence, for $n \geq 2$, P(n) us true.

Q.14 **(a)**

Q.15 (1)

Let $P(n) = 3^{2n} = 9^n$

$\therefore$ $9^n = (1 + 8)^n = 1 + 8\lambda$

$\Rightarrow$ Choice (1) is the correct answer. **Ans.**

Q.16 (1)

$P(n) : n^2 + n + 1$

$P(1) : 1 + 1 + 1 = 3$, which is an odd number = 2 + 1

Let $P(k) : k^2 + k + 1 = 2m + 1$(an odd number)

$\therefore$ $P(k + 1) : (k + 1)^2 + (k + 1) + 1 = 2\lambda + 1$

L.H.S. of $P(k + 1) = (k + 1)^2 + (k + 1) + 1$

$= k^2 + 1 + 2k + k + 1 + 1$

$= (k^2 + k + 1) + 2k + 2$

$= (2m + 1) + 2k + 2$

[Assuming P(k) to be true]

$= 2 (m + k + 1) + 1$

$= 2\lambda + 1$(an odd number)

Hence $P(n) = n^2 + n + 1$ is an odd natural number for $n \in N$. **Ans.**

Q.17 (1)

Since the reason II is obvious, so the greatest positive integer which divides the product

$(n + 11) (n + 12) (n + 13) (n + 14)$ is $4! = 24$. **Ans.**

Q.18 (1)

For every n!, $\forall\ n \geq 5$, ends with 0 and 3^{4n} ends with 1,

$\therefore$ $3^{4n + 3} = 3^{4n}\ 3^3$ whose unit place is 7. **Ans.**

EXERCISE-II

NUMERICAL BASED QUESTIONS

Q.1 120

Given, $a_n = na_{n-1}$

$a_2 = 2a_1 = 2$ $(\because\ a_1 = 1\text{given})$

$a_3 = 3a_2 = 3(2) = 6$

$a_4 = 4(a_3) = 4(6) = 24$

$a_5 = 5(a_4) = 5(24) = 120$

Q.2 7

$2^{3n} - 1 = (2^3)^n - 1$

$= 8^n - 1 = (1 + 7)^n - 1$ $= 1 + {}^nC_1 7 + {}^nC_2 7^2 + ... + {}^nC_n 7^n - 1$ $= 7[{}^nC_1 + {}^nC_2 7 + ... + {}^nC_n 7^{n-1}]$

$\therefore$ $2^{3n} - 1$ is divisible by 7

Q.3 6

Let $P(n) = n(n + 1) (n + 2)$

$P(1) = 1 \cdot 2 \cdot 3 = 6$

$P(2) = 2 \cdot 3 \cdot 4 = 24$

Hence, it is divisible by 6.

Q.4 133

On putting n = 1 in $11^{n+2} + 12^{2n+1}$, we get

$11^{1+2} + 12^{2\times1+1} = 11^3 + 12^3 = 3059$

Which is divisible by 133

Q.5 120

Q.6 2

Given, $n! < \left(\frac{n+1}{2}\right)^n$

At n = 1, $1! \not< 1$

At n – 2, $2! < \left(\frac{3}{2}\right)^2$

$\Rightarrow$ $2 < 2.25$ which is true.

Q.7 2

Let $P(n) \equiv n! > 2^{n-1}$

$P(3) \equiv 6 > 4$

Let $P(k) \equiv k! > 2^{k-1}$ is true.

$\therefore$ $P(k + 1) = (k + 1)! = (k' + 1)k!$

$> (k + 1)2^{k-1}$

$> 2^k$ (as $k + 1 > 2$)

PREVIOUS YEAR'S

MHT CET

Q.1 (3)

COMPLEX NUMBERS AND QUADRATIC EQUATIONS & INEQUALITIES

EXERCISE-I (MHT CET LEVEL)

Q.1 (3)

Since $\frac{1+i}{1-i}=\frac{(1+i)(1+i)}{(1-i)(1+i)}=i$

Therefore $\left(\frac{1+i}{1-i}\right)^{4n+1}=i^{4n+1}=i^{4n}.i^{1}=i \quad (\because i^{4n}=1)$

Q.2 (2)

$$\frac{1+i}{1-i}=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{(1+i)^2}{2}=\frac{2i}{2}=i$$

$\therefore \left(\frac{1+i}{1-i}\right)^{m}=i^{m}=1$ (as given)

So the least value of m = 4 $\{\because i^4=1\}$

Q.3 (2)

$$\frac{i^{584}(i^8+i^6+i^4+i^2+1)}{i^{574}(i^8+i^6+i^4+i^2+1)}-1=\frac{i^{584}}{i^{574}}-1$$

$$=i^{10}-1=-1-1=-2$$

Q.4 **(1)**

Q.5 (1)

Q.6 (4)

Q.7 (1)

Q.8 (2)

$\sqrt{-7-24i}=x-iy$

Squaring both sides, $-7-24i=x^2-y^2-i(2xy)$

Equating real and imaginary parts, we get

$x^2-y^2=-7$ and $2xy=24$

$\therefore\ x^2+y^2=\sqrt{49+576}=\sqrt{625}=25$

Q.9 (1)

$$A+iB=\frac{1-i\alpha}{1+i\alpha}\Rightarrow A-iB=\frac{1+i\alpha}{1-i\alpha}$$

$$\Rightarrow(A+iB)(A-iB)$$

$$=\frac{(1-i\alpha)(1+i\alpha)}{(1+i\alpha)(1-i\alpha)}=1$$

$$\Rightarrow A^2+B^2=1$$

Q.10 (2)

Given that $\bar{z}=\frac{1}{z}\Rightarrow z\bar{z}=1 \Rightarrow |z|^2=1\Rightarrow|z|=1$

Q.11 (3)

Here $z+\bar{z}=(x+iy)+(x-iy)=2x$ (Real)

and $z\bar{z}=(x+iy)(x-iy)=x^2+y^2$ (Real).

Q.12 (D)

Given $z_1=1+2i$, $z_2=3+5i$ and $\bar{z}_2=3-5i$

$$\frac{\bar{z}_2z_1}{z_2}=\frac{(3-5i)(1+2i)}{(3+5i)}=\frac{13+i}{3+5i}$$

$$=\frac{13+i}{3+5i}\times\frac{3-5i}{3-5i}=\frac{44-62i}{34}$$

Then $\mathrm{Re}\left(\frac{\bar{z}_2z_1}{z_2}\right)=\frac{44}{34}=\frac{22}{17}$

Q.13 (2)

Q.14 (3)

$$z=\frac{(2+i)^2}{3+i}=\frac{3+4i}{3+i}\times\frac{3-i}{3-i}=\frac{13}{10}+i\frac{9}{10}$$

Hence conjugate of $\frac{13}{10}+i\frac{9}{10}=\left(\frac{13}{10}-i\frac{9}{10}\right)$

Q.15 (1)

$$\left(\frac{1-i}{1+i}\right)=\frac{1-i}{1+i}\times\frac{1-i}{1-i}=\frac{(1-i)^2}{2}=\frac{-2i}{2}=-i$$

$\mathrm{Im}(z)<0$, Hence amplitude $=-\pi/2$ [$\because \tan\theta=b/a$]

Q.16 (1)

$$\mathrm{amp}\left(\frac{1+\sqrt{3}i}{\sqrt{3}+i}\right)=\mathrm{amp}(1+\sqrt{3}i)-\mathrm{amp}(\sqrt{3}+i)$$

$$=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$

Q.17 (4)

$$z=\frac{1+\sqrt{3}i}{\sqrt{3}-i}=\frac{1+\sqrt{3}i}{\sqrt{3}-i}\times\frac{\sqrt{3}+i}{\sqrt{3}+i}$$

$$=\frac{\sqrt{3}+i+3i-\sqrt{3}}{3+1}=\frac{4i}{4}=i$$

amp $(z)=\pi/2$ $[\because\tan\theta=b/a]$

Q.18 (2)

$$\arg\left(\frac{13-5i}{4-9i}\right)=\arg(13-5i)-\arg(4-9i)$$

$$=-\tan^{-1}\left(\frac{5}{13}\right)+\tan^{-1}\frac{9}{4}=\frac{\pi}{4}$$

Q.19 (2)

Q.20 (4)

$$|z||\omega|=1 \qquad \text{......(i)}$$

and $\arg\left(\frac{z}{\omega}\right)=\frac{\pi}{2}\Rightarrow\frac{z}{\omega}=i\ \left|\frac{z}{\omega}\right|=1$ .....(ii)

From equation (i) and (ii)

$|z|=|\omega|=1$ and $\frac{z}{\omega}+\frac{\bar{z}}{\bar{\omega}}=0;\ \ z\bar{\omega}+\bar{z}\omega=0$

$$\bar{z}\omega=-z\bar{\omega}=\frac{-z}{\omega}\bar{\omega}\omega\ ;\ \bar{z}\omega=-i|\omega|^2=-i.\cdot$$

Q.21 (A)

$$\left(\frac{3+2i}{3-2i}\right)=\left(\frac{3+2i}{3-2i}\right)\left(\frac{3+2i}{3+2i}\right)$$

$$=\frac{9-4+12i}{13}=\frac{5}{13}+i\left(\frac{12}{13}\right)$$

$$\text{Modulus}=\sqrt{\left(\frac{5}{13}\right)^2+\left(\frac{12}{13}\right)^2}=1\,.$$

Q.22 **(3)**

Q.23 (2)

$$\frac{1+2i}{1-(1-i)^2}=\frac{1+2i}{1-(1-1-2i)}=\frac{1+2i}{1+2i}=1+0i$$

Modulus =1

Amplitude θ= $\tan^{-1}\frac{0}{1}=0$

Q.24 (1)

Using De Moivre's theorem

$(\cos\theta+i\sin\theta)^n=(\cos n\theta+i\sin n\theta)$ and putting n = 0 , 1, 2, then we get required roots.

Q.25 (2)

Let $\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}=z$ and $\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}=\frac{1}{z}$

Therefore, $\left(\frac{1-z}{1-\frac{1}{z}}\right)^{10}=\left\{\frac{-(z-1)z}{(z-1)}\right\}^{10}=(-z)^{10}$

$$=z^{10}=\left(\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}\right)^{10}=\cos\pi-i\sin\pi=-1\cdot$$

Q.26 (2)

$$iz^4=-1$$

$$z^4=\frac{-1}{i}\Rightarrow z^4=i\Rightarrow z=(i)^{1/4}$$

$$z=(0+i)^{1/4}$$

$$z=\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)^{1/4}$$

$z=\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}$ (using De Moivre's theorem)

Q.27 (2)

Since imaginary cube root of unity are square of each other.

Q.28 (1)

It is obvious because the cube roots of unity are

$$1,\frac{-1+i\sqrt{3}}{2},\frac{-1-i\sqrt{3}}{2}$$

Q.29 (1)

$$(1+\omega)^3-(1+\omega^2)^3=(-\omega^2)^3-(-\omega)^3$$

$$=-\omega^6+\omega^3=-\omega^3\omega^3+\omega^3=-1+1=0$$

Q.30 (2)

$$(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$$

$$=(-2\omega)^5+(-2\omega^2)^5=-32\omega^3\omega^2-32(\omega^3)^3\omega$$

$$=-32(\omega^2+\omega)=-32(-1)=32$$

Q.31 (3)

$$(1-\omega+\omega^2)(1-\omega^2+\omega)^6=(-2\omega)(-2\omega^2)^6=-128\omega.$$

Q.32 (1)

Q.33 (1)

Q.34 **(4)**

Q.35 (3)

Since the coordinates in complex plane are (2, 3) and $(-1,-1)$ Hence the required distance is 5.

Trick : We know that the distance between z_1 and z_2 is $|z_1-z_2|$ therefore, the required length $|2+3i+1+i|=5$.

Q.36 (d)

$$z-2-3i=x+iy-2-3i=(x-2)+i(y-3)$$

$$\tan^{-1}\left(\frac{y-3}{x-2}\right)=\frac{\pi}{4}\Rightarrow\frac{y-3}{x-2}=\tan\frac{\pi}{4}=1$$

$\Rightarrow x-y+1=0$

Q.37 (a)

z$_1$, z$_2$, 0 are vertice of an equ9ilateral triangle,

so we have $z_1^2+z_2^2=z_1z_2\Rightarrow z_1^2+z_2^2-z_1z_2=0$

Q.38 (c)

Let $z_1=1+i, z_2=-2+3i$ and $z_3=0+\frac{5}{3}i$ Then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\begin{vmatrix} 1 & 1 & 1 \\ -2 & 3 & 1 \\ 0 & \frac{5}{3} & 1 \end{vmatrix}$$

$$=1\left(3-\frac{5}{3}\right)+1(2)+1\left(\frac{-10}{3}\right)-\frac{4}{3}+2-\frac{10}{3}-\frac{4+6-10}{3}=0$$

Q.39 (3)

Given $|8+z|+|z-8|=16$.

Locus is a straight line

Q.40 (2)

Let z_1, z_2, z_3 be three complex numbers in A.P.

Then $2z_2=z_1+z_3$.

Thus the complex number z_2 is the mid-point of the line joining the points z_1 and z_3 So the three points z_1, z_2 and are in a straight line.

Q.41 (3)

$|z-1|=|z+i| \Rightarrow |x-1+iy|^2=|x+i(y+1)|^2$

$\Rightarrow (x-1)^2+y^2=x^2+(y+1)^2$

$\Rightarrow x+y=0$ *i.e.*, a straight line through the origin.

Q.42 (3)

$|z-3i|=2$, let $z=x+iy \Rightarrow |x+i(y-3)|=2$

Squaring both sides, we get $[x^2+(y-3)^2]=4$

$\Rightarrow x^2+y^2-6y+5=0$

Q.43 (3)

Here $|z-zi|=1\Rightarrow|x+iy-i(x+iy)|=1$

$\Rightarrow |(x+y)+i(y-x)|=1\Rightarrow$

$\sqrt{(x+y)^2+(y-x)^2}=1$

$\Rightarrow 2(x^2+y^2)=1$. Hence *z* lies on a circle.

Q.44 (1)

$$\left|\frac{z-3i}{z+3i}\right|=1\Rightarrow|z-3i|=|z+3i|$$

[if $|z-z_1|=|z+z_2|$, then it is a perpen dicular bisector of z_1 and z_2] Hence, perpendicular bi sector of (0,3) and (0,-3) is X -axis.

Q.45 (1)

$|z-2+i|=|z-3-i|$

$\Rightarrow |(x-2)+i(y+1)|=|(x-3)+i(y-1)|$

$\Rightarrow\sqrt{(x-2)^2+(y+1)^2}=\sqrt{(x-3)^2+(y-1)^2}=2$

$\Rightarrow x^2+4-4x+y^2+1+2y=x^2+9-6x+y^2+1-2y$

$\Rightarrow 2x+4y-5=0$

Q.46 (1)

Q.47 (4)

(4) $x+\frac{1}{x}=2\Rightarrow x+\frac{1}{x}-2=0$ $(\because x\neq 0)$

$\Rightarrow x^2-2x+1=0\Rightarrow(x-1)^2=0\Rightarrow x=1,1$.

Q.48 (2)

Let $x=2+\cfrac{1}{2+\cfrac{1}{2+.....\infty}}\Rightarrow x=2+\frac{1}{x}$

(on simplification)

$\Rightarrow x=1\pm\sqrt{2}$

But the value of the given expression cannot be negative or less than 2, therefore $1+\sqrt{2}$ is required answer.

Q.49 (1)

Equation $a(x^2+1)-(a^2+1)x=0$

$\Rightarrow ax^2-(a^2+1)x+a=0$

$\Rightarrow (ax-1)(x-a)=0\Rightarrow x=a,\frac{1}{a}$.

Q.50 (3)

$x^{2/3}+x^{1/3}-2=0$

$\Rightarrow (x^{1/3})^2+1(x^{1/3})-2=0$

Let $a=x^{1/3}$, then $a^2+a-2=0\Rightarrow a=1,-2$

Hence $x=1,-8$ (by $a=x^{1/3}$).

Q.51 (1)

(1) $x^{\log_x(1-x)^2}=9$

$\Rightarrow \log_x(9)=\log_x(1-x)^2 (\because a^x=N\Rightarrow\log_a N=x)$

$\Rightarrow 9 = (1-x)^2 \;\; \Rightarrow 1 + x^2 - 2x - 9 = 0$

$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x+2)(x-4) = 0$

$\Rightarrow x = -2, 4$.

Q.52 (3)

Put $x = 4$ in $x^2 + px + 12 = 0$, we get

$p = -7$

Now second equation $x^2 + px + q = 0$ have equal roots. Therefore $p^2 = 4q \Rightarrow q = \frac{49}{4}$

Q.53 (4)

Given equation

$2(a^2 + b^2)x^2 + 2(a+b)x + 1 = 0$

Let $A = 2(a^2 + b^2), B = 2(a+b)$ and $C = 1$

$B^2 - 4AC = 4(a^2 + b^2 + 2ab) - 4\,.2(a^2 + b^2)1$

$\Rightarrow B^2 - 4AC = -4(a-b)^2 < 0$

Thus given equation has imaginary roots.

Q.54 (4)

Equation $2x^2 - kx + x + 8 = 0$ has equal and real roots, then $D = b^2 - 4ac = 0\cdot$

$\Rightarrow (1-k)^2 - 4.2.8 = 0 \Rightarrow k^2 + 1 - 2k - 64 = 0$

$\Rightarrow k^2 - 2k - 63 = 0 \Rightarrow k = 9, -7$.

Q.55 (1)

Here $(b + c - 2a) + (c + a - 2b) + (a + b - 2c) = 0$

Therefore the roots are rational.

Q.56 (3)

The quadratic is $(k+11)x^2 - (k+3)x + 1 = 0$

Accordingly, $(k+3)^2 - 4(k+11)(1) = 0 \Rightarrow k = -7, 5$.

Q.57 (3)

From options put $k = 3 \Rightarrow x^2 + 8x + 7 = 0$

$\Rightarrow (x+1)(x+7) = 0 \Rightarrow x = -1, -7$

means for $k = 3$ roots are negative.

Q.58 (1)

Given equation $(1+2k)x^2 + (1-2k)x + (1-2k) = 0$

If equation is a perfect square then root are equal

i.e., $(1-2k)^2 - 4(1+2k)(1-2k) = 0$

i.e., $k = \frac{1}{2}, \frac{-3}{10}$. Hence total number of values $= 2$.

Q.59 (3)

Given equation $x^2 - a(x+1) - b = 0$

$\Rightarrow x^2 - ax - a - b = 0 \Rightarrow \alpha + \beta = a, \alpha\beta = -(a+b)$

Now $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= -(a+b) + a + 1 = 1 - b$

Q.60 **(2)**

Q.61 **(3)**

Q.62 (4)

$\alpha + \beta = \frac{2(m^2+1)}{2} = m^2 + 1$ .....(i)

and $\alpha\beta = \frac{m^4 + m^2 + 1}{2}$ .....(ii)

Therefore $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (m^2+1)^2 - 2\frac{(m^4 + m^2 + 1)}{2}$

$= m^4 + 2m^2 + 1 - m^4 - m^2 - 1 = m^2$

Q.63 (2)

α, β be the roots of $x^2 - 2x + 3 = 0$, then $\alpha + \beta = 2$ and $\alpha\beta = 3$. Now required equation whose roots are

$\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ is $x^2 - \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)x + \frac{1}{\alpha^2\beta^2} = 0$

$\Rightarrow x^2 - \left(-\frac{2}{9}\right)x + \frac{1}{9} = 0 \Rightarrow 9x^2 + 2x + 1 = 0\cdot$

Q.64 (4)

Under condition, $-\frac{2}{\lambda} = 3 \Rightarrow \lambda = -\frac{2}{3}$

Q.65 (2)

Let first root $= \alpha$ and second root $= \frac{1}{\alpha}$

Then $\alpha . \frac{1}{\alpha} = \frac{k}{5} \Rightarrow k = 5$.

Q.66 (2)

Let roots are α and $\frac{1}{\alpha}$,than

$\alpha . \frac{1}{\alpha} = \frac{k+2}{2k+1} \Rightarrow 1 = \frac{k+2}{2k+1} \Rightarrow k = 1$

Q.67 (3)

According to condition

$\frac{2m-1}{m} = -1 \Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}$

Q.68 (1)

Given equation $4x^2 + 3x + 7 = 0$, therefore

$\alpha + \beta = -\frac{3}{4}$ and $\alpha\beta = \frac{7}{4}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-3/4}{7/4} = \frac{-3}{4} \times \frac{4}{7} = -\frac{3}{7}$.

Q.69 (2)

Given equation can be written as

$(6k+2)x^2 + rx + 3k - 1 = 0$ .....(i)

and $2(6k+2)x^2 + px + 2(3k-1) = 0$ .....(ii)

Condition for common roots is

$$\frac{12k+4}{6k+2} = \frac{p}{r} = \frac{6k-2}{3k-1} = 2 \text{ or } 2r - p = 0$$

Q.70 (3)

Let roots of $x^2 - cx + d = 0$ be α, β then roots of $x^2 - ax + b = 0$ be α, α

$\therefore \alpha + \beta = c, \alpha\beta = d, \alpha + \alpha = a, \alpha^2 = b$

Hence $2(b+d) = 2(\alpha^2 + \alpha\beta) = 2\alpha(\alpha + \beta) = ac$

Q.71 (1)

Let α is the common root,

so $\alpha^2 + p\alpha + q = 0$ .....(i)

and $\alpha^2 + q\alpha + p = 0$ ....(ii)

from (i) – (ii),

$\Rightarrow (p-q)\alpha + (q-p) = 0 \Rightarrow \alpha = 1$

Put the value of α in (i), $p + q + 1 = 0$.

Q.72 (4)

$x^2 - 4x < 12$

$\Rightarrow x^2 - 4x - 12 < 0 \Rightarrow x^2 - 6x + 2x - 12 < 0$

$\Rightarrow (x-6)(x+2) < 0 \Rightarrow -2 < x < 6$.

Q.73 (1)

According to given condition,

$4a^2 - 4(10 - 3a) < 0 \Rightarrow a^2 + 3a - 10 < 0$

$\Rightarrow (a+5)(a-2) < 0 \Rightarrow -5 < a < 2$.

Q.74 (2)

Given, $x + 2 > \sqrt{x+4} \Rightarrow (x+2)^2 > (x+4)$

$\Rightarrow x^2 + 4x + 4 > x + 4 \Rightarrow x^2 + 3x > 0$

$\Rightarrow x(x+3) > 0 \Rightarrow x < -3$ or $x > 0 \Rightarrow x > 0$.

Q.75 (3)

Putting $x = 5$

$2(5)^5 - 14(5)^4 + 31(5)^3 - 64(5)^2 + 19(5) + 130 = 0$

Hence $x = 5$ satisfies the given equation.

Thus 5 is a root of the equation.

Q.76 (1)

If $x^2 - 6x + 10 = (x-3)^2 + 1$

For real x, least value of $(x-3)^2 + 1$ is 1.

Q.77 (2)

Let $f(x) = 5 + 4x - 4x^2 = y \Rightarrow 4x^2 - 4x - 5 + y = 0$

Since x is real, so $B^2 - 4AC \geq 0$

$\Rightarrow 16 - 4.4(-5 + y) \geq 0$

$\Rightarrow 16 - 16(-5 + y) \geq 0 \Rightarrow -5 + y \leq 1 \Rightarrow y \leq 6$

Hence maximum value of y is 6.

Q.78 (1)

Let $y = x^2 - 6x + 13 \Rightarrow x^2 - 6x + 13 - y = 0$

Its discriminant $D \geq 0 \Rightarrow 36 - 4(13 - y) \geq 0$

$\Rightarrow 36 - 52 + 4y \geq 0 \Rightarrow 4y \geq 16 \Rightarrow y \geq 4$

Hence y is not less than 4.

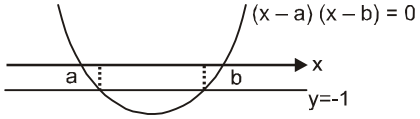
Aliter : $x^2 - 6x + 13 = (x-3)^2 + 4$

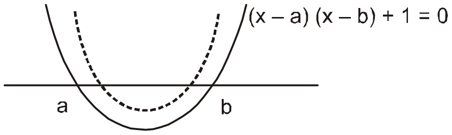
Obviously the minimum value is 4.

Q.79 (1)

b > a

both roots lies in (a, b)





Q.80 (2)

$f(x) = x^2 + (a-1)x + a^2 - 4$

$f(0) < 0$

$a^2 - 4 < 0 \Rightarrow a \in (-2, 2)$.]

EXERCISE-II (JEE MAIN LEVEL)

Q.1 (4)

$S = 1 + i^2 + i^4 + ... + i^{2n} = 1 - 1 + 1 - 1 + ... + (-1)^n$

Obviously it depends on n.

Hence cannot be determined unless is known.

Q.2 (a)

$$i^{57} + \frac{1}{i^{25}} = \left(i^4\right)^{14} \cdot i + \frac{1}{\left(i^4\right)^6 i}$$

$$= i + \frac{1}{i}\left(\because i^4 = 1\right) = i - i\left(\because \frac{1}{i} = -i\right) = 0$$

Q.3 (3)

If $z = x + iy$ is the additive inverse of $1 - i$ then

$(x + iy) + (1 - i) = 0 \Rightarrow x + 1 = 0$, $y - 1 = 0$

$\Rightarrow x = -1$, $y = 1$

$\therefore$ The additive inverse of $1 - i$ is $z = -1 + i$

Trick : Since $(1 - i) + (-1 + i) = 0 + 0i$.

Q.4 (1)

$$\left(\frac{i-1}{i+1} \times \frac{i-1}{i-1}\right)^n = \left(\frac{-2i}{-2}\right)^n = i^n$$

Hence, to make the real number the least positive integer is 2.

Q.5 (1)

$$\frac{(\sqrt{5+12i}+\sqrt{5-12i})(\sqrt{5+12i}+\sqrt{5-12i})}{(\sqrt{5+12i}-\sqrt{5-12i})(\sqrt{5+12i}+\sqrt{5-12i})}$$

$$=\frac{5+12i+5-12i+2\sqrt{5+12i}\sqrt{5-12i}}{5+12i-5+12i}$$

$$=\frac{10+2\times(\pm 13)}{24i}=-\frac{3}{2}i \text{ or } \frac{2i}{3}$$

Q.6 (4)

$$\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$$

$$=\left[\frac{1+2i}{1^2+2^2}+\frac{3-3i}{1^2+1^2}\right]\left[\frac{6-16+12i+8i}{2^2+4^2}\right]$$

$$=\left(\frac{2+4i+15-15i}{10}\right)\left(\frac{-1+2i}{2}\right)$$

$$=\frac{(17-11i)(-1+2i)}{20}=\frac{5+45i}{20}=\frac{1}{4}+\frac{9}{4}i$$

Q.7 (B)

Let $\sqrt{3-4i}=x+iy \Rightarrow 3-4i=x^2-y^2+2ixy$

$\Rightarrow x^2-y^2=3,\quad 2xy=-4$ ......(i)

$\Rightarrow (x^2+y^2)^2=(x^2-y^2)^2+4x^2y^2=(3)^2+(-4)^2=25$

$\Rightarrow x^2+y^2=5$ .....(ii)

From equation (i) and (ii) $x^2=4 \Rightarrow x=\pm 2$,

$y^2=1 \Rightarrow y=\pm 1$. Hence the square root of $(3-4i)$ is $\pm(2-i)$

Q.8 **(b)**

Q.9 (3)

$|z|-z=1+2i$

Let $z=x+iy$, therefore $|x+iy|-(x+iy)=1+2i$

Equating real and imaginary parts, we get

$\sqrt{x^2+y^2}-x=1$ and $y=-2 \Rightarrow x=\frac{3}{2}$

Hence complex number $z=\frac{3}{2}-2i$

Q.10 (a)

$$\left(\frac{z_1}{z_2}\right)^{50}=\left(\frac{\sqrt{3}+i\sqrt{3}}{\sqrt{3}+i}\right)^{50}$$

$$=\left[\left(\frac{\sqrt{3}(1+i)}{\sqrt{3}+i}\right)^2\right]^{25}=\left[\frac{3(2i)}{3-1+2\sqrt{3i}}\right]^{25}$$

$$=\left(\frac{3i}{1+\sqrt{3i}}\right)^{25}=\frac{3^{25}i^{25}}{(-2\omega^2)^{25}}$$

$$=i.\omega\left(\frac{3}{2}\right)^{25}=-i\left(\frac{-1+\sqrt{3i}}{2}\right)\left(\frac{3}{2}\right)^{25}$$

$$=\left(\frac{3}{2}\right)^{25}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$$

Hence, $\left(\frac{z_1}{z_2}\right)^{50}$ lies in the first quadrant asn both real and imaginary parts of this number are positive.

Q.11 **(1)**

Q.12 (3)

$$\left|(1+i)\frac{(2+i)}{(3+i)}\right|=|1+i|\left|\frac{2+i}{3+i}\right|=\frac{\sqrt{2}\times\sqrt{5}}{\sqrt{10}}=1$$

Q.13 (1)

$|z|=1 \Rightarrow |x+iy|=1 \Rightarrow x^2+y^2=1$

$$\omega=\frac{z-1}{z+1}=\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}$$

$$=\frac{(x^2+y^2-1)}{(x+1)^2+y^2}+\frac{2iy}{(x+1)^2+y^2}=\frac{2iy}{(x+1)^2+y^2}$$

$(\because x^2+y^2=1)$

$\therefore\ \mathrm{Re}(\omega)=0$.

Q.14 (3)

Q.15 (4)

Q.16 **(1)**

Q.17 (2)

We have $|z_1|=1$ and z_2 be any complex number.

$$\Rightarrow \left|\frac{z_1-z_2}{1-z_1\bar{z}_2}\right|=\frac{|z_1-z_2|}{\left|1-\frac{\bar{z}_2}{\bar{z}_1}\right|}; \qquad \because z_1\bar{z}_1=|z_1|^2$$

$$=\frac{|z_1-z_2|}{|\bar{z}_1-\bar{z}_2|}|\bar{z}_1|;\ \text{Given that } \because |\bar{z}_1|=1$$

$$=\frac{|z_1-z_2|}{|\overline{z_1-z_2}|}=\frac{|z_1-z_2|}{|z_1-z_2|}=1.$$

Q.18 **(4)**

Q.19 **(a)**

Let z = x + iy

$$\therefore |z+3-i|=|(x+3)-i(y-1)=1|$$

$$\Rightarrow \sqrt{(x+3)^2+(y-1)^2}=1 \qquad \ldots(i)$$

$$\because \arg z=\pi \Rightarrow \tan^{-1}\frac{y}{x}=\pi$$

$$\Rightarrow \frac{y}{x}=\tan\pi=0 \Rightarrow y=0 \qquad \ldots(ii)$$

From equations (i) and (ii), we get

x = –3, y = 0 ∴ z = –3

$$\Rightarrow |z|=|-3|=3$$

Q.20 (a)

Roots

$$=\frac{4(2-i)\pm\sqrt{16(2-i)+8(1+i)(5+3i)}}{4(1+i)}$$

$$=\frac{4-i}{1+i} \text{ or } \frac{-i}{1+i}=\frac{3-5i}{2} \text{ or } \frac{-1-i}{2}$$

Q.21 (a)

$$z=1+2i \Rightarrow |z|=\sqrt{1+4}=\sqrt{5}$$

$$\therefore\ f(z)=\frac{7-z}{1-z^2}=\frac{7-1-2i}{1-(1=2i)^2}$$

$$=\frac{6-2i}{1-(1-4+4i)}=\frac{6-2i}{4-4i}=\frac{3-i}{2-2i}$$

$$\Rightarrow |f(z)|=\left|\frac{3-i}{2-2i}\right|=\frac{|3-i|}{|2-2i|}$$

$$=\frac{\sqrt{9+1}}{\sqrt{4+4}}=\frac{\sqrt{5}}{2}=\frac{|z|}{2}$$

Q.22 (1)

Let $z=\frac{1+i\sqrt{3}}{1+\sqrt{3}}$ ∴ amp(z) or arg(z)

$$=\tan^{-1}\left[\frac{\sqrt{3}/(1+\sqrt{3})}{1/(1+\sqrt{3})}\right]=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$$

Q.23 (c)

$$\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$$

$$=2\sin\frac{\pi}{10}\cos\frac{\pi}{10}+i2\sin^2\frac{\pi}{10}$$

$$=2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)$$

For amplitude, $\tan\theta=\frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}}=\tan\frac{\pi}{10}$

$$\Rightarrow \theta=\frac{\pi}{10}$$

Q.24 (2)

Q.25 (a)

$$\bar{z}_1=\frac{z_1\bar{z}_1}{z_1}=|z_1|^2 z_1^{-1}$$

$$\Rightarrow \arg(z_1^{-1})=\arg(\bar{z}_1)\Rightarrow \arg(z_2)$$

$$\Rightarrow z_2=kz_1^{-1}(k>0)$$

Q.26 (2)

Q.27 **(4)**

Q.28 (c)

Q.29 (a)

Q.30 (4)

$$|z|=\sqrt{4^2+(-3)^2}=5$$

Let z_1 be the new vector obtained by rotating z in the clockwise sense through 180^o, therefore

$z_1=e^{-i\pi}z$

z_1 = (cos π – i sin π) (4 – 3i)

= (–1) (4 – 3i)

= – 4 + 3i

z_2 = 3(z_1) = 3 (–4 + 3i) = – 12 + 9i

Q.31 (1)

New z = $\frac{3}{2}(-4+5i)e^{i\pi}=6-\frac{15}{2}i$

Q.32 (1)

Given that $z=\frac{\sqrt{3}+i}{2}=\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$$\Rightarrow iz=-\frac{1}{2}+i\frac{\sqrt{3}}{2}=\omega$$

Now

$$z^{69} = z^{4(17)}z = (iz)^{4(17)}z = (\omega)^{68}z, \ \ (\because i^{4n} = 1)$$

$$= \frac{\omega^{69}}{i} = \frac{(\omega^3)^{23}}{i} = \frac{1}{i} = -i$$

Aliter : $z = \frac{\sqrt{3}}{2} + i\frac{1}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$$\Rightarrow z^{69} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{69} = \cos\frac{69\pi}{6} + i\sin\frac{69\pi}{6}$$

$$= \cos\left(11\pi + \frac{\pi}{2}\right) + i\sin\left(11\pi + \frac{\pi}{2}\right) = 0 + i(-1) = -i\,.$$

Q.33 (1)

Q.34 (1)

$$\left(\frac{\sqrt{3}+i}{2}\right)^6 + \left(\frac{i-\sqrt{3}}{2}\right)^6 = \left(\frac{-1+\sqrt{3}i}{2i}\right)^6 + \left(\frac{-1-\sqrt{3}i}{2i}\right)^6$$

$$= \frac{1}{i^6}[(\omega)^6 + (\omega^2)^6] = -[(\omega^3)^2 + (\omega^3)^4]$$

$$\left(\because \ \omega = \frac{-1+\sqrt{3}i}{2}, \omega^2 = \frac{-1-\sqrt{3}i}{2}\right)$$

$$= -(1+1) = -2$$

Q.35 (3)

$$(x-y)(x\omega - y)(x\omega^2 - y)$$

$$= (x^2\omega - xy - xy\omega + y^2)(x\omega^2 - y)$$

$$= x^3 - x^2y(1+\omega+\omega^2) + xy^2(1+\omega+\omega^2) - y^3$$

$$= x^3 - y^3 \qquad (\because 1+\omega+\omega^2 = 0)$$

Q.36 (1)

$$(3+\omega^2+\omega^4)^6 = (3+\omega^2+\omega)^6 = (3-1)^6 = 64$$

Q.37 (2)

$\omega^4 = \omega, \omega^8 = \omega^2$ etc. 3rd, 5th, 7th factors are each equal to 1st and 4th, 6th, 8th factors are each equal to 2nd.

L.H.S $= (-2\omega)(-2\omega^2)(-2\omega)(-2\omega^2)$.....to $2n$ factors

$$= (2^2\omega^3)(2^2\omega^3)\text{...... to } n \text{ factors}$$

$$= (2^2)^n = 2^{2n}$$

Q.38 (3)

Given, $\sin\left[(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right] = \sin\left[(\omega + \omega^2)\pi - \frac{\pi}{4}\right]$

$$= \sin\left(-\pi - \frac{\pi}{4}\right) = -\sin\left(\pi + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\,.$$

Q.39 **(3)**

Q.40 (a)

Q.41 (1)

$$\left|\frac{z-5i}{z+5i}\right| = 1 \ \left|\frac{x+i(y-5)}{x+i(y+5)}\right| = 1$$

$$\Rightarrow |x+i(y-5)| = |x+i(y+5)|, \ \left(\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right)$$

$$\Rightarrow \ x^2 + 25 - 10y + y^2 = y^2 + x^2 + 25 + 10y$$

$$\Rightarrow 20y = 0 \Rightarrow \ y = 0$$

Q.42 (4)

$$\arg\{(x-a) + iy\} = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y}{x-a}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{y}{x-a} = \tan\frac{\pi}{4} = 1 \ \Rightarrow \ x - a = y$$

Q.43 (d)

Q.44 (4)

$$z - 2 - 3i = x + iy - 2 - 3i = (x-2) + i(y-3)$$

$$\tan^{-1}\left(\frac{y-3}{x-2}\right) = \frac{\pi}{4} \Rightarrow \frac{y-3}{x-2} = \tan\frac{\pi}{4} = 1$$

$$\Rightarrow \ x - y + 1 = 0.$$

Q.45 (1)

$$\left|\frac{z}{z - \frac{i}{3}}\right| = 1 \Rightarrow |z| = \left|z - \frac{i}{3}\right|$$

Clearly locus of *z* is perpendicular bisector of line joining points having complex number $0 + i0$ and $0 + \frac{i}{3}$.

Hence *z* lies on a straight line.

Q.46 (1)

Let $\omega = -1 + 5z$, then $\omega + 1 = 5z$

$\Rightarrow |\omega + 1| = 5|z| = 5 \times 2 = 10 \quad (\because |z| = 2$, given value)

Thus ω lies on a circle.

Q.47 (2)

We have $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2}$

Therefore $\arg\frac{z-1}{z+1} = \tan^{-1}\frac{2y}{x^2+y^2-1}$

Hence $\tan^{-1}\frac{2y}{x^2+y^2-1} = \frac{\pi}{3}$

$$\Rightarrow \frac{2y}{x^2+y^2-1} = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\Rightarrow x^2+y^2-1 = \frac{2}{\sqrt{3}}y \Rightarrow x^2+y^2-\frac{2}{\sqrt{3}}y-1=0$$

Which is obviously a circle.

Q.48 (2)

Put $z = x+iy$ in $arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{6}$

$$\arg\left(\frac{(x-2)+iy}{(x+2)+iy}\right) = \frac{\pi}{6}$$

$$\arg((x-2)+iy) - \arg((x+2)+iy) = \frac{\pi}{6}$$

$$\tan^{-1}\frac{y}{x-2} - \tan^{-1}\frac{y}{x+2} = \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{\frac{y}{x-2}-\frac{y}{x+2}}{1+\frac{y^2}{x^2-4}}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{xy+2y-xy+2y}{x^2+y^2-4} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$= x^2+y^2-4\sqrt{3y}-4=0$$

Which is equation of a circle.

Q.49 (d)

Real part of $\frac{\overline{z}+2}{\overline{z}-1}$ is given by

$$\frac{1}{2}\left[\frac{\overline{z}+2}{\overline{z}-1}+\left(\overline{\frac{\overline{z}+2}{\overline{z}-1}}\right)\right]=4$$

$$\Rightarrow \frac{\overline{z}+2}{\overline{z}-1}+\frac{z+2}{z-1}=8$$

$$\Rightarrow z\overline{z}-\overline{z}+2z-2+z\overline{z}+2\overline{z}-z-2$$

$$=8(z\overline{z}-\overline{z}-z+1)$$

$$\Rightarrow z\overline{z}-\frac{3}{2}z-\frac{3}{2}\overline{z}+2=0 \quad .....(i)$$

Comparing with the equation

$z\overline{z}+\overline{a}z+a\overline{z}+b=0$, we get $a=-\frac{3}{2}$ and $b=2$.

Thus, the locus of z given by the equation

(i) is a circle with centre $\frac{3}{2}$ and radius $=\frac{1}{2}$

Q. 50 (1)

Given equation is $x^4-2x^3+x-380=0$

$$\Rightarrow (x^2-x-20)(x^2-x+19)=0$$

$$\Rightarrow (x-5)(x+4)(x^2-x+19)=0$$

Hence, the required roots of the equation are

$$5,-4,\frac{1\pm5\sqrt{-3}}{2}$$

Q.51 (4)

Given $x^2+x+1=0$

$$\therefore x=\frac{1}{2}[-1\pm i\sqrt{3}]=\frac{1}{2}(-1+i\sqrt{3}),\frac{1}{2}(-1-i\sqrt{3})=\omega,\omega^2$$

But $\alpha^{19}=\omega^{19}=\omega$ and $\beta^7=\omega^{14}=\omega^2$.

Hence the equation will be same.

Q.52 (3)

We have $4ax^2+3bx+2c=0$ Let roots are α and β

Let $D=B^2-4AC = 9b^2-4(4a)(2c)=9b^2-32ac$

Given that , $(a+b+c)=0 \Rightarrow b=-(a+c)$

Putting this value, we get

$=9(a+c)^2-32ac=9(a-c)^2+4ac$.

Hence roots are real.

Q.53 (4)

Given $|x|^2-3|x|+2=0$

Here we consider two cases $viz.$ $x<0$ and $x>0$

Case I : $x<0$ This gives $x^2+3x+2=0$

$\Rightarrow (x+2)(x+1)=0 \Rightarrow x=-2,-1$

Also $x=-1,-2$ satisfy $x<0$, so $x=-1,-2$ is solution in this case.

Case II : $x > 0$. This gives $x^2 - 3x + 2 = 0$ $\Rightarrow (x-2)(x-1) = 0 \Rightarrow x = 2,1$, so $x = 2$, 1 is solution in this case. Hence the number of solutions are four i.*e*. $x = -1, 1, 2, -2$

Aliter : $|x|^2 - 3|x| + 2 = 0$

$\Rightarrow \quad (|x| - 1)(|x| - 2) = 0$

$\Rightarrow |x| = 1$ and $|x| = 2 \Rightarrow x = \pm 1, x = \pm 2$.

Q.54 (3)

Roots of $x^2 - 8x + (a^2 - 6a) = 0$ are real. So $D \geq 0$

$\Rightarrow \quad 64 - 4(a^2 - 6a) \geq 0 \Rightarrow 16 - a^2 + 6a \geq 0$

$\Rightarrow \quad a^2 - 6a - 16 \leq 0 \Rightarrow (a-8)(a+2) \leq 0$

Now we have two cases:

Case I : $(a-8) \leq 0$ and $(a+2) \geq 0$

$\Rightarrow a \leq 8$ and $a \geq -2$

Case II : $(a-8) \geq 0$ and $(a+2) \leq 0$

$\Rightarrow a \geq 8$ and $a \leq -2$ but it is impossible

Therefore, we get $-2 \leq a \leq 8$

Aliter : Students should note that the expression $(x-a)(x-b)\{a < b\}$ will be less than or equal to zero if $x \in [a, b]$ or otherwise $x \notin [a, b]$.

Therefore $(a-8)(a+2) \leq 0$

i.e., $\{a - (-2)\}(a-8) \leq 0 \Rightarrow a \in [-2, 8]$.

Q.55 (3)

Given expression $x^2 + 2x + 2xy + my - 3$ can be written as $x^2 + 2x(1+y) + (my - 3)$

But factors are rational, so $B^2 - 4AC$ is a perfect square.

Now $4\{(1+y)^2 - (my-3)\} \geq 0$

$\Rightarrow 4\{y^2 + 1 + 2y - my + 3\} \geq 0$

$\Rightarrow \quad y^2 + 2y - my + 4 \geq 0$

Hence $2y - my = \pm 4y$ {as it is perfect square}

$\Rightarrow \quad 2y - my = 4y \Rightarrow m = -2$.

Now taking (–) sign, we get $m = 6$.

Q.56 (d)

The second equation can be rewritten as

$$a\left(\frac{x}{x+1}\right)^2 + b\left(\frac{x}{x+1}\right) + c = 0$$

and hence is roots correspond to $\dfrac{x}{x+1} = \alpha$

and $\dfrac{x}{x+1} = \beta$.

Hence $x = \dfrac{\alpha}{1-\alpha}$ and $\dfrac{\beta}{1-\beta}$

Q. 57 (a)

Given equation is (x – a) (x – b) + (x – b) (x – c) + (x – c) (x – a) = 0

$$\Rightarrow 3x^2 - 2(\mathrm{b+a+c})\,\mathrm{x} + \mathrm{ab+bc+ca} = 0$$

Now, here A = 3, B = –2 (a + b + c) C = ab + bc + ca

$$\therefore D = \sqrt{B^2 - 4AC}$$

$$= \sqrt{(-2(\mathrm{a+b}+c))^2 - 4(3)(\mathrm{ab+bc+ca})}$$

$$= \sqrt{4((\mathrm{a+b}+c)^2 - 12(\mathrm{ab+bc+ca})}$$

$$= 2\sqrt{a^2 + \mathrm{b}^2 + c^2 - \mathrm{ab - bc - ca}}$$

$$= 2\sqrt{\frac{1}{2}\{(a-\mathrm{b})^2 - (\mathrm{b-c})^2 + (\mathrm{c-a})^2\}} \geq 0$$

Q.58 (2)

Sum of roots $\alpha + \beta = -(a+b)$ and $\alpha\beta = \dfrac{a^2 + b^2}{2}$

$\Rightarrow (\alpha+\beta)^2 = (a+b)^2$ and $(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= 2ab - (a^2 + b^2) = -(a-b)^2$

Now the required equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

$x^2 - \{(\alpha+\beta)^2 + (\alpha-\beta)^2\}\, x + (\alpha+\beta)^2(\alpha-\beta)^2 = 0$

$\Rightarrow x^2 - \{(a+b)^2 - (a-b)^2\}\, x - (a+b)^2(a-b)^2 = 0$

$\Rightarrow x^2 - 4abx - (a^2 - b^2)^2 = 0$

Q.59 (4)

$\alpha + \beta = -\dfrac{b}{a}, \alpha\beta = \dfrac{c}{a}$

and $\qquad \alpha^2 + \beta^2 = \dfrac{(b^2 - 2ac)}{a^2}$

Now $\dfrac{\alpha}{a\beta + b} + \dfrac{\beta}{a\alpha + b} = \dfrac{\alpha(a\alpha + b) + \beta(a\beta + b)}{(a\beta + b)(a\alpha + b)}$

$$= \frac{a(\alpha^2 + \beta^2) + b(\alpha + \beta)}{\alpha\beta a^2 + ab(\alpha + \beta) + b^2} = \frac{a\dfrac{(b^2 - 2ac)}{a^2} + b\left(-\dfrac{b}{a}\right)}{\left(\dfrac{c}{a}\right)a^2 + ab\left(-\dfrac{b}{a}\right) + b^2}$$

$$= \frac{b^2 - ac - b^2}{a^2c - ab^2 + ab^2} = \frac{-2ac}{a^2c} = -\frac{2}{a}.$$

Q.60 (3)

Let α and β be two roots of $ax^2 + bx + c = 0$

Then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2}{a^2} - 2\frac{c}{a}$$

So under condition $\alpha + \beta = a^2 + \beta^2$

$$\Rightarrow -\frac{b}{a} = \frac{b^2 - 2ac}{a^2} \Rightarrow b(a+b) = 2ac .$$

Q.61 (3)

As given, if α, β are the roots, then $\alpha + \beta = p$ and $\alpha\beta = 8$

$$\therefore (\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$$

$$\Rightarrow p^2 - 2^2 = 4(8) \Rightarrow p^2 = 36 \Rightarrow p = \pm 6$$

Q.62 (3)

Let α, β are roots of $x^2 + px + q = 0$

So $\alpha + \beta = -p$ and $\alpha\beta = q$

Given that $(\alpha + \beta) = 3(\alpha - \beta) = -p \Rightarrow \alpha - \beta = \frac{-p}{3}$

Now $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$\Rightarrow \frac{p^2}{9} = p^2 - 4q \text{ or } 2p^2 = 9q .$$

Q.63 (4)

Let roots are α and β

$\alpha + \beta = 2$ and $\alpha^3 + \beta^3 = 98$

$$\therefore \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$$

$$\Rightarrow 98 = 2\left[(\alpha + \beta)^2 - 3\alpha\beta\right] \Rightarrow 49 = (4 - 3\alpha\beta)$$

$$\Rightarrow \alpha\beta = -15$$

Thus equation is $x^2 - 2x - 15 = 0$.

Q.64 (3)

Let α, β be the roots of $x^2 + bx + c = 0$ and α', β' be the roots of $x^2 + qx + r = 0$.

Then $\alpha + \beta = -b, \alpha\beta = c, \alpha' + \beta' = -q, \alpha'\beta' = r$

It is given that $\frac{\alpha}{\beta} = \frac{\alpha'}{\beta'} \Rightarrow \frac{\alpha + \beta}{\alpha - \beta} = \frac{\alpha' + \beta'}{\alpha' - \beta'}$

$$\Rightarrow \frac{(\alpha + \beta)^2}{(\alpha - \beta)^2} = \frac{(\alpha' + \beta')^2}{(\alpha' - \beta')^2} \Rightarrow \frac{b^2}{b^2 - 4c} = \frac{q^2}{q^2 - 4r}$$

$$\Rightarrow b^2 r = q^2 c$$

Q.65 (1)

Let roots are α, β so, $\frac{\alpha}{\beta} = \frac{2}{3} \Rightarrow \alpha = \frac{2\beta}{3}$

$$\therefore \alpha + \beta = \frac{m}{12}$$

$$\Rightarrow \frac{2\beta}{3} + \beta = \frac{m}{12} \Rightarrow \frac{5\beta}{3} = \frac{m}{12} \quad .....(i)$$

and $\alpha\beta = \frac{5}{12} \Rightarrow \frac{2\beta}{3}.\beta = \frac{5}{12} \Rightarrow \beta^2 = \frac{5}{8}$

$$\Rightarrow \beta = \sqrt{5/8}$$

Put the value of β in (i), $\frac{5}{3}.\sqrt{\frac{5}{8}} = \frac{m}{12} \Rightarrow m = 5\sqrt{10}$.

Q.66 (d)

$$\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$$

Or $x^2 + (a+b)x + ab = (b-a)x + (b-a)c$

Or $x^2 + 2ax + ab + ca - bc = 0$

Since product of the roots =0

Ab+ca-bc=0; $a = \frac{bc}{b+c}$

Thus sum of roots=-2a $= \frac{2bc}{b+c}$

Q.67 (2)

Q.68 (d)

We have

$\mathrm{x}^3 + 1 \equiv (\mathrm{x}+1)\left(\mathrm{x}^2 - \mathrm{x} + 1\right)$ Therefore, α and β are the complex cube root of -1 so that we may take $\alpha = -\omega$ and

$\beta = -\omega^2$, where $\omega \neq 1$ is a cube root of unity.

Thus $\alpha^{100} = (-\omega)^{100} = \omega$ and $\beta^{100} = \left(-\omega^2\right)^{100} = \omega^2$,

so that the required equation is $\mathrm{x}^2 + \mathrm{x} + 1 = 0$

Q. 69 (3)

Given quadratic eqn. is $\mathrm{x}^2 + \mathrm{px} + \frac{3\mathrm{p}}{4} = 0$

So, $\alpha + \beta = -\mathrm{p}$, $\alpha\beta = \frac{3\mathrm{p}}{4}$

Now, given $|\alpha - \beta| = \sqrt{10} \Rightarrow \alpha - \beta$

$= \pm\sqrt{10}$

$$\Rightarrow (\alpha + \beta)^2 = 10 \Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 10$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 10$$

$$\Rightarrow p^2 - 4\times\frac{3p}{4} = 10 \Rightarrow p^2 - 3p - 10 = 0$$

$\Rightarrow p = -2.5, \Rightarrow p \in \{-2, 5\}$

Q.70 (1)

Let the roots be $\alpha, \beta; \beta, \gamma$ and γ, α respectively.

$\therefore \quad \alpha + \beta = -p,\ \beta + \gamma = -q, \gamma + \alpha = -r$

Adding all, we get $\Sigma\alpha = -(p+q+r)/2$ etc.

Q.71 (4)

Let α be a common root, then

$\alpha^2 + a\alpha + 10 = 0$ …..(i)

and $\alpha^2 + b\alpha - 10 = 0$ …..(ii)

form (i) – (ii),

$$(a-b)\alpha + 20 = 0 \Rightarrow \alpha = -\frac{20}{a-b}$$

Substituting the value of α in (i), we get

$$\left(-\frac{20}{a-b}\right)^2 + a\left(-\frac{20}{a-b}\right) + 10 = 0$$

$$\Rightarrow 400 - 20\,a(a-b) + 10(a-b)^2 = 0$$

$$\Rightarrow 40 - 2a^2 + 2ab + a^2 + b^2 - 2ab = 0$$

$$\Rightarrow a^2 - b^2 = 40\,.$$

Q.72 (2)

Expressions are $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, then

$$\Rightarrow \frac{x^2}{-22a+14a} = \frac{x}{a-2a} = \frac{1}{-14+11}$$

$$\Rightarrow \frac{x^2}{-8a} = \frac{x}{-a} = \frac{1}{-3} \Rightarrow x^2 = \frac{8a}{3} \text{ and } x = \frac{a}{3}$$

$$\Rightarrow \left(\frac{a}{3}\right)^2 = \frac{8a}{3} \Rightarrow \frac{a^2}{9} = \frac{8a}{3} \Rightarrow a = 0,\ 24\,.$$

Trick : We can check by putting the values of a from the options.

Q.73 (4)

$x^2 - 3x + 2$ be factor of $x^4 - px^2 + q = 0$

Hence $(x^2 - 3x + 2) = 0 \Rightarrow (x-2)(x-1) = 0$

$\Rightarrow x = 2, 1,$ putting these values in given equation

so $4p - q - 16 = 0$ .....(i)

and $p - q - 1 = 0$ .....(ii)

Solving (i) and (ii), we get (p, q)=(5, 4)

Q.74 (4)

Q.75 (3)

Q.76 (b)

Q.77 (4)

Q.78 **(4)**

Q.79 (2)

Case I: When $x + 2 \geq 0$ *i.e.* $x \geq -2,$

Then given inequality becomes

$x^2 - (x+2) + x > 0 \quad \Rightarrow x^2 - 2 > 0 \Rightarrow |x| > \sqrt{2}$

$\Rightarrow x < -\sqrt{2}$ or $x > \sqrt{2}$

As $x \geq -2,$ therefore, in this case the part of the solution set is $[-2, -\sqrt{2}) \cup (\sqrt{2}, \infty).$

Case II: When $x + 2 \leq 0$ *i.e.* $x \leq -2,$

Then given inequality becomes $x^2 + (x+2) + x > 0$

$\Rightarrow x^2 + 2x + 2 > 0 \Rightarrow (x+1)^2 + 1 > 0,$ which is true for all real x

Hence, the part of the solution set in this case is $(-\infty, -2]$. Combining the two cases, the solution set is

$(-\infty, -2) \cup ([-2, -\sqrt{2}] \cup (\sqrt{2}, \infty) = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty).$

Q.80 (2)

Given equation is $x^3 - 3x + 2 = 0$

$\Rightarrow x^2(x-1) + x(x-1) - 2(x-1) = 0$

$\Rightarrow (x-1)(x^2 + x - 2) = 0 \Rightarrow (x-1)(x-1)(x+2) = 0$

Hence roots are $1, 1, -2$

Q.81 (4)

Let $y = x^2$. Then $x = \sqrt{y}$

$\therefore\ x^3 + 8 = 0 \Rightarrow y^{3/2} + 8 = 0$

$\Rightarrow y^3 = 64 \Rightarrow y^3 - 64 = 0$

Thus the equation having roots α^2, β^2 and γ^2 is $x^3 - 64 = 0\cdot$

Q.82 (3)

If α, β, γ are the roots of the equation.

$x^3 - px^2 + qx - r = 0$

$$\therefore (\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} = \frac{p^2+q}{pq-r}$$

Given, $p = 0,\ q = 4,\ r = -1$

$$\Rightarrow \frac{p^2+q}{pq-r} = \frac{0+4}{0+1} = 4\,.$$

Q.83 (4)

We know that the roots of the equation

$ax^3 + bx^2 + cx + d = 0$ follows $\alpha\beta\gamma = -d/a$

Comparing above equation with given equation

we get $d = 1, a = 1$

So, $\alpha\beta\gamma = -1$ or $\alpha^3\beta^3\gamma^3 = -1\,.$

Q.84 (3)

Let $y = \dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$\Rightarrow\ (y-1)x^2+3(y+1)x+4(y-1)=0$

For x is real $D \geq 0$

$\Rightarrow\ 9(y+1)^2-16(y-1)^2 \geq 0$

$\Rightarrow\ -7y^2+50y-7 \geq 0 \Rightarrow 7y^2-50y+7 \leq 0$

$\Rightarrow\ (y-7)(7y-1) \leq 0$

Now, the product of two factors is negative if one in $-ve$ and one in $+ve$.

Case I : $(y-7) \geq 0$ and $(7y-1) \leq 0$

$\Rightarrow\ y \geq 7$ and $y \leq \frac{1}{7}$. But it is impossible

Case II : $(y-7) \leq 0$ and $(7y-1) \geq 0$

$\Rightarrow\ y \leq 7$ and $y \geq \frac{1}{7} \Rightarrow \frac{1}{7} \leq y \leq 7$

Hence maximum value is 7 and minimum value is $\frac{1}{7}$

Q.85 (4)

Let $y = \frac{x^2+34x-71}{x^2+2x-7}$

$\Rightarrow\ x^2(y-1)+2(y-17)x+(71-7y)=0$

For real values of x, its discriminant $D \geq 0$

$\Rightarrow 4(y-17)^2-4(y-1)(71-7y) \geq 0$

$\Rightarrow (y^2-3+y+289)-(71y-7y^2-71+7y) \geq 0$

$\Rightarrow y^2-14y+45 \geq 0 \Rightarrow (y-5)(y-9) \geq 0$

It is possible when both $y-5$ and $y-9$ are negative or both positive. Let $y-5 \leq 0 \Rightarrow y \leq 5$ and $y-9 \leq 0 \Rightarrow y \leq 9$.

Hence $y \leq 5$ .....(i)

If $y-5 \geq 0 \Rightarrow y \geq 5$ and $y-9 \geq 0 \Rightarrow y \geq 9$

Hence $y \geq 9$. .....(ii)

Therefore y does not lie between 5 and 9.

Q.86 (b)

As given a and be are the roots of theeqution $x^2 + ax + b = 0$

$\Rightarrow$ sum of roots, a + b = – a

$\Rightarrow$ b = – 2a ..... (1)

and product of roots, ab = b

$\Rightarrow$ ab – b = 0

$\Rightarrow$ b (a – 1) = 0

if b = 0 then a = 0

if b ≠ 0 then a = 1 and b = – 2

so, the expression will be,

$f(x) = x^2 x - 2$

$$= x^2 + 2.\frac{1}{2}x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2$$

$$\Rightarrow f(x) = \left(x+\frac{1}{2}\right)^2 - \frac{9}{4}$$

So, f(x) will be minimum, if $\left(x+\frac{1}{2}\right)^2$

i.e. when $x = -\frac{1}{2}$

$\Rightarrow$ minimum value of function $-\frac{9}{4}$

Q.87 (c)

Since b, c, > 0

Therefore $\alpha+\beta=-b<0$ and $\alpha\beta=-c<0$

Since product of the roots is –ve therefore roots must be of opposite sign.

Q.88 (d)

Let $y = \frac{x}{x^2-5x+9}$

$\Rightarrow x^2y - (5y+1)x + 9y = 0$

for real x, Discriminant = $b^2 - 4ac \geq 0$

$(5y+1)^2 - 36y^2 \geq 0$

$\Rightarrow (5y+1-6y)\,(5y+1+6y) \geq 0$

$\Rightarrow (-y+1)(11\,y+1) \geq 0$

$\Rightarrow (y-1)(11\,y+1) \leq 0 \Rightarrow\ y \in \left[\frac{-1}{11}, 1\right]$

EXERCISE-III

NUMERICAL VALUE BASED

Q.1 **0004**

$z = z + iy$

$\Rightarrow\ z^2 = x^2 - y^2 + 2\,ixy$

$\Rightarrow\ \text{Re}(z^2) = x^2 - y^2, |z| = \sqrt{x^2+y^2}$

$\Rightarrow x^2 - y^2 = 0,\ \ x^2 + y^2 = 3\ \Rightarrow\ x^2 = y^2 = \frac{3}{2}$

$\Rightarrow\ x = \pm\sqrt{\frac{3}{2}},\qquad y = \pm\sqrt{\frac{3}{2}}$

Q.2 0000

We have $\frac{3}{|z_2-z_3|} = \frac{4}{|z_3-z_1|} = \frac{5}{|z_1-z_2|}$

$= k$ (let)

$\Rightarrow\ \frac{9}{|z_2-z_3|^2} = \frac{16}{|z_3-z_1|^2} = \frac{25}{|z_1-z_2|^2} = k^2$

Now $\frac{9}{|z_2 - z_3|^2} = k^2$

$\Rightarrow \frac{9}{z_2 - z_3} = k^2\left(\overline{z}_2 - \overline{z}_3\right)$ ....(1)

[As $|z|^2 = z\overline{z}$]

|||ly $\frac{16}{|z_3 - z_1|^2} = k^2$

$\Rightarrow \frac{16}{z_3 - z_1} = k^2\left(\overline{z}_3 - \overline{z}_1\right)$ ....(2)

|||ly $\frac{25}{|z_1 - z_2|^2} = k^2$

$\Rightarrow \frac{25}{z_1 - z_2} = k^2\left(\overline{z}_1 - \overline{z}_2\right)$ ....(3)

$\therefore$ On adding (1), (2) and (3), we get

$$\frac{9}{z_2 - z_3} + \frac{16}{z_3 - z_1} + \frac{25}{z_1 - z_2} = k^2$$

$\left(\overline{z}_2 - \overline{z}_3 + \overline{z}_3 - \overline{z}_1 + \overline{z}_1 - \overline{z}_2\right) = 0$ **Ans.**

Q.3 **0010**

From the hypothesis we have

$$z = \frac{\sqrt{3}}{2} - \frac{i}{2} = i\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) = i\omega$$

where $\omega = \left(-\frac{1}{2}\right) - \left(\frac{i\sqrt{3}}{2}\right)$ which is a cube root of unity. Now $z^{95} = (i\omega)^{95} = –i\omega^2$ (since $\omega^3 = 1$) and $i^{67} = i^3 = –i$.

Therefore, $z^{95} + i^{67} = –i(1 + \omega^2) = (–i)(–\omega) = i\omega$

$(z^{95} + i^{67})^{94} = (i\omega)^{94} = i^2\omega = –\omega$

Now $–\omega = z^n = (i\omega)^n \Rightarrow i^n \cdot \omega^{n–1} = –1 \Rightarrow n = 2, 6, 10, 14, \ldots\ldots$ and $n – 1 = 3, 6, 9, \ldots\ldots$

Therefore $n = 10$ is the required least positive integer.

Q.4 **0002**

Let $z = x + iy$

$\therefore \quad z + \overline{z} = 2|z - 1|$

$\Rightarrow \quad 2x = 2\sqrt{(x-1)^2 + y^2}$

$\Rightarrow \quad y^2 = 2x – 1$ ……(i)

For z_1 and z_2

$y_1^2 = 2x_1 – 1$ ……(ii)

$y_2^2 = 2x_2 – 1$ ……(iii)

By equation (ii) and (iii)

$y_1^2 - y_2^2 = 2(x_1 – x_2)$

$\Rightarrow (y_1 – y_2)(y_1 + y_2) = 2(x_1 – x_2)$ ……(iv)

Again $\arg(z_1 – z_2) = \frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1}\frac{y_1 - y_2}{x_1 - x_2} = \frac{\pi}{4}$

$\Rightarrow \quad \frac{y_1 - y_2}{x_1 - x_2} = 1$ ……(v)

By equation (iv) and (v)

$y_1 + y_2 = 2$

$\Rightarrow \quad \text{Im}(z_1) + \text{Im}(z_2) = 2$

Q.5 32

$z = \alpha \qquad \alpha \in R$

$\alpha^3 – (3 + i)\alpha + m + 2i = 0$

$\alpha^3 – 3\alpha + m = 0 \quad \& \quad – \alpha + 2 = 0$

$\alpha = 2$

$8 – 6 + m = 0 \quad \Rightarrow \quad m = – 2$

$\Rightarrow \quad \alpha^4 + m^4 = 32$

Q.6 99

$N = (a + ib)^3 – 107i$

$= (a^3 – 3ab^2) + i[3a^2b – b^3] – 107i$ = Positive integer

$\therefore \quad 3a^2b – b^3 – 107 = 0$

$b(3a^2 – b^2) = 107$

$b = 1 \quad 3a^2 – b^2 = 107 \quad$ 107 is prime

$\Rightarrow a = 6$ or $b = 107 \quad 3a^2 – (107)^2 = 1$

a is not integer not possible

$\therefore \quad a = 6 \quad b = 1$

$N = 216 – 3 \times 6 = 216 – 18 = 198.$

Q.7 **0001**

If α and β are the roots of quadratic equation $x^2 – x – 1 = 0$

then $\alpha^2 – \alpha – 1 = 0$ and $\beta^2 – \beta – 1 = 0$

For $\frac{a_{2012} - a_{2010}}{a_{2011}}$

$$= \frac{\frac{(\alpha^{2012} - \beta^{2012})}{(\alpha - \beta)} - \frac{(\alpha^{2010} - \beta^{2010})}{(\alpha - \beta)}}{\frac{(\alpha^{2011} - \beta^{2011})}{(\alpha - \beta)}}$$

$$\Rightarrow \quad \frac{\alpha^{2012} - \beta^{2012} - \alpha^{2010} + \beta^{2010}}{\alpha^{2011} - \beta^{2011}}$$

$$\Rightarrow \quad \frac{\alpha^{2010}(\alpha^2 - 1) - \beta^{2010}(\beta^2 - 1)}{\alpha^{2011} - \beta^{2011}}$$

$$\Rightarrow \quad \frac{\alpha^{2010}(\alpha) - \beta^{2010}(\beta)}{\alpha^{2011} - \beta^{2011}} = 1$$. **Ans.**

Q.8 **0000**

If the equation $(x^2 + 2a + b^2)(x^2 + 2bx + c^2) = 0$ has four distinct real roots then

$4a^2 - 4b^2 > 0$ …(i)

and $4b^2 - 4c^2 > 0$ …(ii)

By (i) & (ii)

$a^2 > b^2 > c^2$ …(iii)

For equation $x^2 + 2cx + a^2 = 0$ then

Discriminant $= 4c^2 - 4a^2$

$= (c^2 - a^2)$

$= -$ ive $(c^2 - a^2 < 0)$

$\therefore$ Since equation has no real roots. Ans.]

Q.9 **0012**

$x^2 - 3x + y = 0$ ... (1)

Let he roots are $\alpha + \beta$,

$x^3 - 4x^2 + qx = 0$ ... (2)

One root of the equation is zro and other root is repeated then roots are

$0, \beta, \beta$, then

Sum of roots $2\beta = 4$

$\therefore \quad \beta = 2$

$\alpha + \beta = 3$ by equation (1)

$\therefore \quad \alpha = 1$

$\therefore \quad b = \alpha\beta = 2$

$q = 0\beta + \beta^2 + \beta\alpha = \beta^2 = 4$

$\therefore \quad 2(b + q) = q(4 + 2) = 12$. **Ans.**

Q.10 **0008**

If $a, b, c \in R$

$a + b + c = 6$ then $(a + b) = 6 - c$

$ab + bc + ca = 9 \;\; ab = 9 - c\,(6 - c)$

Let the euqaiton where roots are a and b then

$x^2 - (6 - c)x + 9 - c\,(6 - c) = 0$

for real x discriminant should be ≥ 0

$(6 - c)^2 - 4[a - c(6 - c)] \geq 0$

$\Rightarrow 3c^2 \leq 12c$

$\Rightarrow c \in [0, 4]$

If exactly one root of equation

$F(x) = x^2 - (m + 2)x + 5m = 0$ lie between [0, 4] then

$f(0)\, f(x) < 0$

$\Rightarrow 5m\,(m + 8) < 0$

$m \in (-8, 0)$

Now for $x = 0$, $x^2 - 2x = 0$

$x = 0, 2$

for $m = -8$, $x^2 + 6x - 40 = 0$

$x = -10, 4$

$\therefore m \in (-8, 0]$

No. of integral values are 8. **Ans.**

Q.11 10

$\alpha.\beta = b\,;\ \gamma\delta = b - 2$

$\Rightarrow \quad \alpha\beta\gamma\delta = b(b - 2) = 24$

$\therefore \quad bx^2 + ax + 1 = 0$ has roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

$$\Rightarrow \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-a}{b}$$

$(b - 2)x^2 - ax + 1 = 0$ has root $\dfrac{1}{\gamma}, \dfrac{1}{\delta}$

$$\Rightarrow \quad \frac{1}{\gamma} + \frac{1}{\delta} = \frac{a}{b-2}$$

$$\frac{1}{\gamma} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{-a}{b} + \frac{a}{b-2} = \frac{5}{6} \quad ;$$

$\dfrac{+2a}{b(b-2)} = \dfrac{5}{6}$; $\dfrac{+2a}{24} = \dfrac{5}{6}$; $a =$ 10.

Q.12 **0003**

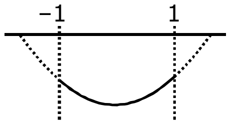
$\cos^2 x + (1 - a)\cos x - a^2 \leq 0 \quad \forall \; x \in R$

Let $t = \cos x \in [-1, 1]$

$f(t) = t^2 + (1 - a)t - a^2$

$f(-1) \leq 0$ & $f(1) \leq 0$

$1 - (1 - a) - a^2 \leq 0$



$\&\ 1 + (1 - a) - a^2 \leq 0$

$a(a - 1) \geq 0 \;\;\&\;\; a^2 + a - 2 \geq 0$

$(a + 2)(a - 1) \geq 0$

finally $a \in (-\infty, -2] \cup [1, \infty)$

$|k_1| + |k_2| = |-2| + |1| = 3$

PREVIOUS YEAR'S

MHT CET

Q.1 (3)

Q.2 (2)

$$\text{Given, } z = \frac{\left(\sqrt{3}+i\right)^2 (3i+4)^2}{(8+6i)^2}$$

$$\text{Now, } \quad |z| = \left|\frac{\left(\sqrt{3}+i\right)^3 (3i+4)^2}{(8+6i)^2}\right|$$

$$= \frac{\left|\left(\sqrt{3}+i\right)^3\right|\left|(3i+4)^2\right|}{\left|(8+6i)^2\right|}\left[\because \left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|\right]$$

$$=\frac{\left|\sqrt{3}+i\right|^3\left|3i+4\right|^2}{\left|8+6i\right|^2}\left[\because\left|z^n\right|=\left|z\right|^n\right]$$

$$=\frac{\left(\sqrt{3+1}\right)^3\left(\sqrt{9+16}\right)^2}{\left(\sqrt{64+36}\right)^2}$$

$$=\frac{(2)^3(5)^2}{(10)^2}=\frac{10^2.2}{(10)^2}=2$$

Q.3 (2)

Given, $\frac{3}{2+\cos\theta+i\sin\theta}=a+ib$

$\Rightarrow$ $\frac{3\left[(2+\cos\theta)-i\sin\theta\right]}{(2+\cos\theta)^2+\sin^2\theta}=a+ib$

$\Rightarrow$ $\frac{3\left[2+\cos\theta-i\sin\theta\right]}{5+4\cos\theta}=a+ib$

$\Rightarrow$ $a=\frac{3(2+\cos\theta)}{5+4\cos\theta}$

and $b=-\frac{3\sin\theta}{5+4\cos\theta}$

$\therefore$ $(a-2)^2+b^2=\left(\frac{6+3\cos\theta}{5+4\cos\theta}-2\right)^2+\frac{9\sin^2\theta}{(5+4\cos\theta)^2}$

$$=\frac{(-4-5\cos\theta)^2+9\sin^2\theta}{(5+4\cos\theta)^2}$$

$$=\frac{16+25\cos^2\theta+40\cos\theta+9\sin^2\theta}{(5+4\cos\theta)^2}$$

$$=\frac{16+25\cos^2\theta+40\cos\theta+9}{(5+4\cos\theta)^2}$$

$$=\frac{(5+4\cos\theta)^2}{(5+4\cos\theta)^2}=1$$

Q.4 (3)

Q.5 (2)

Given, $2\alpha=-1-i\sqrt{3}$ and $2\beta=-1+i\sqrt{3}$

\ $\alpha+\beta=-1$ and $\alpha\beta=1$

Now, $5\alpha^4+5\beta^4+\frac{7}{\alpha\beta}=5[\{(\alpha+\beta)^2-2(\alpha\beta)^2]+\frac{7}{\alpha\beta}$

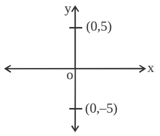
$=5[\{(-1)^2-2\times1\}^2-2(1)^2]+\frac{7}{1}$

$=5[(1-2)^2-2]+7=2$

Q.6 (1)

Given, $\left|\frac{z-5i}{z+5i}\right|=1\Rightarrow|z-5i|=|z+5i|$

$\because$ If $|z-z_1|=|z-z_2|$, then it is a perpendicular bisector of z_1 and z_2.



$\therefore$ Perpendicular bisector of (0,5) and (0,–5) is X-axis.

Q.7 (1)

Let $z_1=x_1+iy_1$, $z_2=x_2+iy_2$ and $z_3=x_3+iy_3$

Given, $Z_1=\frac{\lambda z_2+z_3}{\lambda+1}$

$\Rightarrow$ $x_1+iy_1=\frac{\lambda(x_2+iy_2)+(x_3+iy_3)}{\lambda+1}$

$\Rightarrow$ $x_1+iy_1=\frac{\lambda x_2+\lambda iy_2+x_3+iy_3}{\lambda+1}$

$\Rightarrow$ $x_1+iy_1=\frac{\lambda x_2+x_3}{\lambda+1}+\frac{i(\lambda y_2+y_3)}{\lambda+1}$

$\therefore$ $x_1=\frac{\lambda x_2+x_3}{\lambda+1}$ and $y_1=\frac{\lambda y_2+y_3}{\lambda+1}$

Hence, z_1 (point A) divides z_2 (point B) and z_3 (point C) in the ratio $\lambda:1$.

$\therefore$ B, A and C are collinear.

So, the distance of point A from the line joining points B and C is zero.

Q.8 (2)

We have, $\frac{x+iy}{9}=\left(-2-\frac{1}{3}i\right)^2$

$\Rightarrow$ $\left(\frac{x+iy}{9}\right)=\left[-\frac{1}{3}(6+i)\right]^2$

$\Rightarrow$ $\frac{x+iy}{9}=\frac{1}{9}\left(36+i^2+12i\right)$

$=\frac{1}{9}(36-1+12i)$

$\Rightarrow$ $\frac{x+iy}{9}=\frac{1}{9}(35+12i)$

On equating real adn imaginary parts, we get

$x=35$

and $y=12$

$\therefore$ $x-y=35-12=23$

Q.9 (2)

Given that, $(x-1)^3 + 8 = 0$

$\Rightarrow \quad (x-1)^3 = (-2)^3$

$$\Rightarrow \quad \left(\frac{x-1}{-2}\right)^3 = 1 \Rightarrow \left(\frac{x-1}{-2}\right) = (1)^{1/3}$$

$\therefore$ Cube roots of $\left(\frac{x-1}{-2}\right)$ are 1, ω and ω^2.

Cube roots of $(x-1)$ are -1, $1-2\omega$ and $1-2\omega^2$.

Q.10 (1)
Q.11 (1)
Q.12 (4)
Q.13 (1)
Q.14 (3)

JEE-MAIN PREVIOUS YEAR'S

COMPLEX NUMBERS

Q.1 (2)

$z + \overline{z} = iz^2 + z^2$

Consider $z = x + iy$

$2x = (i+1)(x^2 - y^2 + 2xyi)$

$\Rightarrow 2x = x^2 - y^2 - 2xy$ and $x^2 - y^2 + 2xy = 0$

$\Rightarrow 2x = -4xy$

$\Rightarrow x = 0$ or $y = \frac{-1}{2}$

Case 1 : $x = 0 \Rightarrow y = 0$ so $z = 0$

Case 2 : $y = \frac{-1}{2}$

$\Rightarrow 4x^2 - 4x - 1 = 0$

$(2x-1)^2 = 2$

$2x - 1 = \pm\sqrt{2}$

$x = \frac{1 \pm \sqrt{2}}{2}$

Here $Z = \frac{1+\sqrt{2}}{2} - \frac{i}{2}$ or $Z = \frac{1-\sqrt{2}}{2} - \frac{i}{2}$

Sum of squares of modulus of z

$$= 0 + \frac{(1+\sqrt{2})^2 + 1}{4} + \frac{(1-\sqrt{2})^2 + 1}{4} = \frac{8}{4} = 2$$

Q.2 (1)

Given $1 + x^2 + x^4 = 0$

$(x^2 + x + 1)(x^2 - x + 1) = 0$

$$x^2 + x + 1 = 0 \left\langle \begin{matrix} \omega \\ \omega^2 \end{matrix} \right.$$

$\therefore$ ω, ω^2 are roots of $x^2 + x + 1$

$$x^2 - x + 1 = 0 \left\langle \begin{matrix} -\omega \\ -\omega^2 \end{matrix} \right.$$

$-\omega, -\omega^2$ are roots of $x^2 - x + 1 = 0$

$\therefore \alpha = \omega$

$\alpha^{1011} + \alpha^{2022} - \alpha^{3033} = (\omega)^{1011} + (\omega)^{2022} - (\omega)^{3033}$

$= (\omega^3)^{337} + (\omega^3)^{674} - (\omega^3)^{1011}$

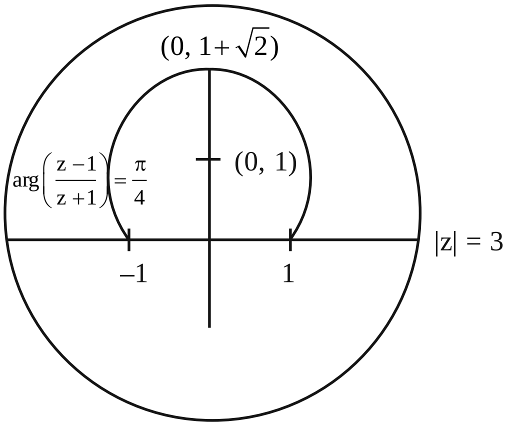
$= 1 + 1 - 1 = 1 \; \{\because \; \omega^3 = 1\}$

Q.3 (3)

complex number z such that $|z| = 3$

$\Rightarrow \arg(z-1) - \arg(z+1) = \frac{\pi}{4}$

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$$



Q.4 [80]

$|7-3| \le|$

$(x-3)^2 + y^2 \le 1$

$7(4+3i)\ \overline{Z}\ (4-3i) \le 24$

$4(Z + \overline{Z}) + 3i(Z - \overline{Z}) \le 24$

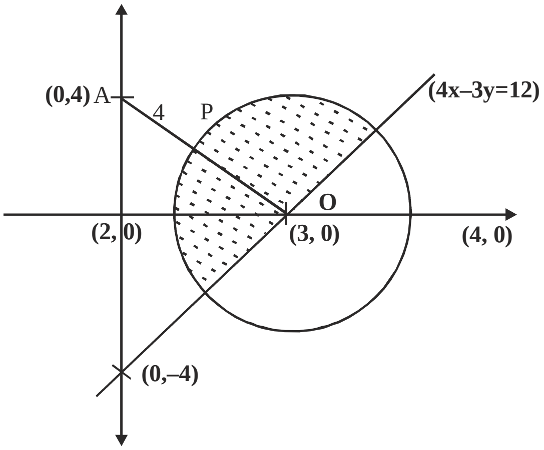
$4(2x) + 3i\,(2iy) \le 24$

$8x - 6y \le 24$

$\boxed{4x - 3y \le 12}$

$$\boxed{\frac{x}{3} + \frac{y}{-4} \le 1}$$

$$\left.\begin{matrix} \text{centre} = 13, 07 \\ \text{rat} \quad = 1 \end{matrix}\right\}$$



P divides AO in 4 : 1

$\alpha, B = \left(\frac{12}{5}, \frac{4}{5}\right)$

$25(\alpha + B)$

$25\left(\frac{12}{5} + \frac{4}{5}\right)$

$25\left(\frac{16}{5}\right)$

$= 80$

Q.5 [2]

$z^2 + z + 1 = 0 \Rightarrow z = \omega, \omega^2$

$$\left|\sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$$

$$= \left|\sum_{n=1}^{15} z^{2n} \sum_{n=1}^{15} \frac{1}{z^{2n}} + \sum_{n=1}^{15} (-1)^n\right|$$

$$= \left|\sum_{n=1}^{15} \omega^{2n} + \sum_{n=1}^{15} \frac{1}{\omega^{2n}} + \sum_{n=1}^{15} (-1)^n\right| \left[\omega = \frac{1}{\omega^2}\right]$$

$$= \left|\sum_{n=1}^{15} \omega^{2n} + \sum_{n=1}^{15} \omega^n + \sum_{n=1}^{15} (-1)^n\right|$$

$= |(\omega^2 + \omega^4 + \omega^6 + ...\omega^{30}) + (\omega + \omega^2 + \omega^3 + ...\omega^{15})$
$+ (-1 + 1 - 1 + 1 ... -1)|$

$= |(\omega^2 (1 + \omega^2 + \omega^3 + ...\omega^{15}) + \omega(1 + \omega + \omega^2 + ...\omega^{14}) - 1|$

$$= \left|\omega^2\left(\frac{1-\omega^{15}}{1-\omega}\right) + \omega\left(\frac{1-\omega^{14}}{1-\omega}\right) - 1\right|$$

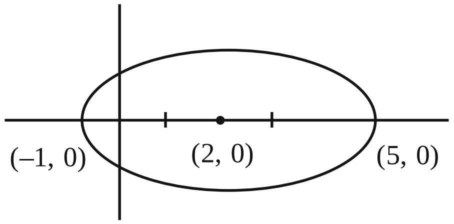
$= |\omega^2(0) + \omega(1 + \omega) - 1|$

$= |\omega^2 + \omega - 1| = |-1-1| = 2$

Q.6 **(3)**

$C : (x-4)^2 + (y-3)^2 = 4$

$E : \frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$



Lower Extremity of vertical diameter of circle → (4, 1)

Put in ellipse $\Rightarrow \frac{(4-2)^2}{9} + \frac{1}{5} - 1$

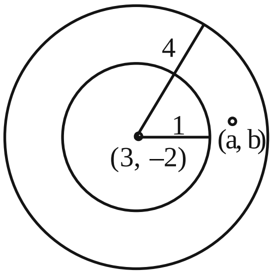
$= \frac{4}{9} + \frac{1}{5} - 1 =$

$= \frac{29}{45} - 1 < 0$

Two solutions

Q.7 [40]

$1 < |Z - 3 + 2i| < 4$



$1 < (a-3)^2 + (b+2)^2 < 16$

$(0, \pm 2), (\pm 2, 0), (\pm 1, \pm 2), (\pm 2, \pm 1)$
$(\pm 2, \pm 3), (3\pm, \pm 2), (\pm 1, \pm 1), (2\pm, \pm 2)$
$(\pm 3, 0), (0, \pm 3), (\pm 3, \pm 1), (\pm 1, \pm 3)$

Q.8 **[3]**

$\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary $\Rightarrow$

So $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} \times$

$\frac{1 - 2i\sin\alpha}{1 - 2i\sin\alpha} = \frac{1 - 2\sin^2\alpha - 3i\sin\alpha}{1 + 4\sin^2\alpha}$

$= \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2 a} - \frac{3i\sin\alpha}{1 + 4\sin^2\alpha}$

$\Rightarrow \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2\alpha} = 0 \Rightarrow \text{Sin}^2\alpha = \frac{1}{2} \Rightarrow \alpha = \pi + \frac{\pi}{4} =$

$\frac{5\pi}{4}$ & $\alpha = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Now $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real

$$\frac{1+i\cos\beta}{1-2i\cos\beta}\times\frac{1+2i\cos\beta}{1+2i\cos\beta} = \frac{1-2\cos^2\beta+3i\cos\beta}{1+4\cos^2\beta}$$

$$\Rightarrow \frac{3\cos\beta}{1+4\cos^2\beta}=0\Rightarrow\cos\beta=0\Rightarrow\beta=\frac{3\pi}{2}$$

Now $z_{\alpha\beta}=\sin 2\alpha+ i\cos 2\beta$

put $\alpha=\frac{5\pi}{4},\beta=\frac{3\pi}{2},\ z_{\alpha\beta}=1-i$

Now put $\alpha=\frac{7\pi}{4},\beta=\frac{3\pi}{2},z_{\alpha b}=-1-i$

$$\sum_{(\alpha,\beta)\in s}\left(iz_{\alpha\beta}+\frac{1}{iz_{\alpha\beta}}\right)= i\,(1-i)\ +\ \frac{1}{i(1-i)}+ i\,(-1-i)\ +$$

$$\frac{1}{i(-1-i)}=(i+1)\ +\ \frac{1-i}{i.2}+-i+1+\frac{-1+i}{i.2}=1$$

Q.9 [6]

$|z^2|=|\overline{z}|\cdot 2^{1-|z|}$

$\Rightarrow |z|=1$

$z^2=\overline{z}\Rightarrow z^3=1$

$\therefore z=\omega$ pr ω^2

$\omega^2=(1+\omega)^n=(-\omega^2)^n$

Least natural value of n is 6

Q.10 (1)

$z^5+(\overline{z})^5=(2+3i)^5+(2-3i)^5$

$=2({}^5C_0 2^5+{}^5C_2 2^3(3i)^2+{}^5C_4 2^1(3i)^4)$

$=2(32+10\times 8(-9)+5\times 2\times 81)=244$

Q.11 (3)

$\left|z_2+|z_2-1|\right|^2=\left|z_2-|z_2+1|\right|^2$

$\Rightarrow\left(z_2+|z_2-1|\right)\left(\overline{z}_2+|z_2-1|\right)$

$=\left(z_2-|z_2+1|\right)\left(\overline{z}_2-|z_2+1|\right)$

$\Rightarrow z_2\,|\,\overline{z}_2+|z_2-1|-z_2(\overline{z}_2-|z_2+1|)+\overline{z}_2(|z_2-1|+|z_2+1|)$

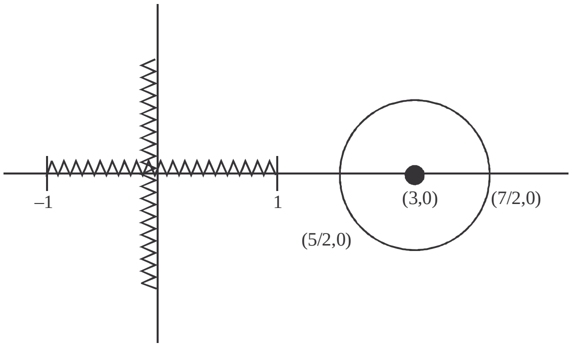
$=|z_2+1|^2-|z_2-1|^2$

$\Rightarrow(z_2+\overline{z}_2)\left(|z_2-1|\right)+\left(|z_2+1|\right)=2\left(z_2+\overline{z}_2\right)$

$\therefore z_2+\overline{z}_2=0$ or $|z_2-1|+|z_2+1|-2=0$

$\therefore z_2$ lie on imaginary axis. Or on real axis with in [–1,1]

Also $|z_1-3|=\frac{1}{2}$ lie on circle having center (3,0) and radius $\frac{1}{2}$.



Clearly $|z_1-z_2|_{\min}=\frac{5}{2}-1=\frac{3}{2}$

Q.12 (1)

$v=|z|^2+|z-3|^2+|z-6i|^2$

$=x^2+y^2+(x-3)^2+y^2+x^2+(y-6)^2$

$=3x^2+3y^2-6x-12y+45$

$=3(x^2-2x+y^2-4y)+45$

$=3\,(x-1)^2+3(y-2)^2+30$

$=3|z-(1+2i)|^2+30$

Minimum at $z_0=(1+2i)$ and min value $v_0=30$

Now, $\left|2z_0^2-\overline{z}_0^3+3\right|^2+v_0^2$

$=|2\,(-3+4i)-(1-2i)^3+3|^2+(30)^2$

$=|-6+8i-(-11+2i)+3|^2+900$

$=|-6+8i+11-2i+3|^2+900$

$=|8-6i|^2+900$

$=64+36+900=1000$

Q.13 [0]

$z^2+\overline{z}=0$

$(x+iy)^2+(x-iy)=0$

$x^2-y^2+2ixy+x-iy=0$

$(x^2-y^2+x)+i(2xy-y)=0$

$x^2+x-y^2=0$ & $y\,(2x-1)=0$ ⟨ $y=0$; $x=\frac{1}{2}$

Case I : $x=\frac{1}{2}$

$\frac{1}{4}+\frac{1}{2}-y^2=0\Rightarrow y^2=\frac{3}{4}\Rightarrow y^2=\frac{3}{4}\Rightarrow y=\pm\frac{\sqrt{3}}{2}$

$z=\frac{1}{2}+\frac{\sqrt{3}i}{2},\frac{1}{2}-\frac{\sqrt{3}i}{2}$

Case II : y = 0

$x^2+x=0\Rightarrow x\,(x+1)=0$

$x = 0, -1$

$z = 0, -1 + 0i$

$$\sum_{z \in s} (\text{Re}(z) + \text{Im}(z)) =$$

$$\left(\frac{1}{2} + \frac{1}{2} + 0 - 1\right) + \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 0 + 0\right)$$

$\Rightarrow 0 + 0 = 0$

QUADRACTIC EQUATIONS

Q.14 (2)

$3x^2 + \lambda x - 1 = 0$

roots are α & β

$$\alpha + \beta = \frac{-\lambda}{3} \;\&\; \alpha\beta = \frac{-1}{3}$$

$$\text{given } \frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$$

$$\frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = 15$$

$$\frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = 15$$

$$\frac{\left(\frac{-\lambda}{3}\right)^2 + \frac{2}{3}}{\left(-\frac{1}{3}\right)^2} = 15 \Rightarrow \frac{\lambda^2}{9} + \frac{2}{3} = \frac{15}{9}$$

$\lambda^2 + 6 = 15$

$\lambda^2 = 9$

$\lambda = \pm 3$

= eq. will be $3x^2 + 3x - 1 = 0$ or $3x^2 3x - 1 = 0$

$$\alpha + \beta = -1,\ \alpha\beta = \frac{-1}{3} \qquad\qquad \alpha + \beta = 1 \;\&\; \alpha\beta = \frac{-1}{3}$$

when λ=3 **when λ = –3**

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \qquad \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$= (-1)^3 - 3\left(\frac{-1}{3}\right)(-1) \qquad\qquad = (1)^3 - 3\left(\frac{-1}{3}\right)(1)$$

$= -1 - 1 = -2$ $\qquad$ $= 1 + 1 = 2$

$\therefore 6(\alpha^3 + \beta^3)^2 = 6(-2)^2$ $\qquad$ $\therefore 6(\alpha^3 + \beta^3) = 6(2)^2$

$= 24$ $\qquad$ $= 24$

Q.15 (2)

$(e^x - 2)(e^x + 2)(3e^x - 1)(2e^x - 1) = 0$

$$e^x = 2, \frac{1}{3}, \frac{1}{2}$$

$$x = \ell n2,\ \ell n\frac{1}{3}, \ell n\frac{1}{2}$$

$x = \ell n2, -\ell n3, -\ell n2$

sum of all root's = $\ell n2 - \ell n3 - \ell n2$

$= -\ell n3$

Q.16 [4]

$P + q = 3$

$\Rightarrow (p + q)^2 = 9$

$\Rightarrow p^2 + q^2 + 2pq = 9$

given $p^4 + q^4 = 369$

$(p^2 + q^2)^2 - 2(pq)^2 = 369$

$\Rightarrow (9 - 2pq)^2 - 2(pq)^2 = 369$

$\Rightarrow 81 - 36pq + 4p^2q^2 - 2p^2q^2 = 369$

$\Rightarrow 2(pq)^2 - 36(pq) - 288 = 0$

$\Rightarrow (pq)^2 - 18(pq) - 144 = 0$

$pq = 24, -6$

if $pq = 24$

$p^2 + q^2 = 9 - 2(pq) =$ –ve(not possible)

take $pq = -6$

$$\left(\frac{1}{p} + \frac{1}{q}\right)^{-2} = \left(\frac{pq}{p + q}\right)^2 = \left(\frac{-6}{3}\right)^2 = 4$$

Q.17 [45]

$$e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0]$$

$$(e^x)^3 - 11(e^x)^2 - 45 + \frac{81e^x}{2} = 0]+$$

$e^x = t$

$2t^3 - 22t^2 + 81t - 90 = 0$

$t_1t_2t_3 = 45$

$e^{x_1}.e^{x_2}.e^{x_3}. = 45$

$e^{x_1 + x_2 + x_3} = 45$

$\log_e (e^{x_1 + x_2 + x_3}) = \log_e 45$

$x_1 + x_2 + x_3 = \log_e 45$

$\log_e P = = \log_e 45$

$P = 45$

Q.18 [36]

x=0 is not the root of this equation so divide it by x^2

$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2 + 2 - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

$$\left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right) = 0$$

$$x - \frac{1}{x} = 0, \qquad\qquad x - \frac{1}{x} = 3$$

$x^2 - 1 = 0,$ $\qquad$ $x^2 - 3x - 1 = 0$

$x = \pm 1$ $\qquad$ $\gamma + \delta = 3$

$\alpha = 1, \beta = -1$ $\qquad$ $\gamma\delta = -1$

$\alpha^3+\beta^3+\gamma^3+\delta^3$
$=1-1+(\gamma+\delta)((\gamma+\delta)^2-3\gamma\delta)$
$=+3(9-3(-1))=3(12)=36$

Q.19 (2)

$$x^4+x^3+x^2+x+1=\underset{\delta}{0}\begin{array}{l}\nearrow\alpha\\ \rightarrow\beta\\ \searrow\gamma\end{array}$$

$(x-1)(x^4+x^3+x^2+x+1)=(x-1).0$

$x^5-1=0\Rightarrow\boxed{x^5=1}\Rightarrow\alpha^5=\beta^5=\gamma^5=\delta^5=1$

Now $S=\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}=\Sigma\alpha^{2021}$
$S=\Sigma\alpha(\alpha^5)^{400}$
$S=\Sigma\alpha(1)$
$S=\Sigma\alpha$
$S=-1$

Q.20 [16]

$P_n=\alpha^n-\beta^n$ $\qquad x^2-x-4=0$

$$\frac{P_{15}P_{16}-P_{14}P_{16}-P_{15}^2+P_{14}P_{15}}{P_{13}P_{14}} \qquad ...(1)$$

As $P_n-P_{n-1}=(\alpha^n-\beta^n)-(\alpha^n-1-\beta^{n-1})$
$=\alpha^{n-2}(\alpha^2-\alpha)-\beta^{n-2}(\beta^2-\beta)$
$=4(\alpha^{n-2}-\beta^{n-2})$
$P_n-P_{n-1}=4P_{n-2}$
Hence, expression (1)

$$\frac{P_{16}(P_{15}-P_{14})-P_{15}(P_{15}-P_{14})}{P_{13}P_{14}}$$

$$=\frac{(P_{15}-P_{14})(P_{16}-P_{15})}{P_{13}P_{14}}=\frac{(4P_{13})(4P_{14})}{P_{13}P_{14}}=16$$

Q.21 [3]

$\alpha^2+\beta^2=(\alpha+\beta)^2-2ab$
$=(3-a)^2-2(1-2a)$
$=a^2-6a+9-2+4a$
$=a^2-2a+7\Rightarrow(a-1)^2\ 6\geq 6$

Q.22 [272]

$(px-q)^2+(qx-r)^2=0$
$px-q=0,\ 2x-r=0$

$$x=\frac{q}{p}=\frac{r}{2}$$

Now root's of equation $x^2+2x-8=0$
$x=-4,\ 2$
$\therefore$ q and p one not of same sign

$$\therefore\ \frac{q}{p}=\frac{r}{q}=-4$$

$$\frac{q^2+r^2}{p^2}=272$$

Q.23 [3]

$x^5(x^3-x^2-x+1)+x(3x^3-4x^2-2x+4)-1=0$
$=(x-1)^2(x+1)(x^5+3x-1)=0$
$=f(x)=x^5+3x-1$
$=f'(x)=5x^4+3$
$=f'(x)>0\ f(x)\uparrow$
Total = 3 distinct real root's

LINEAR INEQUALITIES

EXERCISE-I (MHT CET LEVEL)

Q.1 **(2)**

Since, Ravi has to buy rice in packets only, he may not be able to spend the entire amount of Rs. 200 because 200 is not a multiple of 30. Hence,

$$30x < 200$$

Q.2 **(3)**

Let x denotes the number of registers and y, the number of pens which Reshman buys, then the total amount spent by her is Rs (40x + 20y). In this case, the total amount spent may be Rs. 120, then

$$40x + 20y \le 120$$

Note : The statement 40x + 20y ≤ 120 consists of two statements 40x + 20y < 120 and 40x + 20y = 120, where 40x + 20y < 120 is an inequality and 40x + 20y = 120 is an equation.

Q.3 (d)

$$\frac{2x+3}{5} < \frac{4x-1}{2} \Rightarrow -16x < -11$$

$$\Rightarrow 16x > 11 \Rightarrow x > \frac{11}{16}$$

Hence, $x \in \left(\frac{11}{16}, \infty\right)$

Q.4 **(2)**

Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' forms an inequality. For Example, $30x < 200, 40x + 20y \le 120, 40x + 20y < 120$ etc.

Q.5 **(1)**

3 < 5; 7 > 5 are the examples of numerical inequalitites while x < 5; y > 2; x ≥ 3 are the example of literal inequalitites.

Q.6 **(4)**

3 < 5 < 7 (read as 5 is greater than 3 and less than 7), 3 < x < 5 (read as x is greater than or equal to 3 and less than 5) and 2 < y < 4 are the examples of double inequalities.

Q.7 **(4)**

We state the following rules for solving an inequality.

Rule-1 : Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of inequality.

Rule-2 : Both sides of an inequality can be multiplied (or divided) by the same positive number. But when both sides are mutliplied or divided by a negative number then the sign of inequality is reversed.

Q.8 **(3)**

We have, $4x + 3 < 6x + 7$

or $4x - 6x < 6x + 4 - 6x$

or $-2x < 4$ or $x > -2$

i.e., all the real numbers which are greater than –2, are the solutions of the tgiven inequality. Hence, the solution set is $(-2, \infty)$

Q.9 **(3)**

We have,

$$\frac{5-2x}{3} \le \frac{x}{6} - 5$$

or $2(5-2x) \le x - 30$

or $10 - 4x \le x - 30$

or $-5x \le -40$, i.e., $x \ge 8$

Thus, all real numbers x which are greater than or equal to 8 are the solutiosn of the given ienquality i.e., $x \in [8, \infty)$

Q.10 **(1)**

We have,

$$\frac{3x-4}{2} \ge \frac{x+1}{4} - 1$$

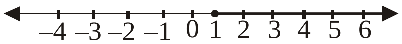
or $\dfrac{3x-4}{2} \ge \dfrac{x-3}{4}$

or $2(3x-4) \ge (x-3)$

or $6x - 8 \ge x - 3$

or $5x \ge 5$ or $x \ge 1$

The graphical representation of solutions is given in figure.



Q.11 **(1)**

Let x be the marks obtained by student in the annual examination. Then,

$$\frac{62+48+x}{3} \ge 60$$

or $110 + x \ge 180$

or $x \ge 70$

Thus, the student must obtain a minimum of 70 mariks to get an average of atleast 60 marks.

Q.12 **(3)**

Let x be the smaller of the two consecutive odd natural number, so that the other one is x + 2. Then, we should have

$x > 10$ ......(i)

and $x + (x + 2) < 40$ ......(ii)

Solving (ii), we get

$2x + 2 < 40$

i.e., $x < 19$ .....(iii)

From (i) and (iii), we get

$10 < x < 19$

Since x is an odd number, x can take the values 11, 13, 15, and 17. So, the required possible pairs will be

(11, 13), (13, 15), (15, 17), (17, 19)

Q.13 **(1)**

We have, $3x + 8 > 2$

Adding –8 on both sides,

$3x + 8 - 8 > 2 - 8$

$\Rightarrow 3x > -6$

Dividing by 3 on both sides,

$$\Rightarrow \frac{3x}{3} > \frac{-6}{3}$$

$\Rightarrow x > -2$

(i) When x is an integer, the solution of the given inequality is {–1, 0, 1, 2, ...}.

(ii) When x is a real number, the solution of the given inequality is (–2, ∞). i.e., all the numbers lying between –2 and ∞ but –2 and ∞ are not included.

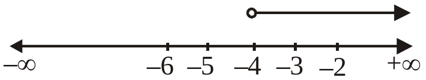
Q.14 **(1)**

We have, $4x + 3 < 5x + 7$

Transferring the term 5x to LHS and the term 3 to RHS,

$4x - 5x < 7 - 3 \Rightarrow -x < 4 \Rightarrow x > -4$

With the help of number line, we can easily look for the numbers greater than – 4.



∴ Solution set is (–4, ∞) i.e., all the numbers lying between –4 and ∞ but –4 and ∞ are not included as x > –4.

Q.15 **(1)**

We have, $3x - 7 > 5x - 1$

Transferring the term 5x to LHS and the term – 7 to RHS.

Dividing both sides by 2,

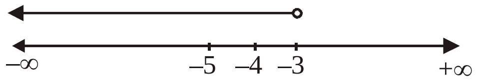
$3x - 5x > -1 + 7$

$\Rightarrow -2x > 6$

$$\Rightarrow \frac{2x}{2} < -\frac{6}{2}$$

$\Rightarrow x < -3$

With the help of numebr line, we ca easily look for the numbers less than –3



∴ Solution set is (–∞, –3) i.e., all the numebrs lying between –∞ and –3 and but –∞ and –3 are not included as x < –3.

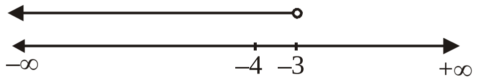
Q.16 **(4)**

We have, $3x - 3 \le 2x - 6$

Transferring the term 2x to LHS and the term – 3 ot RHS,

$3x - 2x \le -6 + 3$

$\Rightarrow x \le -3$



∴ Solutions et is (–∞, –3].

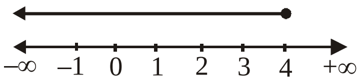
Q.17 **(2)**

We have, $3(2 - x) \ge 2(1 - x) = 6 - 3x \ge 2 - 2x$

Transferring the term 2 to LHS and the term –3x to RHS.

$6 - 2 \ge -2x + 3x$

$\Rightarrow 4 \ge x \Rightarrow x \le 4$



∴ Solution set is (–∞, 4].

Q.18 **(3)**

We have, $2(2x + 3) - < 6(x - 2)$

$\Rightarrow 4x + 6 - 10 < 6x - 12$

$\Rightarrow 4x - 4 < 6x - 12$

Transferring the term 6x to LHS and (– 4) to RHS,

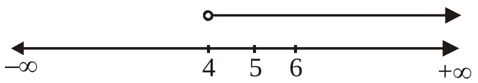
$4x - 6x < -12 + 4$

$\Rightarrow -2x < -8$

Dividing both sides by –2

$$\frac{-2x}{-2} > \frac{-8}{-2}$$

$\Rightarrow x > 4$



∴ Solution set is (4, ∞)

Q.19 **(3)**

We have $37 - (3x + 5) \ge 9x - 8(x - 3)$

$(37 - 3x - 5) \ge 9x - 8x + 24$

$\Rightarrow 32 - 3x \ge x + 24$

Transferring the term 24 to LHS and the term (– 3x) to RHS,

$32 - 24 \ge x + 3x$

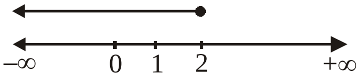
$\Rightarrow 8 \ge 4x$

$\Rightarrow 4x \le 8$

Dividing both sides by 4,

$$\frac{4x}{4} \le \frac{8}{4}$$

$\Rightarrow\ x \geq 2$



∴ Solution set is (– ∞,2].

Q.20 **(1)**

We have, $\dfrac{x}{4} < \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

$\dfrac{x}{4} < \dfrac{5(5x-2)-3(7x-3)}{15}$

$\Rightarrow\ 15x < 4\,[(25x-10)-(21x-9)]$

$\Rightarrow\ 15x < 4\,[(25x-10)-21x+9]$

$\Rightarrow\ 15x < 4\,[4x-1]$

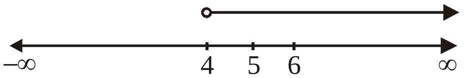
$\Rightarrow\ 15x < 16x-4$

Transferring the term 16x to LHS.

$15x - 16x\ < -4 \Rightarrow -x < -4$

Multiplying by – 1 both side, we get

$x > 4$



∴ Solution set is (4, ∞).

Q.21 **(2)**

We have, $\dfrac{2x-1}{3} \geq \left(\dfrac{3x-2}{4}\right) - \left(\dfrac{2-x}{5}\right)$

Taking LCM in RHS,

$\dfrac{2x-1}{3} \geq \dfrac{5(3x-2)-4(2-x)}{20}$

$\Rightarrow\ \dfrac{2x-1}{3} \geq \dfrac{(15x-10)-(8-4x)}{20}$

$\Rightarrow\ \dfrac{2x-1}{3} \geq \dfrac{15x-10-8+4x}{20}$

$\Rightarrow\ \dfrac{2x-1}{3} \geq \dfrac{19x-18}{20}$

$\Rightarrow\ 20\,(2x-1) \geq 3\,(19x-18)$

$\Rightarrow\ 40x - 20 \geq\ 57x - 54$

Transferring the term 57x to LHS and the term – 20 to RHS,

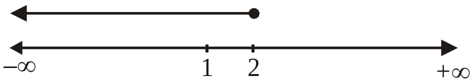
$40x - 57x \geq -54 + 20$

$\Rightarrow\ -17x \geq -34$

Dividing both sides by – 17,

$\dfrac{-17x}{-17} \leq \dfrac{-34}{-17}$

$\Rightarrow\ x \leq \dfrac{-34}{-17} \Rightarrow x \leq 2$



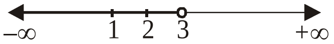
∴ Solution set is (– ∞, 2].

Q.22 **(1)**

We have, $3x - 2 < 2x + 1$

Transferring the term 2x to LHS and the term (–2) to RHS,

$3x - 2x < 1+2\ \Rightarrow\ < 3$



All the numbers on the left side of 3 will be less than it.

∴ Solution set is (– ∞, 3).

Q.23 **(1)**

We have, $\dfrac{x}{2} \geq \dfrac{(5x-2)}{3} - \dfrac{(7x-3)}{5}$

Taking LCM in RHS,

$\dfrac{x}{2} \geq \dfrac{5(5x-2)-3(7x-3)}{15}$

$\Rightarrow\ \dfrac{x}{2} \geq \dfrac{25x-10-21x+9}{15}$

$\Rightarrow\ \dfrac{x}{2} \geq \dfrac{(25x-21x)-(10-9)}{15}$

$\Rightarrow\ \dfrac{x}{2} \geq \dfrac{4x-1}{15}$

$\Rightarrow\ 15x \geq 2\,(4x-1)$

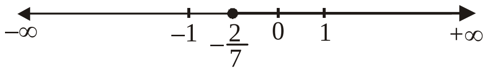
$\Rightarrow\ 15x \geq 8x - 2$

Transferring the term 8x to LHS,

$15x - 8x \geq -2\ \Rightarrow\ 7x \geq -1$

Dividing both sides by 7,

$\dfrac{7x}{7} \geq \dfrac{-2}{7}\ \ \Rightarrow\ \ x \geq -\dfrac{2}{7}$



∴ Solution set is $\left[\dfrac{-2}{7}, \infty\right)$.

Q.24 **(2)**

Let Ravi got x marks in third unit test.

∴ Average markes obtained by Ravi

$= \dfrac{\text{Sum of markes in all test}}{\text{Number of test}}$

$= \dfrac{70+75+x}{3} = \dfrac{145+x}{3}$

Now, it is given that he wants to obtain an average of atleast 60 marks.

Atleast 60 marks means that the marks should be greater than or equal to 60.

i.e., $\dfrac{145+x}{3} \geq 60$

$\Rightarrow\ 145 + x \geq 60 \times 3$

$\Rightarrow\ 145 + x \geq 180$

Now, transferring the term 145 to RHS,

$x \geq 180 - 145$

$\Rightarrow\ x \geq 35$

i.e., Ravi should get treater than or equal to 35 marks in third unit test to get an average of atleast 60 marks.

∴ Minimum marks Ravi should get = 35

Q.25 **(2)**

Let Sunita got x marks in the fifth exam.

$\therefore$ Average markes obtained by

$$\text{Sunita} = \frac{\text{Sum of marks in all exams}}{\text{Number of exams}}$$

$$= \frac{87+92+94+95+x}{5} = \frac{368+x}{5}$$

Now, it is given that Sunita wants to obtain grade A for that her average marks should be greater than or equal to 90.

i.e., $\frac{368+x}{5} \geq 90 \Rightarrow 368+\geq 450$

Transferring the term 368 to RHS,

$x \geq 450-368$

$\Rightarrow x \geq 82$

i.e., Sunita should get greater than or equal to 82 marks in fifth exam to get grade A.

$\therefore$ Minimum marks = 82

Q.26 **(1)**

Let the numbers are 2x + 1 and 2x +3. Then, according to the question,

$2x+1<10$ and $2x+3<10$

$\Rightarrow 2x<9$ and $2x<7$

$\Rightarrow x<\frac{9}{2}$ and $x<\frac{7}{2}$

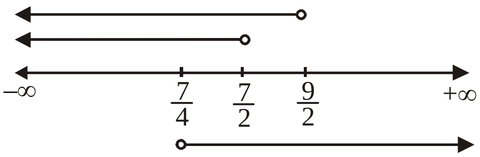
Also, $2x+3+2x+1>11$

$\Rightarrow 4x+4>11$

$\Rightarrow 4x>11-4$

$\Rightarrow x>\frac{7}{4}$

Now, plotting all the values of x on line.



From the graph, it is clear that $x \in \left(\frac{7}{4}, \frac{7}{2}\right)$ in which integer values are x = 2 and 3.

When x = 3, the numbers are (2 × 3 + 1, 2 × 3 + 3) = (7, 9)

When x = 2, the numbers are (2 × 2 + 1, 2 × 2 + 3) = (5, 7)

$\Rightarrow$ Required pairs are (5, 7) and (7, 9).

Q.27 (2)

Let numbers are 2x and 2x + 2

Then, according to the question,

$2x>5 \Rightarrow x>\frac{5}{2}$

and $2x+1>5 \Rightarrow 2x>5-2$

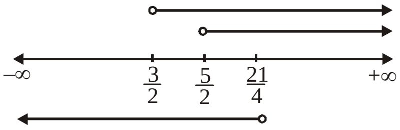
$\Rightarrow 2x>3 \Rightarrow x>3/2$

and $2x+2x+2<23 \Rightarrow 4x<23-2$

$\Rightarrow 4x<21$

$\Rightarrow x<\frac{21}{4}$

Now, ploting all these values on number line.



From above graph, it is clear that $x \in \left(\frac{5}{2}, \frac{21}{4}\right)$ in which integer values are x = 3, 4, 5

When x = 3 pair is (2 × 3, 2 × 3 + 2) = (6, 8)

When x = 4 pair is (2 × 4, 2 × 4 + 2) = (8, 10)

When x = 3 pair is (2 × 5, 2 × 5 + 2) = (10, 12)

$\therefore$ Required pairs are (6, 8), (8, 10), (10, 12).

Q.28 **(3)**

Let the shortest side be x cm.

Then, according to the given condition,

Longest side = 3x cm and third side = (3x – 2)cm

Now, perimeter of triangle ≥ 61 i.e., such of all sides ≥ 61

$\Rightarrow x+3x+3x-2 \geq 61$

$\Rightarrow 2+7x-2 \geq 61+2$

$\Rightarrow 7x \geq 63$

$\Rightarrow \frac{7x}{7} \geq \frac{63}{7}$

$\Rightarrow x \geq 9$

$\therefore$ Minimum length of the shortest side is 9 cm.

Q.29 **(2)**

Let the shortest side be x cm.

Then, by given condition, second length = x + 3cm

Third length = 2x cm

Also given, total length = 91

Hence, sum of all the three lengths should be less than equal to 91.

$x+x+3+2x \leq 91$

$\Rightarrow 4x+3 \leq 91$

Subtracting (–3) to each term,

$-3+4x+3 \leq 91-3$

$\Rightarrow 4x \leq 88$

$\Rightarrow \frac{4x}{4} \leq \frac{88}{4}$

$\Rightarrow x \leq \frac{88}{4}$

$\Rightarrow x \leq 22$ cm ...(i)

Third length ≥ second length +5

Again, given that 2x ≥ (x + 3) + 5

$2x \geq x+(3+5)$

Transferring the term x to LHS,

$2x-x \geq 8 \Rightarrow x \geq 8$ ...(ii)

From Eqs. (i) and (ii), length of shortest board should be greater than or equal to 8 but less than or equal to 22 i.e., $8 \leq x \leq 22$.

Q.30 **(2)**
A line divides the cartesian plane into two parts.

Q.31 **(1)**
Each part in which a line divides the cartesian plane is known as half plane.

Q.32 **(1)**
A vertical line will divides the palen in left and right half planes and a non- vertical line will divide the palen into lower and upper half planes.

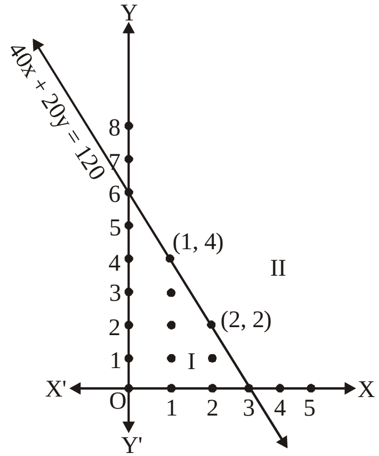
Q.33 **(3)**
A point in the cartesian plane will either lie on a line or will lie in either of the hafl planes I or II.

Q.34 **(1)**
All points (x, y) satisfying $ax + by = c$, lie on the line it represent.

Q.35 **(3)**
If a point P (α, β) lie on the line $ax + by = c$, then it will satisfy $ax + by = c$, i.e., $a\alpha + b\beta = c$. All the points lying on the line, $ax + by = c$ is the solution to the given equation.

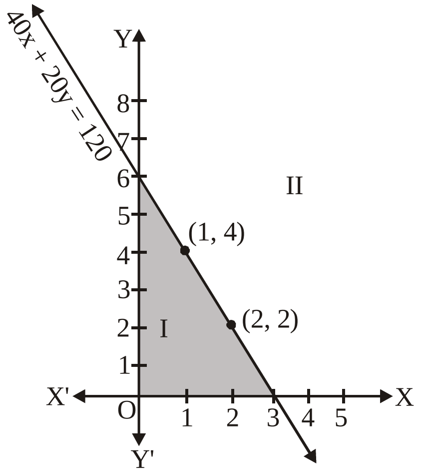
Q.36 **(3)**
The graph of the inequality $ax + by > c$ will be half plane I (called solution region) and represented by shading half plane/not including the point on the line $ax + by = c$.

Q.37 **(1)**
Given, $40x + 20y \le 120$, x and y are whole numbers.
To start eith, let x = 0. then, LHS of given inequality is
$40x + 20y = 40 (0) + 20y = 20y$
Thus, we have $20y \le 120$
or $y \le 6$
For x = 0, the corresponding values of y can be 0, 1, 2, 3, 4, 5, 6 only. In this case, the solutions of given inequality are (0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), and (0, 6).



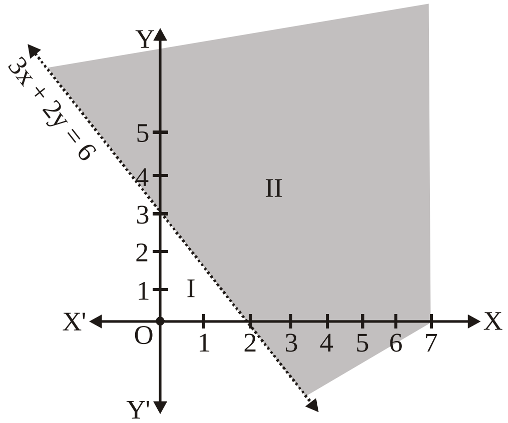
Similarly, other solutions of given inequality when x = 1, 2 and 3 are (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (2, 0), (2, 1), (2, 2), (3, 0).
This is shown in above figure. Statement (b) is incorrect, since x and y are whole numbers. x and y can only take values {0, 2, 3 ...∞}. Given shaded region shows negative as well as fractional values of x and y.

Q.38 **(4)**
We have, $40x + 20y \le 120, \ge 0y \ge 0$ ...(i)
In order to draw the graph of the inequality (i), we take one point say (0,0), in half plane I and check whether values of x and y satisfy the inequality or not.



We observe that x = 0, y = 0 satisfy the inequality. Thus, we say that the half plane I is the graph of the inequality. Since, the points on the line also satisfy the inequality (i) above, the line is also an part of the graph.
Thus, the graph of the given inequality is half plane I including the line itself. Clearly, half plane II is not the part of the graph. Hence, solutions of inequality (i) will consist of all the points of its graph (half plane Iinculding the line).
Also, since it is given $x > 0, y > 0$, x and y can only takd positive values in half plane I.

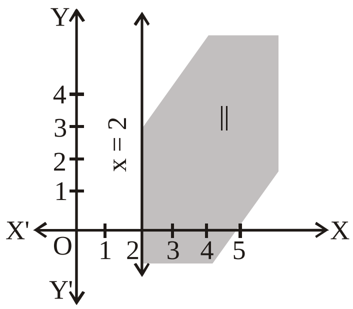
Q.39 **(2)**
Graph of $3x + 2y = 6$ is given as dotted line in the figure.
This line divides the xy-plane in two half planes I and II. We select a point (not on the line), say (0, 0), which lies in one of the half planes (see figure). Now, determine whether this point satisfies the given inequality or not. We note that $3(0) +2(0) > +$ or > 6, which is false.



Hence, half plane I is not the solution region of the given inequality. Clearly, any point on the line does not satisfy the given strict inequality. In other words, the shaded half plane II excluding the points on the line is the solution region of the inequality.

Q.40 (1)

Graph of 3x - 6 = 0 is given in the figure.



We select a point say (0, 0) and subtituting it in given inequality, we see that

$3(0) - 6 \geq 0$ or $-6 \geq 0$ which is false

Thus, the solution region is the shaded region on the right hand side of the line x = 2.

Q.41 (2)

We have the given inequality, $2x + y \geq 6$ ......(i)

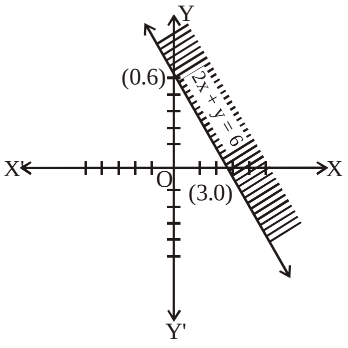
Step I Consider the inequation as a strict equation i.e., $2x + y = 6$

Step II Find the point on X-axis and Y-axis i.e.,

x	3	0
y	0	6

Step III Plot the graph using the above table.

Step IV Take a point (0, 0) and put it in the given inequaiton (i), we get $0 + 0 \geq 6$, which is false, so shaded region will be away from the origin.



Here, shaded region shows the inequality $2x + y \geq 6$

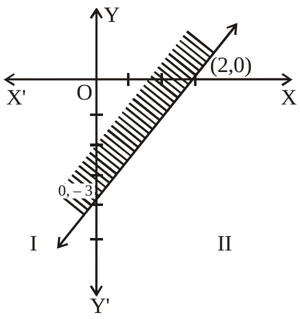
Note The region is said to be bounded, if it is enclosed otherwise it is unbounded.

Q.42 (3)

The given inequation $-3x + 2y \geq -6$

Step I Considere the inequation as a stric equation i.e., $-3x + 2y = -6$

x	0	2
y	-3	0



Step II Find the point on the X-axis and Y-axis i.e.,

Step III Plot graph using the above table.

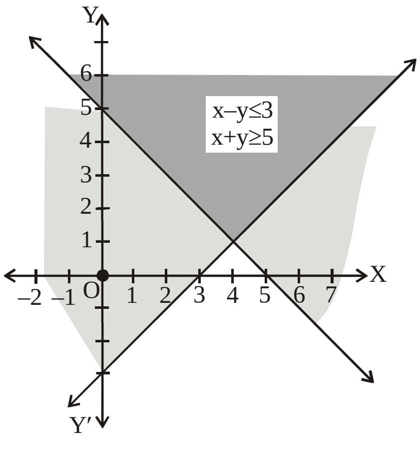
Step IV Take a point (0, 0) and put it in the given inequation (i), we get $0 + 0 \geq -6$

Which is true, so the shaded region will be towards the origin.

Hence, the shaded half plane I including the point on the line is the solution of given inequality

Q.43 **(3)**

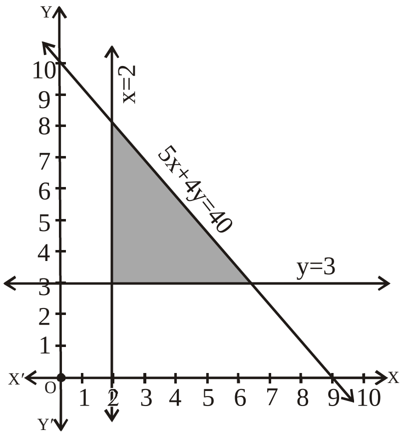
The graph of linear equation x + y = 5 is drawn figure. We note that solution of inequation (i) is represent by the shaded region above the line x + y = 5, including point on the line.



On the same set of axes, we draw the graph of the equation x - y = 3as shown in figure. Then, we note that inequality represents the shaded region above the line x - y = ... including the points on the line Clearly, the double shaded region, common to the above two shaded region is the required solution region of the given sytem of inequalities.

Q.44 (1)

We first draw the graph of the lines 5x + 4y = 40, x = 2 and y = 3



Then, we note that the inequality (i) represents shaded region below the line 5x + 4y = 40 and inequality (ii) represents the shaded region right of line x = 2 but inequality (iii) represents the shaded region above the line y = 3. Hence shaded region (figure including all the point on the lines are also the solution of the given system of the linear inequalities.

In many practical situations involving system of inequalitie the variable x and y often represent quantities that cannot have negative values, for example, number of unit produced, number of articles purchased, number of hour worked etc.

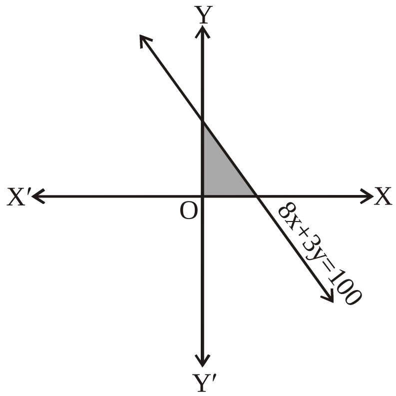
Clearly, in such cases, $x \geq 0$, $y \geq 0$ and the solution region lies only in the first quadrant.

Since, the region is enclosed, so it is bounded.

Q.45 **(2)**

Consider 8x + 3y = 100. We observe that the shaded region and the origin lie on the same side of the line and (0, 0) satisfies $8x + 3y \leq 100$. Therefore, we must have $8x + 3y \leq 100$ as linear inequality corresponding to the line 8x + 3y = 100. Also, shaded region lies in the first quadrant only. Therefore, $x \geq 0$, $y \geq 0$, Hence, the linear inequalities corresponding to the given solution are

$8x + 3y \leq 100, x \geq 0, y \leq 0.$



Q.46 (4)

The given system of inequalities

$2x + y \geq 4$ ....(i)

$x + y \leq 3$ ....(ii)

$2x - 3y \leq 6$ ....(iii)

Step I Consider the inequations as strict equations i.e., 2x + y = 4

x + y = 3

2x – 3y = 6

Step II Find the points on the X-axis and Y-axis for, 2x + y = 4

x	0	2
y	4	0

and x + y = 3

x	0	3
y	3	0

and 2x – 3y = 6

x	0	3
y	–2	0

Step III Plot the graph using the above tables.

Step IV Take a point (0, 0) and put it in the inequation (i), (ii), and (iii), we get

$0 + 0 \geq 4$ (false)

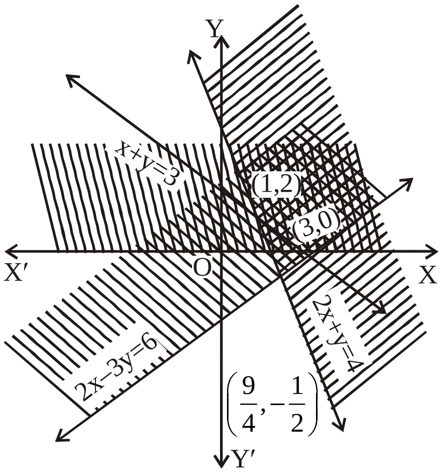
So, the shaded region will be away from origin

$0 + 0 \leq 3$ (true)

So, the shaded region will be towards origin

$0 - 0 \leq 6$ (true)

So, the shaded region will be towards origin



Thus, common shaded region shows the solution of the inequalities.

Q.47 **(1)**

The given system of inequalities

$x - 2y \leq 3$ ....(i)

$3x + 4y \geq 12$ ....(ii)

$x \geq 0$ .....(iii)

$y \geq 1$ .....(iv)

Step I Consider the inequations as strict equations

i.e., $x - 2y = 3$, $3x + 4y = 12$ $x = 0$, $y = 1$

Step II Find the point on the X-axis and Y-axis

for $x - 2y = 3$

x	0	3
y	–3/2	0

and $3x + 4y = 12$

x	0	4
y	3	0

Step III Plot the graph using the above tables

(i) For $x - 2y = 3$ and $3x + 4y = 12$ use the above tables.

(ii) Graph of $x = 0$ will be Y = axis

(iii) Graph of $y = 1$, will be line parallel to X-axis, intersecting Y-axis at 1.

Step IV Taking a point (0, 0) and put it in the the inequations (i) and (ii),

$0 - 0 \leq 3$ (true)

So, the shaded region will be towards origin and

$0 + 0 \geq 12$ (false)

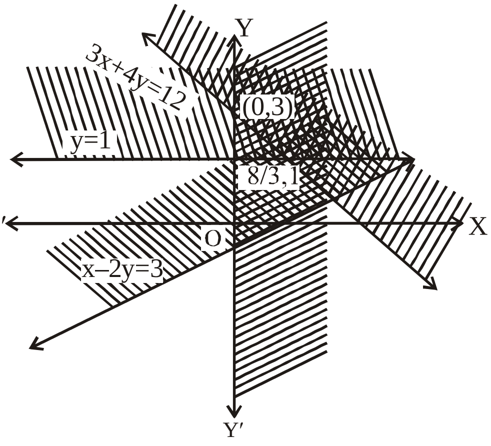
So, the shaded region will be away from the origin.

Again take a point (1, 0)

put it in the inequations (iii), we get

$1 \geq 0$ (true)

So, the shaded region will be towards origin.



And a point (1, 0) put it in the inequations (iv), we get

$0 \geq 1$ (false)

So, the shaded region will be away from the origin.

Thus, common shaded region shows the solution of the inequalities.

Q.48 **(2)**

The given system of inequalities

$4x + 3y \leq 60$ ...(i)

$y \geq 2x$ ...(ii)

$x \geq 3$ ...(iii)

$x, y \geq 0$ ...(iv)

Step I Consider the inequations as strict equations

i.e., $4x + 3y = 60$

$y = 2x$

$x = 3$

Step II Find the point on the X-axis and Y-axis for

$4x + 3y = 60$

x	0	15
y	20	0

and $y = 2x$

x	0	5
y	0	10

Step III Plot the graph using the above tables,

(i) For $4x + 3y = 60$ use the above tables

(ii) For $y = 2x$ use the tables

(iii) Graph of $x = 3$ will be straight parallel to Y-axis intersectin ghe X-axis at 3.

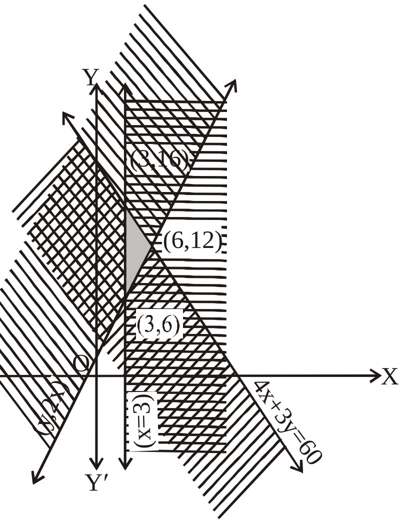
Step IV Take a point (0, 0). Put it in the inequation (i) and (ii)

$0 + 0 \leq 60$ (true)

$0 \geq 3$ (false)

and a point (1, 1), putting it in the inequation (ii),

$1 \geq 2$ (flase)



Thus, common shaded region shows the solution of the inequalities.

Q.49 (3)

(i) Consider, $x + 4y = 80$. We observe that the shaded region and the origin lie on the same side of this line and (0, 0) satisfies $x + 4y \leq 80$. Therefore, we must have $x + 4y \leq 80$ as linear inequality corresponding to the line $x + 4y = 80$.

(ii) Consider, $3x + 2y=150$. We observe that the shaded region and the origin lie on the same side of this line and (0, 0) satisfies $3x + 2y \leq 150$. Therefore $3x + 2y \leq 150$ is the linear inequality correspondin to the line 3x

$+ 2y = 150$.

(iii) Consider, $x = 15$. We observe that the shadedregion and origin lie on the same side and (0, 0) satisfies $x \le 15$. Therefore, $x \le 15$ is the linear inequality corresponding to the line $x = 15$.

(iv) Also, the shaded region lies in the first quadrant only Therefore, $x \ge 0, y \ge 0$.

Hence, in view of (i), (ii), (iii), (iv) above, the linear inequalities corresponding to the given solution set are $3x + 2y \le 150, x + 4y \le 80, x \le 15, x, y \le 0$.

Q.50 **(4)**

The given system of inequalities

$x + 2y \le 10$

$x + y \ge 1$

$x - y \le 0$

$x \ge 0, y \ge 0$

Step I Consider the given inequations as strict equations i.e., $x + 2y = 10, x + y = 1, x - y = 0$ and $x = 0, y = 0$

Step II Find the points on the X-axis and Y-axis for $x + 2y = 10$

x	0	10
y	5	0

and $x + y = 1$

x	0	1
y	1	0

For $x - y = 0$

x	1	2
y	1	2

Step III Plot the graph of $x + 2y = 10$

$x + y = 1$, $x - y = 0$ using the above tables

Step IV Take a point' (0, 0)and put it in the inequations (i) and (ii),

$0 + 0 \le 10$ (true)

So, the shaded region owill be towards origin

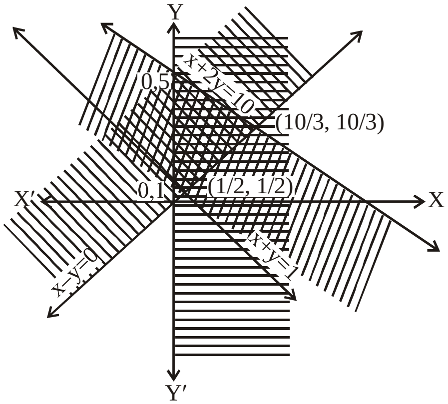
And $0 + 0 \ge 1$ (flase)

So, the shaded region will be away from the origin

Again, take a point (2, 2) and put it inthe inequation (iv), we get

$2 \ge 0, 2 \ge 0$ (true)

So, the shaded region will be towards point (2, 2).



And take a point (0, 1) and put it in the inequation (iii), we get

$0 - 1 \le 0$

EXERCISE-II (JEE MAIN LEVEL)

INEQUALITIES

Q.1 **(3)**

Since, maximum number of computers shopkeeper can sell, is 250. Therefore,

$$x + y \le 250$$

Q.2 **(3)**

Given, $3 < 3t - 18 \le 18$

Adding 18 to each term,

$3 + 18 < 3t - 18 + 18 \le 18 + 18$

$\Rightarrow 21 \le 3t \le 36$

Dividing by 3 to each term,

$$\frac{21}{3} \le \frac{3t}{3} \le \frac{36}{3}$$

$\Rightarrow 7 \le t \le 12$

Adding 1 to each term,

$7 + 1 \le t + 1 \le 12 + 1$

$\Rightarrow 8 \le t + 1 \le 13$

Q.3 **(2)**

The given inequality $6 \le -3(2x - 4) < 12$

$6 \le -6x + 12 < 12$

Adding (–12) to each term,

$$\frac{-6}{-6} \ge \frac{6x}{-6} > \frac{0}{-6}$$

$\Rightarrow 1 \ge x > 0$

$\Rightarrow 0 \le x < 1$

$\therefore$ Solution set is (0, 1]

Q.4 **(3)**

The given inequality $-3 \le 4 - \frac{7x}{2} \le 18$

Adding (– 4) to each term,

$$-3 - 4 \le 4 - \frac{7x}{2} - 4 \le 10 - 4$$

$$\Rightarrow -7 \le \frac{-7x}{2} \le 14$$

Multiplying by $\left(\frac{-2}{7}\right)$ to each term,

$$-7 \times \left(\frac{-2}{7}\right) \ge -\frac{7}{2} x \times \left(\frac{-2}{7}\right) \ge \times \left(\frac{-2}{7}\right)$$

$\Rightarrow 2 \ge x \ge -4 \Rightarrow -4 \le x \le 2$

$\therefore$ Solution set is [– 4, 2].

Q.5 **(4)**

I. The given inequality $2 \le 3x - 4 \le 5$

$\Rightarrow 2 + 4 \le 3x \le 5 + 4$

$\Rightarrow 6 \le 3x \le 9$

Dividing by 3 in each term,

$$\frac{6}{3} \le \frac{3x}{3} \le \frac{9}{3}$$

$\Rightarrow 2 \le x \le 3$

$\therefore$ Solution set is [2, 3]

II. The given inequality

$$-12 < 4 - \frac{3x}{-5} \le 2$$

$$\Rightarrow \quad -12 < 4 + \frac{3x}{5} \le 2$$

Adding (– 4) to each term,

$$-12 - 4 < 4 + \frac{3x}{5} - 4 \le 2 - 4$$

$$\Rightarrow \quad -16 < \frac{3x}{5} \le -2$$

Multiplying by $\frac{5}{3}$ to each term,

$$-16\frac{5}{3} < \frac{3x}{5} \times \frac{5}{3} \le -2 \times \frac{5}{3}$$

$$\Rightarrow \quad -\frac{80}{3} < x \le -\frac{10}{3}$$

$\therefore$ Solution set is $\left(-\frac{80}{3}, -\frac{10}{3}\right]$ or $\left]-\frac{80}{3}, \frac{-10}{3}\right]$.

Q.6 **(2)**

Given inequality is $5x + 1 > -24$.

Adding (– 1) on both sides,

$-1 + 5x + 1 > -24 - 1$

$\Rightarrow 5x > -25$

Dividing both sides by 5,

$$x > \frac{-25}{5}$$

$\Rightarrow x > -5$

Also, $5x - 1 < 24$ ....(i)

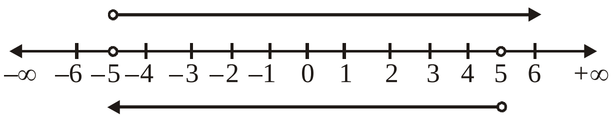
Adding 1 on both sides,

$1 + 5x - 1 < 24 + 1$

$\Rightarrow 5x < 25$

$\Rightarrow \quad x < \frac{25}{5} \Rightarrow x < 5$ ....(ii)

Draw the graph of inequations (i) and (ii) on the number line.



Hence, solutions of the system are real numbers x lying between –5 and 5 excluding –5 and 5 i.e., $-5 < x < 5$.

$\therefore$ Solution set is (–5, 5) or]– 5, 5[.

Q.7 **(3)**

We have the given inequalities as

$2(x - 1) < x + 5$ and $3(x + 2) > 2 - x$

Now, $2x - 2 < x + 5$

Transferring the term x to LHS and the term – 2 to RHS,

$2x - 2 < x + 5$

$\Rightarrow x < 7$ ...(i)

and $3(x + 2) > 2 - x$

$\Rightarrow 3x + 6 > 2 - x$

Transferring the term (– x) to LHS and the term 6 to RHS,

$\Rightarrow 3x + x > 2 - 6$

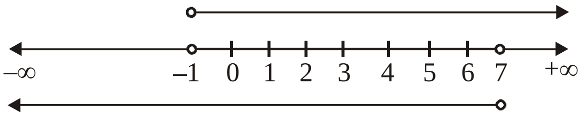
$\Rightarrow 4x > -4$

Dividing both sides by 4,

$$x > \frac{-4}{4}$$

$\Rightarrow x > -1$ ...(ii)

$\Rightarrow$ Draw the graph of inequalities (i) and (ii) on the number line.



Hence, solution set of the inequalities are real numbers, x lying between – 1 and 7 exculding – 1and 7.

i.e., $-1 < x < 7$

$\therefore$ Solution set is (–1, 7) or]– 1, 7[.

Q.8 **(3)**

We have, $2x - 7 < 11$ and $3x + 4 < -5$

Now, $2x - 7 < 11$

Adding 7 on both sides,

$2x < 11 + 7 \Rightarrow 2x < 18$

Dividing by 2 both sides,

$x < 9$ ...(i)

and $3x + 4 < -9$

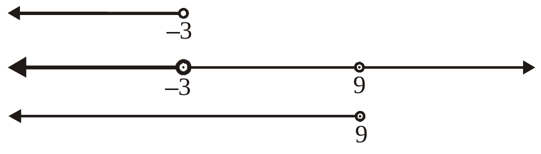
Adding (–4) on both sdies,

$3x < -9$

Dividing by 3 both sides,

$x < -3$ ...(ii)

Draw the graph of inequations (i) and (ii) on the number line



Hence, solution of the system of linear inequations are ral numbers less than –3 excluding – 3.

$\therefore$ Solution set is $(-\infty, -3)$.

Q.9 **(2)**

We have, $|x| < 3$

$\Rightarrow -3 < x < 3$ $(\because |x| < a \Rightarrow -a < x < a)$

Q.10 **(4)**

We have, $|x| > b, b > 0$

$\Rightarrow\ x < -b$ and $x > b$

$\Rightarrow\ x \in (-\infty, -b) \cup [b, \infty)$

Q.11 **(3)**

We have, $\frac{x-2}{x+5} - 2 > 0$

$$\Rightarrow\ \frac{x-2-2(x+5)}{x+5} > 0$$

$$\Rightarrow\ \frac{x-2-2x-10}{x+5} > 0$$

$$\Rightarrow\ \frac{-(x+12)}{x+5} > 0$$

Multiply by (– 1) on both sides,

$$\frac{x+12}{x+5} < 0$$

Given inequality will be less than 0 in two cases.

Case I

$(x+12) > 0$ and $x+5 < 0$

$\Rightarrow\ x > -12$ and $x < -5$

$$\left(\because \frac{a}{b} > 0 \Rightarrow \text{a and b are of opposite signs}\right)$$

Case II

$(x+12) < 0$ and $x+5 > 0$

or $x < -12$ and $x > -5$ (not possible)

$\therefore\ -12 < x < -5$

i.e., $x \in (-12, -5)$

Q.12 **(3)**

We have, $|3-4x| \geq 9$

$\Rightarrow\ 3-4x \leq -9$ or $3-4x \geq 9$

$(\because |x| \geq a \Rightarrow x \leq -a \text{ or } x \geq a)$

Subtracting 3 from both sides of each inequality,

$3-4x-3 \leq -9$ or $3-4x-3 \geq 9-3$

$-4x \leq -12$ or $-4x \geq 6$

Dividing by (–4) on both sides of each inequality,

$$\frac{-4x}{-4} \geq \frac{-12}{-4} \quad \text{or} \quad \frac{-4x}{-4} \leq \frac{6}{-4}$$

$$x \geq 3 \text{ or } x \leq -\frac{3}{2}$$

$$x \in \left(-\infty, -\frac{3}{2}\right) \cup [3, \infty)$$

Q.13 **(2)**

We have, $\frac{|x-2|}{x-2} \geq 0$

Case I If $x < 2$

$$\frac{|x-2|}{x-2} \geq 0$$

$$\Rightarrow\ \frac{-|x-2|}{x-2} \geq 0 \quad (\because |x-2| = -(x-1), \text{ if } x < 2)$$

$\Rightarrow\ -1 \geq 0$

Which is not true.

Case II If $x > 2$

$$\frac{|x-2|}{x-2} \geq 0$$

$$\Rightarrow\ \frac{|x-2|}{x-2} \geq 0 \quad (\because |x-2| = x-2, \text{ if } x > 2)$$

$\Rightarrow\ 1 \geq 0$

Which is true.

Also, given expression is not defined at x = 2, since denominator at x = 2 is 0. There is no solution at x = 2. Hence, x > 2 is the required solution. i.e., $x \in (2, \infty)$

Q.14 **(2)**

We have,

$a < b$

and $c < 0$

Dividing both sides of a < b by c. Since, c is a negative number, sign at inequality will get reversed.

Hence, $\frac{a}{c} > \frac{b}{c}$

Q.15 **(2)**

We have, p> 0, q < 0

$\because\ q < 0$

Adding p on both sides,

$p + q < p$

Q.16 **(1)**

We have, $-3x + 17 < -13$

Subtracting 17 from both sides,

$-3x < -13 - 17$

$\Rightarrow\ -3x < -30$

Dividing both sides by (– 3),

$x > 10$

$\Rightarrow\ x \in (10, \infty)$

Q.17 **(2)**

The number line represents

$$x \geq \frac{9}{2} \text{ i.e., } x \in \left[\frac{9}{2} \infty\right)$$

Q.18 **(2)**

To earn some profit, we must have

$R(x) > C(x)$

$\Rightarrow\ 60 + 2000 > x + 4000$

Subtracting 20x from both sides,

$40x + 2000 > 4000$

Subtracting 2000 from both sides,

$40x > 2000$

Dividing both sides by 40,

$$x > \frac{2000}{40} \Rightarrow x > 50$$

Q.19 **(3)**

We have, $|x+3| \geq 10$

$\Rightarrow\ x+3 \leq -10$ or $x+3 \geq 10$ ($\because |x| \geq a \Rightarrow x < -a$ or $x > a$)

$\Rightarrow\ x+3-3 \leq -10-3$ or $x+3-3 \geq 10-3$ $\Rightarrow x \leq 13$ or $x \geq 7$

$\Rightarrow\ x \in (-\infty, -13] \cup [7, \infty)$

Q.20 **(3)**

Let breadth of rectangle be x cm.

$\therefore$ Length of rectangle = 3x

Perimeter of rectangle = 2 (Length + Breadth)

$= 2(x+3x) = 8x$

Given, Perimeter ≥ 160 cm

$8x \geq 160$ cm

Dividing both sides by 8,

$x \geq 20$ cm

Q.21 **(3)**

We know that, $|3-x| \geq 0 \forall x \in R$

Adding 7 on both sides,

$|3-x| + 7 \geq 7$

$\Rightarrow\ f(x) \geq 7$

$\therefore$ Minimum value of f (x) is 7.

Q.22 **(1)**

We have, $2 \leq |x-3| < 4$

Case I If $x < 3$, then

$2 \leq |x-3| < 4$

$\Rightarrow\ 2 \leq -(x-3) < 4$

$\Rightarrow\ 2 \leq -x+3 < 4$

Subtracting 3 from both sides,

$-1 \leq -x < 1$

Multiplying (–1) on both sides,

$-1 < x \leq -1$

$\Rightarrow\ x \in (-1, 1]$

Case II If $x > 3$, then

$2 \leq |x-3| < 4$

$\Rightarrow\ 2 \leq -x+3 < 4$

Adding 3 on both sides,

$\Rightarrow\ 5 \leq x < 7$

Hence, the solution set of given inequality is

$x \in (-1, 1] \cup [5, 7)$

Q.23 **(1)**

Let x be the smaller of two consecutive even positive integers. Then, the other even integer is x + 2.

Given, $x > 8$ and $x + x + 2 < 25$

$\Rightarrow\ x > 8$ and $2x + 2 < 25$

$\Rightarrow\ x > 8$ and $2 < 25$

$\Rightarrow\ x > 8$ and $x < \frac{23}{2}$

$\Rightarrow\ x = 10$

Hence, there exists only one pair of even integer (10, 12).

Q.24 (1)

The shaded region in the figure lies between x = – 3 and x = 3 not including the line x = – 3 and x = 3 (lines are dotted)

Therefore , $-3 < x < 3$

$\Rightarrow\ |x| < 3$ ($\because |x| < a \Leftrightarrow -a < x < a$)

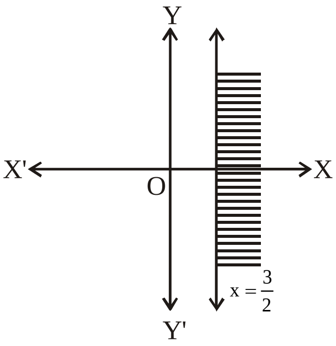
Q.25 (2)

Given, $2x - 3 \geq 0$

......(i)

$\Rightarrow\ 2x \geq 3$

$\Rightarrow\ x \geq \frac{3}{2}$



First, draw the graph of the line $x = \frac{3}{2}$. Consider the point O(0, 0). Putting x = 0 in Eq. (i),

$2(0) - 3 \geq 0$

$\Rightarrow\ 0 - 3 \geq 0$

$\Rightarrow$ 3 ³ 0, which is not true

Hence, given, inequality represent the half plane made by the line $x = \frac{3}{2}$, which does not contains origin.

Q.26 (3)

Given inequalities are

$3x + 4y \geq 12$ ...(i)

$y \geq 1$ ...(ii)

$x \geq 0$ ...(iii)

The line corresponding to (i) is

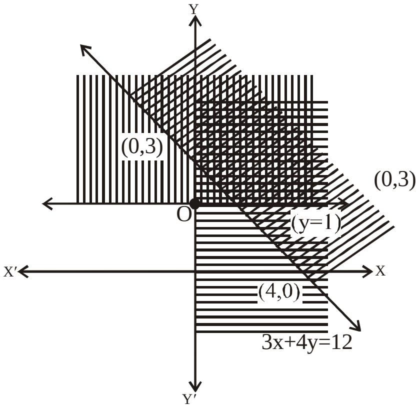
$3x + 4y = 12$ ...(iv)

Table for $3x + 4y = 12$

x	4	0
y	0	3

Draw the graph of the line 3x+4y=12

$\because$ 3(0) + 4(0) $\geq$ 12 i.e., 0 $\geq$ 12, which is not true.
$\therefore$ Inequality (i) represent the half plane made by the line (iv) which does not contain origin .
Inequality y $\geq$ 1 represent the half plane made by the line y = 1 which does not contain origin ($\because$ 0 $\geq$ 1 is not true)



Inequality x $\geq$ 0 represent the right side of the Y-axis. Hence, the shaded part is common to all the given inequalities.

Q.27 (1)

Given inequalities are
x – y $\leq$ 1 ...(i)
x + 2y $\leq$ 8 ...(ii)
2x + y $\geq$ 2 ...(iii)
x $\geq$ 0 ...(iv)
y $\geq$ 0 ...(v)
The lines corresponding to (i), (ii) and (iii) are
x – y = 1 ...(vi)
x + 2y = 8 ...(vii)
2x + y = 2 ...(viii)
Table for x – y + 1

x	1	0
y	0	–1

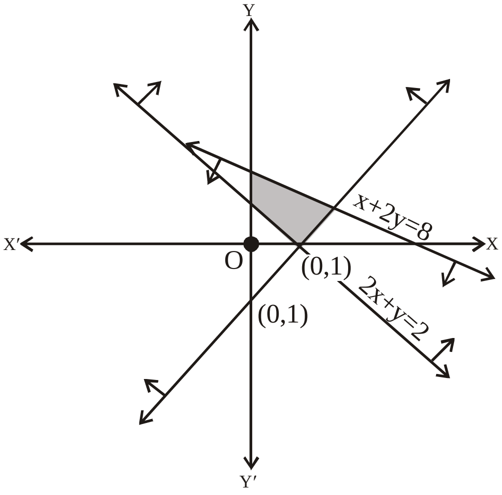
Table for x + 2y = 8

x	8	0
y	0	4

Table for 2x + y = 2

x	1	0
y	0	2

Draw the graph of line (vi), (vii) and (viii).



$\because$ 0 – 0 $\leq$ 1 i.e., 0 $\leq$ 1, which is true

Therefore, inequality (i) represent the half plane made by the line (vi), which contains origin
Now, 0 + 2(0) $\leq$ 8 i.e., 0 $\leq$ 8, which is true. So, inequality (ii) represent the half plane made by line (vii) which contain origin.
Also, x $\geq$ 0, y $\geq$ 0 represent the region in 1 quadrant.
The region common to (i), (ii), (iii), (iv) and (v) is the shaded region which is bounded.

Q.28 (2)

(i) Consider the line x + y = 8 We observe that the shaded region and origin lie on the same side of this line and (0, 0) satisfies x + y $\leq$ 8. Therefore, x + y $\leq$ 8 is the linear inequality corresponding to the line x + y = 8
(ii) Consider x + y = 4. We observe that shaded region and origin are on the opposite side of this line and (0,0) satisfies x + y $\leq$ 4. Therefore, we must have x + y $\geq$ 4 as linear inequalities corresponding to the line x + y = 4
(iii) Shaded portion lie below the line y = 5. So, y $\leq$ 5 is the linear inequality corresponding to y=5
(iv) Shaded portion lie on the left side of the line x = 5. So x $\leq$ 5 is the linear inequality corresponding to x=5.
(v) Also, the shaded region lies in the first quadrant only. Therefore, x $\geq$ 0, y $\geq$ 0.
In view of (i), (ii), (iii), (iv) and (v) above the linear inequalities correspondinfg to the given solutions are x + y $\leq$ 8, x + y $\geq$ 4, y $\leq$ 5, x $\leq$ 5, x $\geq$ 0 and y $\geq$ 0